Shear load design

Comparison between EC 2, EHE-08, FIB Model Code 2010 and the new proposal of Dr. Antoni Cladera Bohigas

Diseño a cortante

Comparativa entre el EC 2, EHE-08, FIB Model Code 2010 y la nueva propuesta del Dr. Antoni Cladera Bohigas

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ABSTRACT

The objective of the study is to carry out a comparative analysis of technical terms between the normative of shear design of reinforced concrete in Spain, Austria, the FIB Model Code and the new proposal of Dr. Antoni Cladera Bohigas.

For this, firstly a study on the historical development of knowledge and rules used from the beginning of the common use of structural concrete until today has been conducted, mainly dwelving into the current state of knowledge. Subsequently, starting from beam dimensions and specific parameters, the stress bearing ablity of the beams has been calculated, in accordance to each of the standards aforementioned.

The main conclusions obtained from the analysis are, the significant difference observed in relation to the cracking angles of the piece considered for the design and the contribution of the concrete to shear stress, which shows considerable variation in the resistance of the workpiece. Thus, it appears that the latest regulations included in the FIB Model Code minimize the ratio of breaking angles. In addition to that the proposal of Dr. Antoni Cladera Bohigas also determines a particular breaking angle, depending on the depth of the neutral axis and the effective depth to obtain more exact figures. Furthermore the contribution of the concrete along the shear reinforcement is taken into consideration, which is also taken into account in the EHE-08.

Thus, one can conclude that the line of development of the shear design takes into account the resistance provided by the concrete, as the cracking angle of the piece tends to be more accurately defined.

CONTENTS

1. INTRODUCTION

1.1 Rationale of the work

Within the field of structural design, the shear stress is one of the issues in which even today knowledge is much more limited that it may be in other fields, such as the calculation of compression, traction or flexion.

Therefore, in order to check the main differences between shear force design in Spain and Austria, it is very interesting to compare current regulations in both countries and analyze the results obtained by the comparison of the variables used.

In addition, the international regulations have been analyzed, as well as a new proposal of shear calculation to observe and compare the current regulations with the new contributions and new concepts on which the new studies are based have been presented. Furthermore a check whether the difference is notable or not has been also conducted.

The regulations that have been used for this analysis were the legislation currently in force in Spain, EHE-08, the legislation used in Austria, the Euro Code 2 adapted into the country by the National Annex, and FIB Model Code, which has the objective to serve as a basis for future codes for concrete structures and present new developments with regard to concrete structures.

In addition to the aforementioned regulations, a new proposal for calculating shear stress is also extensively analysed.

Thus a comparison has been made between the methodologies used in the two European countries, with a distinctly different construction culture; a standard developed by a large number of experts from different countries and continents;

and a new proposal, innovative and with the possibility of providing new insights, developed by Dr. Antoni Cladera Bohigas, a current piece of the European Working Group for the drafting of the new Euro Code 2 scheduled for 2018.

1.2. Approaches, aims and objectives

To carry out the study the following steps and methodology was conducted:

1. Study and analysis of existing bibliographical documentation.
2. Usage of an excel file to calculate the shear resistance according to each standard.
3. Use of graphics for the display of concrete resistance.
4. Comparison of the Spanish standard with the other examples.
5. Overall contrast between the solutions presented in the different standards.
6. Development of new proposals based on the conclusions of the study.

The main goal of this study is to check the differences between the Spanish legislation (the EHE-08), the Austrian legislation (Eurocode 2, with the National Annex of Austria included), the international model (FIB Model Code 2010) and the new proposal by Dr. Antoni Cladera Bohigas.

For this a rectangular section of a beam with specific characteristics has been used and calculated upon, in accordance with the regulations and the proposal mentioned above, in order to assess the resistance observed by using each of the standards.

Thus, it has been observed that the results of the tests on the beam according to the various legislations vary considerably as do the variables used in each regulation.

2. STATE OF THE ART IN SPAIN- SHEAR LOAD DESIGN

2.1 Introduction to shear load design

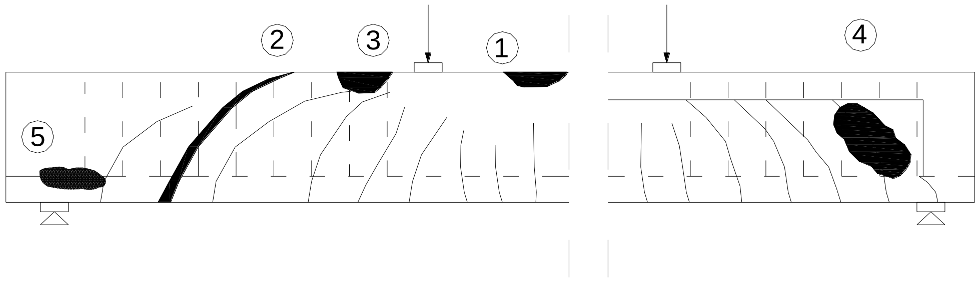
The scope inside of structural calculates which is developed in this thesis is shear load design. The main difference between a piece submitted to shear load (cross efforts) and other which is submitted to normal efforts is that, in the last case, if the reinforced concrete piece is submitted to the same efforts belong whole piece, it is only necessary to study one section to know the general state of the piece.

In contrast, in shear-load design it is necessary to study the whole piece because resistance mechanisms are not flat but spatial.

Some important aspects to define the shear load design are the section form and the variation throughout the piece, the provision of lengthwise and cross reinforcements, the slenderness of the piece, the adherence between steel and concrete, the type of loads and supports and the status of the ones, etc.

The main characteristic of cross efforts in general and shear load in particular, is the inclination of principal tensile stress regarding the guideline of the piece. When loads are minor, the tensile stress produced does not exceed the tensile strength of concrete, so it is necessary to increase loads to see the cracks produced in concrete and the corresponding tension adjustment between concrete and reinforcement, which varies as cracks increase until the break.

The different ways in which this phenomenon may be produced are defined schematically in picture 2.1.1.



Picture 2.1.1 Break forms in a beam. [1]

1- Pure bending breaking: this occurs when tensile reinforcements have excessive deformation which causes a rise of neutral fibre until compressed concrete is not able to balance tractions. If tensile reinforcements were high it is possible the concrete was broken without the steel reaching its yield strength.

2- Failing in shear: this is produced when the cross reinforcements are insufficient. The concrete compression zone ought to resist an important part of shear stress, and if it increases the crack can appear on the upper edge.

3- Bending and shear breaking: Even though the moment was not the maximum, if cross reinforcements are insufficient, the cracks go up more in the zone submitted to bend and shear than in zones with pure bending doing a decrease of resistance capacity of compressed concrete.

4- Core compression failure: this kind of break may occur in T-shaped sections or double T-shaped sections with fine core if principal compression stress overtakes the concrete resistance.

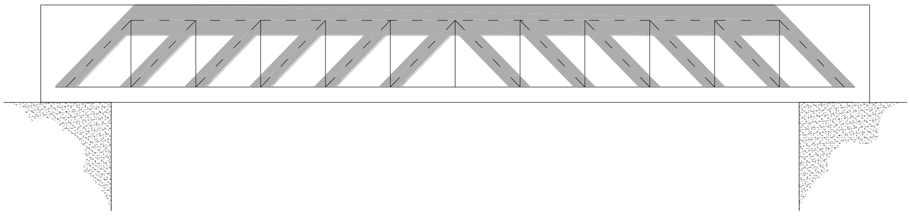
5- Reinforcement slipping failure: the stress of the tensile reinforcement increase towards the middle of the beam and this increase is caused by adherence between concrete and steel. If shear stress increases and lengthwise reinforcements are fixed insufficiently, their slipping could occur near the support, where shear is the maximum.

Shear load design is used to maintain these cracks under permissible levels, providing security on cracks.

[1] Pedro Jiménez Montoya et ál., *Hormigón armado*. Barcelona: Gustavo Gili, 2009, pp. 350-351

2.2 Historical development

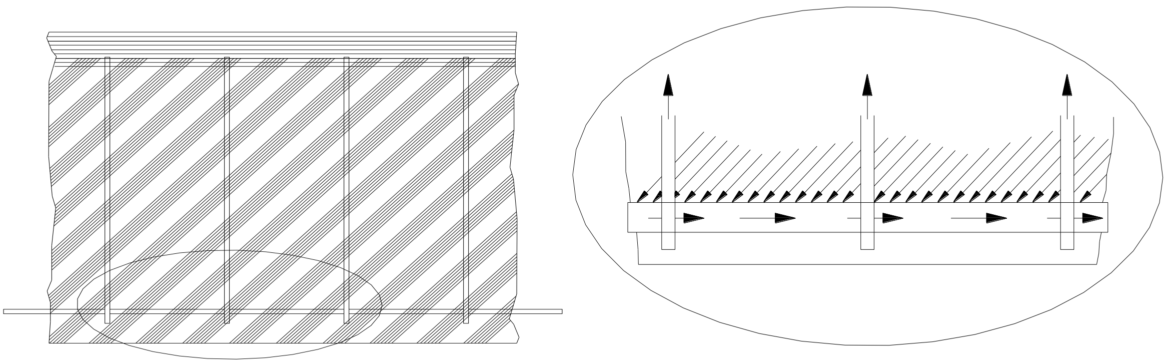
Going back to the nineteenth century, shear failure in reinforced concrete was considered, incorrectly, as pure shear; and it was in the last decade of the century, 1899, that Ritter introduced the concept of diagonal traction and proposed an analogy with lattice. Ritter's theory[2] considered that fences contributed to shear resistance of reinforced concrete by traction and without resisting tangential stresses.



Picture 2.2.1.Ritter's model

These purposes were not accepted among technicians of the time, so there were two currents; one of them believed that fences resisted tangential stresses and the other one supported the new concepts which Ritter had suggested.

Afterwards, in 1909, Mörsch[3] demonstrated that an element submitted to pure tangential stresses presents a diagonal tension with an inclination of 45º. This, together with the fact that the tensile strength of concrete is less than the compressive strength, supposed the break was carried out by diagonal tension of the core.



Picture 2.2.2. Mörsch's model[4]

[2] Wilhelm Ritter. Die bauweise hennebique. [PDF] Available online in: http://dx.doi.org/10.5169/seals-21308.

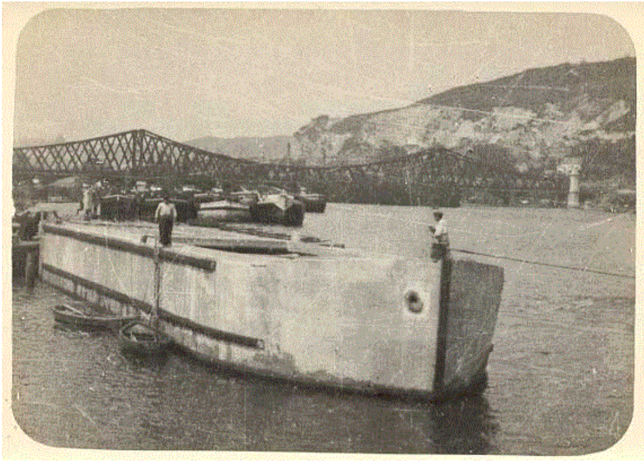
[3] Emil Mörsch. *Concrete steel construction.* New york: McGraw-Hill, 1909.

[4] David Fernández-Ordoñez Hernández. "Mecanismos de respuesta frente al esfuerzo cortante en vigas prefabricadas". Tesis doctoral. Madrid: Universidad Politécnica de Madrid, 2001. pp. 3.42.

Mörsch also introduced the concept of shear stress as nominal size of diagonal tension of the core and consolidated Ritter's idea of the contribution of fences to shear strength resisting tensile stresses, although he only considered the compressive strength of the concrete to shear design.

In 1910 specifications of shear load design were developed in the USA and they believed that this stress depended only on the compressive strength, so the maximum allowable shear force was considered 2% of concrete compression strength, $0.02f_c'$[5].

During the First World War, there were lots of trials carried out on shipbuilding and they concluded that the compressive strength of concrete is not able to define the shear stress and it was Moretto, in the late 40's, who introduced the lengthwise reinforcement quantity together with the compressive strength of concrete to obtain shear stress.



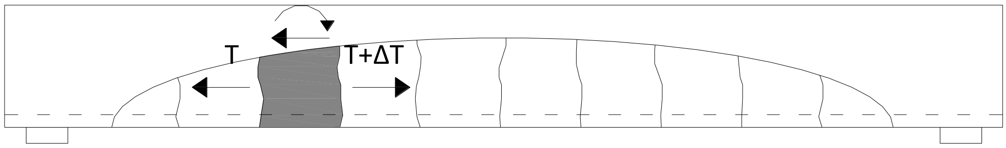
Reinforced concrete barge used in France for the First World War
http://www.exdya.com/barcos-de-hormigon/

In the following decades there were lots of investigations an5d they concluded that different variables were necessary to calculate shear stress and, in the 50's, Clark[6] added a new variable, the connection between thickness-edge. This was a great evolution in shear design.

In 1964 Kani[7] proposed a comparison between a reinforced concrete beam and a comb. In this comparison he supposes that the teeth of the comb are the

[5] As indicated below, current legislations as EHE-08, Euro Code 2 and FIB Model Code use more variables as angle of shear reinforcement, quantity of reinforcement and resistance of the steel among others although the compression strength continue being an important variable.
[6] Arthur P. Clark, Diagonal Tension in reinforced concrete beams. ACI Journal N.48. pp 145-156.
[7] G.N.J. Kani, The Riddle of Shear Failure and its Solution, ACI Structural Journal N. 61. pp 441-467

concrete between bending cracks and, on these teeth, shear is acting due to lengthwise reinforcing.



Picture 2.2.3. Kani's model

Afterwards, this model was studied by many investigators but it was Taylor[8] who, in 1974, added as conclusions to his investigation that each resistance mechanism varied among:

- 20-40% to shear stress where the concrete is not cracked (compression flange of beam).
- 30-50% to aggregate interlock o crack friction.
- 15-25% to dowel action.

On the other hand, in 1978, Collins[9] proposed other model starting from Tension Field Theory which is based on Compression Field Theory.

These models were carried out until late twentieth century although it was Zsutty[10] who proposed the formula that nowadays regulations are based on.

The development of knowledge of shear load design was reflected in different regulations, which continue changing today.

In the case of Spain, the development of structural concrete knowledge was highly influenced by France and Germany because, although the knowledge

[8] H.P.J. Taylor*, Further Test to Determine Shear Stresses in Reinforced concrete Beams*, Cement and Concrete Asociation, London, 1970.
H.P.J.Taylor, The Fundamental Behavoir of Reinforced Concrete Beams in Bending and Shear, ACI SP-42. pp. 43-47.
H.P.J. Taylor*, Investigation of the Dowel Shear Forces Carried by The Tensile Steel in Reinforced Concrete Beams*, Cement and Concrete Asociation, London, 1969.
H.P.J. Taylor*, Investigation of the Forces carried Across Cracks in Reinforced Concrete Beams in Shear by Interlock of Aggregate*, Cement and Concrete Asociation, London, 1970.
H.P.J. Taylor, Shear Stresses in Reinforced Concrete Beams Without Shear Reinforcement, Cement and Concrete Association, London, 1968.
[9] M.P. Collins, Toward a rational theory for RC pieces in shear, ASCE Structural Journal N. 104.
pp. 649-666.
[10] T.C. Zsuty, *Shear strength prediction for separate categories of simple beams test,* ACI Structural Journal N. 68. pp. 138-143.

arrived to the country at the end of the nineteenth century, it was in 1906 that they began to take foreign rules and apply them in Spain.

The first regulation used unofficially in Spain was *“Circulare du minister des Travaux Publics, des postes et des telegraphs aux ingenieurs enchef des ponts en chausses”* which provided the basis for the criteria of the reinforced concrete, together with German law of 1904.

Subsequently, in 1910, Juan Manuel de Zafra published the first Spanish Treatise which was titled *“Mecánica del hormigón armado”* and in 1912 the first regulatory instructions were developed by military engineer’s corps.

In 1917, the Ministry of Development creates a Commission to write the bases for next Instructions to concrete in Public Works and in 1939 the First reinforcement concrete instruction had been provisionally adopted until 1944, when it was adopted definitely.

After this, new instructions were written and modified like HE-61, HE-68, the Spanish Actions Instructions in 1972 and the Instruction about projects, materials and performance in 1973.

Instructions about pre-stressed concrete arrived to Spain in 1977 with EP-77 which was followed by EP-80, modified in 1985, and subsequently by EP-93.

In the eighties, three instructions of reinforced concrete were written in Spain, but it was in 1998 when appeared a common instruction for pre-stressed concrete and reinforced concrete. It was the Instruction of structural concrete, well-known like EHE-98, which was modified by EHE-08 that is usually adopted nowadays in Spain.

2.3 State of shear load design

Nowadays, from a theoretical point of view, it has been demonstrated that it is not possible to study only shear stress in a section to determine the traction force. This occurs because if the piece is subjected to important bending moments, the section is broken and the stress-strain relationship is non-linear.

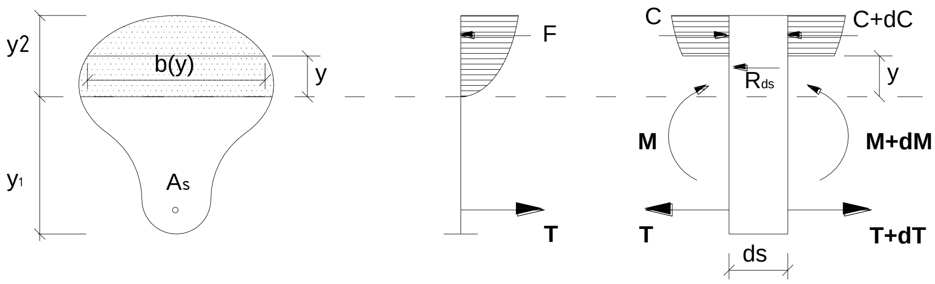


Figure 2.3.1.Stress diagram in a section piece[11]

Thus, it is not possible to know the shear force distribution or tangential tension in ultimate limit state in points above the bend allowance because the diagram of compression consists of parabola-rectangle. However, below this, the shear force is constant, and tangential tension is defined by:

$$\tau = \frac{\frac{dM}{ds}}{b\,z} \qquad (1.1)$$

Therefore, it is necessary to study the whole piece and, depending on whether or not the shear reinforcement is utilised, apply the “lattice assimilation method” or “direct analysis method.”

2.3.1 Lattice assimilation method

The Lattice assimilation method is only used when a piece of concrete has shear reinforcement and it begins from the Ritter-Mörsch premise previously referred to in chapter 2.2.

Starting from a lattice of type Pratt with the characteristics shown below, of stanchions and diagonals of 45º, and generalizing this to a lattice of type

[11] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 39.

Warren, diagonals compressed with cordons at an angle θ and tensile diagonals at an angle α with the same cordons are obtained. Thus, compressed diagonals of such a lattice are like concrete at the piece of reinforced concrete and tensile diagonals are equal to shear reinforcement.

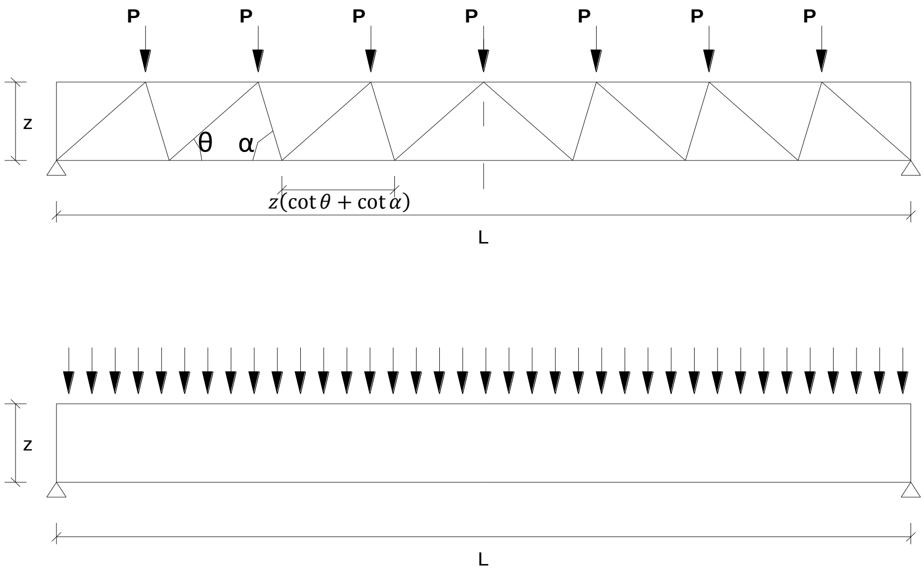


Figure 2.3.2[12]

In this method a segment, like in Figure 2.3.3, can be analyzed to understand the meaning:

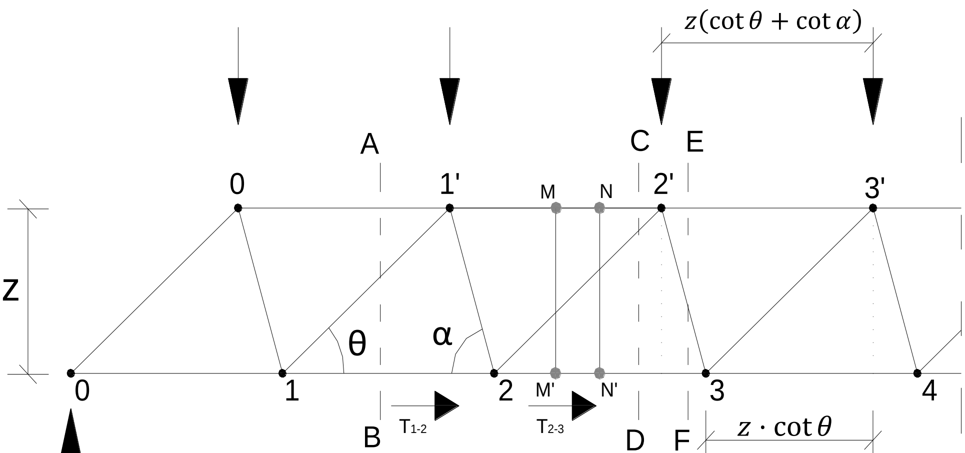


Figure 2.3.3[11]

[12] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 48.

In order to obtain the tensile strength in node 2, one must take into account tensile strengths T_{1-2} and T_{2-3} by cutting A-B for $M_{1'}$ and C-D for $M_{2'}$. The following moments taken at node 1' and 2' are obtained:

$$T_{1-2} \cdot z = M_{1'} \tag{1.2}$$

$$T_{2-3} \cdot z = M_{2'} \tag{1.3}$$

Furthermore, it is necessary to consider the average of these strengths in this node:

$$T_2 = \frac{1}{2}(T_{1-2} + T_{2-3}) = \frac{1}{2}\frac{M_{1'} + M_{2'}}{z} \tag{1.4}$$

But, in this case, the average obtained by this equation is the bending moment, M, and this is not in the node analyzed but rather the point half-way between 1' and 2'. Therefore the bending moment obtained corresponds to another section advanced 2-M' as the bending moment increases:

$$2 - M' = \frac{z(\cot\theta + \cot\alpha)}{2} - \mathrm{z} \cdot \cot\alpha \tag{1.5}$$

If 2-M' is replaced by $k_t{\cdot}z$ the following relation can be obtained:

$$k_t = \frac{1}{2}(\cot\theta - \cot\alpha) \tag{1.6}$$

In the same way, at the top chord compression, if the piece is cut off at C-D, then the moment is calculated at node 2 by:

$$C_{1\prime-2\prime} = \frac{M_2}{z} \tag{1.7}$$

$$C_{2\prime-3\prime} = \frac{M_3}{z} \tag{1.8}$$

Therefore:

$$C_{2\prime} = \frac{1}{2}\frac{M_2 + M_3}{z} \tag{1.9}$$

Thus, considering that $\frac{M_2+M_3}{z}$ is the moment at the point half-way between 2 and 3, called N', the compression strength in 2' is the bending moment located at a distance of 2'-N along the axis in which the bending moment decreases.

$$2' - N = \frac{z(\cot\theta + \cot\alpha)}{2} - z\cot\alpha \qquad (1.10)$$

And, if 2'-N is replaced by $k_c \cdot z$, the following is obtained:

$$k_c = \frac{1}{2}(\cot\theta - \cot\alpha) \qquad (1.11)$$

Therefore, according to this analysis, moments' displacement occurs of the same magnitude but in opposite directions in both chords, and as with these displacements, there are variations in shear reinforcements and compressed concrete.

Assuming that F is the shear reinforcement strength of diagonal 2'-3, if the beam is cut in E-F and projected onto the normal of the directrix is derived:

$$F = \frac{V_{3'}}{\sin\alpha} \qquad (1.12)$$

Therefore, shear tensile in diagonal length 2'-3 is the shear strength in 3', separated by z·cotgθ from 3. The connecting rod 2-2' is projected as:

$$C = \frac{V_{2'}}{\sin\theta} \qquad (1.13)$$

So, the compression in 2-2' is the same as shear strength in 2'.

2.3.2 Direct analysis method

In this method, it is possible to find two variants, analyse pieces with shear reinforcement or pieces without shear reinforcement.

In the first moment, the piece is not broken but is submitted to a tension increasing in shear reinforcements (Figure 2.3.4), it has the sum of the load which the piece is submitted to and the dorsal component of shear stress (Vd).

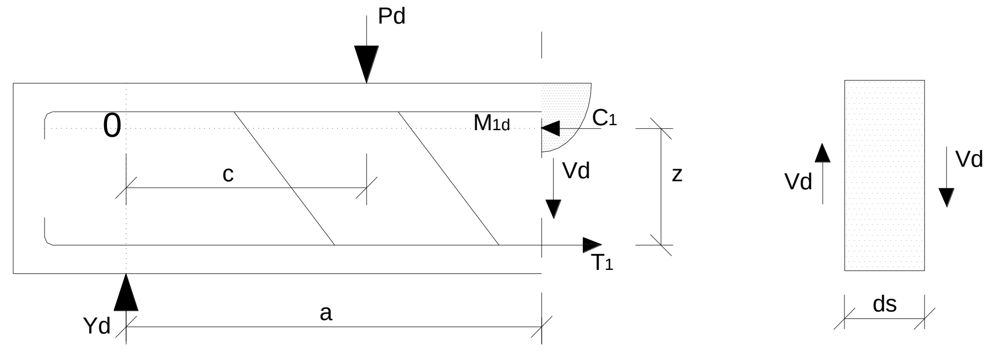


Figure 2.3.4[13]

So, if moments are taken relative to point 0, the compression resulting in the concrete and compressed reinforcement will be:

$$T_1 \cdot z - P_d \cdot c - V_d \cdot a = 0 \qquad (1.14)$$

And bearing in mind:

$$V_d = Y_d - P_d \qquad (1.15)$$

The following is obtained:

$$M_{1d} = T_1 \cdot z = Y_d \cdot a - P_d(a - c) \qquad (1.16)$$

2.3.2.1 Cracked piece without shear reinforcement

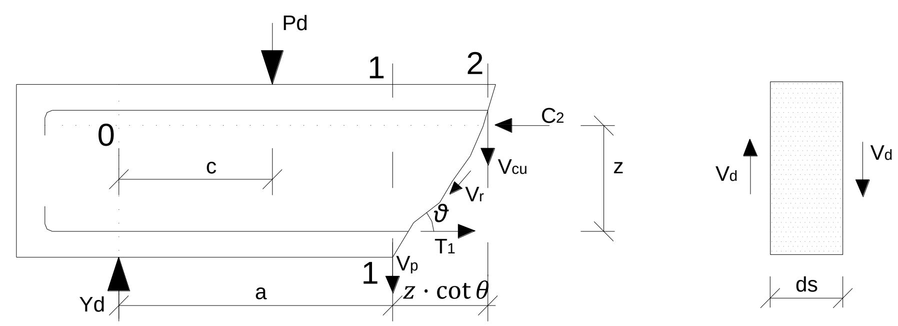


Figure 2.3.5[14]

[13] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 51.

[14] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 52.

When the crack is formed, stress at the direction orthogonal to any point of this is zero and in parallel to it will be σ_{CI}. Thus, according to classic formulas about elasticity, tensile stress is:

$$\sigma_{CII} = \frac{\sigma_x}{2} + \sqrt{\frac{\sigma_{x^2}}{4} + \tau_{xy}^2} = 0 \qquad (1.17)$$

So, parallel compression stress will be:

$$\sigma_{CI} = -2\sqrt{\frac{\sigma_{x^2}}{4} + \tau_{xy}^2} \qquad (1.18)$$

And, like throughout the crack σ_{CII}=0, σ_{CI}=τ_{xy}=2τ_c, compression stress in the connecting rod when there is no shear reinforcement is:

$$\sigma_{CI} = 2\frac{V}{b \cdot z} \qquad (1.19)$$

Thus, taking bending moments in point 0 and assuming that there is no friction between the broken pieces and that transversal strength from compression and tensile reinforcement are zero:

$$T_1 \cdot z - V_{cu}(a + z \cdot \cot\theta) - P_d \cdot c = 0 \qquad (1.20)$$

And, knowing that $V_{cu} = V_d$ and bearing in mind (1.15):

$$T_1 \cdot z = Y_d(a + z \cdot \cot\theta) - P_d(a + z \cdot \cot\theta - c) = M_{2d}$$

$$T_1 = \frac{M_{2d}}{z} \qquad (1.21)$$

So, through this equation it is possible to observe $T_1 \cdot z = M_{1d} + V_d \cdot \cot\theta$, that is, that the increase in tensile because of the crack due to shear strength is $\Delta T = V_d \cdot \cot\theta$.

2.3.2.2 Cracked piece with shear reinforcement

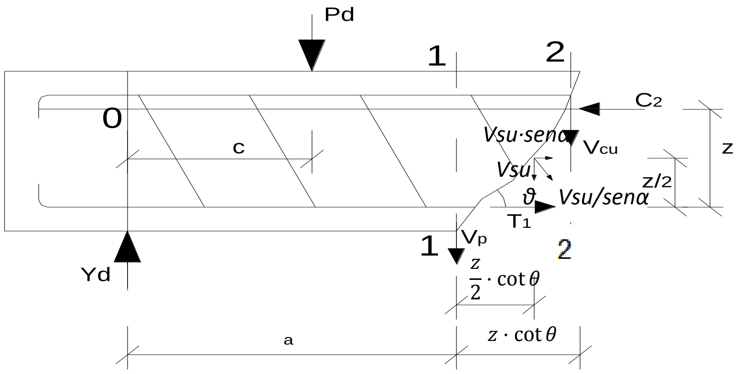


Figure 2.3.5[15]

In this case, V_p are strengths due to the effect of pin and V_{su} is the tensile in shear reinforcement. V_p are considered zero because the main shear transfer mechanisms is the friction between the contact surfaces, so the effect of pin is only important when the piece is heavily reinforced. The remaining strengths are equal to those previously considered.

On 2-2, shear strength V_{cu} will be absorbed by concrete and shear stress V_{su}, which is resisted by reinforcement, can be replaced by a unique strength z/2 from tension reinforcement. So, taking moments in 0:

$$T_1 \cdot z = a(V_{cu} + V_{su}) + P_d \cdot c + V_{su}\frac{z}{2}(\cot\theta - \cot\alpha) + V_{cu} \cdot z \cdot \cot\theta \qquad (1.22)$$

Whereas $V_d = V_{su} + V_{cu}$ and considering *[Eq 1.16]*:

$$T_1 \cdot z = M_{1d} + z\left[\frac{V_{su}}{2}(\cot\theta - \cot\alpha) + V_{cu} \cdot \cot\theta\right] \qquad (1.23)$$

And making $\frac{V_{su}}{V_d} = \lambda$, it is obtained that the tensile increase in reinforcements due to the crack by shear strength is:

$$\Delta T_1 = V_d\left[\cot\theta - \frac{\lambda}{2}(\cot\alpha + \cot\theta)\right] \qquad (1.24)$$

If the formula inside the square bracket is substituted by k'$_t$, by applying (1.15) or (1.16) we obtain the following:

[15] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 54.

$$M_d = M_{1d} + (Y_d - P_d) \cdot k'_t \cdot z \qquad (1.25)$$

$$M_d = Y_d(a + k'_t) - P_d(a + k'_t - c) \qquad (1.26)$$

Therefore, tensile reinforcement in section 1-1 is corresponding to another section located at the distance $k'_t \cdot z = z\left[\cot\theta - \frac{\lambda}{2}(\cot\alpha + \cot\theta)\right]$ in the direction which the bending moment increases.

Thus, if the piece has uniform section and is submitted to pure bending, reinforcement must be in the section previous to point k_{td} in the direction which increases bending moment.

2.3.2.3 Compressive force decrease in compressed head

When the piece is cracked and has shear reinforcement (Figure 2.3.5), if the forces over the directrix project:

$$C_2 \cdot z = M_{1d} + z \cdot V_d(k'_t + \lambda\cot\alpha) \qquad (1.27)$$

And substituting with (1.16):

$$C_2 \cdot z = Y_d(a + k'_z) - P_d(a - c + k'_z) \qquad (1.28)$$

Thus, compression C_2 in section 2-2 is the moment in other section moved k'_z to 1-1 in the direction of the increase of the bending moment.

2.3.2.4 Section to consider for shear reinforcement design and checking compressed concrete

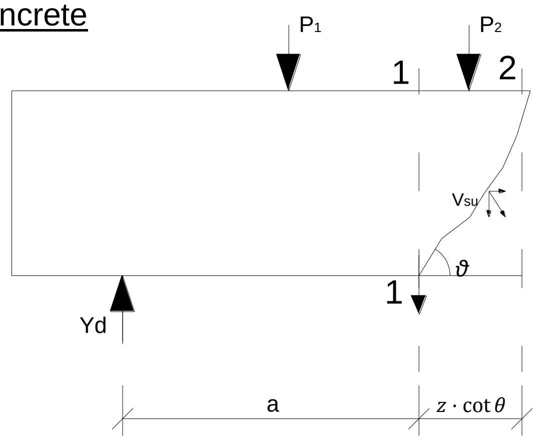


Figure 2.3.6[16]

[16] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 62.

Shear reinforcement in section like 1-1 in *Figure 2.3.6* will be calculate depending on shear stress in section 2-2, which is moved z·cotgθ in the direction of the decrease of shear stress. In the same way, to calculate compressed concrete, it is necessary to do it by considering the shear stress in section moved z·cotgθ in same direction.

2.4 State of shear load design in Spain

Usually, θ=45º is used to calculate shear stress because concrete tensile stresses are insignificant when adopting a 45º lattice hypothesis. Nevertheless, EHE-08 accepts a variation range in the tilt strength between:

$$0.5 \leq \cot\theta \leq 2 \quad (1.29)$$

That is equal to:

$$63.4º \geq \theta \geq 26.6º \quad (1.30)$$

In general, except for linear elements such as slabs and panels, in every section under a plane P with actions that produce tangential tension, it has to be pierced by cross reinforcements and fixed on both sides of the plane to avoid future cracks.

If reinforcements have section A_{st} with distance *s,* and depletion is produced with tilt angle θ regarding to plane *P*, the equilibrium is obtained with tangential tension, traction reinforcements, and compressions between cracks.

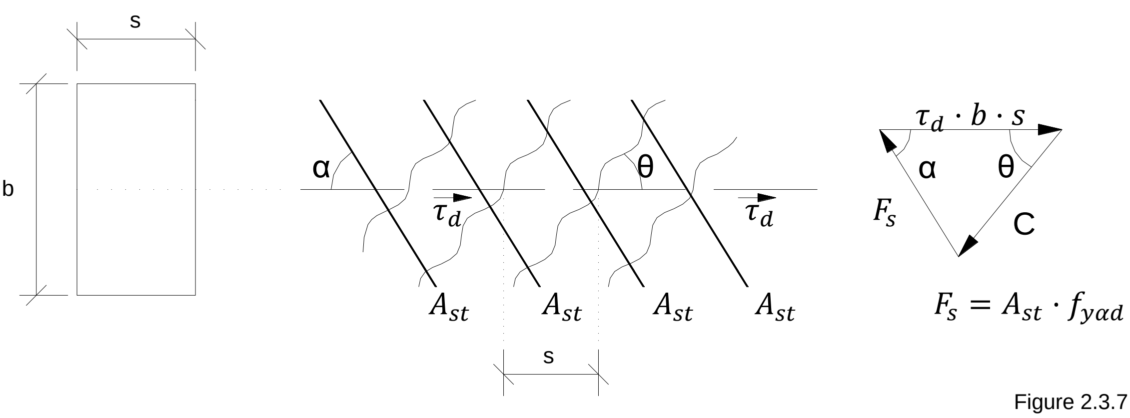


Figure 2.3.7

From which it follows:

$$\frac{\tau_d \cdot b \cdot s}{\sin(\theta+\alpha)} = \frac{F_s}{\sin\theta} \quad (1.31)$$

Substituting and modifying:

$$\tau_d \leq \frac{A_{st}}{b \cdot s} f_{y\alpha d} \cdot \sin\alpha\,(\cot\alpha + \cot\theta) \quad (1.32)$$

Thus it is possible to calculate the necessary reinforcement with τ_d or $\tau_d$from reinforcement.

If it is calculated with $\theta = 45º$, usually in reinforced concrete:

$$\tau_d \leq \frac{A_{st}}{b \cdot s} f_{yd} \sin\alpha\,(1 + \cot\alpha) \qquad (1.33)$$

So, after this, it depends on tilt angle of shear reinforcement, and it is a condition for EHE-08 that $45º \leq \alpha \leq 90º$::

If $\alpha = 45º$

$$\tau_d \leq \sqrt{2}\frac{A_{st}}{b \cdot s} f_{y\alpha d} \qquad (1.34)$$

If $\alpha = 90º$

$$\tau_d \leq \frac{A_{st}}{b \cdot s} f_{y\alpha d} \qquad (1.35)$$

Therefore bent bars 45º are 41% more effective than brackets.

And it is important to know that EHE-08 does not allow the use of steel with characteristic limit of elasticity more than 400 N/mm^2 and does not consider concrete action.

Straightaway it is necessary to check concrete compression and from Figure 2.3.7 and (1.31) follows:

$$C = \frac{\tau_d \cdot b \cdot s \cdot \sin\alpha}{\sin(\theta + \alpha)} \qquad (1.36)$$

And the compressive stress of the connecting rod is:

$$\sigma_c = \frac{C}{b \cdot s \cdot \sin\theta} \qquad (1.37)$$

So:

$$\sigma_c = \frac{\tau_d}{\sin^2\theta\,(\cot\alpha + \cot\theta)} \qquad (1.38)$$

Also, EHE-08 has a limitation:

$$\sigma_c \leq K \cdot f_{1cd} \qquad (1.39)$$

To avoid micro-cracking damages, the result must be:

$$\tau_d \leq K \cdot f_{1cd} \cdot \sin^2\theta\,(\cot\alpha + \cot\theta) \qquad (1.40)$$

Where f_{1cd} is the connecting rod concrete compression strength and it has different values depending on characteristic resistance of concrete:

$$f_{1cd} = 0.60 f_{cd} \qquad \text{If } f_{ck} \leq 60\ MPa \qquad (1.41)$$

$$f_{1cd} = \left(0.9 - \frac{f_{ck}}{200}\right) f_{cd} \not< 0.5 f_{cd} \qquad \text{If } f_{ck} > 60\ MPa \qquad (1.42)$$

And K is the coefficient for axial force applied on the piece and it depends on concrete effective stress:

$$K=1 \qquad \text{If there is not axial force} \qquad (1.43)$$

$$K = 1 + \frac{\sigma'_{cd}}{f_{cd}} \qquad \text{If } 0 \leq \sigma'_{cd} \leq 0.25 f_{cd} \qquad (1.44)$$

$$K=1.25 \qquad \text{If } 0.25 f_{cd} \leq \sigma'_{cd} \leq 0.5 f_{cd} \qquad (1.45)$$

$$K = 2.5\left(1 - \frac{\sigma'_{cd}}{f_{cd}}\right) \qquad \text{If } 0.5 f_{cd} \leq \sigma'_{cd} \leq f_{cd} \qquad (1.46)$$

Where:

$$\sigma'_{cd} = \frac{N_d - A'_s \cdot f_{yd}}{A_c} \qquad (1.47)$$

This varies according to:

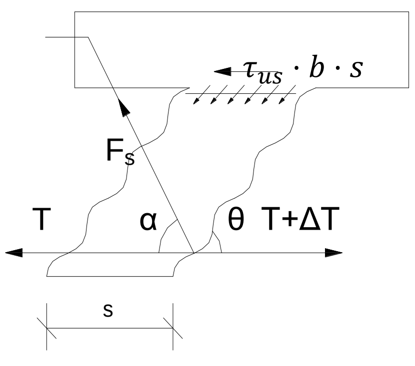
N_d= axial force

A'_s= compressive reinforcement area

f_{yd}= resistance of reinforcement

A_c= concrete section area

To calculate, it is necessary to select a piece of beam between two consecutive cracks starting from forces seen in Figure 2.3.5 which would be like Figure 2.3.8.



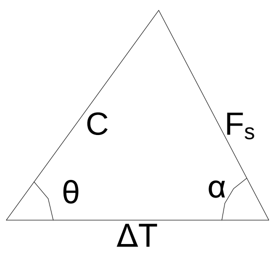


Figure 2.3.8[17]

[17] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 69.

2.4.1. Shear stress resisted by concrete

According to EHE-08, shear stress resisted by concrete is:

$$V_{cu} = \left[\frac{0.18}{\gamma_c}\xi(100\rho_1 f_{ck})^{1/3} + 0.15\alpha_1\sigma'_{cd}\right] b_0 \cdot d \qquad (1.48)$$

And it has to have a minimum value:

$$V_{cu} \geq \left[\frac{0.075}{\gamma_c}\xi^{3/2} f_{ck}^{1/2} + 0.15\sigma'_{cd}\right] b_0 \cdot d \qquad (1.49)$$

Where:

$\xi = 1 + \sqrt{\frac{200}{d}} \not> 2$ shear strength increases as edge decreases

$\rho_1 = \frac{A_s}{b_0 \cdot d} \not> 0.02$ this is when the steel is B400S, but if other kinds of steel with more resistance are used, it would be better to multiply the result by 1.25 and decrease the limit to 0.016.[18]

$b_0 =$ figure width if the core width changes, b_0 is the smaller width within the three-fourths of effective depth from the tension reinforcement.

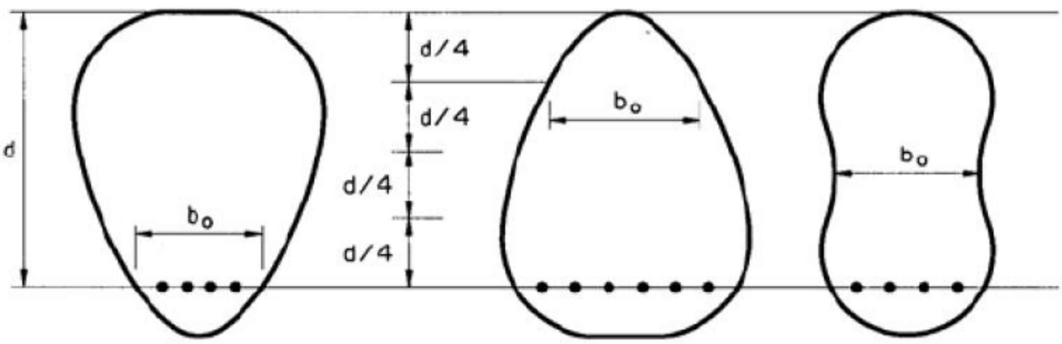


Figure 2.3.9[19]

[18] José Calavera Ruiz, *Proyecto y cálculo de estructuras de hormigón: en masa, armado, pretensado.Tomo II. 2º Ed.* Madrid: Intemac, 2008. p. 70.
[19] EHE-08, *Instrucción española de hormigón estructural.* B.O.E 22-Agosto-2008. art. 44.2.1.

According to EHE-08, f_{ck} has to be smaller than 60MPa, or if the concrete is with reduced control[20] it will be fewer than 15MPa.

2.4.2. Shear stress resisted by tension reinforcement

According to Figure 2.3.7, reinforcement force, compressions in connecting rod between cracks, and tangential tensions have to be in balance:

$$\frac{F_s}{\sin\theta} = \frac{C}{\sin\alpha} = \frac{\tau_{us} \cdot b \cdot s}{\sin(\theta + \alpha)} \quad (1.50)$$

Where:

$$\tau_{us} = \frac{V_{su}}{b \cdot z} \approx \frac{V_{su}}{b \cdot 0.9d} \quad (1.51)$$

So:

$$F_s = A_{st} \cdot f_{y\alpha d} = \frac{V_{su} \cdot \sin\theta}{0.9\frac{d}{s}(\sin\theta \cdot \cos\alpha + \cos\theta \cdot \sin\alpha)} \quad (1.52)$$

And modifying:

$$V_{su} = 0.9\frac{d}{s}A_{st} \cdot f_{y\alpha d} \cdot \sin\alpha\,(\cot\alpha + \cot\theta) \quad (1.53)$$

2.4.3. Maximum allowable compression

Starting from (1.50) and using Vd to designate shear stress:

$$\tau_d \cdot b \cdot 0.9d = V_d = \sigma_c \cdot b \cdot d \cdot \sin^2\theta\,(\cot\alpha + \cot\theta) \quad (1.54)$$

Thus, bearing in mind (1.39) to avoid micro cracking in concrete:

$$V_{u1} = K \cdot b \cdot d \cdot f_{1cd}\frac{(\cot\alpha + \cot\theta)}{1 + \cot^2\theta} \quad (1.55)$$

[20] In Spain, according to EHE-08, it is necessary to check throughout fulfilment characteristic resistance of concrete and it has to be equal or greater than the one specified in the project. There are different kinds of check and reduce control is only possible to use in small engineering works and buildings where, moreover, concrete has a class of exposure different of III and IV.

2.4.4. Check

Broadly speaking, it should be necessary to make four different checks because, for the Spanish legislation, the beam should be considered a lattice. Thus, it should be necessary to check the top and bottom chord, uprights and diagonals. Firstly, it would be necessary to check if the compression top chord can bear the load but, as in this area there are no cracks, it is not necessary to make the check because tangential tensions appear to be contributing to hold the shear force.

Secondly, it would be necessary to check the bottom chord, which has to bear an increase of traction as following:

$$\Delta T = V_{Rd} \cdot \cot\theta - \frac{V_{su}}{2}(\cot\theta + \cot\alpha) \qquad (1.56)$$

But according to the Spanish legislation, this can be fulfilled shifting the bending moment diagram a magnitude equal to:

$$s_d = z\left(\cot\theta - \frac{1}{2}\frac{V_{su}}{V_{rd}}(\cot\theta + \cot\alpha)\right) \qquad (1.57)$$

in the most adverse sense as it can be seen in the figure 2.3.10.

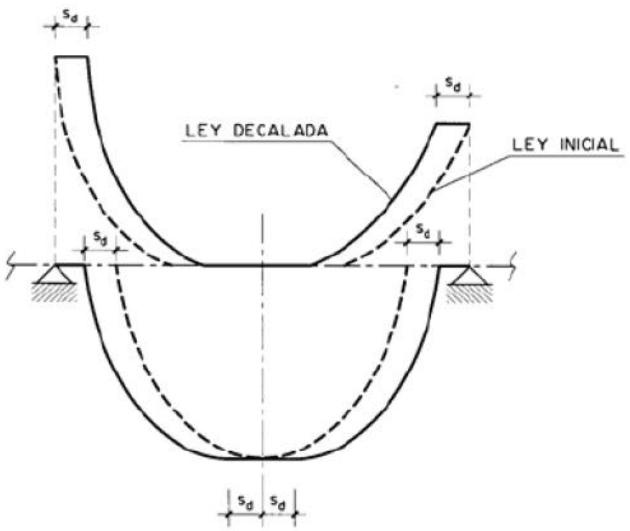


Figure 2.3.10[21]

[21] EHE-08, *Instrucción española de hormigón estructural.* B.O.E 22-Agosto-2008. art. 44.2.3.4.2.

Thus, in general it is only necessary to make two checks and, henceforward $V_{rd} = V_d$ when the section piece is constant.

2.4.4.1. Check of depletion due to diagonal compression core

$$V_{rd} \leq V_{u1} \tag{1.58}$$

Where V_{u1} is (1.55) that must be calculated over the support and, if the check is wrong, it can be solved with:

- Using a different crack tilt angle
- Rising *b* or *d*, although it is better rising *d* because it produces decreased reinforcement.
- Improving concrete resistance.

2.4.4.2. Check of depletion due to tension core, reinforcement and concrete

$$V_{rd} \leq V_{u2} \tag{1.59}$$

Where according to EHE-08:

$$V_{u2} = V_{cu} + V_{su} \tag{1.60}$$

And V_{su} is (1.48) but in this case:

$$V_{cu} = \left[\frac{0.15}{\gamma_c}\xi(100\rho_1 f_{ck})^{1/3} + 0.15\alpha_1\sigma'_{cd}\right]\beta \cdot b_0 \cdot d \tag{1.61}$$

Where:

$$\beta = \frac{2\cot\theta - 1}{2\cot\theta_e - 1} \qquad \text{If } 0.5 \leq \cot\theta \leq \cot\theta_e \tag{1.62}$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} \qquad \text{If } \cot\theta_e \leq \cot\theta \leq 2 \tag{1.63}$$

And:

$$\cot\theta_e = \frac{\sqrt[2]{f_{ctm}^2 - f_{ctm}(\sigma_{xd} + \sigma_{yd}) + \sigma_{xd} \cdot \sigma_{yd}}}{f_{ctm} - \sigma_{yd}} \tag{1.64}$$

Bearing in mind:

$$f_{ctm} = 0.3 f_{ck}^{2/3} \qquad \text{If } f_{ck} \leq 50\ N/mm^2 \tag{1.65}$$

$$f_{ctm} = 0.58 f_{ck}^{1/2} \qquad \text{If } f_{ck} > 50\ N/mm^2 \tag{1.66}$$

$$\sigma_{xd} = \frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}} \quad (1.67)$$

$$\sigma_{yd} = \frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}} \quad (1.68)$$

If the check is wrong, the best way to solve it is by increasing tension reinforcement, although the solutions specified in 2.4.4.1 can be adopted if they are more economical.

2.4.4.3. θ values

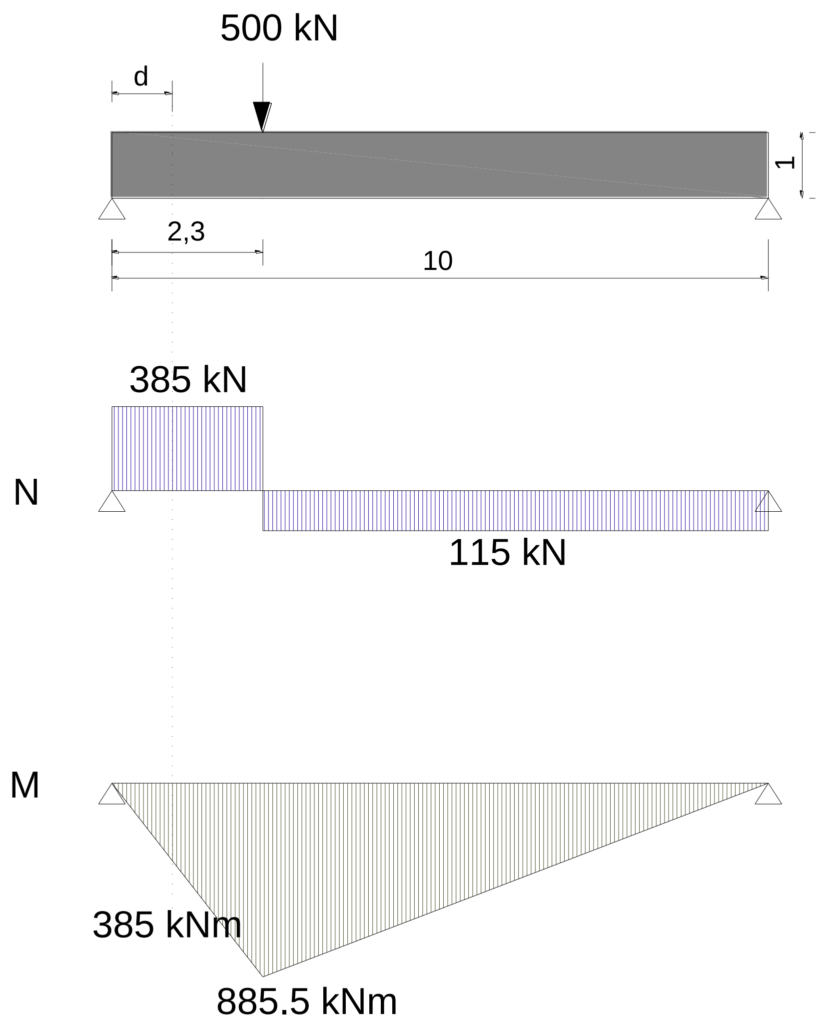
Although θ=45º is normally used, θ values must be used to obtain the best cost optimization, and the designer is free to select the angle according to EHE-08. Within the range of variation, low values of θ decrease cross reinforcement and increase traction of flexural reinforcement due to shear stress and with high values otherwise occurring.

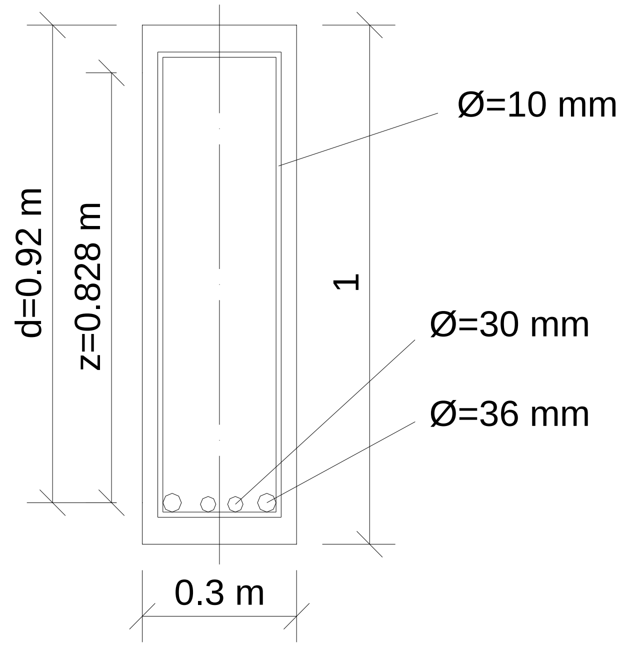
3. CALCULATION OF RC MEMBER

3.1. Baseline data

To better analyse shear load design, one must calculate a piece with specific dimensions, reinforcement, and concrete.

The characteristics are as follows:





$\theta = 45º$

$\alpha = 90º$

$f_{ck} = 30\ N/mm^2$

$\gamma_c = 1.5$

$f_{yk} = 550\ N/mm^2$

$\gamma_s = 1.15$

$s = 120\ mm$

Figure 3.1.1

3.2. Calculation of RC member according to EHE-08

According to EHE-08, the RC member described in head 2.1 has to meet:

$$V_{Rd} \leq V_{u1} \quad (2.1)$$

$$V_{Rd} \leq V_{u2} \quad (2.2)$$

Where:

$$V_{Rd} = V_d + V_{pd} + V_{cd} \quad (2.3)$$

And:

V_{u1}: shear failure due to diagonal compression core.

V_{u2}: shear failure due to traction in the core.

V_{Rd}: design value of effective shear force.

V_d: design value of the shear force produced by external actions.

V_{pd}:design value of the parallel component of the prestressed section.

V_{cd}:design value of the parallel component of the normal stresses resulting from compression and tension in the passive reinforcement, on the longitudinal fibers of concrete.

Firstly it is important to check if the piece complies with equation (2.1) about compression core on the edge of the support:

$$V_{u1} = K \cdot f_{1cd} \cdot b_0 \cdot d \cdot \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta} \tag{2.4}$$

Where, according to EHE-08:

$f_{1cd} = 0.6f_{ck}$=0.6·30=12 N/mm² (2.5)

b_0: in this case, as the core width does not change it is the figure width

$K = 1$

Thereby:

$$V_{u1} = 12 \cdot 300 \cdot 920 \cdot \frac{1+0}{1+1}$$

$$\boxed{V_{Rd} = 385\ kN < V_{u1} = 1656\ kN} \tag{2.6}$$

And then it is necessary to check if, at a distance of the effective depth from the edge of the support, the beam can bear the traction in the core without shear reinforcement. Thus, according to EHE-08, when the beam has not shear reinforcement:

$$V_{u2} = \left[\frac{0.18}{\gamma_c} \cdot \xi \cdot (100 \cdot \rho_1 \cdot f_{cv})^{1/3} + 0.15\sigma'_{cd}\right] b_0 \cdot d \tag{2.7}$$

Where:

$$\xi = 1 + \sqrt[2]{\frac{200}{d}} = 1 + \sqrt[2]{\frac{200}{920}} = 1.466 < 2 \tag{2.8}$$

$$\sigma'_{cd} = \frac{N_d}{A_c} = 0 < 0.3f_{cd} \tag{2.9}$$

$$\rho_1 = \frac{A_s + A_p}{b_0 \cdot d} = \frac{2\pi \cdot 18^2 + 2\pi \cdot 15^2}{300 \cdot 920} = 0.0125 \leq 0.02 \tag{2.10}$$

A_s: area of the tensile reinforcement

A_p: area of the prestressed reinforcement

b_0: as the core width does not change it is the figure width

Therefore, substituting in (2.7):

$$V_{u2} = 162.54\ kN \tag{2.11}$$

And it must comply the minimum amount according to EHE-08:

$$V_{u2,min} = \left[\frac{0.075}{\gamma_c} \cdot \xi^{3/2} \cdot f_{cv}{}^{1/2} + 0.15\sigma'_{cd}\right] b_0 \cdot d \tag{2.12}$$

That substituting:

$$V_{u2,min} = 134.20\ kN \tag{2.13}$$

Thus:

$$\boxed{V_{u2} = 162.54\ kN > V_{u2,min} = 134.20\ kN} \tag{2.14}$$

And:

$$\boxed{V_{u2} = 162.54\ kN < V_{Rd} = 385\ kN} \tag{2.15}$$

So, the beam without shear reinforcement cannot bear the shear force and needs shear reinforcement. To check if the reinforcement in *Figure 3.1.1* is enough according to EHE-08, one must be checked using equation (1.58).

Bearing in mind (1.53) where:

$$A_{st} = 2\pi \cdot 5^2 = 157.08\ mm^2$$

$$f_{y\alpha d} = \frac{550}{1.15} = 478.26\ N/mm^2$$

As EHE-08 establish $f_{y\alpha d} \leq 400N/mm^2$ and the characteristic of the steel used has $f_{y\alpha d} > 400N/mm^2$, it will be adopted $f_{y\alpha d} = 400N/mm^2$, the maximum allowed. Thus:

$$V_{su} = 828 \cdot 1(0+1) \cdot \frac{157.08}{120} \cdot 400$$

$$\boxed{V_{su} = 433.54\ kN} \tag{2.16}$$

And substituting according to *Figure 3.1.1* in (1.61) where:

$$f_{ct,m} = 0.3 \cdot 30^{2/3} = 2.896\ N/mm^2$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} = \frac{2 \cdot 1 - 1}{2 \cdot 1.25 - 1} = 1$$

Because:

$$\cot\theta_e = \frac{\sqrt[2]{f_{ctm}^2 - f_{ctm}\left(\sigma_{xd} + \sigma_{yd}\right) + \sigma_{xd}\cdot\sigma_{yd}}}{f_{ctm} - \sigma_{yd}} = \frac{2.896}{2.896} = 1$$

$$\cot\theta_e \leq \cot\theta \leq 2$$

Bearing in mind that:

$$\sigma_{xd} = \frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}} = 0$$

$$\sigma_{yd} = \frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}} = 0$$

The conclusion, substituting in (1.61) is:

$$V_{cu} = \left[\frac{0.15}{1.5}1.466(100\cdot 0.0125\cdot 30)^{1/3} + 0\right]1\cdot 300\cdot 920$$

$$\boxed{V_{cu} = 135.45\ kN} \qquad (2.16)$$

And finally, according to (1.60):

$$V_{u2} = 433.5 + 135.4$$

$$\boxed{V_{u2} = 568.99\ kN} \qquad (2.17)$$

Thus, the check proves that:

$$\boxed{V_{Rd} = 385\ kN < V_{u2} = 568.99\ kN} \qquad (2.18)$$

4. COMPARISON TO EC 2. NATIONAL ANNEX AUSTRIA

4.1 Calculation of RC member according to EC 2 National Annex Austria.

According to Eurocode-2 (EC 2), the RC member described above has to meet:

$$V_{Rd} \geq V_{Ed} \tag{3.1}$$

Where:

$$V_{Rd} = V_{Rd,s} + V_{ccd} + V_{td} \tag{3.2}$$

Which:

V_{Ed}: design value of the shear force in the studied section from external influence.

$V_{Rd,s}$: design value of the shear force that can be sustained by the yielding shear reinforcement, the contribution of the longitudinal reinforcement and the concrete contribution of a piece with shear reinforcement.

V_{ccd}: design value of the shear reinforcement of the force in the compression area, in the case of an inclined compression chord.

V_{td}: design value of the shear component of the force in the tensile reinforcement, in the case of an inclined tensile chord.

Also, in the pieces with shear reinforcement it is important to calculate:

$V_{Rd,max}$: design value of the maximum shear force which can be sustained by the piece, limited by crushing of the compression struts.

To calculate the resistance of the RC member firstly, it is necessary to check if:

$$V_{Ed} \leq V_{Rd,c} \tag{3.3}$$

And $V_{Rd,c}$ is the design shear resistance of the piece without shear reinforcement, the equivalent of V_{u2} in EHE-08 when it is calculated without shear reinforcement:

$$V_{Rd,c} = \left[C_{Rd,s} \cdot K \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + K_1 \cdot \sigma_{cp}\right] b_w \cdot d \tag{3.4}$$

Where:

$$f_{ck} = 30 N/mm^2$$

$$K = 1 + \sqrt[2]{\frac{200}{d}} = 1 + \sqrt[2]{\frac{200}{920}} = 1.466 \leq 2 \tag{3.5}$$

$$\rho_1 = \frac{A_{sl}}{b_w \cdot d} = \frac{2\pi \cdot 18^2 + 2\pi \cdot 15^2}{300 \cdot 920} = 0.012498 \leq 0.02 \tag{3.6}$$

A_{sl}: area of the tensile reinforcement

b_w: the smallest cross-sectional width in the tensile zone of the cross section.

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} = 0 < 0.2 f_{cd} \tag{3.7}$$

$$C_{Rd,s} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12 \tag{3.8}$$

And substituting in (3.4):

$$V_{Rd,c} = \left[0.12 \cdot 1.466 \cdot (100 \cdot 0.012498 \cdot 30)^{1/3} + 0\right] 300 \cdot 920$$

$$\boxed{V_{Rd,c} = 162.54\ kN < V_{Ed} = 385\ kN} \tag{3.9}$$

Thus, as the result does not comply with (3.3), the piece needs shear reinforcement to be able to bear V_{Ed}. Therefore, it is necessary to check that the reinforcement the RC member has is sufficient, and in this case, what is the maximum shear force it can sustain.

According to (3.2), to calculate the effective shear force one needs to know $V_{Rd,s}$, defined by (3.9):

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \sin\alpha \tag{3.10}$$

Where A_{sw} is the cross sectional area of shear reinforcement and “s” is the space between stirrups. Moreover in this case $\theta = 45º$ is used although, in Austria the minimum angle is usually used ($\theta = 30.96º$) which presents a greater resistance:

$$V_{Rd,s} = \frac{2\pi \cdot 5^2}{120} \cdot 828 \cdot 478.26 \cdot (1+0)1$$

$$\boxed{V_{Rd,s} = 518.36 \text{ kN}} \tag{3.11}$$

That:

$$\boxed{518.36\ kN = V_{Rd,s} > 385\ kN = V_{Ed}} \tag{3.12}$$

And (3.3) is fulfilled.

Thus, it is important to check the maximum shear force that the beam can sustain, limited by the crushing of the compression struts, the equivalent of V_{u1}in EHE-08:

$$V_{Rd,max} = b_w \cdot z \cdot \nu \cdot f_{cd} \cdot \frac{1}{\cot\theta + \tan\theta} \tag{3.13}$$

Where:

$$\nu = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.6\left(1 - \frac{30}{250}\right) = 0.528 \tag{3.14}$$

$$f_{cd} = \frac{30}{1.5} = 20\ N/mm^2 \tag{3.15}$$

Thus, substituting in (3.13):

$$V_{Rd,max} = 300 \cdot 828 \cdot 0.528 \cdot 20 \cdot \frac{1}{1+1}$$

$$\boxed{V_{Rd,max} = 1311.55\ kN} \tag{3.16}$$

4.2 Comparison EHE-08 to EC 2.

In general, the main differences between EHE-08 and EC 2 are the breaking angle, which can be considered in designing the structure, and the value of the yield of the strengthened steel.

The different break angles that are allowed to consider is:

EHE-08	EC 2
$\mathbf{0.5 \leq cot\,\theta \leq 2}$	$1 \leq cot\,\theta \leq 2.5$
$\mathbf{63 \geq \theta \geq 26.6}$	$45 \geq \theta \geq 21.8$

In connection with the yield of the strengthened steel, the definition by Euro Code 2 is $f_{y\alpha d} = \frac{f_{yk}}{\gamma_s}$. In EHE-08 this value is restricted to be no more than 400 N/mm^2. In the case it surpasses this threshold, by the Spanish legislation; the beam has to be redesigned as to yield an amount of 400 N/mm^2.

This difference in results is the most prevalent, but more minor differences are discussed further on.

4.2.1. Shear force without shear reinforcement.

Beam shear resistance without shear reinforcement in EC 2 is calculated exactly as in EHE-08. Thus, the comparison is not possible because variables are the same in both specifications and the results are also equal.

4.2.2. Shear force with shear reinforcement.

The shear force that the beam can bear varies from EHE-08 to EC 2.

According to EHE-08, the design is made taking into consideration that shear reinforcement can bear and the stress that the concrete can contribute, while EC 2 only keeps in mind the stress which the shear reinforcement can bear.

Thus, shear strength will be less in EC 2 than EHE-08 and in case the concrete is modified, the result in EC 2 will be unchanged, as opposed to EHE-08, where the results vary depending on the concrete used.

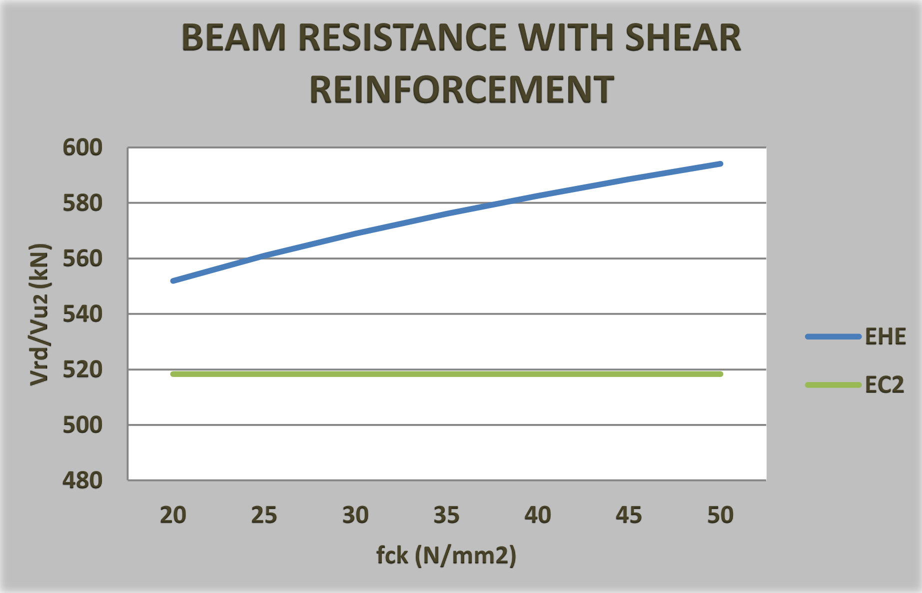


Diagram 4.1

4.2.3. Maximum shear force limited by crushing of the compression struts.

To calculate this different variables are used in Spain and in Austria:

EHE-08	EC 2
d	z
$\boldsymbol{f_{1cd} = 0.6 f_{ck}}$	$f_{cd} = \frac{f_{ck}}{\gamma_c}$
K=1	-
-	$\nu = 0.6(1 - \frac{f_{ck}}{250})$

Because of this, the maximum shear force will be less in EC2 than in EHE-08 although the compressive resistance of the concrete was different because “z”, the inner lever arm, is always less than “d”, the effective depth.

Moreover, "ν" will be always less than 1 and it is reversely proportionate to the compressive resistance. So, the difference between maximum shear force calculated, limited by crushing of the compression struts, will increase as the compressive resistance of the concrete is higher:

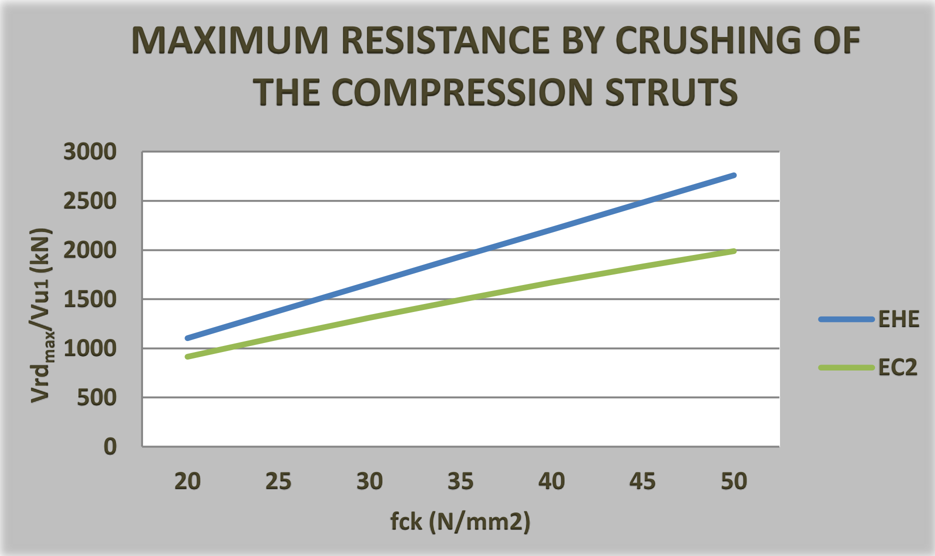


Diagram 4.2

5. COMPARISON TO FIB MODEL CODE 2010

5.1 Calculation of RC member according to FIB Model Code for Concrete Structures 2010.

According to FIB Model Code, the RC member described in chapter 3.1 has to meet:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed} \tag{4.1}$$

Where:

V_{Rd}: is the design shear resistance.

$V_{Rd,c}$: is the design shear resistance attributed to the concrete.

$V_{Rd,s}$: is the design shear resistance provided by shear reinforcement.

V_{Ed}: is the design shear force.

Firstly, to calculate the resistance of the RC member, it is necessary to check if it can bear the forces without shear reinforcement, as in EHE-08. To verify if this is possible, according to FIB Model Code, it is necessary to check $V_{Rd,c}$ in this case in *Level II of approximation*:

$$V_{Rd,c} = K_v \frac{\sqrt{f_{ck}}}{\gamma_c} z \cdot b_w \tag{4.2}$$

Where:

$$K_v = \frac{0.4}{1 + 1500\varepsilon_x} \cdot \frac{1300}{1000 + k_{dg} \cdot z} = 0.25 \tag{4.3}$$

Bearing in mind that:

$$\varepsilon_x = \frac{1}{2 \cdot \varepsilon_s \cdot A_s}\left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed} \cdot \left(\frac{1}{2} \pm \frac{\Delta e}{z}\right)\right) = 6.16 \cdot 10^{-4} \tag{4.4}$$

$$k_{dg} = \frac{32}{16 + d_g} = \frac{32}{32} = 1 > 0.75 \tag{4.5}$$

Thus, substituting in (4.2):

$$V_{Rd,c} = 0.25 \frac{\sqrt{30}}{1.5} 828 \cdot 300 = 226.39\ kN \tag{4.6}$$

And, as:

$$\boxed{V_{Rd,c} = 226.39\ kN < V_{Ed} = 385\ kN} \tag{4.7}$$

The RC member needs to be calculated with shear reinforcement according to the same regulation but, in this case, *Level III of approximation* is used and the below formula is necessary to be implemented:

$$V_{Rd} = V_{Rds} + V_{Rdc} \leq V_{Rd,\max(\theta min)} \tag{4.8}$$

Where:

$$V_{Rd,\max(\theta min)} = k_c \frac{f_{ck}}{\gamma_c} b_w \cdot z \cdot \sin\theta_{min} \cdot \cos\theta_{min} \tag{4.9}$$

Bearing in mind:

$$b_w = 300mm$$

$$k_c = k_\varepsilon \cdot \eta_{fc} = 0.55 \tag{4.10}$$

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} = \left(\frac{30}{30}\right)^{1/3} = 1 \tag{4.11}$$

$$k_\varepsilon = \frac{1}{1.2 + 55\varepsilon_1} = \frac{1}{1.2 + 55 \cdot 0.0085} = 0.55 < 0.65 \tag{4.12}$$

$$\varepsilon_x = 0.000616 \tag{4.13}$$

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.002)\cot^2\theta = 0.0085 \tag{4.14}$$

$$\theta_{min} = 20º + 10000\varepsilon_x = 26.16º \tag{4.15}$$

Thus, substituting in (4.9):

$$V_{Rd,\max(\theta min)} = 1074.16\ kN \tag{4.16}$$

And:

$$V_{Rd,s} = \frac{A_{sw}}{s_w} z \cdot f_{ywd} \cdot \cot\theta = \frac{2\pi 5^2}{120} 828 \cdot 478.26 \cdot 1.73 = 1055.30\ kN \quad (4.17)$$

Thus:

$$V_{Rd,c} = K_v \frac{\sqrt{f_{ck}}}{\gamma_c} b_w \cdot z \quad (4.18)$$

Where:

$$K_v = \frac{0.4}{1 + 1500\varepsilon_x}\left(1 - \frac{V_{Ed}}{V_{Rd,\max\,(\theta min)}}\right) = 0.13 \quad (4.19)$$

And substituting according to (4.18):

$$V_{Rd,c} = 0.13 \frac{\sqrt{30}}{1.5} 300 \cdot 828 = 120.98\ kN \quad (4.20)$$

Thus:

$$V_{Rd} = 1055.30 + 120.98 = 1176.28\ kN$$

$$\boxed{V_{Rd} = 1176.28\ kN > V_{Rd,\max\,(\theta min)} = 1074.16\ kN} \quad (4.21)$$

And:

$$\boxed{V_{Rd,\max\,(\theta min)} = 1074.16\ kN > V_{Ed} = 385\ kN} \quad (4.22)$$

5.2 Comparison EHE to FIB MODEL CODE 2010.

In general, the main difference between EHE-08 and FIB Model is, like in the case of Euro Code 2, the yield of strengthened steel in tension considered. The FIB Model defines it as $f_{y\alpha d} = \frac{f_{yk}}{\gamma_s}$ while in EHE-08 this value is restricted to be no more than 400 N/mm^2. In the case it surpasses this threshold, by the Spanish legislation; the beam has to be redesigned as to yield an amount of 400 N/mm^2.

Moreover, while EHE-08 presents a defined breaking angle, the FIB Model Code considers that the limits of the compressive stress field inclination relative to the longitudinal axis of the piece are:

$$\theta_{min} \leq \theta \leq 45º$$

Where θ_{min} varies and depends on the longitudinal reinforcement and the stress it has to bear and it is possible to have a negative θ_{min} which in this case has to be taken as zero.

5.2.1. Shear force without shear reinforcement.

Beam shear resistance without shear reinforcement in FIB Model Code is calculated keeping in mind the maximum aggregate size, maximum bending moment, maximum shear and longitudinal reinforcements. Moreover, it is a direct function of the mechanical arm instead of the effective depth that is used in EHE-08.

Thus, there are many differences between the variables used in both norms, as the beam shear resistance increases much more in the FIB Model Code than in the EHE-08 while at the same time concrete resistance increases steadily.

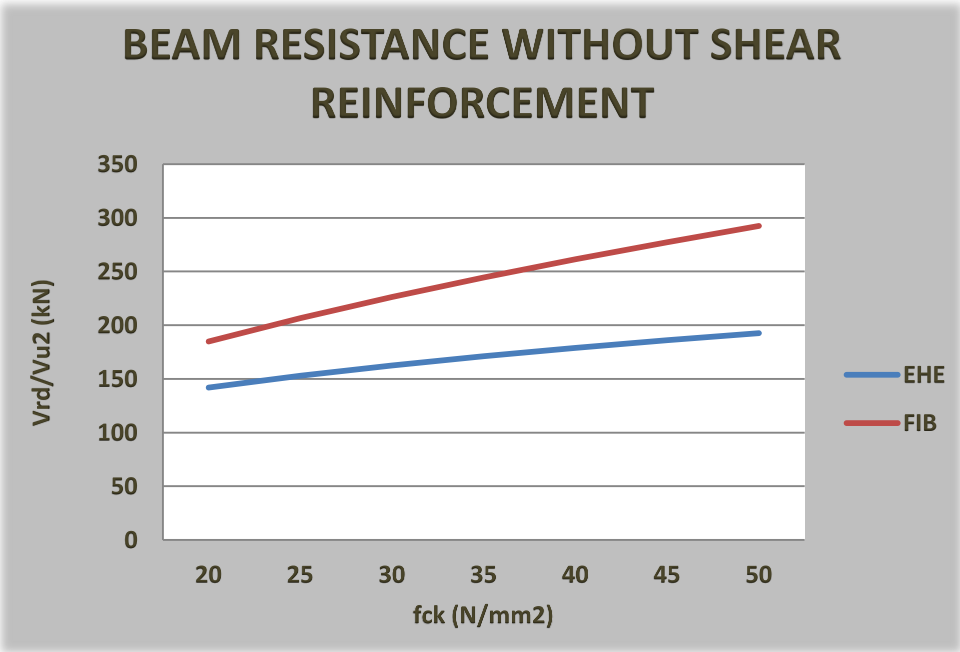


Diagram 5.1

5.2.2. Shear force with shear reinforcement.

The shear force that the beam can bear varies a lot from the EHE-08 to the FIB Model Code 2010, although in both cases they consider the stress that shear reinforcement can bear and the stress that concrete can contribute.

This occurs due to the FIB Model Code considering the minimum inclination of the compressive stress to design as opposed to EHE-08 where it is not necessary to choose the minimum inclination. Thus, the angle to design shear force in the FIB Model Code is less than what is used in the EHE-08 so accordingly the cotangent is greater and as a result is directly proportional to cotangent, and the result will be proportionally greater.

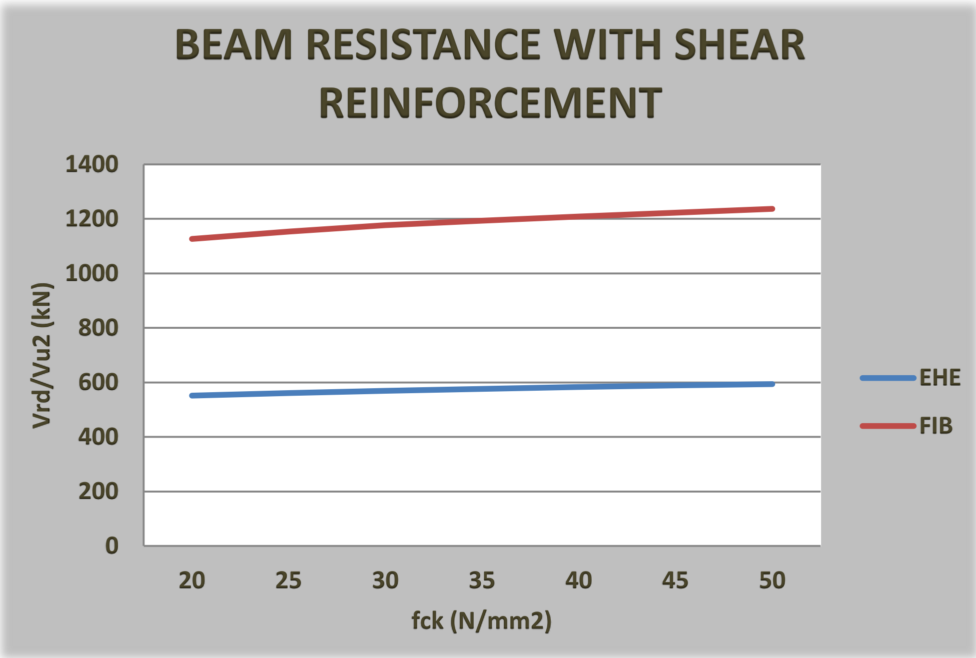


Diagram 5.2

5.2.3. Maximum shear force limited by crushing of the compression struts.

To calculate the maximum shear force limited by crushing of the compression struts in the third level, the FIB Model Code takes the lever arm instead of the effective depth, the minimum angle and a coefficient (k_v) bearing in mind length tension at mid-height and the characteristic strength of concrete.

Thus, the maximum shear force according to the EHE-08 will be greater than according to the FIB Model Code because the coefficient used in the FIB Model Code always will be less than 1 while in EHE-08, for structures without prestressed reinforcement and axial force the coefficient is 1 and, in the same way the lever arm is always less than the effective depth.

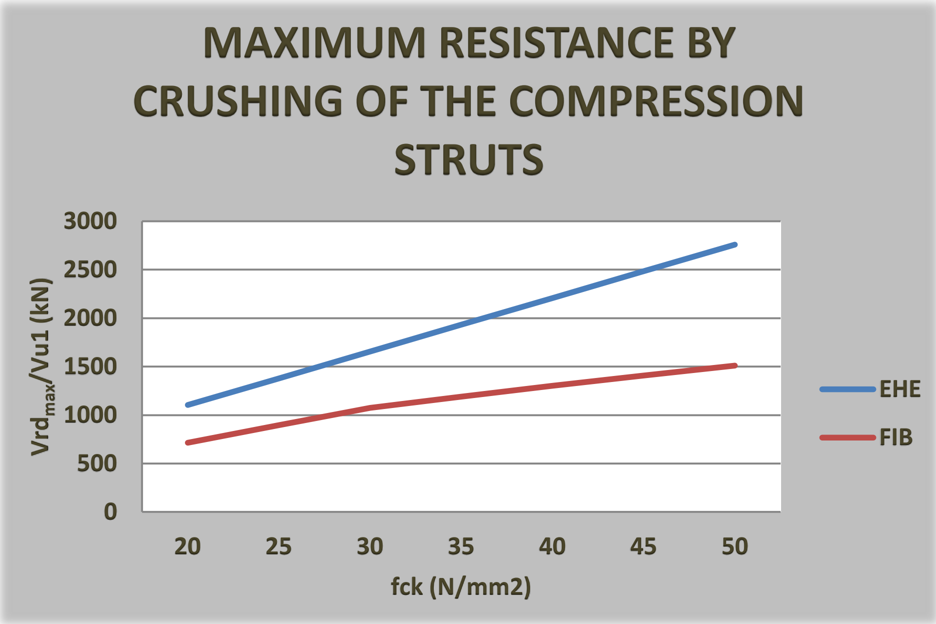


Diagram 5.3

5.2.4. Shear force the beam can bear.

Besides above, in the FIB Model Code the maximum resistance that occurs by crushing of the compression struts is lower than the beam resistance calculated with shear reinforcement when a concrete with $f_{ck} \leq 40\ N/mm^2$ is used. Thus, according to FIB Model Code, the resistance that the beam should resist is the minimum between the results with shear reinforcement and the maximum resistance which occurs also in EHE-08, but in the Spanish legislation the maximum resistance is greater than the resistance calculated with shear reinforcement. So the *Diagram 5.4* reflects the resistance that the beam actually can bear according to both legislations and it shows that the FIB Model Code with concretes has a characteristic resistance that is greater than 40 N/mm2 and yields results nearly twice as strong as though it were designed through the EHE-08.

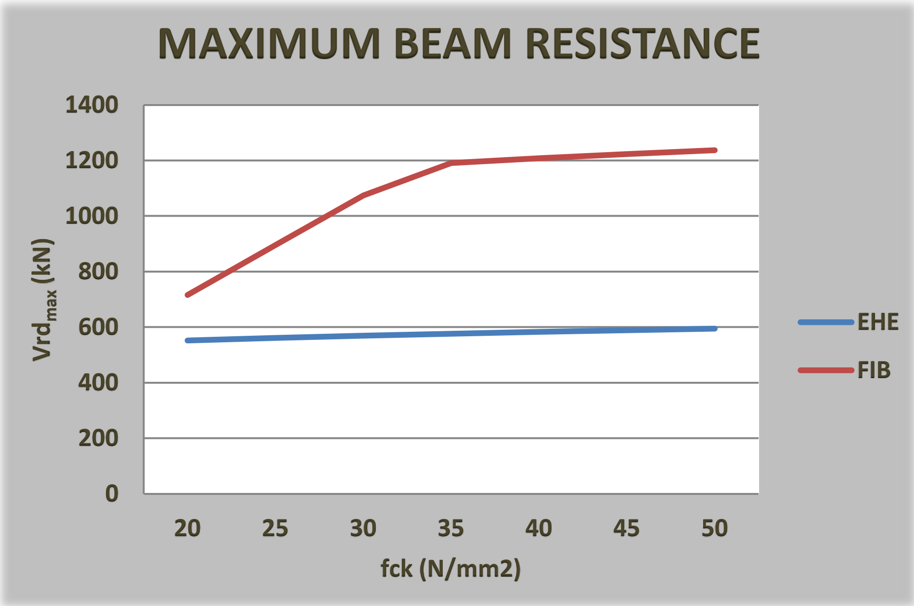


Diagram 5.4

6. COMPARISON TO PROPOSAL OF DR. CLADERA

6.1 Calculation of RC member according to Proposal of Dr.Cladera.

According to the Proposal of Dr. Antoni Cladera Bohigas, the shear resistance of a RC member with shear reinforcement is equal to:

$$V_{Rd} = V_{Rd,s} + V_{ccd} + V_{td} \geq V_{Ed} \qquad (5.1)$$

Where:

$V_{Rd,s}$: is the design value of the shear force that can be sustained by the yielding shear reinforcement, the contribution of the longitudinal reinforcement and the concrete contribution of a piece with shear reinforcement.

V_{ccd}: is the design value of the shear component of the force in the compression area, in the case of an inclined compression chord.

V_{td}: is the design value of the shear component of the force in the tensile reinforcement, in the case of an inclined tensile chord.

But, first it is necessary to calculate the resistance of the RC member according to:

$$V_{Rd} = V_{Rd,c} \qquad (5.2)$$

And, if:

$$V_{Rd,c} > V_{Ed} \qquad (5.3)$$

The piece does not need shear reinforcement because $V_{Rd,c}$ is the design shear resistance of the piece without shear reinforcement.

Thus, according to the RC member described in *chapter 3.1*, according to Dr. Antoni Cladera it is necessary to check if shear reinforcement is needed in the following way:

$$V_{Rd,c} = \nu_{g,0} \cdot f_{ct,d} \cdot b \cdot d \tag{5.4}$$

Where:

$$\nu_{g,0} = \zeta \nu_{c,0} + \nu_w \tag{5.5}$$

And:

$$\zeta = 1.2 - 0.2\left(\frac{a}{d}\right)d = 1.2 - 0.2 \cdot 2.5 \cdot 0.92 = 0.74 > 0.65 \tag{5.6}$$

$$\nu_{c,0} = \left(0.88\frac{x}{d} + 0.02\right)(0.94 + 0.3\mu) = 0.302 \tag{5.7}$$

$$\frac{x}{d} = \alpha_\varepsilon \rho\left(-1 + \sqrt{1 + \frac{2}{\alpha_\varepsilon \rho}}\right) = 0.31 \tag{5.8}$$

$$\alpha_\varepsilon = \frac{E_s}{E_{cm}} = \frac{200000}{33000} = 6.06 \tag{5.9}$$

$$\rho = \frac{A_s}{b \cdot d} = \frac{34.49}{30 \cdot 92} = 0.0125 \tag{5.10}$$

$\mu = 0.2$ for rectangular sections without prestressing. (5.11)

$$\nu_w = 167\frac{f_{ct,d}}{E_{cm}}\left(1 + \frac{2E_{cm} \cdot G_f}{f_{ct,d}^2 \cdot d_0}\right) = 167\frac{1.33}{33000}\left(1 + \frac{2 \cdot 33000 \cdot 0.13}{1.33^2 \cdot 920}\right) = 0.04 \tag{5.12}$$

$$f_{ct,d} = \frac{f_{ctk\,0.05}}{\gamma_c} = \frac{2}{1.5} = 1.33 \tag{5.13}$$

$$E_{cm} = 33\ GPa = 33000\ MPa \tag{5.14}$$

$$f_{cm} = 38 \tag{5.15}$$

$$G_f = 0.028 \cdot f_{cm}^{0.18} \cdot d_g^{0.32} = 0.028 \cdot 38^{0.18} \cdot 16^{0.32} = 0.13 \tag{5.16}$$

Thus:

$$\nu_{g,0} = 0.74 \cdot 0.302 + 0.04 = 0.266 \tag{5.17}$$

And:

$$V_{Rd,c} = 0.266 \cdot 1.33 \cdot 300 \cdot 920 = 97.91\ kN \tag{5.18}$$

Then:

$$\boxed{V_{Rd,c} = 97.91\ kN < V_{Ed} = 385\ kN} \tag{5.19}$$

So, the beam cannot bear the forces without shear reinforcement and it is necessary to check if it can support the structure with the shear reinforcement described method shown above in harmony with the proposal of Antoni Cladera where:

$$V_{Rd,s} = \left(\nu_g + \nu_l + \nu_s\right) f_{ct,d} \cdot b \cdot d \tag{5.20}$$

And:

$$\cot\theta = \frac{0.85}{1-\frac{x}{d}} = \frac{0.85}{1-0.32} = 1.25 \tag{5.21}$$

Thus, bearing in mind:

$$\nu_l = 0.25\frac{x}{d} - 0.05 = 0.25 \cdot 0.32 - 0.05 = 0.030 \tag{5.22}$$

$$\nu_s = \frac{A_{sw}}{s \cdot b}\frac{f_{ywd}}{f_{ct,d}}\cot\theta\left(1-\frac{x}{d}\right) = \frac{2\pi 5^2}{120 \cdot 300}\frac{478.26}{1.33} 2.10(1-0.32) = 1.33 \tag{5.23}$$

And:

$$\nu_g = \zeta \nu_c + \nu_w \tag{5.24}$$

Which, according to equations (5.6), (5.12) and calculating:

$$\nu_c = \left[(0.88 + 0.76\,\nu_s)\frac{x}{d} + 0.02\right](0.94 + 0.3\mu) = 0.63 \tag{5.25}$$

The result is:

$$\nu_g = 0.74 \cdot 0.63 + 0.04 = 0.51 \tag{5.26}$$

And, in accordance with (5.18):

$$V_{Rd,s} = (0.51 + 0.03 + 1.33)1.33 \cdot 300 \cdot 920 = 686.91\ kN$$

$$\boxed{V_{Rd,s} = 686.91\ kN > V_{Ed} = 385\ kN} \tag{5.27}$$

Also, it is necessary to check that:

$$V_{Rd,max} > V_{Rd,s} \tag{5.28}$$

According to:

$$V_{Rd,max} = \alpha_{cc} \cdot b_w \cdot z \cdot \nu_1 \frac{f_{cd}}{\cot\theta + \tan\theta} \tag{5.29}$$

Where:

$$\nu_1 = \eta_{fc} \cdot \eta_{\varepsilon} = 0.55 \tag{5.30}$$

Because:

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} = \left(\frac{30}{30}\right)^{1/3} = 1 \leq 1 \tag{5.31}$$

$$\eta_{\varepsilon} = 0.55 \tag{5.32}$$

Thus, substituting in (5.29):

$$V_{Rd,max} = 1332.54kN \tag{5.33}$$

And, finally, checking in (5.28):

$$\boxed{V_{Rd,max} = 1332.54\ kN > V_{Rd,s} = 686.91\ kN} \tag{5.34}$$

6.2 Comparison EHE to Proposal of Dr. Cladera.

Broadly speaking, the main difference between EHE-08 and the proposal of Antoni Cladera is the inclination of the shear crack, which is defined as:

$$\cot\theta = \frac{0.85}{1 - \frac{x}{d}}$$

So, according to the new proposal it is not possible to choose the inclination, as in EHE-08, if not it depends on the neutral axis depth of the cracked section and the effective depth of the cross-section.

Moreover, this proposal uses other characteristics of concrete like the design tensile strength of concrete, mean value of the compressive and tensile strength of the concrete which are not bore in mind in the EHE-08.

6.2.1. Shear force without shear reinforcement.

Comparing the variables used in the formula for the design of the shear force in the proposal and the EHE-08, their only common point is the overall width and the effective depth of the cross section. So, the rest of the variables used in the proposal are the coefficient, which consider the contributions of the un-cracked concrete chord, the shear resisted along the crack length and the design tensile strength of concrete and it does not bear in mind longitudinal reinforcement.

Thus, the *Diagram 6.1* reflects yield results according to EHE-08 that are nearly twice as strong as though it were designed through the proposal of Dr. Antoni Cladera.

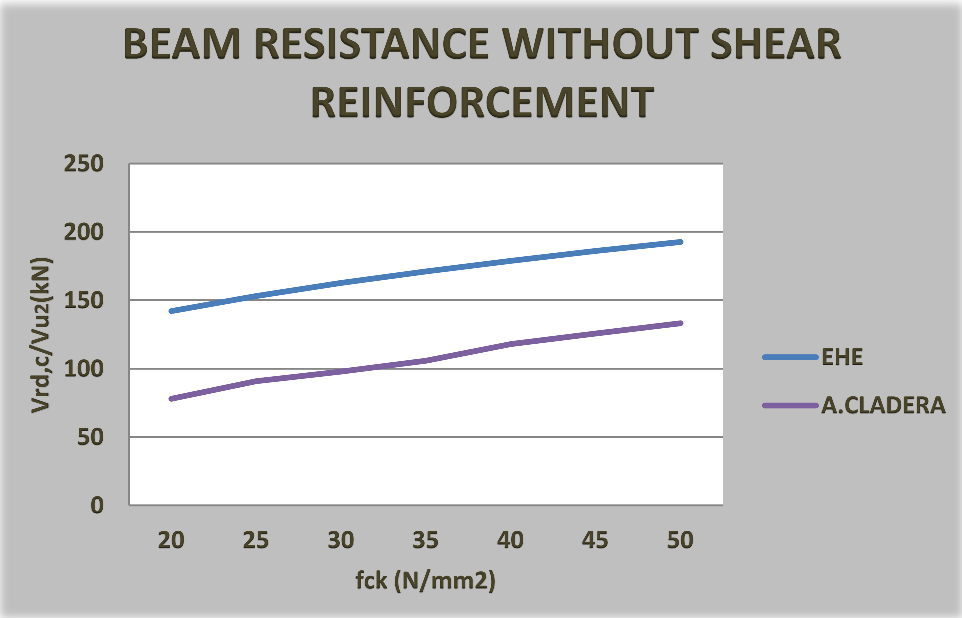


Diagram 6.1

6.2.2. Shear force with shear reinforcement.

In this case, when the beam has shear reinforcement, the proposal of Dr. Cladera uses the shear resistance of the piece without shear reinforcement mentioned above but introducing the resistance that shear reinforcement adds to the beam twice.

Because of this, the *Diagram 6.2* shows that if shear strength is designed through the proposal of Dr. Antoni Cladera the results are greater than if it is designed through the Spanish legislation, EHE-08.

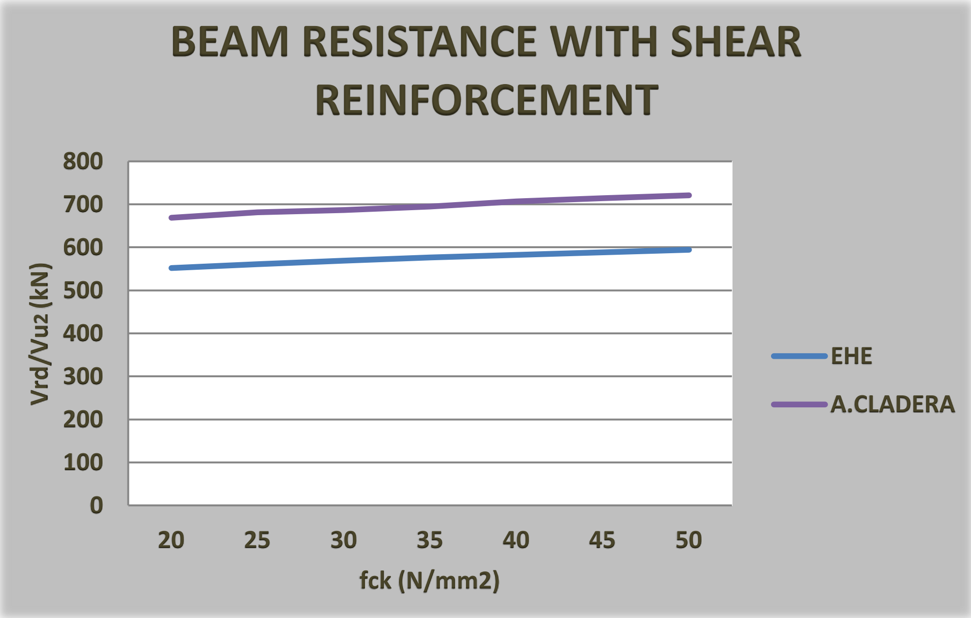


Diagram 6.2

6.2.3. Maximum shear force limited by crushing of the compression struts.

In the proposal of Dr. Cladera, to calculate shear force limited by crushing of the compression struts, the cracking angle is considered and not the angle of shear reinforcement, concrete strength and long term adverse effects. Moreover, there is a coefficient taking into account, as well as the influence of concrete brittleness for high strength concrete and the influence of cracking on the strength of compression struts, which is calculated multiplying a strength reduction factor accounting for the brittleness of concrete and a strength reduction accounting for the influence of cracking on the compressive strength of concrete.

Then, comparing this with the design in the EHE-08, none of the above is considered, but there is only a coefficient which depends on the axial force. It is important to know that the proposal uses the mechanical arm instead of the effective depth, which is used in the EHE-08.

Thus, if the results are compared, the EHE-08 gives a greater maximum resistance which increases as concrete resistance increases much more than the design proposed by Antoni Cladera.

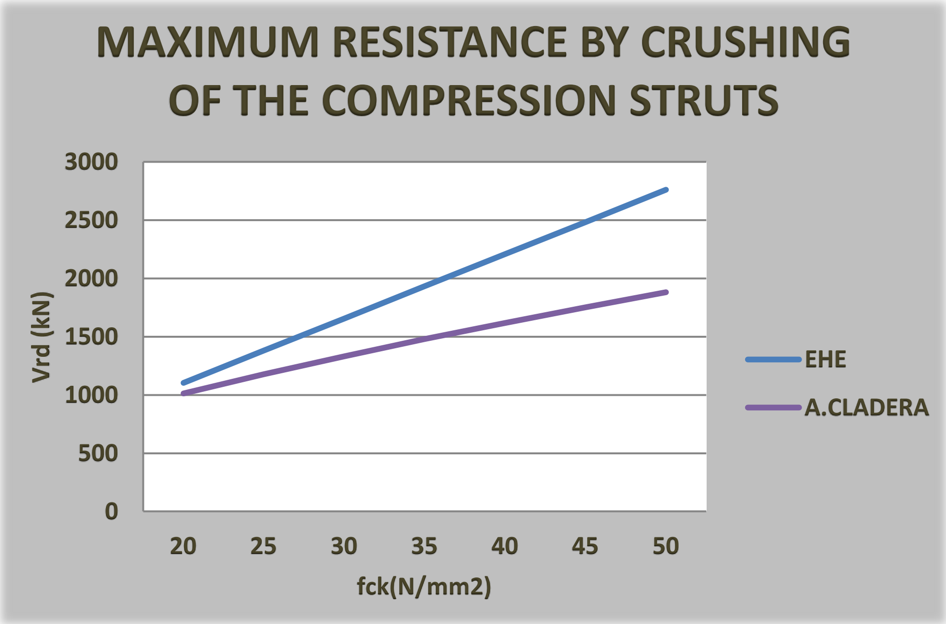

MAXIMUM RESISTANCE BY CRUSHING OF THE COMPRESSION STRUTS
Vrd (kN)
3000
2500
2000
1500
1000
500
0
20
25
30
35
40
45
50
fck(N/mm2)
EHE
A.CLADERA

Diagram 6.3

7. CONCLUSIONS AND FUTURE WORKS

The conclusions obtained after the in depth analysis of the results of the shear design carried out in the previous chapters, are as follows.

The highlight of the calculations developed is given in the calculation of the piece with shear reinforcement. In this case, according to the *Diagram 7.1*, it is observed that according to the Euro Code 2, the shear strength of the element does not vary with the characteristic resistance of the concrete because it does not bear in mind the contribution of it for shear calculation while the other regulations discussed consider it.

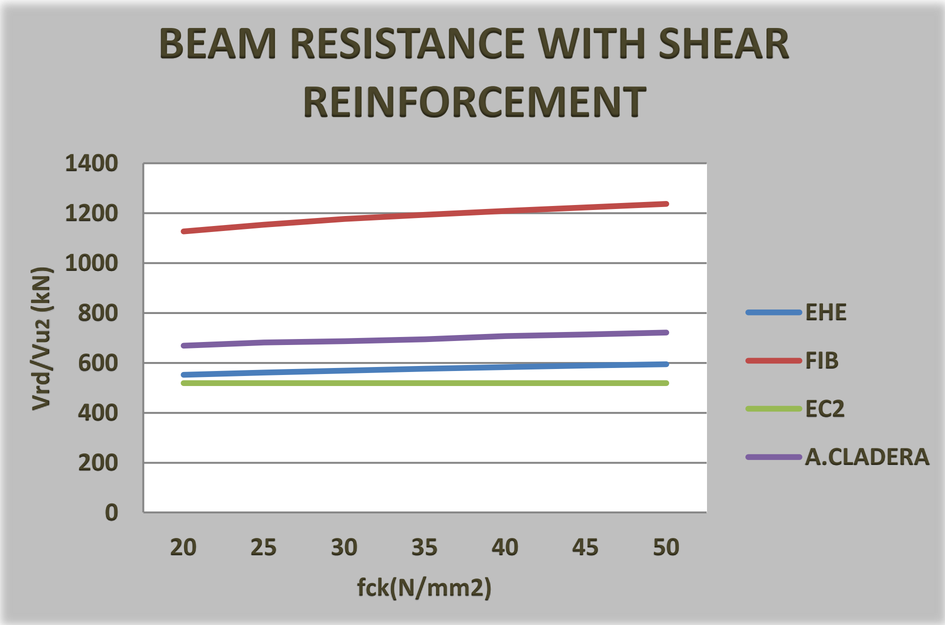


Diagram 7.1

Furthermore, there is a great difference between the studied regulations, mainly due to the breaking angle considered in each of the standards. Both the Spanish and Austrian legislation establish ratios within which the engineer can choose the angle that suits, although usually in Spain the calculation starting from $\theta = 45º$ is performed and in Austria usually the calculation starting from the minimum angle is performed. In contrast, in the FIB Model Code, despite setting the ratio (much smaller), the angle used for the design is the minimum. Also, in the proposal of Dr. Antoni Cladera there is no ratio set, but variables are directly determined by a specific angle depending of the characteristics dimensions of the piece, such as the depth of the neutral axis and the effective depth.

Thus it can be assumed that by taking determined angles based on the geometric characteristics of each piece, and keeping in mind the concrete for shear design, higher strengths are obtained.

In relation to the calculation of the resistance that the piece can bear without shear reinforcement is noteworthy that both the Austrian and Spanish regulations have the same results, while the FIB Model Code and the new approach presented in the previous chapter, consider new variables as the maximum aggregate size, the effective depth of the piece or the modulus of elasticity of the longitudinal reinforcement. This results in a small increase in resistance which becomes more apparent as the concrete strength increases. It should be noted that this increase, compared with the strengths obtained with shear reinforcement is of a very low order, as steel has considerably better features than concrete, but may be important in some specific circumstances.

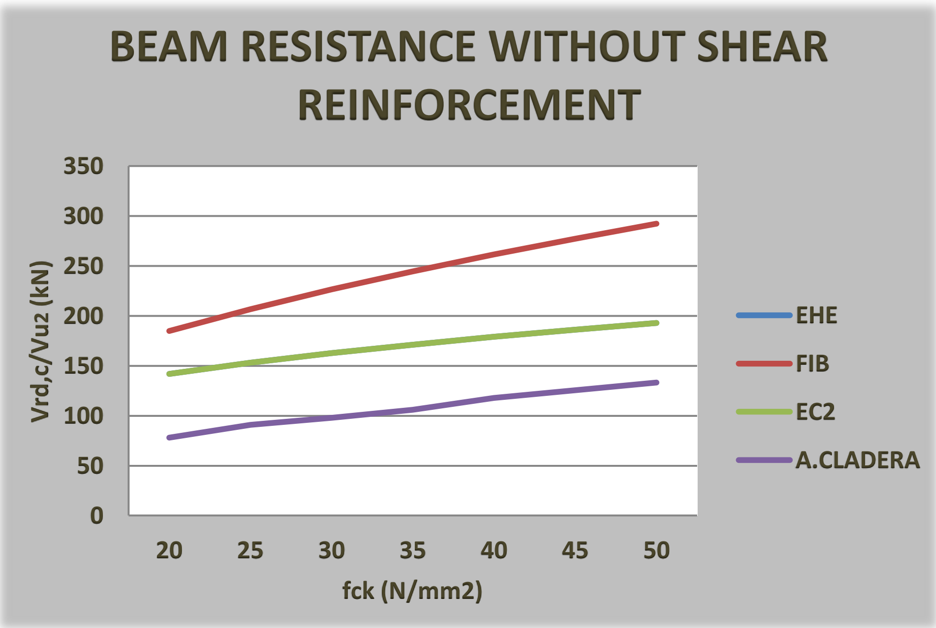


Diagram 7.2

In relation to the high resistance of the concrete compressive failure is noted that it is not usually decisive in determining the strength of the supporting piece, although there are clear differences between the resistances achieved according to the Spanish legislation in comparison to all others, it is important to note that in the case of the new proposal of Dr. Antoni Cladera values obtained are almost double compared to the others.

Thus by using and analysing the aforementioned formulas, it is observed that the Spanish legislation is only considering the shear reinforcement angle when calculating the compression breakage strength of the concrete.

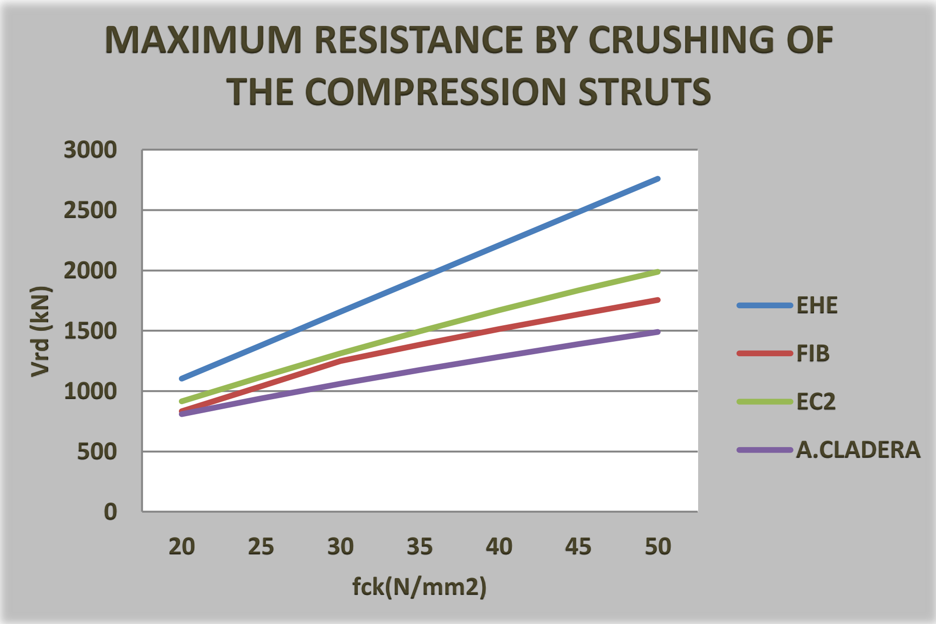


Diagram 7.3

Thereby, new works can analyse the exact breaking angle in shear load design due to the current legislation in Austria and Spain that allows the engineer to choose within a range. In the new proposals, the engineer chooses only one angle but each one depends on different variables.

Moreover, new works can also study how important the concrete actually is in a beam with reinforced concrete because every legislation has different ways to add the contribution of the concrete, even though Euro Code 2 does not include it.

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