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Abstract

Perfect knowledge of future driving conditions can be rarely assumed on real applications when optimally splitting power demands among different energy sources in a Hybrid Electric Vehicle (HEV). Since performance of a control strategy in terms of fuel economy and pollutant emissions is strongly affected by vehicle power requirements, accurate predictions of future driving conditions are needed.

This paper proposes different methods to model driving patterns with a stochastic approach. All the addressed methods are based on the statistical analysis of previous driving patterns to predict future driving conditions, some of them employing standard vehicle sensors while others require non conventional sensors (for instance GPS or IRS). The different modelling techniques to estimate future driving conditions are evaluated with real driving data and optimal control methods, trading off model complexity with performance.

Keywords: HEV; PHEV; hybrid electric vehicle; energy management; optimal control

1 Introduction

The Energy Management Problem in HEV, hereinafter EMP, consists in finding the control policy which minimises the fuel consumption of the vehicle under a set of restrictions over a defined driving cycle. The EMP has been extensively addressed in the literature [12, 4], generally from the Optimal Control perspective, applying methods such as Dynamic Programming (DP) [17, 16], Pontryagin’s Minimum Principle (PMP) [13, 15, 14] or ad hoc methods, being Equivalent Consumption Minimisation Strategy (ECMS) [9, 8] the most popular among them.

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Despite being exhaustively studied during the last decade, the EMP is still a challenging problem. First of all, the complexity of the system to optimise makes difficult to develop models accurate enough to capture the system behaviour with affordable computational cost. The backward quasi-static approach is generally employed to model the vehicle dynamics, because of its compromise between accuracy and computational burden [11].

The EMP is also rather hard to solve because of the difficulty which involves the charge-sustainability condition [3]. This is usually addressed by a hard constraint, imposing the integral of the battery power to be zero over the driving cycle [4], which may be too restrictive in some cases. To avoid the use of such a hard constraint, some authors propose different approaches [7, 10].

Finally, another important issue defining the EMP is the driving cycle itself. Driving style, road profile or traffic conditions have a strong impact on vehicle fuel consumption and optimal control [18] making specific cycle optimisations useless with other cycles. Although Optimal Control techniques require \textit{a priori} knowledge of the driving cycle very limited future information is available on-line. The present paper is aimed to propose and describe different methods to estimate future driving conditions in order to address the EMP from an Optimal Control approach.

The paper is organised as follows: in section 3 the control problem is formulated, followed by section 4, which provides the theoretical description of the proposed methods to estimate the driving profile. All the techniques employed to estimate the driving cycle will be combined with the ECMS optimisation technique, while the DP will be used to provide a benchmark solution. Section 5 shows the simulation results, comparing the performance of proposed cycle estimation techniques, and analysing the effect of the driver’s behaviour on the fuel consumption and the optimal control strategy. Finally, conclusions are presented in section 6.

2 Case study

A series hybrid vehicle is presented as a simple case study to illustrate the impact of the driving behaviour on the EMP. The powertrain consists of a genset (internal combustion engine and generator both coupled together), a Ni-MH battery and an electric motor that provides traction to wheels.

The HEV has been modelled with a backwards model. The battery model consists of a resistance and a global efficiency (both variable with SoC) [14], while genset and electric motor are modelled with quasi-static maps. Vehicle dynamics are computed via a resistance equation assessing aerodynamic drag (drag coefficient of 0.32), rolling resistance and vehicle inertia (vehicle mass of 1262 kg).

Figure 1 depicts the cited HEV series architecture studied in the present paper as well as the actual power flow that occurs along the powertrain. As shown, power required by the vehicle ($P_{req}$) is only satisfied by the mechanical power produced by the motor MG1 ($P_{m,1}$). This motor is fed by either the
generator MG2 or the batteries ($P_{m,2}$ and $P_{el}$ respectively). The generator MG2 is driven by the engine mechanical power ($P_{icc}$) at the expense of consuming fuel ($P_f$), meanwhile batteries release or absorb energy modifying their internal stored energy ($E_b$). Note that the variation of the energy stored in the battery ($-P_b$) and the electrical power provided by it ($P_{el}$) may be positive (driving the vehicle) or negative (absorbing energy), while the electrical power provided by the generator MG2 ($P_{m,2}$) is always positive.

Given the vehicle speed profile the power required by the vehicle becomes an input whereby the powertrain has a single degree of freedom, namely the power split between the battery and the generator. Accordingly, in the present paper, the relative power provided by the generator ($P_{m,2}/P_{m,2}^\text{max}$) has been chosen as control variable ($u$).

The described HEV model was run with urban and highway real driving cycles where two non-professional drivers did the same route.

### 3 Problem statement

In general, the EMP consists on finding the control law $u(t)$ to be applied over a defined driving run with duration $t_f - t_0$ in order to minimise the following cost function:

$$J = \int_{t_0}^{t_f} P_f(u(t)) \, dt$$  \hspace{1cm} (1)

The only dynamic equation in the problem is that governing the evolution of the energy stored in the battery:

$$-\dot{E}_b = P_b$$  \hspace{1cm} (2)

The problem is constrained as the powertrain must produce the mechanical power demanded by the vehicle to follow the driving cycle. The formulation of this constraint depends on the powertrain layout. In the case at hand a series
HEV architecture is considered, where the vehicle wheels are exclusively driven by the electric motor:

\[ P_{\text{req}}(t) = P_{m,1}(t) \]  

(3)

Additional constraints should also be included in order to take into account the limitations in the power ranges of the powertrain elements:

\[ P_{\xi,i}^{\text{min}} \leq P_{\xi,i}(u(t)) \leq P_{\xi,i}^{\text{max}} \]

(4)

where subscript \( \xi \) refers to the internal combustion engine, battery or electric motor.

Finally, the battery is subject to limitations on the energy which is able to store, but also, the net battery charge variation in a long enough cycle should be roughly zero to ensure the battery charge sustainability. Then, the following constraints should be also considered:

\[ \int_{t_0}^{t_f} P_b(u(t), E_b(t)) \, dt = 0 \]

(5)

For the sake of reasonableness, the last constraint will be relaxed so that the energy stored in the battery should reach the reference level within a certain time horizon. This approach is consistent with the limitations in the information about future driving conditions. Both the methods employed to estimate the driving conditions within a reduced time horizon and the corresponding charge sustainability constraints will be described in the following section.

## 4 Methods to estimate driving behaviour

The formulation of the methods proposed in this paper is oriented so that they may be combined with the ECMS to solve the EMP. The ECMS was presented in [9] and its main contribution is that under certain conditions, the integral problem presented in (1) can be replaced by a set of problems to be solved at each instant. To cope with this objective, the energy level of the battery should be included in the cost function in order to take into account the potential of discharging the batteries in the current moment and recharging them in the future or vice versa. Accordingly, the cost may be re-defined as:

\[ f(u(t), E_b(t), t) = P_f(u(t), t) + s P_b(u(t), E_b(t), t) \]

(6)

where the parameter \( s \) is an equivalence factor to transform electrical into an equivalent fuel power. According to [13] the ECMS is a simplification of the Pontryagin’s Minimum Principle approach in which the Lagrangian parameter \( s = s(t) \) is considered constant over the driving cycle. Of course, the solution provided by the ECMS depends on the value of the corresponding \( s \) parameter. For a given cycle, the optimal \( s \) value may be obtained by means of shooting methods [14] or by analysing the optimal solution previously calculated [3]. Note
that in any case, the application of the $s$ parameter to other driving cycles will jeopardise the optimality, charge sustainability and robustness of the solution.

To deal with this issue, the present paper proposes different methods to obtain an optimal estimated $s$ value which minimises the fuel consumption guaranteeing the charge sustainability within a limited time horizon in which the operating conditions will be estimated.

The estimation of the driving conditions should be addressed by means of the analysis of past driving patterns. Several studies have been done to characterise the driving patterns with mean values and standard deviations of performance parameters such as velocity, acceleration or demanded power amongst others [2, 1]. Most of those factors are related with power demands and speed level, which leads to the approach followed in [7, 5]. In addition, information about road conditions, trip distance or traffic provided by ITS may be also combined with historical data to reduce the uncertainty in the driving profile estimation [18].

In the present work, three main methods of driving cycle estimation are addressed. First, Markov chain characterisation is combined with ECMS. The second method is based on the estimation of future driving power requirements in an stochastic fashion by using probability distributions (histograms) of demanded power. The third method is an upgrade in which the histograms of power demands are mapped with the vehicle location. All three methods are described in the following subsections. Of course, the reader should keep in mind that those methods only provide an estimated $s$ value so the EMP stated in equations 1 to 5 must be subsequently solved iteratively using the ECMS algorithm referenced above.

4.1 Markov chains

Markov chains stand for a particular discrete stochastic event in which the current state only depends on the previous one, behaviour known as Markov Property. These have been used to solve automotive problems in the past [6] since they allow a system to be characterised into a probability matrix. In this paper, as shown in figure 2, current vehicle speed and power demand are used as Markov chain states to estimate future power requirements, like approached in [7].

Once a probability matrix (or a set of matrices) is trained with past driving information it is possible to build as many random driving cycles as desired by means of the Montecarlo method. All these driving cycles contain the same driving pattern and, if Markov Property is satisfied, they also represent the behaviour of the original driving cycle. In this case, despite being fictitious, at the limit these cycles have the same EMP solution in average than the original EMP problem and, therefore, their corresponding optimal $s$ value converges to the optimal value for the expected driver’s style.

The set of random cycles may be optimally solved using several different methods, however ECMS with shooting technique is selected since it gives a direct calculation of the optimal $s$ value for each cycle. Afterwards, the averaged
Figure 2: Markov probability matrix \((P_t)\) for a vehicle speed of 40 km/h, where \(P_t\) is probability, subscript \(n\) refers to the present time and \(n+1\) to the next time step.

\(s\) value obtained from the set of driving cycles randomly generated with the Markov chain is used as optimal \(s\) value for the driver. Then, the obtained \(s\) value is used to apply plain ECMS in future driving cycles without \textit{a priori} knowledge to obtain the optimal EMP solution, as stated in the introduction of section 4.

Nevertheless, the method suffers of a lack of feedback. Since small drifts between expected and simulated powertrain behaviours may be present, their integration along time can carry to an unstable solution.

### 4.2 Histogram-based models (S-ECMS)

The EMP solution for a zero-order system depends only on the actual state of it. Therefore, assuming a quasi-static powertrain behaviour, the optimal power splitting policy is only determined by instantaneous power requirements. This simplification allows a driving pattern characterisation by just power demand probability distributions (histograms) which deals not only with driving style but also with traffic and road related information. In [10] probability distributions have demonstrated their potential recognising driving patterns as well as different kinds of roads. Thus, the main target of histogram-based methods (hereinafter S-ECMS) is to calculate average predictions to estimate the \(s\) parameter for the ECMS method with a little computational burden.

Since power requirement probability distribution is known, an average delivered battery power may be calculated:

\[
\overline{P_b}(s, E_b) = \sum_n P_t(P_{req, n}) \cdot P_b(P_{req, n}, s, E_b)
\]  

\(\overline{P_b}(s, E_b)\)
In addition, the effect of the SoC (state of charge), and accordingly the battery stored energy \((E_b)\), may be negligible within operation limits, so a quasi-static approach might be addressed:

\[
\mathcal{T}_b(s) = \sum_n P_r(P_{req,n}) \cdot P_b(P_{req,n}, s)
\]

so \(P_b(P_{req,n}, s)\) could be mapped offline for computational economy. Then, since a particular SoC at the end of the optimisation horizon is aimed, the optimal \(s\) estimation must verify that average delivered battery power reaches that state:

\[
\hat{P}_b = E_b(t) - E_b^* h(t)
\]

where \(\hat{P}_b\) is the estimated battery power consumption, \(E_b^*\) is the desired value for the stored battery energy and \(h(t)\) is the control horizon. Therefore, the optimal \(s^*\) selection for a particular histogram is a value such that fulfils the previous constraint:

\[
\mathcal{T}_b(s^*) = \hat{P}_b
\]

In figure 3 an example of \(\mathcal{T}_b(s)\) may be appreciated, where lower \(s\) values carries to higher battery consumptions since it gives battery energy a lower cost compared to fuel as seen in (6). Finally, this algorithm is performed online each step to estimate a set of continuously updated \(s^*\) value for its use in the ECMS optimisation method.

### 4.3 Geotagging (Geo-S-ECMS)

A power demand histogram contains information regarding driving style and road type, but it can only store one pattern (or several mixed together). However, very different styles may be found in just one trip. As an example, in figure 4 power requirements histograms have been calculated along the route each 150 meters and grouped attending to their similarities showing, at least,
two different driving styles. Therefore, this paper proposes a geotagging technique (hereinafter Geo-S-ECMS) to store different driving patterns since they are mostly linked to vehicle location (due to traffic lights, speed limits, etc.).

A set of geographically located power requirement histograms can be easily built by means of in-car sensors and a GPS receiver. For this purpose, the relevant geographic area is discretised into a grid. As depicted in figure 5, while the vehicle drives through a grid element its power requirements are stored in that element forming an histogram. As the grid is filled, node histograms are computed as the average of its four adjacent element histograms. Finally, element histograms are discarded and only node histograms are permanently stored in memory.

Later, with this set of node histograms a unique histogram for any point inside the grid may be calculated as a distance-pondered average of its four closest node histograms. Thus, if a route is preprogrammed in the vehicle (i.e. via in-car navigation system) a set of geotagged histograms might be computed a priori for that particular trip.

Hence, the S-ECMS method may be applied with slight differences. On one hand, each time there will be an energy drift in battery state with respect to the desired final stored energy, expressed in 9, where the horizon, \(h(t)\), may be estimated and updated by navigation system. On the other hand, the expected average battery power consumption for a particular \(s\) value during the rest of the trip can be calculated based on the geotagged histograms:

\[
\overline{P_b}(s) = \sum_m \Pr(\ell_m) \left( \sum_n \Pr_m(P_{req,n}) \cdot P_b(P_{req,n}, s) \right)
\]

(11)

where \(m\) is the histogram index, \(n\) refers to the power requirements, \(\ell_m\) is the geographic position, and \(P_b(P_{req,n}, s)\) is mapped like in 4.2. The selection of optimal \(s^*(t)\) value is such that the estimated battery power delivery (\(\overline{P_b}(s)\)) fulfils the final constraint average power requirement (\(\overline{P_b}\)), so equation 10 is
Figure 5: Geotagging technique for Geo-S-ECMS. Power requirements are stored in elements (grey boxes), then averaged into nodes (white circles) (5(a)) and finally pondered with distance to vehicle \((d_{k,i})\) to get time and position dependent histograms (5(b)).

used at each calculation step. Finally, a plain ECMS optimisation method is performed using that estimated \(s(t)\) value.

Since Geo-S-ECMS is able to reach a particular SoC at the end of a route while locally optimising the energy management in the power train, this method is specially useful for Plug-In HEVs, where the optimal EMP solution should fully discharge the battery at the end of the trip.

5 Results

In first place DP was applied for both cycles in order to obtain a benchmark solution. Following, the presented methods were implemented and calculated online (i.e. without the use of future information during simulation). Note that Markov chains and histograms have been trained with four real driving cycles belonging to the same route than the simulated trip.

Markov method estimated \(s\) value was calculated from ten random driving cycles, previously built from the trained Markov chain, and then ECMS method solved the EMP giving the results shown in figure 6. These show that \(s\) value estimations are not accurate enough for the highway cycle since the final SoC value is appreciably higher than desired (same as at the beginning). On the other hand, urban estimation is quite fine with an almost optimal solution. Therefore, Markov estimations work well under certain situations but they are not robust enough.

The S-ECMS method was implemented as explained in the previous section. The trained histogram as well as the mapped average battery power consumption, \(\overline{P_b}(P_{req}, s)\), were applied and an estimated \(s\) value was calculated online for each step. Results in 6 show a quite good fit with DP results, mostly when running in urban cycles. Final constraints are always satisfied since S-ECMS method is able to correct misestimations on real time to reach the desired final value.

The Geo-S-ECMS method was implemented and simulated similarly to S-
Figure 6: Optimisation results for all four methods in both urban (left) and highway (right) cycles. From dark to light: DP, Markov chain, S-ECMS and Geo-S-ECMS.

Figure 7: Optimisation results for two different drivers running in the same route in both urban (left) and highway (right) cycles.

ECMS. The trained set of geotagged histograms and the $P_t$ map were used to estimate $s$ value as explained in 4.3. Then, the EMP solution was calculated by applying that estimation to ECMS. Results for Geo-S-ECMS method are close to S-ECMS due to the fact that histogram diversity is not great enough to appreciate a big difference between both methods because just one or two histograms are able to represent all the presented driving conditions. Anyway, since Geo-S-ECMS is an S-ECMS upgrade, constraints at the end of the trip are also verified.

Total fuel consumption for each driving cycle ($m_f$ in kg of fuel per driving cycle) is shown in figure 6. Slight differences may be appreciated among methods. However, due to its lack of robustness, in some cases Markov method shows a considerably greater fuel consumption than others. Therefore, histogram-based methods (i.e. S-ECMS and Geo-S-ECMS) are demonstrated to show fuel efficiencies close to DP solution.

In addition, a drivers comparative is presented in figure 7. Results belong to Geo-S-ECMS method and, according to them, optimal EMP solutions for two different drivers, driving the same cycle, are absolutely different. Differences between drivers are also reflected in total fuel consumption since they are greater than those from different control policies. This fact suggests that energy management optimality and EMP solution are based mostly on driver behaviour instead of just on driving cycle or road type.
6 Conclusions

Driver’s style and behaviour was thought to play an important role in the EMP solution, i.e. in the optimal control policy for a HEV. In fact, the solution depends mostly in those human-behaviour parameters as was briefly explained in 5. Then, the importance in an accurate driving behaviour estimation model has motivated the evaluation of several stochastic methods for HEVs. These methods are pretended to predict future driving behaviour with just one parameter (s-value) for its use with ECMS local optimisation method. Despite that fact, all the evaluated stochastic optimisation methods are simulated in an on-line fashion, i.e. without using a priori information during simulation.

In first place, a Markov chain-based estimation model was presented and described with a direct application in a plain ECMS control policy. However, the robustness of the method depends in hard-to-assure conditions such chains irreducibility or Markov Property verification, the main drawback is the lack of feedback to provide battery energy traceability fulfilling the target SoC at the end of the trip. This lack of robustness carries the EMP solution to an overcharging or depleting policy which deviates from the optimal management.

On the other hand, since power demand histograms have demonstrated efficient when identifying different driving styles, a histogram-based estimation method is proposed. Using a quasi-static battery approach, those methods predict average power requirements and battery depletion in a time horizon to verify final constraints. Two different approaches in this topic have been suggested: an average histogram for a driver and road type (S-ECMS) and several histograms geographically located (Geo-S-ECMS).

Both approaches shown promising results in fuel efficiency terms since total fuel consumption differs a very little from DP solution. Also, charge sustain-ability final constraints (i.e. same SoC at the end than at the beginning) are satisfied with this method because the estimated s-value (for ECMS optimisation method) is corrected in live-time to reach the final desired value. In addition, histogram-based methods, specially that geographically located, are suitable for PHEV, since they are able to verify any final constraint in a particular distance. Thus, when Geo-S-ECMS is combined with in-car navigation systems, a EMP solution may be calculated to deplete batteries at the end of the trip.

Nevertheless, Geo-S-ECMS has not shown any improvement over S-ECMS method. The main advantage in geotagging histograms stands in the fact of identifying a particular driving style among many others when traveling through very different roads and situations. Despite being this the usual case when driving a vehicle in real conditions, the presented simulations have taken place in areas with homogeneous road types and driving styles so just one histogram fits quite well in most of the cases. Anyway, a better fuel efficiency is expected for such that heterogeneous driving cycles.
References


