Fuzzy goal programming for material requirements planning under uncertainty and integrity conditions**

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Abstract

In this paper, we formulate the material requirements planning problem of a first-tier supplier in an automobile supply chain through a fuzzy multi-objective decision model which considers three conflictive objectives to optimize: minimization of normal, overtime and subcontracted production costs of finished goods plus the inventory costs of finished goods, raw materials and components; minimization of idle time; minimization of backorder quantities. Lack of knowledge or epistemic uncertainty is considered in the demand, available and required capacity data. Integrity conditions for the main decision variables of the problem are also considered. For the solution methodology, we use a fuzzy goal programming approach where the importance of the relations among the goals is considered fuzzy instead of using a crisp definition of goal weights. For illustration purposes, an example based on modifications of real-world industrial problems is used.

Keywords: Material requirements planning; uncertainty; fuzzy methods; integer programming.

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1 Introduction

In Mula et al. (2006a, b), some of the approaches which predominate in the production planning and control field are evaluated, such as material requirements planning, hierarchical production planning (HPP), just-in-time (JIT) and optimized production technology (OPT), and their pros and cons are stressed. From this analysis, the need for new systems which simultaneously consider the planning requirements of materials and capacities is concluded, and which enable to model the various uncertainty elements present in all the planning phases owing to the complex and dynamic nature of the relations between different supply chain members. In such contexts, where planning decisions involve resources and information from different supply chain entities, there are two main aspects which the decision maker must face: (1) conflictive objectives, which can arise given the nature of the operations (for example, minimization of costs and, simultaneously, higher level of customer service) and the supply chain structure, in which aligning the

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objectives of various participants is generally a complex matter; (2) fuzziness at the aspiration and/or epistemic uncertainty levels or lack of knowledge of data (for instance, in demand). Therefore, it is important to design models which deal with problems in this area to enable the management of these two types of complexity (Torabi and Hassini 2008). A recent historical review of material requirements planning systems can be found in Olhager (2013).

According to Mula et al. (2006b) and Peidro et al. (2009a), the literature provides various planning models under uncertainty. Those models defined by analytical approaches (Lee and Billington 1993; Leung et al. 2006; Sabri and Beamon 2000), simulation approaches (Chiang and Feng 2007; Hung and Chang 1999; Koh 2004; Suwanruji and Enns 2006, Li Sun et al. 2009) or hybrid approaches (based on the integration of analytical and simulation models) (Bookbinder et al. 1989; Jung et al. 2004; Lee et al. 2002) represent uncertainty in the supply chain based on probability distributions, which are generally based on historic data. Normally however, a material requirements planning model operates in an uncertainty scenario in which statistical data are not very reliable, or are not even available. It can scarcely admit that the future values of certain parameters, like demand and capacity, have a frequentistic nature and are, therefore, likely to be treated by a stochastic approach. So when statistical data are not very reliable or are not available, the determination-based models of these probability distributions may not be the best option. In this context, fuzzy mathematical programming can prove to be an alternative approach to model the different types of uncertainty inherent to production planning processes. Two main fuzzy mathematical applications can be highlighted, which have been extensively dealt with by the authors of this article. On the one hand, fuzzy mathematical programming can be employed to incorporate epistemic uncertainty or lack of knowledge in the input parameters to analytical models or fuzziness in their objectives (Mula et al. 2006c; Mula et al. 2007; Mula et al. 2008; Peidro et al. 2009b; Peidro et al. 2010b, Mula and Díaz-Madroñero 2012). On the other hand, fuzzy optimization can be used as a resolution technique of multiple-objective programming models (Peidro et al. 2009a, Torabi and Moghaddam, 2012). The literature contains other approaches to contemplate uncertainty in material requirements planning systems, such as fuzzy logic (Grabot et al. 2005; Barba-Gutiérrez and B. Adenso-Díaz 2008, Guillaume et al. 2011), the stochastic control of inventories (Inderfurth 2009) or parameterization (Hnaien at al. 2008, Louly et al. 2008, Louly and Dolgui 2012). We refer readers to Dolgui et al. (2013) and Aloulou et al. (2013) for additional reviews of uncertainty modeling in material requirements planning systems.

For the purpose of contributing to the state of the art and application of models for production planning under uncertainty, this work solves a fuzzy multi-objective decision model for the material requirements planning problem in a first-tier supplier of an automobile supply chain. The need for this work arises mainly from the difficulty for some firms to properly quantify idle time costs and backorder costs (as required in the previous models by Mula et al. 2006c, 2007 and 2008) and to measure these aspects in idle time units and delayed product units. Thus, the objectives in conflict to simultaneously optimize are to: (i) minimize production (normal, overtime and subcontracting) and inventory costs; (ii) minimize the fuzzy idle time of production resources; (iii) minimize fuzzy backorder quantities. The aspiration levels of these three objectives can be considered fuzzy in nature because of the incompleteness and/or unavailability of the data required throughout the planning horizon, which could be subjectively obtained depending on the planner's experience. In addition, due to the uncertain nature of demand and the flexible characteristic of the available and required capacity in a mass customization manufacturing context, both are considered uncertain in terms of lack of knowledge or epistemic uncertainty.

The differences of this research to previous ones are mainly related to the application of a multi-objective approach for measuring different aspects of material requirements planning with other measure units rather than costs, which has been the traditional approach. This multi-objective approach also establishes fuzzy importance relationships among the different objectives instead of crisp relationships based on a weight method, which prevents the required weight being defined for each objective. Furthermore, the proposal provides a formulation to overcome infeasibility problems, which may arise from the use of equality constraints with integrity conditions and fuzzy numbers for the right-hand side. In order to address the equality or inequality constraints with the fuzzy numbers of the proposed model, we firstly propose the approach by Jiménez (1996) and Jiménez et al. (2007) based on ranking fuzzy numbers which, by using fuzzy numbers for uncertain parameters, transforms the initial model into a parametric model with fuzzy objectives and crisp constraints. Diaz-Madroñero et al. (2012) show how the approach by Jiménez (1996) and Jiménez et al. (2007) works efficiently for fuzzy linear programming problems. Yet for fuzzy integer linear programming problems with equality constraints, where the integrity condition is imposed on the solution, the solution involves an unfeasibility problem. To face it, we propose a modified version of the approach of Jiménez et al. (2007). Next to solve the fuzzy multi-objective model, we use the approach by Akoz and Petrovic (2007), which establishes the relative preferences of goals through fuzzy preference relations. For other solution methodologies to solve fuzzy multiple objective approaches, readers are referred to Zimmerman (1978), Lai and Hwang (1993), Li et al. (2006), Torabi and Hassini (2008) and Selim and Ozkarahan (2008). Other multi-objective applications for production planning may also be found in Noori et al. (2008), Peidro et al. (2012) and Mohapatra et al. (2013).

The rest of this article is arranged as follows: Section 2 formulates a multi-objective model for the material requirements planning problem in a first-tier supplier of an automobile supply chain. Section 3 describes its solution methodology. Section 4 validates the proposed model by using an example based on a real-world problem. Section 5 offers conclusions and further research.

2 Fuzzy multi-objective model formulation for material requirements planning

2.1 Assumptions and nomenclature

This section formulates the multi-objective model for material requirements planning. This model considers the following assumptions:

- A multi-product manufacturing environment. By the term product, we refer to the finished goods, components, raw materials and subassemblies structured in a bill of materials.
- A multi-level production where the subsets of components are assembled independently.
- A multi-period planning horizon comprising a set of consecutive and integer time periods of the same length.
- The lead time of a product is the number of the consecutive and integer periods required for their finalization.
- The inventory of each product (finished good, raw materials and components) is the available volume at the end of a given period.
- The backlog of the demand of a product at the end of a period is defined as the non-negative difference between cumulated demand and the volume of available product.
- The master production schedule (MPS) which specifies the quantity to produce of each finished good in each planning horizon period, and the material requirements planning which provides the net requirements of raw materials and components for each planning period, are solved jointly.
- Programmed receptions are contemplated.
- Production capacity constraints.
- Overtime limits.
- Subcontracted products are assumed to ready exactly when required without lead time changes.
- Fuzzy right-hand-side numbers for the demand, $\tilde{d}_{it} = \{d_{it1}, d_{it2}, d_{it3}, d_{it4}\}$, fuzzy technological coefficients for required capacity, $\widetilde{AR}_{ir} = \{AR_{ir1}, AR_{ir2}, AR_{ir3}, AR_{ir4}\}$ and fuzzy technological coefficients for available capacity, $\widetilde{CAP}_{rt} = \{CAP_{rt1}, CAP_{rt2}, CAP_{rt3}, CAP_{rt4}\}$.

The nomenclature defines the indices, sets of indices, parameters and decision variables (Table 1).

Table 1. *Nomenclature.*

Sets of Indices	
T	Number of periods in the planning horizon ($t = 1\ldots T$)
I	Number of products ($i = 1\ldots I$)
J	Number of parent products in the bill of materials ($j = 1\ldots J$)
R	Number of resources ($r = 1\ldots R$)
Decision Variables	
P_{it}	Quantity of product i to be produced during period t
$INVT_{it}$	Inventory of product i at the end of period t
B_{it}	Backlog of product i at the end of period t
Tun_{rt}	Undertime hours of resource r during period t
Tov_{rt}	Overtime hours of resource r during period t
$Tsub_{rt}$	Subcontracting hours of resource r during period t
Objective Function Cost Coefficients	
cp_{it}	Variable cost of the normal production of a finished good unit or the purchase of a unit of raw material or component i
ci_{it}	Inventory cost of a unit of product i
$ctov_{rt}$	Overtime hour cost of resource r during period t
$csub_{rt}$	Subcontracting hour cost of resource r during period t
Data	
$\tilde{d}_{it}$	Fuzzy market demand of product i during period t
α_{ij}	Required quantity of i to produce one unit of product j
LT_i	Lead time of product i
ov_{rt}	Overtime limits (in terms of percentage of normal production) of resource r during period t
SR_{it}	Scheduled receipts of product i during period t
$INVT_{i0}$	Inventory of product i during period 0
B_{i0}	Backlog of product i during period 0
Technological Coefficients	
$A\tilde{R}_{ir}$	Fuzzy required time of resource r for one unit of production of product i
$CA\tilde{P}_{rt}$	Available capacity of resource r during period t

The model formulation is as follows:

$$Min \quad z_1 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}\left(cp_{it}P_{it} + ci_{it}INVT_{it}\right) + \sum_{r=1}^{R}\sum_{t=1}^{T}\left(ctov_{rt}Tov_{rt} + csub_{rt}Tsub_{rt}\right) \quad (1)$$

$$Min \quad z_2 \cong \sum_{i=1}^{I}\sum_{t=1}^{T} B_{it} \quad (2)$$

$$Min \quad z_3 \cong \sum_{r=1}^{R}\sum_{t=1}^{T} Tun_{rt} \quad (3)$$

Subject to

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} = \tilde{d}_{it} \quad \forall i \forall t \ (4)$$

$$\sum_{i=1}^{I} P_{it}A\tilde{R}_{ir} + Tun_{rt} - Tov_{rt} - Tsub_{rt} = CA\tilde{P}_{rt} \quad \forall r \forall t (5)$$

$$Tov_{rt} \leq \sum_{i=1}^{I} ov_{rt} \cdot P_{it} \cdot A\tilde{R}_{ir} \quad \forall r \forall t (6)$$

$$B_{iT} = 0 \quad \forall i \ (7)$$

$$P_{it}, INVT_{it}, B_{it}, Tun_{it}, Tov_{it}, Tsub_{it} \geq 0 \quad \forall i \forall r \forall t \ (8)$$

$$P_{it}, INVT_{it}, B_{it} \in Z \qquad \forall i \forall t \quad (9)$$

There are three conflictive objectives to simultaneously optimize: z_1 minimizes the total costs over the time periods that have been computed. Total costs include: the normal production costs, which consider the purchasing costs for raw material and components and the manufacturing costs for finished goods; inventory holding costs of finished goods; overtime production costs; and subcontracted production costs of finished goods; z_2 minimizes deferred demand throughout the planning horizon, expressed in product units; and z_3 minimizes the idle time expressed in time units (hours). The model should minimize the resulting idle time by adjusting demand and by minimizing storage costs.

The decision maker has imprecise aspiration levels for each objective function. Symbol "$\cong$" is the fuzzified version of "=" and refers to the fuzzification of the aspiration levels related to each objective. Accordingly, Eqs. (1), (2) and (3) are fuzzy, and it is necessary for the decision maker to simultaneously optimize these conflicting objectives within the framework of imprecise aspiration levels by using a fuzzy goal programming (FGP) approach.

The constraints originally proposed by Mula et al. (2006c, 2007 and 2008) have been extended in this proposal and are described as follows: Constraint (4) is the inventory balance equation for all the products. These equations take into account the backlogs which behave as negative inventory. It is important to highlight the consideration of programmed receptions, SR_{it}, which guarantees the continuity of the production plan throughout the successive runs carried out in a given planning horizon. Demand is considered a fuzzy number. Constraint (5) establishes the available capacity for normal, overtime and subcontracted production. Both required and available capacities are considered fuzzy numbers. Constraint (6) provides the overtime limits in terms of a percentage of the normal production working time. A constraint has also been added (Constraint 7), which does away with delays during the last planning horizon period (T). The model also contemplates the nonnegativity constraints (8) for the decision variables. In this case, we consider $P_{it}, INVT_{it}, B_{it}$ to be integers, but they could change depending on the real-world problem to which the model is applied. A deterministic version of this model would imply all the objective functions (1)-(3) and constraints (4)-(9) by considering the $\tilde{d}_{it}$, $A\tilde{R}_{ir}$ and $CA\tilde{P}_{rt}$ crisp values, as follows: $\tilde{d}_{it} = d_{it3}$; $A\tilde{R}_{ir} = AR_{ir3}$ and $CA\tilde{P}_{rt} = CAP_{rt3}$.

3 Solution methodology

3.1 Addressing the constraints with fuzzy numbers according to Jiménez et al. (2007)

In this section, in order to address the equality or inequality relations among the fuzzy numbers of fuzzy constraints (4), (5) and (6), we propose the ranking fuzzy numbers approach by Jiménez (1996) and Jiménez et al. (2007) by considering trapezoidal fuzzy numbers in a similar manner to that applied in Peidro et al. (2010b). Evidently, it would simpler to defuzzify the fuzzy numbers intervening in the constraints from the beginning. However in our opinion, this would not be a genuine fuzzy approach, but would just be another way of assigning a crisp value to the magnitudes intervening in the problem. Accordingly, Dubois (2011) indicates that "the problem with fuzzy set methods extending existing ones is that, more often than not, the proposed handling of fuzzy intervals is itself *ad hoc* or so approximate that the benefit of fuzzification is lost. Moreover the thrust of fuzzy interval analysis is to provide information about the uncertainty pervading the results of the decision process. Some authors make an unjustified use of defuzzification that erases all information of this type. For instance the decision maker is asked for some figures in the form of fuzzy intervals so as to account for the difficulty to provide precise ratings, and then these ratings are defuzzified right away. Or the fuzzy intervals are propagated through the decision process but the final results are defuzzified in order to make the final decision ranking step easier. In such procedures it is not clear why fuzzy intervals were used in the first stand. The uncertainty pervading the ratings should play a role in the final decision-making process, namely to warn the decision maker when information is not sufficient for justifying a clear ranking of alternatives".

Let us consider the following multi-objective linear programming problem with fuzzy parameters in the constraints:

$$\text{Min } \tilde{z} = (z_1, z_2, ..., z_k)$$
$$\text{s.t. } x \in N(\tilde{A},\tilde{b}) = \{x \in R^n / \tilde{a}_i x \leq \tilde{b}_i, i = 1,...,m, x \geq 0\} \tag{10}$$

The possibility distribution of the fuzzy parameters is assumed to be characterized by fuzzy trapezoidal numbers. The expected interval of a fuzzy trapezoidal number *(a_1,a_2,a_3,a_4)*, denoted $EI(\tilde{a})$, is calculated as follows (Heilpern, 1992):

$$EI(\tilde{a}) = [E_1^a, E_2^a] = \left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_3 + a_4)\right] \tag{11}$$

We use a fuzzy relation to compare the fuzzy numbers according to (Jiménez, 1996). For any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree σ in which $\tilde{a}$ is bigger than $\tilde{b}$ is the following:

$$\mu(\tilde{a},\tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \tag{12}$$

where $[E_1^a, E_2^b]$ and $[E_1^b, E_2^b]$ are the expected intervals of $\tilde{a}$ and $\tilde{b}$. When $\mu(\tilde{a},\tilde{b}) \geq \sigma$ we can say that $\tilde{a}$ is bigger than, or equal, to $\tilde{b}$, at least in a degree *σ,* and we can represent it by $\tilde{a} \geq_\sigma \tilde{b}$. When $\mu(\tilde{a},\tilde{b}) = 0.5$ we can say that $\tilde{a}$ and $\tilde{b}$ are indifferent.

Thus by applying the approach described by Jiménez et al. (2007), the problem (10) is transformed into the crisp equivalent parametric problem defined in (13).

$$\text{Min } \tilde{z} = (z_1, z_2, ..., z_k)$$
$$\text{s.t. } [(1-\sigma)E_1^{a_i} + \sigma E_2^{a_i}]x \leq \sigma E_1^{b_i} + (1-\sigma)E_2^{b_i}, i = 1,...,m, x \geq 0, \sigma \in [0,1] \tag{13}$$

where *σ* represents the degree to which at least all constraints $\tilde{a}_i x \leq \tilde{b}_i$ are fulfilled; that is, *σ* is the feasibility degree of a decision *x*.

If an equality type constraint is $\tilde{a}_i x = \tilde{b}_i$, given that $\tilde{a}_i x \geq_{0.5} \tilde{b}_i$ and $\tilde{a}_i x \leq_{0.5} \tilde{b}_i$ mean indifference, it could be represented by two inequalities, $\tilde{a} \geq_{\sigma/2} \tilde{b}$ and $\tilde{a} \leq_{\sigma/2} \tilde{b}$, where σ represents the degree to which equality is fulfilled. Thus, similarly to what was done in (13), the fuzzy equality constraint: $\tilde{a}_i x = \tilde{b}_i$ could be transformed into two equivalent crisp constraints:

$$\left[\left(1-\frac{\sigma}{2}\right)E_2^{a_i} + \frac{\sigma}{2}E_1^{a_i}\right]x \geq \frac{\sigma}{2}E_2^{b_i} + \left(1-\frac{\sigma}{2}\right)E_1^{b_i}, i = 1,...,m, x \geq 0, \sigma \in [0,1]$$
$$\left[\left(1-\frac{\sigma}{2}\right)E_1^{a_i} + \frac{\sigma}{2}E_2^{a_i}\right]x \leq \frac{\sigma}{2}E_1^{b_i} + \left(1-\frac{\sigma}{2}\right)E_2^{b_i}, i = 1,...,m, x \geq 0, \sigma \in [0,1] \tag{14}$$

where σ represents the degree to which the equality is fulfilled.

Consequently, by applying this approach to the previously defined fuzzy multi-objective model for material requirements planning, and by considering trapezoidal fuzzy numbers for the uncertain parameters, we obtain the following parametric model in which parameter σ represents the degree of fulfillment of the fuzzy constraints (4), (5) and (6).

Eq. (1)-(3)

Subject to

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I} \alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} \leq \frac{\sigma}{2} \cdot \frac{d_{it1} + d_{it2}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{d_{it3} + d_{it4}}{2} \quad \forall i \forall t \text{ (15)}$$

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I} \alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} \geq \frac{\sigma}{2} \cdot \frac{d_{it3} + d_{it4}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{d_{it1} + d_{it2}}{2} \quad \forall i \forall t \text{ (16)}$$

$$\sum_{i=1}^{I} \left[P_{it} \cdot \left(\frac{\sigma}{2} \cdot \frac{AR_{ir1} + AR_{ir2}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{AR_{ir3} + AR_{ir4}}{2} \right) \right] + Tun_{rt} - Tov_{rt} - Tsub_{rt} \leq \frac{\sigma}{2} \cdot \frac{CAP_{rt1} + CAP_{rt2}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{CAP_{rt3} + CAP_{rt4}}{2} \quad \forall r \forall t \text{ (17)}$$

$$\sum_{i=1}^{I} \left[P_{it} \cdot \left(\frac{\sigma}{2} \cdot \frac{AR_{ir3} + AR_{ir4}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{AR_{ir1} + AR_{ir2}}{2} \right) \right] + Tun_{rt} - Tov_{rt} - Tsub_{rt} \geq \frac{\sigma}{2} \cdot \frac{CAP_{rt3} + CAP_{rt4}}{2} + \left(1 - \frac{\sigma}{2}\right) \cdot \frac{CAP_{rt1} + CAP_{rt2}}{2} \quad \forall r \forall t \text{ (18)}$$

$$Tov_{rt} \leq \sum_{i=1}^{I} ov_{rt} \cdot P_{it} \cdot \left(\sigma \cdot \frac{AR_{ir1} + AR_{ir2}}{2} + (1 - \sigma) \cdot \frac{AR_{ir3} + AR_{ir4}}{2} \right) \quad \forall r \forall t \text{ (19)}$$

and Eq. (7)-(9)

3.2 Modified approach of Jiménez et al. (2007) for equality constraints

Diaz-Madroñero et al. (2012) show that the approach by Jiménez et al. (2007) applied to fuzzy integer linear programming problems, where the integrity condition is imposed on the solution, can present an unfeasibility problem of the solution. Here it happens because the right-hand side of Constraints (15) and (16) are equal fractional values, while the left-hand side of these constraints must be integer values, which could be infeasible for certain σ values. To face this, we propose substituting the right-hand-side terms of the constraints (15-18) for the corresponding closest integer values. Therefore, we have to add new auxiliary decision variables to ensure that the right-hand side of the constraints (15-18) can be transformed into integer and fractional values in order to achieve the closest integer values. The previous model is modified as follows:

Eq. (1)-(3), and

$$Min \quad z_4 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}\left(d1_{it}^{ABS} + d2_{it}^{ABS}\right) \tag{20}$$

Subject to

$$d1_{it}^{INT} + d1_{it}^{DEC} = \frac{\sigma}{2}\cdot\frac{d_{it1}+d_{it2}}{2} + \left(1-\frac{\sigma}{2}\right)\cdot\frac{d_{it3}+d_{it4}}{2} \qquad \forall i \forall t \; (21)$$

$$d2_{it}^{INT} + d2_{it}^{DEC} = \frac{\sigma}{2}\cdot\frac{d_{it3}+d_{it4}}{2} + \left(1-\frac{\sigma}{2}\right)\cdot\frac{d_{it1}+d_{it2}}{2} \qquad \forall i \forall t \; (22)$$

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} \leq d1_{it}^{INT} \qquad \forall i \forall t \; (23)$$

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} \geq d2_{it}^{INT} \qquad \forall i \forall t \; (24)$$

$$d1_{it}^{DEC} \leq d1_{it}^{ABS} \qquad \forall i \forall t \; (25)$$

$$-d1_{it}^{DEC} \leq d1_{it}^{ABS} \qquad \forall i \forall t \; (26)$$

$$d2_{it}^{DEC} \leq d2_{it}^{ABS} \qquad \forall i \forall t \; (27)$$

$$-d2_{it}^{DEC} \leq d2_{it}^{ABS} \qquad \forall i \forall t \; (28)$$

$$d1_{it}^{INT}, d2_{it}^{INT} \geq 0, \text{ integer} \qquad \forall i \forall t \; (29)$$

$$d1_{it}^{ABS}, d2_{it}^{ABS} \leq 0.5 \qquad \forall i \forall t \; (30)$$

$$d1_{it}^{ABS}, d2_{it}^{ABS} \geq 0 \qquad \forall i \forall t \; (31)$$

And Eq. (7)-(9)
Eq. (17)-(19)

where the right-hand side coefficients of Constraints (15) and (16) are represented by the sum of an integer variable and a real variable. Thus, the right-hand-side coefficient of Constraint (15) is the equivalent to the sum of $d1_{it}^{INT}$ and $d1_{it}^{DEC}$. The same applies to Constraint (16) and $d2_{it}^{INT}$ and $d2_{it}^{DEC}$. Then, the right-hand-side coefficients of Constraints (15) and (16) are replaced with these integer variables in Constraints (23) and (24). Hence, $d1_{it}^{DEC}$ and $d2_{it}^{DEC}$ represent the deviation from the original values in Constraints (15) and (16) to the integer values in Constraints (23) and (24), and will be lower than 1. These deviations are expressed in a linear form of absolute value in Constraints (25-28) by incorporating variables $d1_{it}^{ABS}$ and $d2_{it}^{ABS}$. Finally, the total sum of the absolute value of these deviations is added as a new objective function, z_4, to be minimized.

3.3 Fuzzy goal programming as a solution method

The fuzzy aspiration levels of each objective must be established by bearing in mind the decision maker's preferences. Nonetheless, and to help determine them, parameter σ or the degree that the constraints are fulfilled can be taken into account. The set of all the solutions which the constraints fulfil by at least one σ degree can be represented by $F_\sigma(x)$. By assuming that, for example, the minimum degree of fulfilment that the decision maker is willing to admit is σ =0.1, from this point, several values are given to parameter σ; for example σ={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1} (32)

By considering $\underline{z}_k$ and $\overline{z_k}$ as the lower and upper limits for the fuzzy goals provided by the decision maker (DM), the fuzzy aspiration levels of each objective can be modelled by the fuzzy sets whose membership function is the following:

$$\mu_k = \begin{cases} 1 & if \quad z_k \le \underline{z_k} \\ \dfrac{\overline{z_k} - z_k}{\overline{z_k} - \underline{z_k}} & if \quad \underline{z_k} \le z_k \le \overline{z_k} \\ 0 & if \quad z_k \ge \overline{z_k} \end{cases} \tag{33}$$

After determining the fuzzy aspiration levels for the objectives, it is important to highlight that the objective function (33) adopts the approach by Akoz and Petrovic (2007), which establishes imprecise priority relations between the goals rather than using weights to represent the relative importance of the goals. We adopt this approach for two reasons: (1) the relation between the weights and solutions is not always as expected. According to Jones (2011), "we have to point out that the relation between the changes in weights and the changes in decision space is not smooth. Especially in the linear case where the solution jumps from one extreme point of the feasible set to another. So small change in weights may lead to a large change in decisions space. Similarly large change in weights can lead to no change in decisions space because they can be insufficient to move to a new extreme point"; (2) determination of the weights are not easy in many cases. The approach by Akoz and Petrovic (2007) establishes the relative preferences of the goals through the fuzzy preference relations of type: $\widetilde{R}_1(k,l)$ "*k*-th goal is slightly more important than *l*-th goal"; $\widetilde{R}_2(k,l)$ "k-th goal is moderately more important than *l*-th goal" or $\widetilde{R}_3(k,l)$ "*k*-th goal is significantly more important than *l*-th goal". The membership functions corresponding to these linguistic terms are given in (34)-(36), respectively.

$$\mu_{\widetilde{R}_1(k,l)} = \begin{cases} \mu_k - \mu_l + 1, & if\ -1 \le \mu_k - \mu_l \le 0 \\ 1 & if\ 0 \le \mu_k - \mu_l \le 1 \end{cases} \tag{34}$$

$$\mu_{\widetilde{R}_2(k,l)} = \frac{\mu_k - \mu_l + 1}{2}, \quad -1 \le \mu_k - \mu_l \le 1 \tag{35}$$

$$\mu_{\widetilde{R}_3(k,l)} = \begin{cases} 0 & if\ -1 \le \mu_k - \mu_l \le 0 \\ \mu_k - \mu_l & if\ 0 \le \mu_k - \mu_l \le 1 \end{cases} \tag{36}$$

In order to incorporate the DM's preference structure, we built a GP model in which the objectives are the aspiration levels' achievement and the imprecise preferences' fulfilment. Thus, following Aköz and Petrovic (2007), we propose an achievement function that is a convex combination of goal achievement and the fulfilment of the fuzzy binary relations. Therefore, the following GP model with fuzzy hierarchies is formulated:

$$\text{Max } \lambda \cdot \left(\sum_{k=1}^{K} \mu_k \right) + (1-\lambda) \cdot \left(\sum_{k=1}^{K} \sum_{l=1}^{K} b_{kl} \cdot \mu_{\widetilde{R}(k,l)} \right)$$

s.t.

$$\mu_k \le 1 \quad \forall k$$
$$\mu_k \ge 0 \quad \forall k$$

$$\mu_k - \mu_l + 1 \geq \mu_{\tilde{R}_1(k,l)} \quad \textit{for all } b_{kl} = 1 \textit{ and } \tilde{R}(k,l) = \tilde{R}_1$$
$$\frac{\mu_k - \mu_l + 1}{2} \geq \mu_{\tilde{R}_2(k,l)} \quad \textit{for all } b_{kl} = 1 \textit{ and } \tilde{R}(k,l) = \tilde{R}_2$$
$$\mu_k - \mu_l \geq \mu_{\tilde{R}_3(k,l)} \quad \textit{for all } b_{kl} = 1 \textit{ and } \tilde{R}(k,l) = \tilde{R}_3$$
$$\mu_{\tilde{R}(k,l)} \leq 1 \quad \textit{for all } b_{kl} = 1$$
$$\mu_{\tilde{R}(k,l)} \geq 0 \quad \textit{for all } b_{kl} = 1$$
$$ax \leq b$$
$$x \geq 0$$
$$\lambda \in [0,1] \tag{37}$$

where $b_{kl,}\ k,l = 1,...,K$ are binary variables, taking a value of 1 if an importance relation is defined between goals k and l, μ_k denotes the membership function of the *kth* objective and $\mu_{\tilde{R}(k,l)}$ corresponds to the fuzzy priority relation between objectives k and l. λ ($0 \leq \lambda \leq 1$) is a parameter which allows to obtain different solutions, which are more or less balanced between goals achievement (λ= 1) and the fulfilment of the preference relations (λ=0) . As λ decreases, the relative preference relations gain more importance and, consequently, more solutions that satisfy them are obtained.

In our problem, we have established a relation of type $\tilde{R}_1$ between the objective of minimizing the sum of integer deviations, z_4, and the minimization of total costs, z_1; a relation of type $\tilde{R}_1$ between z_4, and the objective of minimizing backorder units, z_2; and a relation of type $\tilde{R}_3$ between z_1 and the objective of minimizing total idle time, z_3, as well as between z_2 and z_3. These relations can be established in a particular manner in each context. By giving values to σ (see (32)), different models can be obtained whose solutions for the different parameter λ values offers the decision maker a set of solutions from which to choose according to his/her preferences.

The equivalent formulation based on the previous approach of our model in section 3.2 is presented as follows:

$$Max\ \lambda \cdot (\mu_1 + \mu_2 + \mu_3 + \mu_4) + (1-\lambda) \cdot (\mu_{\tilde{R}_1(4,1)} + \mu_{\tilde{R}_1(4,2)} + \mu_{\tilde{R}_3(1,3)} + \mu_{\tilde{R}_3(2,3)}) \tag{38}$$
$$\mu_1 \leq 1 \tag{39}$$
$$\mu_2 \leq 1 \tag{40}$$
$$\mu_3 \leq 1 \tag{41}$$
$$\mu_4 \leq 1 \tag{42}$$
$$\mu_4 - \mu_1 + 1 \geq \mu_{\tilde{R}_1(4,1)} \tag{43}$$
$$\mu_4 - \mu_2 + 1 \geq \mu_{\tilde{R}_1(4,2)} \tag{44}$$
$$\mu_1 - \mu_3 \geq \mu_{\tilde{R}_3(1,3)} \tag{45}$$
$$\mu_2 - \mu_3 \geq \mu_{\tilde{R}_3(2,3)} \tag{46}$$
$$\mu_{\tilde{R}_1(4,1)} \leq 1 \tag{47}$$
$$\mu_{\tilde{R}_1(4,2)} \leq 1 \tag{48}$$
$$\mu_{\tilde{R}_3(1,3)} \leq 1 \tag{49}$$

$$\mu_{\widetilde{R}_3(2,3)} \leq 1 \tag{50}$$

$$\mu_{\widetilde{R}_1(4,1)} \geq 0 \tag{51}$$

$$\mu_{\widetilde{R}_1(4,2)} \geq 0 \tag{52}$$

$$\mu_{\widetilde{R}_3(1,3)} \geq 0 \tag{53}$$

$$\mu_{\widetilde{R}_3(2,3)} \geq 0 \tag{54}$$

Eq. (7)-(9), (17)-(19) and (21)-(31).

4 Application to a first-tier supplier of an automobile supply chain

This section uses an example based on Mula et al. (2006c, 2007 and 2008) to validate and evaluate the results of our proposal. It is based on a representative finished good or part, dubbed as RPN, of a first-tier supplier of the automobile sector. The fuzzy market demand of part RPN for each weekly period is provided in Table 9: d_{it3} is considered to be the market demand information received by the firm; d_{it1} is obtained by decreasing the value of d_{it3} by 25%; d_{it2} is obtained by decreasing the value of d_{it3} by 20%; and d_{it4} is obtained by increasing the value of d_{it3} by 10%.

Figure 1 schematically depicts the list of RPN materials, where α_{ij} is represented with black boxes and corresponds to the units of child product *i* needed to produce one unit of parent product *j*.

Lead time is considered null for part RPN. The lead times for its components are indicated in Table 10. Lead times are composed of the supplier's production time, transport time and safety time. As the intention is to consider a weekly planning period, it is necessary to convert lead times into weeks by considering weeks of 5 working days and by approaching values of entire weeks in accordance with the firm's criterion. The costs employed for this problem are approximate and are determined based on the consultations made to the firm, which is the object of the application. For confidentiality reasons, these costs are distorted, although a relation among them is maintained. The production unit costs of part RPN and the acquisition unit costs (cp_{it}) of the components that form it are provided in Table 6. The unit costs for maintaining inventories (ci_{it}) are assumed to be 10% of the product value each week.

Table 11 provides the time required (in hours) to produce one unit of product *i* in resource *r*, $\widetilde{AR}_{ir} = \{AR_{ir1}, AR_{ir2}, AR_{ir3}, AR_{ir4}\}$, in the fuzzy sense. In this case, resource *r* is considered the assembly line. The fuzzy capacity available on the assembly line hours per week is indicated in $\widetilde{CAP}_{rt} = \{CAP_{rt1}, CAP_{rt2}, CAP_{rt3}, CAP_{rt4}\}$. Besides, Table 11 provides the costs of each subcontracting hour on the assembly line, $csub_{rt}$, and of each overtime hour, $ctov_{rt}$.

The firm can employ these overtime hours or can resort to external subcontracts sporadically when needed. This problem contemplates overtime lines of 5% of normal production. Overtime costs ($ctov_{rt}$) are 1.5 times normal production time. Subcontracting costs ($csub_{rt}$) are 1.2 times extra production time.

Let us assume that there are no delays in the demand of part RPN at the beginning of the planning horizon. To go about this, programmed deliveries, RP_{it}, of RPN of 356 units for the first period and of 1,102 units for the second planning period are contemplated.

The assumptions to carry out the computational experiment are summarized as follows:

- The study considers a representative finished good called RPN (Representative Part Number) assembled by a first-tier supplier of an automobile maker.
- Decision variables, P_{it}, $INVT_{it}$ and B_{it} are considered integer.
- A 6-monthly planning horizon with weekly period planning has been considered.
- Only the finished good RPN has external demand.
- Firm orders from the automobile maker cannot be rejected, although a backlog for the finished good RPN is considered.
- The firm uses safety lead times, assumed as constant values, for some of the components to be supplied to them.

– A single productive resource restricts production: the assembly line.

The proposed model has been done in the MPL language, V4.2. The resolution has been carried out with optimisation solver Gurobi 5.0.0. The input data and the model solution values were processed with a Microsoft Access database (2010). The experiment was run on a PC with a 2.80 GHz processor and 6 GB of RAM.

4.1 Evaluation of results

In this section, the results obtained by the multi-objective decision model for material requirements planning and the methodology of the proposed solution are validated and evaluated. Table 2 provides the upper and lower limits set arbitrarily for each objective.

Table 2. *Limits for each objective.*

$\overline{z_1}$	1000000
$\underline{z_1}$	400000
$\overline{z_2}$	600
$\underline{z_2}$	0
$\overline{z_3}$	100
$\underline{z_3}$	0
$\overline{z_4}$	30
$\underline{z_4}$	0

The values of objectives z_1, z_2 and z_3 corresponding to the minimization of total costs, backlogged demand and idle time, respectively, for different feasibility degrees and various λ are provided in Tables 3, 4 and 5. Table 6 shows the sum of the achievement degrees obtained for all the fuzzy goals and Table 7 offers the sum of the fulfilment degrees of the imprecise relative importance relations among them for different combinations of σ and λ. As seen in Table 6, higher sums of goal fulfilment degrees are obtained with higher λ and lower σ values. As the σ value increases, the sum of the fulfilment degrees of fuzzy goals reduces. Likewise, the sum of the fulfillment degrees lowers if the λ *v*alue falls. In this case, the fuzzy relative importance of the goals is more weighted, and higher values for the sum of the fulfilment degrees of the imprecise relative importance relations among the objectives are obtained (Table 7). Accordingly, and as we can see in Table 3, the results obtained for the approach proposed for z_1 are better for low λ and σ values, and a better solution is achieved for λ=0.1 and σ=0.1. As the feasibility degree and parameter σ rise, the z_1 value increases to reach the maximum for λ=0.9 and σ=1. As in z_1, z_2 increases as parameter σ rises. Nevertheless, according to Table 4, z_2 remains invariable in relation to parameter λ for the feasibility degrees between 0.1 and 0.4, and also between 0.6 and 1, and the highest σ=1 value is obtained. Moreover, the values acquired for z_3 are lower for high λ and low σ values, with a minimum for λ=0.9 and σ=0.1, and a maximum for λ, between 0.1 and 0.5, and the sum of σ (Table 5). It is worth remembering that, as previously mentioned, when it comes to defining the priority relations of the goals, they become more important when total costs are obtained along with shorter backlogged demand than shorter idle times. When comparing the results with those obtained by the deterministic fuzzy goal programming model, solved by the approach of Aköz and Petrovic (2007), this new material requirements planning proposal provides a better set of solutions in terms of production and inventory costs, and backlogged demand. Specifically, the deterministic model generates a total cost of €629,539 with 136 units of backlogs and 100 idle time hours when λ=0.1 (see Table 8). All the proposed models always provided the best total costs and backlogged demand values, while similar values ranging from λ=0.1 to λ=0.5 and better values ranging from λ=0.7 to λ=0.9 are generated for idle time. Likewise, more flexibility is provided to the DM when addressing the model solution, which helps establish different priority relations among the objectives considered. It depends on the DM's criteria to decide which combination of parameters better fulfils his/her requirements. Thus from a DM point of view, and according to the risk level, we can choose among risky solutions with $\sigma \leq 0.5$, average or neutral solutions with $\sigma = 0.5$, and conservative solutions which are not willing to admit high risks for $\sigma \geq 0.5$. The DM also has the possibility of adjusting the solutions obtained by modifying the values of λ to highlight the priority relationships among the objectives. For instance, and from Table 3, if we adopt a neutral risk level ($\sigma = 0.5$) and the desire to obtain the lowest values of the total costs combined with the lowest value of backorders, a value of λ=0.3 is selected. However if a neutral DM wishes to reduce the idle time with priority, a value of λ=0.9 ise chosen despite the higher total costs and backorders.

Table 3. *Sets of solutions for objective z_1*

z_1 (€)		λ				
		0.1	0.3	0.5	0.7	0.9
Degree of feasibility (σ)	0.1	409977.07	410690.98	410309.75	415236.52	453237.57
	0.2	420075.39	420716.90	420626.60	425339.47	466752.17
	0.3	432078.30	431285.42	430999.42	437712.35	480887.85
	0.4	443341.59	441317.79	441235.70	449031.56	498139.43
	0.5	443341.59	441317.79	452328.53	461416.78	503640.45
	0.6	465572.81	464285.08	463371.49	472726.27	500684.98
	0.7	479093.97	477382.23	476206.88	484123.26	520762.39
	0.8	491483.11	491490.58	491483.39	496452.15	514011.82
	0.9	508504.39	508509.79	508503.00	509627.93	524004.16
	1	528556.23	528559.34	528556.23	523391.14	526541.72

Table 4. *Sets of solutions for objective z_2*

z_2 (units)		λ				
		0.1	0.3	0.5	0.7	0.9
Degree of feasibility (σ)	0.1	0	0	0	0	0
	0.2	0	0	0	0	0
	0.3	0	0	0	0	0
	0.4	0	0	0	0	0
	0.5	0	0	6	6	6
	0.6	17	17	17	17	17
	0.7	28	28	28	28	28
	0.8	40	40	40	40	40
	0.9	51	51	51	51	51
	1	62	62	62	62	62

Table 5. *Sets of solutions for objective z_3*

z_3 (h)		λ				
		0.1	0.3	0.5	0.7	0.9
Degree of feasibility (σ)	0.1	100	100	100	19.3428	8.8729
	0.2	100	100	100	23.4474	11.1883
	0.3	100	100	100	25.6059	14.1337
	0.4	100	100	100	28.1450	16.3566
	0.5	100	100	100	30.9560	21.2139
	0.6	100	100	100	33.9746	26.7915
	0.7	100	100	100	37.1922	29.0000
	0.8	100	100	100	40.5806	35.8611
	0.9	100	100	100	44.2554	40.0833
	1	100	100	100	48.7103	46.7522

Table 6. *Sets of solutions for the sum of the fulfilment degrees for all the fuzzy goals*

$\sum_{k=1}^{K} \mu_k$		λ				
		0.1	0.3	0.5	0.7	0.9
Degree of feasibility (σ)	0.1	2.7900	2.7888	2.7895	3.5878	3.6292
	0.2	2.7565	2.7555	2.7556	3.5133	3.5669
	0.3	2.7665	2.7679	2.7683	3.5011	3.5438
	0.4	2.7378	2.7411	2.7413	3.4468	3.4829
	0.5	2.7378	2.7411	2.7028	3.3781	3.4051
	0.6	2.6524	2.6545	2.6560	3.3007	3.3259
	0.7	2.6015	2.6044	2.6063	3.2212	3.2421
	0.8	2.5909	2.5908	2.5909	3.1768	3.1947
	0.9	2.5275	2.5275	2.5275	3.0831	3.1008
	1	2.4324	2.4324	2.4324	2.9539	2.9682

Table 7. *Sets of solutions for the sum of the fulfilment degrees of the imprecise relative importance relations among objectives*

$\sum_{k=1}^{K}\sum_{l=1}^{K} b_{kl}\mu_{\tilde{R}(k,l)}$		λ				
		0.1	0.3	0.5	0.7	0.9
Degree of feasibility (σ)	0.1	3.6133	3.6133	3.6133	2.0002	1.7908
	0.2	3.5800	3.5800	3.5800	2.0489	1.8038
	0.3	3.6400	3.6400	3.6400	2.1521	1.9227
	0.4	3.6200	3.6200	3.6200	2.1829	1.9471
	0.5	3.6200	3.6200	3.6000	2.2191	2.0243
	0.6	3.5800	3.5800	3.5800	2.2595	2.1158
	0.7	3.5600	3.5600	3.5600	2.3038	2.1453
	0.8	3.6200	3.6200	3.6200	2.4316	2.3372
	0.9	3.5867	3.5867	3.5867	2.4718	2.3883

Table 8. *Sets of solutions for deterministic fuzzy goal programming*

	λ				
	0.1	0.3	0.5	0.7	0.9
z_1 (€)	629539.22	629547.32	629544.85	600178.67	600175.62
z_2 (units)	136	136	136	136	136
z_3 (h)	100	100	100	100	33.3626
$\sum_{k=1}^{K} \mu_k$	1.3908	1.3908	1.3908	2.1059	2.1061
$\sum_{k=1}^{K}\sum_{l=1}^{K} b_{kl}\mu_{\tilde{R}(k,l)}$	1.3908	1.3908	1.3908	0.1072	0.1070

5 Conclusions

This paper has addressed the material requirements planning problem under uncertainty in parameters such as demand, required capacity and available capacity through a fuzzy multi-objective decision model. We have shown the real necessity of multi-objective models given the difficulty for firms to define production parameters, such as backlog costs or idle time costs, which usually appear in traditional uni-objective material requirements planning models. For the purpose of solving the multi-objective model, we propose a solution methodology based on FGP.

The solution methodology firstly addresses the fuzzy parameters in constraints according to the Ranking Fuzzy Numbers Approach by Jiménez (1996) and Jiménez et al. (2007). This approach is improved in order to be feasibly applied to integer programmes with equality constraints. Next the FGP approach by Aköz and Petrovic (2007), based on establishing the fuzzy importance relations among objectives, is proposed in order to obtain a set of solutions to the problem. This proposal is applied in a real-world problem of a first-tier supplier of an automobile maker. It is proved that in relation to the alternative mixed integer linear programming model, this FGP model can achieve better performance in terms of minimizing the total production and inventory costs and backorder levels without considerably increasing the levels of idle time and computational efficiency. Thus, the validity of our model has been demonstrated by comparing its results with those obtained by the deterministic fuzzy goal programming model, solved by the approach of Aköz and Petrovic (2007) by using a numerical example inspired in Mula et al. (2006c, 2007 and 2008) related to the automobile industry. This new model presented better performance given the reduction in production and inventory costs of about 34.88% and of 100% for backlogged demands (for a balance between goal achievements of λ=1, and a degree of feasibility of σ=1). Idle time presents similar values, or even better ones, for $\lambda \geq 0.7$ and for each degree of feasibility.

This new material requirements planning proposal provides a better set of solutions in terms of production and inventory costs, and backlogged demand. Specifically, the deterministic model generates a total cost of €629,539 with 136 units of backlogs and 100 idle time hours when λ=0.1 (see Table 8). All the proposed models always provided the best total costs and backlogged demand values, while similar values ranging from λ=0.1 to λ=0.5 and better values ranging from λ=0.7 to λ=0.9 are generated for idle time. Likewise, more flexibility is provided to the DM when addressing the model solution, which helps establish different priority relations among the objectives considered. It depends on the DM's criteria to decide which combination of parameters better fulfils his/her requirements.

Furthermore, our proposal implies more modelling complexity time, but it provides more flexibility to the DM to help obtain a fuzzy solution that matches his/her preferences.

The advantages of this proposal are related to: (1) modelling and establishing priorities for production objectives, which are traditionally measured through costs estimated, which proves difficult for firms; (2) the application and improvement of a ranking fuzzy numbers approach to an integer programme with equality constraints; and (3) the opportunity of determining, in terms of parameter σ, the value of the minimum fulfilment of the constraints, and in terms of parameter λ, the degrees of the maximum fulfilment of the fuzzy goals and the maximum fulfilment of the fuzzy importance relations among the goals considered.

As regards the limitations of this proposal, we describe them through further research proposals: (1) developing a decision support system to systemize model configuration and running; (2) exploring the effect of the size of the decision variables for the deviation variables proposed in this work; and (3) using metaheuristics for improving the efficiency of the solution methodology as a forthcoming work.

For components, the firm has no data, no required capacity and no available capacity of its suppliers to produce all the components supplied. The firm can only do a bill of explosion of materials in its production planning process without performing mid-term capacity planning. Nonetheless, a study of its suppliers' capacities and restrictions is an interesting line for future research which could help cut the supply times contemplated and, therefore, costs throughout the supply chain.

Finally, the material requirements planning system is the central core of our model. Alternative JIT optimization models can be found in Alfieri and Matta (2012). Our fuzzy goal programming approach could also be investigated and compared for these JIT optimization models as a further research work. In general terms, some relevant changes could be related to the calculation of the inventory balance equation and to the introduction of constraints to calculate kanban cards (Lage and Godinho, 2010).

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Annex

Table 9. *RPN demand per period.*

T	d_{it1}	d_{it2}	d_{it3}	d_{it4}
1	609	643	813	893
2	702	741	936	1030
3	582	614	777	854
4	322	340	430	472
5	483	510	645	708
6	377	398	503	552
7	0	0	0	0
8	0	0	0	0
9	207	219	278	305
10	873	922	1166	1282
11	0	0	0	0
12	0	0	0	0
13	1420	1499	1894	2082
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	1339	1413	1786	1964
18	0	0	0	0
19	0	0	0	0
20	0	0	0	0
21	1195	1261	1594	1752
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
25	0	0	0	0
26	0	0	0	0
27	0	0	0	0
28	0	0	0	0
29	0	0	0	0
30	0	0	0	0

Table 10. *Lead times, production/acquisition costs, inventory costs and initial inventories.*

I	LT_i	cp_{it}	ci_{it}	$INVT_{i0}$
1	0	6.34	5.3919	0
2	1	6.1	0.61	891
3	3	0.0096	0.001	13032
4	3	0,03	0,003	13832
5	2	0,187	0,0187	9249
6	2	0,0478	0,0048	4306
7	2	0,1156	0,0116	1244
8	3	0,012	0,0012	50050
9	2	0.0321	0.0032	10543
10	2	0.0291	0.0029	13078
11	2	0.0987	0.0099	1931
12	1	2.087	0.2087	1348
13	3	0.0135	0.0014	2166
14	2	0.0508	0.0051	4992
15	2	0.0539	0.0054	4797
16	2	0.0099	0.001	8654
17	2	0.0307	0.0031	9464
18	2	0.0244	0.0024	17726
19	2	0.0539	0.0054	3632
20	1	0.0377	0.0038	100672
21	2	0.0311	0.0031	10018
22	1	0.0252	0.0025	5275
23	0	14	1.4	0
24	1	0.0016	0.0002	2685951
25	0	0.1006	0.0101	15739
26	0	0,5285	0,0528	3906
27	0	0,5713	0,0571	476
28	0	6,632	0,6632	258
29	0	5,5214	0,5521	421
30	3	0,0177	0,0018	34826
31	2	2,89	0,289	355
32	1	0.6819	0.0682	321
33	1	0.6571	0.0657	598
34	1	1.773	0.1773	709
35	1	0.222	0.0222	1186
36	1	0.222	0.0222	357

I	LT_i	cp_{it}	ci_{it}	$INVT_{i0}$
37	0	0.001	0.0001	0
38	0	0.001	0.0001	0
39	0	0.001	0.0001	0
40	0	2.29	0.229	0
41	1	2.1576	0.2158	1356
42	3	0.1449	0.0145	16606
43	2	0.001	0.0001	38402
44	1	0.001	0.0001	989891
45	2	0.042	0.0042	13772
46	2	0.0409	0.0041	3380

Table 11. *Fuzzy required and available capacities (hours), subcontracted and overtime costs (€/hour).*

r	*i*	AR_{rt1}	AR_{rt3}	AR_{rt3}	AR_{rt4}	CAP_{rt1}	CAP_{rt2}	CAP_{rt3}	CAP_{rt4}	$csub_{rt}$	$ctov_{rt}$
1	1	0.003889	0.008889	0.01389	0.023889	0.98	2.17	3.36	12.04	821.64	684.72

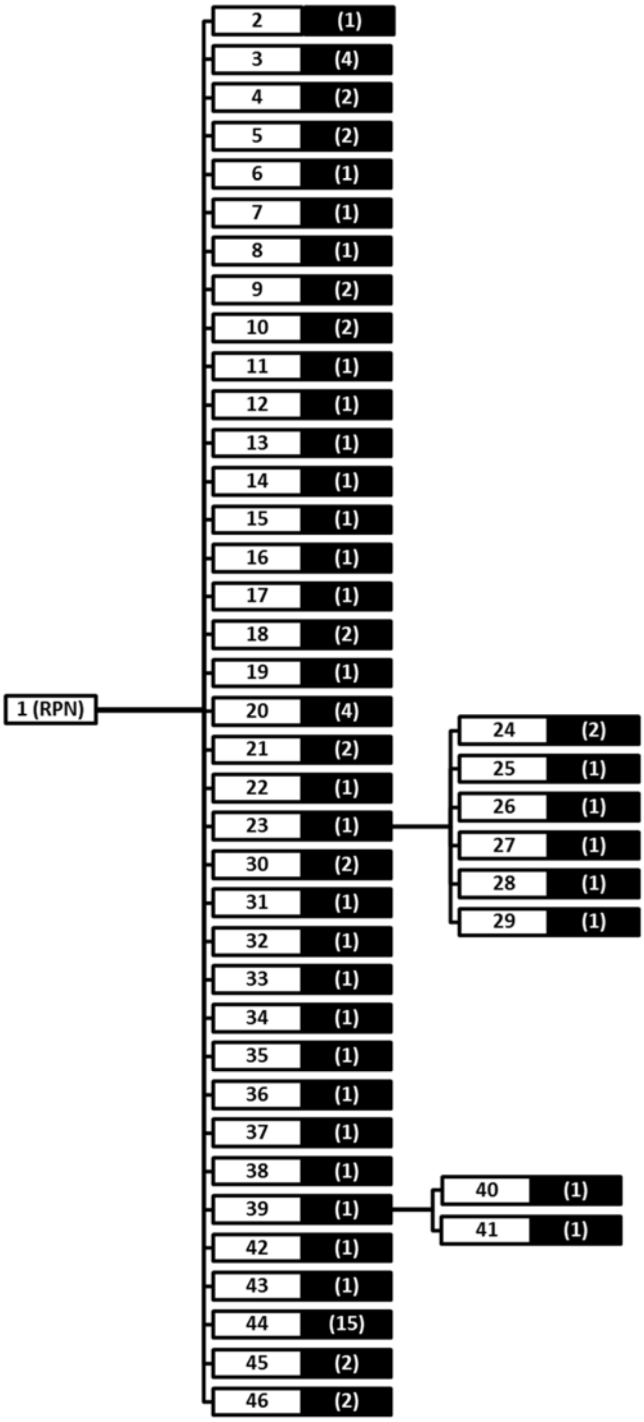

Fig. 1. *Schema of the RPN bill of materials*