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Title: **Unknown signal detection by one-class detector based on Gaussian copula**

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Abstract

One-class detector is an option to deal with the problem of detecting an unknown signal in a background noise, as it is only necessary to know the noise distribution. Thus a Gaussian copula is proposed to capture the dependence among the noise samples, meanwhile the marginals can be estimated using well known methods. We show that classical energy detectors are particular cases of the proposed one-class detector, when Gaussian noise distribution is assumed, but are inappropriate in other cases. Experiments combining simulated noise and real acoustic events have confirmed the superiority of the proposed detectors when noise is non-Gaussian. An interpretation of the methods in terms of the Edgeworth expansion is also included.

Keywords: signal detection, one-class detector, copula, energy detector

Table captions:

Figure captions:

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Abstract

One-class detector is an option to deal with the problem of detecting an unknown signal in a background noise, as it is only necessary to know the noise distribution. Thus a Gaussian copula is proposed to capture the dependence among the noise samples, meanwhile the marginals can be estimated using well known methods. We show that classical energy detectors are particular cases of the proposed one-class detector, when Gaussian noise distribution is assumed, but are inappropriate in other cases. Experiments combining simulated noise and real acoustic events have confirmed the superiority of the proposed detectors when noise is non-Gaussian. An interpretation of the methods in terms of the Edgeworth expansion is also included.

1. Introduction

Signal detection in a random background noise is a classical problem in detection theory. Starting from the popular matched filter [1], which requires perfect knowledge of the signal waveform, different methods exist to deal with the practical problem of partial or null knowledge about the signal. Thus, energy detector [2] has been shown to be optimal if both the noise and the signal are independent zero-mean Gaussian random processes. On the other hand, subspace matched filter is optimal if noise is independent Gaussian and the signal lies in a known subspace. Different extensions of the energy detector [3] [4] and the subspace matched filter [2] [5] exist for the non-independent and/or non-Gaussian cases.

A different approach comes from recognizing that unknown signal detection is conceptually a one-class classifier problem [6], where the "noise" class may be learned (both in the Gaussian and non-Gaussian scenarios), but there is no possibility to learn the "signal" class. This can be also considered inside the so called novelty detection problem [7]. Both approaches converge under the likelihood ratio framework. It is well-known [1] that the optimum test is given by (hypothesis H_1 indicates presence of signal and noise, hypothesis H_0 that only noise is present)

$$\Lambda(\mathbf{x}) = \frac{f(\mathbf{x}/H_1)}{f(\mathbf{x}/H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \quad . \tag{1}$$

where $f(\mathbf{x}/H_i)$ is the multidimensional probability density function (PDF) of the observation vector $\mathbf{x} = [x_1\, x_2 \ldots x_N]^T$ conditioned to hypothesis H_i, and λ is a threshold selected to fit an acceptable probability of false alarm (Neyman-Pearson criteria) or to minimize a defined cost (Bayes approach). $\Lambda(\mathbf{x})$ is the likelihood ratio (*LR*) whose computation obviously requires knowledge of both $f\left(\mathbf{x}/H_1\right)$ and $f\left(\mathbf{x}/H_0\right)$. However, if the signal is totally unknown, a simpler option is to assume that $f\left(\mathbf{x}/H_1\right)$ is constant [6] leading to the one-class test

$$f\left(\mathbf{x}/H_0\right)^{-1} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \quad \Leftrightarrow \; -\ln f\left(\mathbf{x}/H_0\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln\lambda \quad . \tag{2}$$

Notice that $f\left(\mathbf{x}/H_0\right)^{-1}$ is a measure of the degree of departure of $\mathbf{x}$ from the distribution of the "noise" class.

In this paper we will focus in the one-class test of equation (2). Thus, the basic problem will be the estimation of the multidimensional PDF $f\left(\mathbf{x}/H_0\right)$. In case of statistical independence among the components of the noise, $f\left(\mathbf{x}/H_0\right)$ will simply be the product of the marginals, so that available parametric and nonparametric unidimensional methods are applicable. However, the most difficult aspect regarding estimation of $f\left(\mathbf{x}/H_0\right)$ is capturing the possible statistical dependence of the noise. There exist multidimensional nonparametric methods [8], but parametric extensions are not so obvious, except in the multivariate Gaussian model, where a correlation matrix parameterizes the dependence. A more flexible possibility is based on the use of copulas [9]. Although they have been a matter of research in the financial area since long time ago [10], [11], they only are recently being applied in signal detection problems [12],[13],[14], mainly in the context of fusion of heterogeneous detectors. Copula model factorizes $f\left(\mathbf{x}/H_0\right)$ in the marginals and a multidimensional PDF of uniformly distributed variables (copula density) which captures the dependence. Copulas allow defining a variety of parametric dependence models. On the other hand copula densities may be combined with both parametric and nonparametric estimation of the marginals.

The main contribution of this paper is to propose a new detector for unknown signal detection having general applicability in non-Gaussian and non-independent noise scenarios. It is based on approaching the problem as a one-class or novelty detection problem so that only the multivariate noise distribution is to be required. This latter is estimated assuming a Gaussian copula model. This particular type of copula has been selected due to its simplicity of implementation, general applicability, and because it leads to a natural extension of classical methods, based upon the energy computation, which are only optimum in Gaussian scenarios. As far as we know, this is the first time that a one-class copula approach has been proposed for unknown signal detection.

In the next section of this paper we present the copula-based one-class detector. In particular, a Gaussian copula is proposed to capture possible dependences. We show that classical energy detectors are particular cases of the proposed one-class detector for both independent and non-independent noise. Experimental results are presented in Section 3, where real acoustic events are corrupted by simulated noises, having different probability densities, to illustrate the improved performance of the new proposed detector. An interpretation, in terms of the

Edgeworth expansion is given in Section 4 about this superior performance, to reinforce the general interest of the Gaussian copula. Conclusions end the communication.

2. One-class detector based on Gaussian copula

Let us focus in the problem of detecting an unknown signal vector $\mathbf{s} = [s_1 \dots s_N]^{\boldsymbol{T}}$ in a noise background vector $\mathbf{w} = [w_1 \dots w_N]^{\boldsymbol{T}}$. Under H_1, $\mathbf{x} = \mathbf{s} + \mathbf{w}$, and under H_0, $\mathbf{x} = \mathbf{w}$. We are going to use a copula to model the multidimensional noise density $f(\mathbf{w}) = f(\mathbf{x}/H_0)$. Let $F(\mathbf{w})$ be the corresponding multidimensional cumulative distribution function (CDF) i.e., $f(\mathbf{w}) = \frac{\partial^N(F(\mathbf{w}))}{\partial w_1 \partial x w_2, \dots, \partial w_N}$. The Sklar's theorem [15] stays that there exists a unique copula function such that

$$F(\mathbf{w}) = C\big(F_1(w_1), F_2(w_2), \dots, F_N(w_N)\big) \qquad . \tag{3}$$

Where $F_n(w_n)$ is the marginal CDF of random variable w_n, so the random variable $u_n = F_n(w_n)$ is uniformly distributed in the interval [0,1]. Deriving (3) we may express $f(\mathbf{w})$ in the form:

$$f(\mathbf{w}) = \frac{\partial^N}{\partial w_1 \partial w_2, \dots, \partial w_N} C\big(F_1(w_1), F_2(w_2), \dots, F_N(w_N)\big) =$$

$$= f_1(w_1) f_2(w_2) \dots f_N(w_N) \times c\big(F_1(w_1), F_2(w_2), \dots, F_N(w_N)\big). \tag{4}$$

Where $c(\mathbf{u}/H_0) = \frac{\partial^N(C(\mathbf{u}/H_0))}{\partial u_1 \partial u_2, \dots, \partial u_N}$ is the copula density. There are a number of possible parametric copulas with its corresponding copula densities, but we will focus on the Gaussian copula [16] due to its simplicity, general applicability and straightforward connection with classical energy detectors. A Gaussian copula assumes that if the uniform random variables $u_n = F_n(w_n)$ are transformed into standard Gaussian variables (ones having zero mean and unit variance), then the multidimensional PDF in the transformed domain is multivariate Gaussian. Let us call v_n to the transformed variable, i.e., $v_n = \Phi^{-1}(u_n)$, where $\Phi(\cdot)$ is the CDF of a standard Gaussian random variable. Then, it is assumed that

$$f_v(\mathbf{v}) = \frac{1}{2\pi^{N/2} |\mathbf{R}_v|^{1/2}} \cdot \exp\left(-\frac{\mathbf{v}^T \mathbf{R}_v^{-1} \mathbf{v}}{2}\right) \qquad . \tag{5}$$

Where $\mathbf{R}_v = E[\mathbf{v}\mathbf{v}^{\mathrm{T}}]$ is a standard correlation matrix $\mathbf{R}_v(n,m) = E(v_n v_m) = \begin{cases} 1 & n = m \\ < 1 & n \neq m \end{cases}$.

The multivariate Gaussian model assumed in (5) for $f_v(\mathbf{v})$ leads straightforwardly [16] to a particular copula model for $f(\mathbf{w})$, the so called Gaussian copula

$$f(\mathbf{w}) = \frac{1}{|\mathbf{R}_v|^{1/2}} \cdot \exp\left(-\frac{g(\mathbf{w})^T\left(\mathbf{R}_v^{-1} - \mathbf{I}\right)g(\mathbf{w})}{2}\right)\prod_{n=1}^{N} f_n(w_n) \qquad , \qquad (6)$$

where $g(\mathbf{w}) = [g(w_1)g(w_2) \dots g(w_N)]^T$ and $g(w_n) = \Phi^{-1}(F_n(w_n))$. Thus $g(.)$ is a nonlinear function that transforms the original noise components in standard Gaussian random variables.

Finally, considering in (2) that $f(\mathbf{x}/H_0) = f(\mathbf{w})$, we may define from (6) the new proposed (Gaussian) Copula-based One-Class Detector (COCD)

$$g(\mathbf{x})^T\left(\mathbf{R}_v^{-1} - \mathbf{I}\right)g(\mathbf{x}) - 2\sum_{n=1}^{N} lnf_n(x_n/H_0) \gtrless_{H_0}^{H_1} ln\left(\frac{\lambda^2}{|\mathbf{R}_v|}\right) \qquad . \qquad (7)$$

Notice that COCD is a new detector which clearly separate the marginals from the joint statistical properties of the noise. The second term is the only one present in case of independent components and amounts to compute the product of the marginals. The first term captures the possible dependences among the transformed noise components and it will be present only if $\mathbf{R}_v \neq \mathbf{I}$.

It is straightforward to show that (7) leads to classical energy detectors if the noise is assumed multivariate Gaussian. So let us consider that the components of vector **w** are zero-mean Gaussian random variables having the same variance σ^2 and a joint probability density function $f(\mathbf{w}) = \frac{1}{\sqrt{|\mathbf{R}_w|(2\pi\sigma)^N}} \cdot \exp\left(-\frac{\mathbf{w}\mathbf{R}_w^{-1}\mathbf{w}}{2\sigma^2}\right)$ where $\mathbf{R}_w = \frac{\mathbf{1}}{\sigma^2}E[\mathbf{w}\mathbf{w}^{\mathrm{T}}]$ is a standard correlation matrix. In this case the marginals will be $f(w_n) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{{w_n}^2}{2\sigma^2}\right)$ and $\mathbf{v} = g(\mathbf{w}) = \frac{\mathbf{w}}{\sigma}$ and $\mathbf{R}_v = \mathbf{R}_w$. Substituting in (7), we arrive to

$$\mathbf{x}^T\mathbf{R}_w^{-1}\mathbf{x} \gtrless_{H_0}^{H_1} \sigma^2 ln\left(\frac{\lambda^2}{|\mathbf{R}_w|(2\pi\sigma^2)^N}\right) \qquad , \qquad (8)$$

which is an extension of the energy detector for the case of non-independent Gaussian noise [3], requiring a whitening transformation $\mathbf{R}_w^{-\frac{1}{2}}\mathbf{x}$ before computing the energy. Let us call preprocessed energy detector (PED) to (8) as it is done in [3]. In the case of independent Gaussian noise $\mathbf{R}_w = \mathbf{I}$**,** and we obtain the classical energy detector (ED)

$$\mathbf{x}^T\mathbf{x} \gtrless_{H_0}^{H_1} \sigma^2 ln\left(\frac{\lambda^2}{(2\pi\sigma^2)^N}\right) \qquad . \qquad (9)$$

So we see that ED, in spite of the optimality properties reported [2] under noise Gaussian model, is rather limited from the perspective of one-class detection as it constraints the noise PDF to be multivariate Gaussian.

In the next section we are going to test the behavior of COCD with respect to ED and PED in a set of Gaussian/non-Gaussian and independent/non-independent noise scenarios. An interpretation in terms of the Edgeworth expansion will be given in Section 4.

3. Experiments

To verify the relative performance of the different detectors we have generated real acoustic events and combined them with different kinds of synthetic background noise. In that way we perform a hybrid real/simulated verification, where the noise model is under control, but where the events to detect are real sounds. Four types of representative sounds were considered: broken glass, shout, slap and walkie-talkie, having different time and/or spectral characteristics. Five consecutive events were generated for every type of sound, and the whole twenty digitalized events (frequency sampling 44,1 KHz) were sequentially disposed as indicated in figure 1, thus forming a sequence of $6x10^6$ samples. Then synthetic noises were superimposed on that sequence. Different noise models were synthesized varying the marginals PDF, the signal to noise ratio (SNR) and the autocorrelation matrix as indicated in Table I.

A moving window of 256 samples (length N of the observation vector $\mathbf{x}$) was shifted trough the noisy sequence, and tests were made at every shift with a scanning of thresholds. As we know the location of the events, we may determine false alarms or misdetections to obtain the Receiver Operating Characteristics (ROC) representing the probability of detection (P_d) in terms of the probability of false alarm (P_f). We show in Table II a pseudocode description of the different steps of the procedure, including a training part to estimate the noise model information required for the implementation of the detectors.

We can see in the figure 2 (left) the ROC curves obtained for ED, PED and COCD with independent Gaussian noise, for the three different SNR. All the detectors behave the same since ED is a particular case of PED and COCD for independent Gaussian noise. In figure 2 (right), the noise is non-independent Gaussian and thus, those methods which consider the possible presence of dependence (PED and COCD) are better than ED, for the same SNR. On the other hand, being the noise Gaussian, PED and COCD get the same performance.

In the figure 3, we show the case of two non-Gaussian independent noises: uniform (left) and Gamma (right). Uniform noise is distributed between -1 and 1, this distribution is representative of highly non-Gaussian behaviors. On the other hand the two parameters of the Gamma distribution were fitted so that the mean was 7 and the variance 9.8. Thus we obtain a non-Gaussian noise closer to Gaussian than the uniform noise. The levels of the acoustic signals of figure 1 were adjusted to fit the same three different SNR of figure 2. As expected, COCD gives the best results in both cases due to the non-Gaussianity of the noise, while PED and ED behave the same (noise is independent). Moreover, notice that the improvement of COCD is higher for uniform noise than for Gamma distributed noise.

Finally figure 4 shows the same two cases of non-Gaussian noises: uniform (left) and Gamma (right) but now they are non-independent. Again COCD gives the best results, but now PED is better than ED due to the non-independent noise. In the next section we are making and interpretation of the methods in terms of the Edgeworth expansion, in an effort to gain insights into the generalization of the superiority of COCD, as well as finding possible limitations and further extensions of the new detector.

4. Discussion

We have seen that the expectations of Section 2 have been verified in the experiments of Section 3: the use of the Gaussian copula improves the performance of classical detectors when noise is non-Gaussian and non-independent. However experiments are limited to some particular types of non-Gaussian noises, so one may wonder if the Gaussian copula would have general interest in the wide range of applications where a diversity of non-Gaussian noises may appear (see for example [17] and references therein) . Specifically one question may be posed: would COCD is expected to always improve the performance of ED and PED in presence of non-Gaussian and non-independent noise? Let us resort to the multivariate Edgeworth expansion [18], [19], [20]. Given a zero-mean random vector $\mathbf{y}$ with correlation matrix $\mathbf{R}_y$, its multivariate PDF may be expressed in the form

$$f_x(\mathbf{y}) = \frac{1}{2\pi^{N/2}|\mathbf{R}_y|^{1/2}} \cdot \exp\left(-\frac{\mathbf{y}\mathbf{R}_x^{-1}\mathbf{y}}{2}\right) A(\mathbf{y}, \mathbf{R}_y) \quad . \tag{10a}$$

$$A(\mathbf{y}, \mathbf{R}_y) = 1 + \frac{1}{6}\boldsymbol{\kappa}^{i,j,k}\bar{h}_{ijk}(\mathbf{y}, \mathbf{R}_y) + \frac{1}{24}\boldsymbol{\kappa}^{i,j,k,l}\bar{h}_{ijkl}(\mathbf{y}, \mathbf{R}_y) + \frac{1}{72}\boldsymbol{\kappa}^{i,j,k}\boldsymbol{\kappa}^{l,m,n}\bar{h}_{ijklmn}(\mathbf{y}, \mathbf{R}_y) \ldots \tag{10b}$$

Where $\boldsymbol{\kappa}^{i,j,\ldots}$ are cumulant matrices and $\bar{h}_{ij\ldots}$ are generalized Hermite tensors (see [19] for a detailed description of the definitions). ED and PED assume first-order approximations ($A(\mathbf{y}, \mathbf{R}_y) \approx 1$) of the Edgeworth expansion to model the multivariate PDF of the noise. Thus, referring to (10), in ED $\mathbf{y} = \mathbf{w}$, $\mathbf{R}_y = \mathbf{I}$, and in PED $\mathbf{y} = \mathbf{w}$, $\mathbf{R}_y = \mathbf{R}_w$. Apart from the truncation error, these first-order approximations are not able to capture the possible non-Gaussianity of the marginals, as the marginals of a multivariate Gaussian are necessarily Gaussian.

On the other hand, as explained in Section 2, the Gaussian copula has an equivalent interpretation given by equation (5). A Gaussian copula model is equivalent to consider that, once the original non-Gaussian noise random variables w_n are transformed into standard Gaussian noise random variables v_n by the nonlinear function $v_n = g(w_n) = \Phi^{-1}(F(w_n))$, the multidimensional PDF (dependence model) of the elements of the noise vector $\mathbf{v}$ is multivariate Gaussian. Hence, COCD also assumes (implicitly) a first-order approximation (see (5)) but on the transformed domain $\mathbf{v} = g(\mathbf{w})$, i.e., in (10) $\mathbf{y} = \mathbf{v}$, $\mathbf{R}_y = \mathbf{R}_v$. Now the marginals of $\mathbf{v}$ are correctly captured by the first-order approximation, even though the marginals of the original noise **w** can be arbitrary. Therefore COCD should work better than ED and PED for any type of non-Gaussian noise.

Approaching COCD from this new perspective, some improvement should be expected by adding more terms in the Edgeworth expansion of **v** while keeping the Gaussianity of the marginals $f_n(v_n)$. In relation with this, it is noticeable that some works have proposed finite multivariate expansions inspired in the Edgeworth´s in an effort to deal with practical implementation problems. Thus in [21] it is proposed the expansion (we apply it directly to **v**)

$$f_v(\mathbf{v}) = \frac{1}{2\pi^{N/2}|\mathbf{R}_v|^{1/2}} \cdot \exp\left(-\frac{\mathbf{v}^T\mathbf{R}_v^{-1}\mathbf{v}}{2}\right) + \prod_{n=1}^{N} f_n^g(v_n) \sum_{n=1}^{N}\sum_{q=1}^{Q} d_q H_q(v_n) \quad . \quad (11)$$

Where, $f_n^g(v_n)$ is a standard univariate Gaussian PDF, d_q are constants depending on the higher-order moments of the random variable v, and $H_q(.)$ is the Hermite polynomial of order q. It is shown in [21] that the corresponding marginals are given by

$$f_n(v_n) = f_n^g(v_n)\left[1 + \sum_{q=1}^{Q} d_q H_q(v_n)\right] \quad . \quad (12)$$

So, imposing that the marginals are standard Gaussians ($f_n(v_n) = f^g(v_n)$) implies that $d_q = 0\ \forall q$ and that $f_v(\mathbf{v}) = \frac{1}{2\pi^{N/2}|\mathbf{R}_v|^{1/2}} \cdot \exp\left(-\frac{\mathbf{v}^T\mathbf{R}_v^{-1}\mathbf{v}}{2}\right)$, thus adding more terms in the expansion will be incompatible with keeping the Gaussianity of the marginals, or stayed in a more favorable way for the Gaussian copula: no better approximation than the first-order approximation exists if the marginals are constrained to be Gaussian in the finite expansion (11) . In any case, working on (10), it should be possible to find 8extensions of the Gaussian copula.

5. Conclusions

We have presented a one-class detector appropriate for the practical scenario in which there is total ignorance about the signal model. The general case of non-Gaussian non-independent noise has been considered.

A Gaussian copula model has been proposed to capture the possible dependence, meanwhile conventional methods may be used to estimate the marginals. We have seen that classical energy detectors are particular cases of the Gaussian copula-based detector, when the original noise PDF is assumed multivariate Gaussian. Thus energy detectors degrade in presence of non-Gaussian noise, being the one-class detector based on the Gaussian copula a more appropriate alternative. Experiments combining simulated noise and real acoustic events have confirmed the later statement.

Some discussion has been done in the framework of PDF expansions to justify the general applicability of the new proposed detector, and to open some possible extensions of the Gaussian copula.

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References

[1] L. L. Scharf, *Statistical Signal Processing*, Addison-Wesley, Reading, Mass, USA, 1991.

[2] S.M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice-Hall, Englewood Cliffs, NJ, 1998.

[3] J. Moragues, L. Vergara, J. Gosálbez, and I. Bosch, "An extended energy detector for non- Gaussian and non-independent noise," *Signal Process.*, vol. 89, no. 4, pp. 656-661, 2009.

[4] Y.Chen, "Improved Energy Detector for Random Signals in Gaussian Noise," *IEEE Trans. On Wireless Comm.*, vol. 9, no. 2, pp. 558-563, 2010.

[5] J. Moragues, L. Vergara, J. Gosálbez, "Generalized Matched Subspace Filter for Nonindependent Noise Based on ICA,", *IEEE Trans. on Signal Processing*, vol.59, no. 7, pp. 3430-3434, 2011.

[6] O. Mazhelis, "One-Class Classifiers: A Review and Analysis of Suitability in the Context of Mobile-Masquerader Detection," *ARIMA/SACJ*, no. 36, pp. 29-48, 2006.

[7] M. Markou, S.Sameer, "Novelty Detection: A review-Part 1, Statistical Signal Processing," *Signal Processing*, vol.83 , pp. 2481-2497, 2003.

[8] D.W. Scott, S.R. Sain, Multidimensional density estimation, in: C.R. Rao, E.J. Wegman (Eds.), Handbook of Statistics, vol. 23: Data Mining and Computational Statistics, Elsevier, Amsterdam, 2004.

[9] R. B. Nelsen, *An Introduction to Copulas*, Springer, 2006.

[10] R. T. Clemen y T. Reilly, "Correlations and Copulas for Decision and Risk Analysis," *Management Science*, vol. 45, nº. 2, pp. 208–224, 1999.

[11] U. Cherubini, E. Luciano, y W. Vecchiato, *Copula Methods in Finance*, 1.ª ed. Wiley, 2004.

[12] S. G. Iyengar, P. K. Varshney, T. Damarla, "A Parametric Copula-Based Framework for Hypothesis Testing Using Heterogeneous Data," *IEEE Trans. on Signal Processing*, vol.59, no.5, pp. 2308–2319, 2011.

[13] A. Sundaresan, P. K. Varshney, y N. S. Rao, "Copula-Based Fusion of Correlated Decisions," *IEEE Transactions on Aerospace and Electronic Systems*, vol.47, no.1, pp. 454–471, 2011.

[14] S. G. Iyengar, R.Niu, P. K. Varshney, "Fusing dependent decisions for hypothesis testing with heterogeneous sensors," *IEEE Trans. on Signal Processing*, vol.60, no.9, pp. 4888-4896, 2012.

[15] A. Sklar, "Fonctions de répartition à n dimensions et leurs marges," *Publ. Inst. Statist. Univ. Paris*, vol. 8, pp. 229–231, 1959.

[16] P. Song, "Multivariate dispersion models generated from Gaussian copula," *Scand. Journal of Stat*., vol. 27, no.2, pp. 305-320, 2000.

[17] L.M. Garth, H.M. Poor: "Detection of Non-Gaussian Signals: A Paradigm for Modern Statistical Signal Processing", *Proceedings of the IEEE*, vol.82, no.7, pp. 1061-1095, 1994

[18] A. Stuart, J.K. Ord: *Kendall's Advance Theory of Statistics, vol 1*. Edward Arnold, New York, 1994.

[19] P. McCullagh: "Tensor notation and cumulants of Polynomials," *Biometrika*, vol.71, no.3, pp.461-476, 1984.

[20] J. Chambers: "On Methods of Asymptotic Approximation to Multivariate Distributions" *Biometrika*, vol. 54, pp.367-383, 1969.

[21] J. Perote, "The Multivariate Edgeworth-Sargan Density". *Spanish Economic Review* , vol.6, pp. 77-96, 2004.

Noise marginal PDF: $f(w)$	Gaussian, Uniform, Gamma
SNR: $10log\frac{Signal\ energy}{Noise\ mean\ power}$	0 dB, -1 dB, -2 dB
Noise correlation matrix	*Independent noise:* $\mathbf{R}_v = \mathbf{I}$
	Non-independent noise: $\mathbf{R}_v$ *obtained from real air conditioning noise records*

Table I. Description of the different synthetic noise models considered

<table>
<tr><td colspan="3">Given w_i $i = 1 \dots I$ noise samples for estimating the noise model
Given $\mathbf{x}_k$ $k = 1 \dots K$ labelled observation vectors (Nx1) for testing</td></tr>
<tr><td>ED</td><td>PED</td><td>COCD</td></tr>
<tr>
<td>-Estimate the variance of the noise
$\hat{\sigma}^2 = \frac{1}{I}\sum_{i=1}^{I} w_i^2$

-For $\lambda = 0 \dots \lambda_{max}$
For $k = 1 \dots K$
$\mathbf{x}_k^T\mathbf{x}_k \gtrless_{H_0}^{H_1} \hat{\sigma}^2 ln\left(\frac{\lambda^2}{(2\pi\hat{\sigma}^2)^N}\right)$
end
Count the percentage of true positives and false positives to estimate the couple $P_d(\lambda),\ P_f(\lambda)$
end

-Plot the ROC curve</td>
<td>-Estimate the variance of the noise
$\hat{\sigma}^2 = \frac{1}{I}\sum_{i=1}^{I} w_i^2$

-Form noise vectors $\mathbf{w}_l$ $l = 1 \dots L$ of size Nx1

-Estimate the standard correlation matrix of the noise (NxN)
$\hat{\mathbf{R}}_w(n,m) = \frac{\frac{1}{L}\sum_{l=1}^{L}\mathbf{w}_l\mathbf{w}_l^T}{\hat{\sigma}^2}$

-For $\lambda = 0 \dots \lambda_{max}$
For $k = 1 \dots K$
$\mathbf{x}_k^T\hat{\mathbf{R}}_w^{-1}\mathbf{x}_k \gtrless_{H_0}^{H_1} \hat{\sigma}^2 ln\left(\frac{\lambda^2}{\lvert\hat{\mathbf{R}}_w\rvert(2\pi\hat{\sigma}^2)^N}\right)$
end
Count the percentage of true positives and false positives to estimate the couple $P_d(\lambda),\ P_f(\lambda)$
end

-Plot the ROC curve</td>
<td>-Estimate the noise marginal density $\hat{f}(w) = \hat{f}(x/H_0)$ and distribution function $\hat{F}(w) = \hat{F}(x/H_0) = \int_{-\infty}^{\infty}\hat{f}(y/H_0)dy$
(many parametric or nonparametric methods are available)

-Transform the training samples in standard Gaussian random variables
$v_i = \hat{g}(w_i) = \Phi^{-1}(\hat{F}(w_i))$ $i = 1 \dots I$

-Form transformed noise vectors $\mathbf{v}_l$ $l = 1 \dots L$ of size Nx1

-Estimate the standard correlation matrix of the transformed noise (NxN)
$\hat{\mathbf{R}}_v(n,m) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{v}_l\mathbf{v}_l^T$
-For $\lambda = 0 \dots \lambda_{max}$
For $k = 1 \dots K$
$g(\mathbf{x}_k)^T(\hat{\mathbf{R}}_v^{-1} - \mathbf{I})g(\mathbf{x}_k) - 2\sum_{n=1}^{N} ln\hat{f}(x_{kn}/H_0) \gtrless_{H_0}^{H_1} ln\left(\frac{\lambda^2}{\lvert\hat{\mathbf{R}}_v\rvert}\right)$
end

Count the percentage of true positives and false positives to estimate the couple $P_d(\lambda),\ P_f(\lambda)$
end

-Plot the ROC curve</td>
</tr>
</table>

Table II. Description of the different steps for the implementation of the detectors

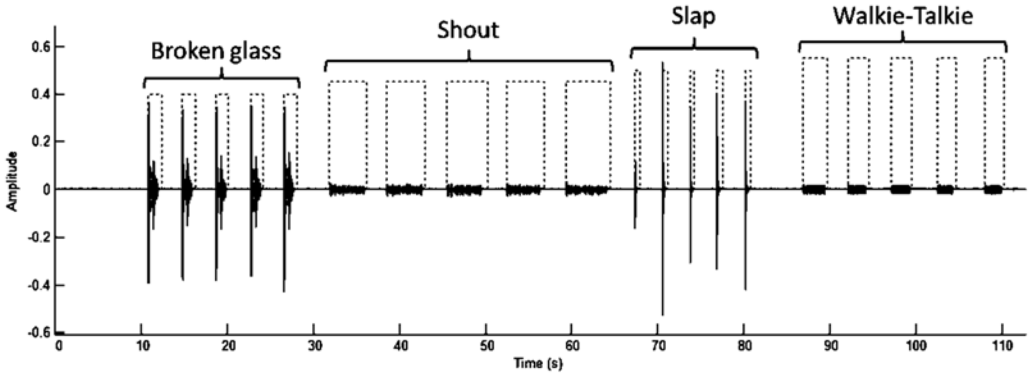

Figure 1. Sequence of real acoustic events generated to verify the different detectors.

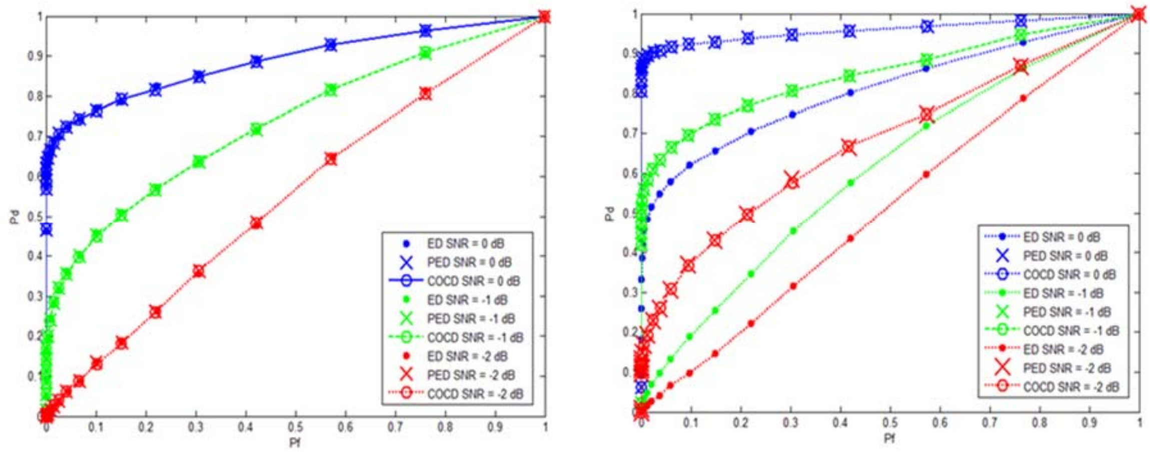

Figure 2. ROC curves: Independent Gaussian noise (left) and non-independent Gaussian noise (right)

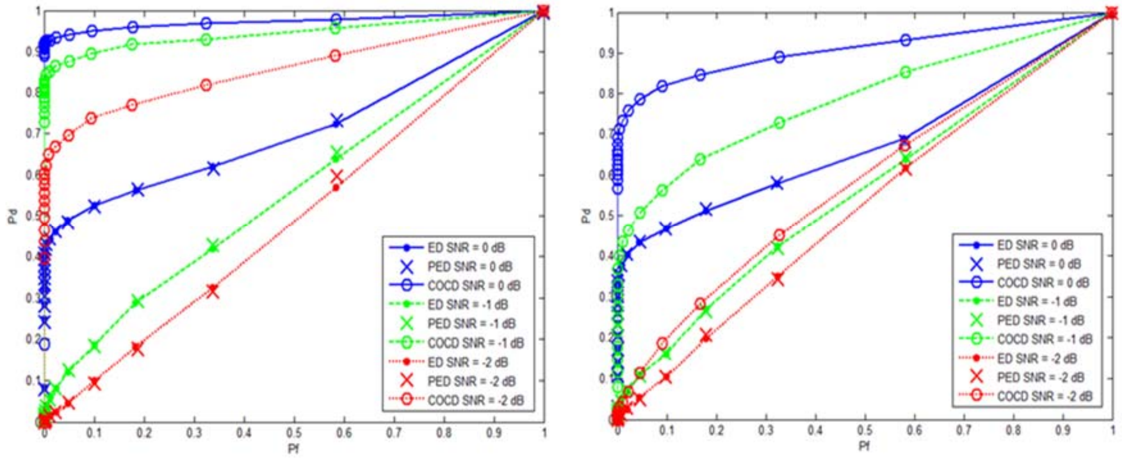

Figure 3. ROC curves: Independent uniform noise (left) and independent Gamma noise (right)

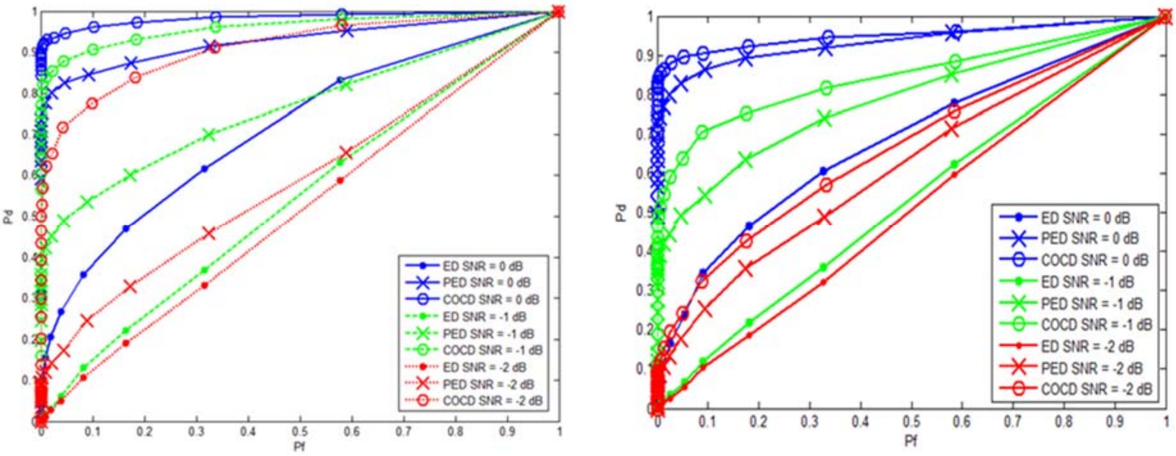

Figure 4. ROC curves. Non-independent uniform noise (left) and non-independent gamma noise (right)