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Additional Information

FACTORIZATION OF STRONGLY (p, σ)-CONTINUOUS MULTILINEAR OPERATORS

D. ACHOUR, E. DAHIA, P. RUEDA AND E. A. SÁNCHEZ PÉREZ *

Abstract. We introduce the new ideal of *strongly (p, σ)-continuous linear operators* in order to study the adjoints of the (p, σ)-absolutely continuous linear operators. Starting from this ideal we build a new multi-ideal by using the composition method. We prove the corresponding Pietsch domination theorem and we present a representation of this multi-ideal by a tensor norm. A factorization theorem characterizing the corresponding multi-ideal —which is also new for the linear case— is given. When applied to the case of the Cohen strongly p-summing operators, this result gives also a new factorization theorem.

1. Introduction

The interpolated operator ideal $\Pi_{p,\sigma}$ of the (p, σ)-absolutely continuous operators —where $1 \leq p < \infty$ and $0 \leq \sigma < 1$— was defined by Matter [18]. Essentially, it is defined to be an intermediate operator ideal between the ideal Π_p of the absolutely p-summing linear operators (see [12, 22]) and the ideal of all continuous operators (see [13]). In the nineties, several papers describing and analyzing the properties and applications of this interpolated class appeared. They were mainly devoted to the study of the factorization properties and the trace duality for these operators, finding in particular the class of tensor norms that represent these operator ideals (see [16, 17, 29]). It must be said that, due to the interpolative procedure that was used for defining them, (p, σ)-absolutely continuous operators preserve some of the characteristic properties of the absolutely p-summing operators. However, the emergence of new classes whenever $0 < \sigma < 1$, different from absolutely p-summing operators, yields that the theory of p-summing operators cannot be applied. Therefore, these new classes are a useful tool to deal with summability properties of operators weaker than absolutely p-summability (see for instance [17, 29, 30]).

On the other hand, the ideal of strongly p-summing operators was introduced by Cohen in [9] to study the space of operators whose adjoint maps are absolutely p^*-summing, with the aim of analyzing the duality properties of this important operator ideal. Motivated by the same objective, the concept of Cohen strongly summing multilinear operator was introduced and studied by Achour and Mezrag in [4] (see also [19, 20]). Actually, in the linear case it is a particular instance of the ideals of (q, ν, p, σ)-dominated operators introduced in [16], for the case $\nu = 1$. In the same line, the present paper is basically a detailed study of the ideal of the *strongly (p, σ)-continuous linear operators* and the composition multi-ideal generated by this ideal. We show the corresponding domination theorem and we prove that the biadjoint of an (p, σ)-absolutely continuous linear operator is also (p, σ)-absolutely continuous. We analyze also the multilinear version of this notion —the multi-ideal obtained by the composition method— and we show that it preserves

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the main properties of the linear case, namely, the Pietsch domination theorem, and the characterization of the adjoint operators. Our main result is a factorization theorem for the multilinear maps of our class, that is also new when $\sigma = 0$, i.e. in the case of the Cohen strongly p-summing multilinear operators that was intensively studied in [4]. Moreover, it must be mentioned that this factorization result is new even for the linear case. Our main motivation is to show that the results that work for the case of the (multi)-ideals of operators involving summability can be extended to the more general setting that is constructed starting from the interpolation method introduced by Jarchow and Matter in [13]. We explore in this way the fundamental aspects of the theory that can be abstracted from the case of p-summing operators.

Our results are presented as follows. After this introductory section, in the second one we recall some properties concerning Banach spaces and definitions regarding operator ideals and multi-ideals, as well as tensor norms. The third section is devoted to study the notion of *strongly* (p, σ)*-continuous linear operator.* We present a characterization given by a summability property and an integral domination. In Section 4 we construct a new multi-ideal by the composition method starting from the ideal of strongly (p, σ)-continuous linear operators. We give an analogue of the Pietsch domination theorem and we characterize the adjoint operators of strongly (p, σ)-continuous m-linear operators. In Section 5 we find the trace duality representation of the strongly (p, σ)-continuous m-linear operators by presenting a tensor norm $g_{p,\sigma}$ on $X_1 \otimes \cdots \otimes X_m \otimes X$ that satisfies that the topological dual of $g_{p,\sigma}$ on $X_1 \otimes \cdots \otimes X_m \otimes Y^*$ is isometric to the space of strongly (p, σ)-continuous m-linear operators from $X_1 \times \cdots \times X_m$ into Y. Finally, in Section 6 we present the factorization theorem for the strongly (p, σ)-continuous m-linear operators (Theorem 6.2). Two particular cases of this result are relevant and new: the linear case, that is given in Theorem 6.4, and Theorem 6.5 which provides the factorization theorem for the Cohen strongly p-summing operators.

2. Preliminaries

We use standard Banach space notation. If X, Y are Banach spaces, we will denote by $\mathcal{B}(X \times Y)$ the Banach space of all continuous bilinear forms on $X \times Y$ under the norm $\|B\| = \sup_{(x,y)\in B_X \times B_Y} |B(x, y)|$, where B_X and B_Y are the closed unit balls of X and Y, respectively. If $1 \le p < \infty$ we write p^* for the extended real number that satisfies $1/p + 1/p^* = 1$, and we denote by $\ell_p^n(X)$ the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\left\|(x_i)_{i=1}^n\right\|_p = (\sum_{i=1}^n \|x_i\|^p)^{\frac{1}{p}},$$

and by $\ell_{p,\omega}^n(X)$ the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\left\|(x_i)_{i=1}^n\right\|_{p,\omega} = \sup_{\|x^*\|_{X^*}\le 1} (\sum_{i=1}^n |\langle x_i, x^*\rangle|^p)^{\frac{1}{p}},$$

where X^* denotes the topological dual of X. We know (see [5, Theorem 2.1]) that $\left(\ell_p^n(X)\right)^* = \ell_{p^*}^n(X^*)$ isometrically i.e.,

$$\left\|(x_i)_{i=1}^n\right\|_p = \sup\left\{\left|\sum_{i=1}^n \langle x_i, x_i^*\rangle\right| : (x_i^*)_{i=1}^n \subset X^*, \left\|(x_i^*)_{i=1}^n\right\|_{p^*} \le 1\right\}. \quad (1)$$

Let $(y_i^*)_{i=1}^n \subset Y^*$. Then it is also known (see [21, Lemma 2.1]) that

$$\left\|(y_i^*)_{i=1}^n\right\|_{p,\omega} = \sup_{\beta\in B_{Y^{**}}} (\sum_{i=1}^n |\beta(y_i^*)|^p)^{\frac{1}{p}} = \sup_{y\in B_Y} \left\|(y_i^*(y))_{i=1}^n\right\|_p. \quad (2)$$

Fix $(x_i)_{i=1}^n$ in X. If $0 \leq \sigma < 1$, we define

$$\delta_{p\sigma}((x_i)_{i=1}^n) = \sup_{x^* \in B_{X^*}} \left(\sum_{i=1}^n \left(|\langle x_i, x^* \rangle|^{1-\sigma} \|x_i\|^\sigma \right)^{\frac{p}{1-\sigma}} \right)^{\frac{1-\sigma}{p}}.$$

The following inequalities are easy to check.

$$\|(x_i)_{i=1}^n\|_{\frac{p}{1-\sigma},\omega} \leq \delta_{p\sigma}((x_i)_{i=1}^n) \leq \|(x_i)_{i=1}^n\|_{\frac{p}{1-\sigma}}. \tag{3}$$

For the extreme cases $\sigma = 1$ and $p = \infty$, we define also for all $0 \leq \tau \leq 1$ and $1 \leq \eta \leq \infty$

$$\delta_{\eta 1}((x_i)_{i=1}^n) = \delta_{\infty\tau}((x_i)_{i=1}^n) = \sup_{1 \leq i \leq n} \|x_i\| = \|(x_i)_{i=1}^n\|_\infty.$$

If μ is a regular Borel probability measure on B_{X^*} (with the weak star topology) and $p = \infty$ or $\sigma = 1$, the expression

$$\left(\int_{B_{X^*}} (|\langle x, x^* \rangle|^{1-\sigma} \|x\|^\sigma)^{\frac{p}{1-\sigma}} d\mu \right)^{\frac{1-\sigma}{p}}$$

must be understood as $\|x\|$.

Let (Π_p, π_p) be the ideal of p-absolutely summing operators for $1 \leq p \leq \infty$. The following definition is due to Matter [18] (see also [13]).

Definition 2.1. *Let $0 \leq \sigma < 1$ and X, Y be Banach spaces. We say that $T \in \mathcal{L}(X,Y)$ is a (p,σ)-absolutely continuous operator if there exist a Banach space G and an operator $S \in \Pi_p(X,G)$ such that*

$$\|Tx\| \leq \|x\|^\sigma \|Sx\|^{1-\sigma}, \quad x \in X. \tag{4}$$

In such case, we put $\pi_{p,\sigma}(T) = \inf \pi_p(S)^{1-\sigma}$, taking the infimum over all Banach spaces G and $S \in \Pi_p(X,G)$ such that (4) holds.

We denote by $(\Pi_{p,\sigma}, \pi_{p,\sigma})$ the Banach ideal of (p,σ)-absolutely continuous linear operators [18]. Clearly, $\Pi_{p,0}$ coincides with the ideal Π_p. We put also $\Pi_{\infty,\sigma} = \Pi_{p,1} = \mathcal{L}$ for $0 \leq \sigma \leq 1$ and $1 \leq p \leq \infty$ (see [28]). The following characterization holds.

Theorem 2.2. [18] *Let $1 \leq p \leq \infty$ and $0 \leq \sigma \leq 1$. For an operator $T \in \mathcal{L}(X,Y)$ the following statements are equivalent.*
(i) $T \in \Pi_{p,\sigma}(X,Y)$.
(ii) There are a constant $C > 0$ and a regular Borel probability measure μ on B_{X^*} (with the weak star topology) such that

$$\|T(x)\| \leq C \left(\int_{B_{X^*}} (|\langle x, x^* \rangle|^{1-\sigma} \|x\|^\sigma)^{\frac{p}{1-\sigma}} d\mu(x^*) \right)^{\frac{1-\sigma}{p}}, \quad x \in X. \tag{5}$$

(iii) There is a constant $C > 0$ such that for every finite sequence $(x_i)_{i=1}^n$ in X,

$$\left\|(Tx_i)_{i=1}^n\right\|_{\frac{p}{1-\sigma}} \leq C.\delta_{p\sigma}((x_i)_{i=1}^n). \tag{6}$$

In addition, $\pi_{p,\sigma}(T)$ is the smallest number C for which (ii) and (iii) holds.

The ideal $\Pi_{p,\sigma}$ is a particular case of the family $\mathcal{D}_{q,\nu,p,\sigma}$ of operator ideals introduced in [16], which generalizes the classical ideal $\mathcal{D}_{q,p}$ of (q,p)-dominated operators [23]. They are defined as follows. Let $1 \leq r, p, q < \infty$ and $0 \leq \sigma, \nu < 1$ such that

$$\frac{1}{r} + \frac{1-\sigma}{p^*} + \frac{1-\nu}{q} = 1.$$

Definition 2.3. *An operator $T \in \mathcal{L}(X,Y)$ is said to be (q,ν,p,σ)-dominated if there exist Banach spaces G,H, operators $R \in \Pi_q(X,G), S \in \Pi_{p^*}(Y^*,H)$ and a constant $C>0$ such that*

$$|\langle Tx, y^*\rangle| \leq C\,\|x\|^{\nu}\,\|Rx\|^{1-\nu}\,\|y^*\|^{\sigma}\,\|Sy^*\|^{1-\sigma} \qquad x\in X, \quad y^*\in Y^*. \tag{7}$$

In such case, we put

$$d_{q,p}^{\nu,\sigma}(T)=\inf\left\{C\pi_q(R)^{1-\nu}\pi_{p^*}(S)^{1-\sigma}\right\},$$

taking the infimum over all $C>0$, $R\in\Pi_q(X,G)$ and $S\in\Pi_{p^*}(Y^*,H)$ such that (7) holds.

We denote by $(\mathcal{D}_{q,\nu,p,\sigma}, d_{q,p}^{\nu,\sigma})$ the Banach ideal of (q,ν,p,σ) -dominated linear operators. The following result is a characterization of this ideal (see [16, Theorem 2.4]).

Theorem 2.4. *Let $1\leq r,p,q\leq\infty$ and $0\leq\sigma,\nu\leq 1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}+\frac{1-\nu}{q}=1$. The following assertions are equivalent.*
(i) $T\in\mathcal{D}_{q,\nu,p,\sigma}(X,Y)$
(ii) There exist a constant $C>0$ and regular probability measures μ and τ on B_{X^*} and $B_{Y^{**}}$, respectively, such that for every $x\in X$ and $y^*\in Y^*$, the following inequality holds

$$|\langle Tx,y^*\rangle|\leq C\left(\int_{B_{X^*}}(|\langle x,x^*\rangle|^{1-\nu}\,\|x\|^{\nu})^{\frac{q}{1-\nu}}d\mu\right)^{\frac{1-\nu}{q}}\left(\int_{B_{Y^{**}}}(|\langle y^*,y^{**}\rangle|^{1-\sigma}\,\|y^*\|^{\sigma})^{\frac{p^*}{1-\sigma}}d\tau\right)^{\frac{1-\sigma}{p^*}}. \tag{8}$$

(iii) There exist a constant $C>0$ such that for every $(x_i)_{i=1}^n\subset X$ and $(y_i^*)_{i=1}^n\subset Y^*$ the following inequality holds

$$\|(\langle Tx_i,y_i^*\rangle)_{i=1}^n\|_{r^*}\leq C\delta_{q\nu}((x_i)_{i=1}^n)\delta_{p^*\sigma}((y_i^*)_{i=1}^n). \tag{9}$$

Moreover, $d_{q,p}^{\nu,\sigma}(T)=\inf C$, where the infimum is taken over all constants C either in (ii) or in (iii).

The family of tensor norms associated to these operator ideals were defined in [16]. They generalize the tensor norms α_{pq} of Lapresté (see [11, p.150]).

Definition 2.5. *Let X,Y be Banach spaces and let $1\leq p,r<\infty, 0\leq\sigma<1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$. The tensor norm $g_{p,\sigma}$ in $X\otimes Y$ is defined by*

$$g_{p,\sigma}(z)=\inf\left\|(x_i)_{i=1}^n\right\|_r\delta_{p^*\sigma}\left((y_i)_{i=1}^n\right)$$

where the infimum is taken over all representations of the simple tensor z of the form $z=\sum\limits_{i=1}^{n}x_i\otimes y_i$ with $x_i\in X, y_i\in Y, i=1,\ldots,n$ and $n\in\mathbb{N}$.

Remark 2.6. The definition of the tensor norms $g_{p,\sigma}$ by means of an infimum over all finite representations of the tensor makes clear that they are finitely generated. This is relevant in order to apply Theorem 2.8 below.

The following proposition can be found in [16].

Proposition 2.7. *Let X,Y be Banach spaces and let $1\leq p,r<\infty, 0\leq\sigma<1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$. An operator $T\in\mathcal{L}(X,Y^*)$ defines a bounded linear functional on $X\widehat{\otimes}_{g_{p,\sigma}}Y$ if and only if T is (p^*,σ)-absolutely continuous. Furthermore, the norm of T in $\left(X\widehat{\otimes}_{g_{p,\sigma}}Y\right)^*$ coincides with $\pi_{p,\sigma}(T)$.*

We recall that every bounded bilinear form on $X\times Y$ has an extension to a bounded bilinear form $B^\sharp$ on $X^{**}\times Y^{**}$ with the same norm. If $B\in\mathcal{B}(X\times Y)$ is defined by the operator $T:X\to Y^*$ by $B(x,y):=\langle y,Tx\rangle$, $x\in X$, $y\in Y$, we may define

$$B^\sharp(x^{**},y^{**}):=\langle T^*y^{**},x^{**}\rangle,\quad x^{**}\in X^{**},\ \ y^{**}\in Y^{**}.$$

Theorem 2.8. [27, Theorem 6.5] *Let α be a (finitely generated) tensor norm, let X, Y be Banach spaces and let $B \in \left(X\widehat{\otimes}_\alpha Y\right)^*$. Then the canonical extension of B is a bounded linear functional on $X^{**}\widehat{\otimes}_\alpha Y^{**}$ with the same norm as B.*

Let us recall now the definition of Cohen of strongly p-summing linear operators (see [9]).

Definition 2.9. *An operator T between two Banach spaces X, Y is strongly p-summing for $1 < p \leq \infty$ if there is a positive constant C such that for all $n \in \mathbb{N}$, $x_1, \ldots, x_n \in X$ and $y_1^*, \ldots, y_n^* \in Y^*$ we have*

$$\|(\langle T(x_i), y_i^*\rangle)_{i=1}^n\|_1 \leq C \,\|(x_i)_{i=1}^n\|_p \,\|(y_i^*)_{i=1}^n\|_{p^*,\omega} \,. \tag{10}$$

The space $\mathcal{D}_p(X,Y)$ of all strongly p-summing linear operators from X into Y which is a Banach space with the norm

$$d_p(T) := \inf\{C > 0 : C \text{ verifying the inequality (10)}\}, \qquad T \in \mathcal{D}_p(X,Y).$$

The multilinear version of the strongly p-summing linear operators has been recently introduced and studied (see [4]). In [19, 20], inclusions between the class of Cohen strongly summing multilinear operators and other classes of operators were systematically analyzed. A related concept and a new generalizations of the concept of Cohen strongly summing multilinear operators have also been recently studied in [8, 7, 2, 3]). For more details concerning the nonlinear theory of summing operators and recent developments and applications we refer to [1, 10].

Let $m \in \mathbb{N}$ and $X_1, \ldots, X_m, Y$ be Banach spaces over $\mathbb{K}$ (real or complex scalars fields). We denote by $\mathcal{L}\left(X_1, \ldots, X_m; Y\right)$ the Banach space of all continuous m-linear mappings from $X_1 \times \cdots \times X_m$ to Y, under the norm $\|T\| = \sup_{x^j \in B_{X_j}} \left\|T(x^1, \ldots, x^m)\right\|$. If $Y = \mathbb{K}$, we write $\mathcal{L}\left(X_1, \ldots, X_m\right)$. In the case $X_1 = \cdots = X_m = X$, we will simply write $\mathcal{L}\left({}^m X; Y\right)$.

By $X_1\widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_m$ we denote the completed projective tensor product of $X_1, \ldots, X_m$. The projective norm is defined by

$$\pi(v) = \inf\left\{\sum_{i=1}^n \prod_{j=1}^m \left\|x_i^j\right\|, n \in \mathbb{N}, v = \sum_{i=1}^n x_i^1 \otimes \cdots \otimes x_i^m\right\}$$

If $X_1 = \cdots = X_m = X$ we write $\widehat{\otimes}_\pi^m X$. For the general theory of tensor products we refer to [11, 27].

Given $T \in \mathcal{L}(X_1, \ldots, X_m; Y)$, consider its linearization $T_L : X_1\widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_m \to Y$, given by $T_L(x^1 \otimes \cdots \otimes x^m) = T\left(x^1, \ldots, x^m\right)$ and extended by linearity, for all $(x^1, \ldots, x^m) \in X_1 \times \cdots \times X_m$. It is well known that $\|T_L\| = \|T\|$ (see [15]).

We denote by $\mathcal{L}_f(X_1, \ldots, X_m; Y)$, the space of all m-linear mappings of finite type, which is generated by the mappings of the special form

$$x_1^* \otimes \cdots \otimes x_m^* \otimes y : \left(x^1, \ldots, x^m\right) \mapsto x_1^*\left(x^1\right) \cdots x_m^*\left(x^m\right) y$$

for some non-zero $x_j^* \in X_j^*$ $(1 \leq j \leq m)$ and $y \in Y$.

The following notion of ideal of multilinear mappings (multi-ideals) goes back to Pietsch [24].

Definition 2.10. *An ideal of multilinear mappings (or multi-ideal) is a subclass $\mathcal{M}$ of all continuous multilinear mappings between Banach spaces such that for all $m \in \mathbb{N}$ and Banach spaces $X_1, \ldots, X_m$ and Y, the components*

$$\mathcal{M}(X_1, \ldots, X_m; Y) := \mathcal{L}(X_1, \ldots, X_m; Y) \cap \mathcal{M}$$

satisfy:

(i) *$\mathcal{M}(X_1,\ldots,X_m;Y)$ is a linear subspace of $\mathcal{L}(X_1,\ldots,X_m;Y)$ which contains the m-linear mappings of finite type.*
(ii) *The ideal property: If $T\in\mathcal{M}(G_1,\ldots,G_m;F)$, $u_j\in\mathcal{L}(X_j;G_j)$ for $j=1,\ldots,m$ and $v\in\mathcal{L}(F;Y)$, then $v\circ T\circ(u_1,\ldots,u_m)$ is in $\mathcal{M}(X_1,\ldots,X_m;Y)$.*
If $\|.\|_{\mathcal{M}}:M\to\mathbb{R}^+$ satisfies
(i') *$(\mathcal{M}(X_1,\ldots,X_m;Y),\|.\|_{\mathcal{M}})$ is a normed (Banach) space for all Banach spaces $X_1,\ldots,X_m$ and Y and all m,*
(ii') $\left\|T^m:\mathbb{K}^m\to\mathbb{K}:T^m\left(x^1,\ldots,x^m\right)=x^1\cdots x^m\right\|_{\mathcal{M}}=1$ *for all m,*
(iii') *If $T\in\mathcal{M}(G_1,\ldots,G_m;F)$, $u_j\in\mathcal{L}(X_j;G_j)$ for $j=1,\ldots,m$ and $v\in\mathcal{L}(F,Y)$, then*
$\|v\circ T\circ(u_1,\ldots,u_m)\|_{\mathcal{M}}\leq\|v\|\,\|T\|_{\mathcal{M}}\,\|u_1\|\cdots\|u_m\|$,
then $(\mathcal{M},\|.\|_{\mathcal{M}})$ is called a normed (Banach) multi-ideal.

In [24], Pietsch introduced a technique to generate ideals of multilinear mappings starting from an operator ideal $\mathcal{I}$. Let us recall a particular case of this procedure that produces what are called composition ideals (see [6]).

Definition 2.11. (Composition Ideals). *Let $\mathcal{I}$ be an operator ideal. An m-linear mapping $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$ belongs to $\mathcal{I}\circ\mathcal{L}$ if there are a Banach space G, an m-linear mapping $R\in\mathcal{L}(X_1,\ldots,X_m;G)$ and an operator $u\in\mathcal{I}(G;Y)$ such that $T=u\circ R$. In this case we write $T\in\mathcal{I}\circ\mathcal{L}(X_1,\ldots,X_m;Y)$.*
If $\mathcal{I}$ is a normed operator ideal and $T\in\mathcal{I}\circ\mathcal{L}(X_1,\ldots,X_m;Y)$ we define

$$\|T\|_{\mathcal{I}\circ\mathcal{L}}:=\inf\left\{\|u\|_{\mathcal{I}}\|R\|:T=u\circ R,R\in\mathcal{L}(X_1,\ldots,X_m;G),u\in\mathcal{I}(G;Y)\right\}.$$

In [6] it is proved that, if $(\mathcal{I},\|.\|_{\mathcal{I}})$ is a Banach operator ideal then $(\mathcal{I}\circ\mathcal{L},\|.\|_{\mathcal{I}\circ\mathcal{L}})$ is a Banach multi-ideal.

Theorem 2.12. [6, Proposition 3.2] *Let $\mathcal{I}$ be an operator ideal. The following are equivalent for $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$.*
(a1) $T\in\mathcal{I}\circ\mathcal{L}(X_1,\ldots,X_m;Y)$.
(a2) $T_L\in\mathcal{I}\left(X_1\widehat{\otimes}_\pi\cdots\widehat{\otimes}_\pi X_m;Y\right)$.

3. Strongly (p,σ)-Continuous Linear Operators

This section is devoted to analyze the linear ideal of strongly (p,σ)-continuous operators. This ideal can be obtained as a particular class of (q,ν,p,σ)-dominated operators, for $\nu=1$ (see Definition 2.3).

Definition 3.1. *Let $1<p,r<\infty$ and $0\leq\sigma<1$, such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$. A mapping $T\in\mathcal{L}(X,Y)$ is strongly (p,σ)-continuous if there are Banach spaces H, an operator $S\in\Pi_{p^*}(Y^*,H)$ and a constant $C>0$ such that for all $x\in X$ and $y^*\in Y^*$ we have*

$$|\langle T(x),y^*\rangle|\leq C\,\|x\|\,\|y^*\|^{\sigma}\,\|S(y^*)\|^{1-\sigma}. \tag{11}$$

The class of all strongly (p,σ)-continuous linear operators from X into Y is denoted by $\mathcal{D}_p^\sigma(X,Y)$ and by $d_p^\sigma(T)$ the strongly (p,σ)-continuous norm which is defined by

$$d_p^\sigma(T)=\inf\left\{C\pi_{p^*}(S)^{1-\sigma}\right\}.$$

where the infimum is taken over all $C>0$ and $S\in\Pi_{p^*}(Y^*,H)$ such that the inequality (11) holds.

We can use Theorem 2.4 in order to obtain the subsequent characterization of strongly (p,σ)-continuous linear operators in terms of a summability property and an integral domination. This is a particular case of the general characterization of (q,ν,p,σ)-dominated operators (see [16]); the equivalence with *(iii)* is new. We write the proof for the aim of completeness.

Theorem 3.2. *Let* $1 < p, r < \infty$ *and* $0 \leq \sigma < 1$, *such that* $\frac{1}{r} + \frac{1-\sigma}{p^*} = 1$. *For* $T \in \mathcal{L}(X, Y)$, *the following statements are equivalent.*
(i) $T \in \mathcal{D}_p^\sigma(X, Y)$.
(ii) There exist a constant $C > 0$ *and regular Borel probability measure* μ *on* $B_{Y^{**}}$, *such that for every* $x \in X$ *and* $y^* \in Y^*$ *the following inequality holds*

$$|\langle T(x), y^* \rangle| \leq C \|x\| \left(\int_{B_{Y^{**}}} (|\langle y^*, \varphi \rangle|^{1-\sigma} \|y^*\|^\sigma)^{\frac{p^*}{1-\sigma}} d\mu \right)^{\frac{1-\sigma}{p^*}}.$$

(iii) There exist a constant $C > 0$ *such that for every* $(x_i)_{i=1}^n \subset X$ *and* $(y_i^*)_{i=1}^n \subset Y^*$ *the following inequality holds*

$$\|(\langle T(x_i), y_i^* \rangle)_{i=1}^n\|_1 \leq C \left\|(x_i)_{i=1}^n\right\|_r \delta_{p^*\sigma}((y_i^*)_{i=1}^n).$$

(iv) There exist a constant $C > 0$ *such that for every* $(x_i)_{i=1}^n \subset X$ *and* $(y_i^*)_{i=1}^n \subset Y^*$ *the following inequality holds*

$$\|(\langle T(x_i), y_i^* \rangle)_{i=1}^n\|_{\frac{p^*}{1-\sigma}} \leq C \|(x_i)_{i=1}^n\|_\infty \delta_{p^*\sigma}((y_i^*)_{i=1}^n).$$

Moreover, $d_p^\sigma(T) = \inf C$, *where the infimum is taken over all constants* C *either in (ii) or (iii) or in (iv).*

Proof. The equivalence (i)$\Longleftrightarrow$(ii) and the implication (iv)$\Longrightarrow$(i) is given by Theorem 2.4 .

(ii)$\Longrightarrow$(iii) Let $(x_i)_{i=1}^n \subset X$ and $(y_i^*)_{i=1}^n \subset Y^*$. An application of Hölder's inequality reveals that

$$\begin{aligned} &\left\|(\langle T(x_i), y_i^* \rangle)_{i=1}^n\right\|_1 \\ \leq\ & C \sum_{i=1}^n \|x_i\| \left(\int_{B_{Y^{**}}} (|\langle y_i^*, \varphi \rangle|^{1-\sigma} \|y_i^*\|^\sigma)^{\frac{p^*}{1-\sigma}} d\mu \right)^{\frac{1-\sigma}{p^*}} \\ \leq\ & C \|(x_i)_{i=1}^n\|_r \left(\sum_{i=1}^n \int_{B_{Y^{**}}} (|\langle y_i^*, \varphi \rangle|^{1-\sigma} \|y_i^*\|^\sigma)^{\frac{p^*}{1-\sigma}} d\mu \right)^{\frac{1-\sigma}{p^*}} \\ =\ & C \|(x_i)_{i=1}^n\|_r \left(\int_{B_{Y^{**}}} \sum_{i=1}^n (|\langle y_i^*, \varphi \rangle|^{1-\sigma} \|y_i^*\|^\sigma)^{\frac{p^*}{1-\sigma}} d\mu \right)^{\frac{1-\sigma}{p^*}} \\ \leq\ & C \|(x_i)_{i=1}^n\|_r \delta_{p^*\sigma}((y_i^*)_{i=1}^n). \end{aligned}$$

(iii)$\Longrightarrow$(iv) For $(x_i)_{i=1}^n \subset X$ and $(y_i^*)_{i=1}^n \subset Y^*$ we have

$$\begin{aligned} \|(\langle T(x_i), y_i^* \rangle)_{i=1}^n\|_{r^*} &\leq \left(\sum_{i=1}^n \|x_i\|^{r^*} \|T^*(y_i^*)\|^{r^*} \right)^{\frac{1}{r^*}} \\ &\leq \left\|(x_i)_{i=1}^n\right\|_\infty \|(T^*(y_i^*))_{i=1}^n\|_{r^*}. \end{aligned}$$

On the other hand, by the equality (1) we have

$$\|(T^*(y_i^*))_{i=1}^n\|_{r^*} = \sup \left\{ \left| \sum_{i=1}^n \langle T^*(y_i^*), z_i \rangle \right| : (z_i)_{i=1}^n \subset X, \left\|(z_i)_{i=1}^n\right\|_r \leq 1 \right\}.$$

Thus, for all $(z_i)_{i=1}^n \subset X$ we obtain

$$\begin{aligned} \left| \sum_{i=1}^n \langle T^*(y_i^*), z_i \rangle \right| &\leq \sum_{i=1}^n |\langle y_i^*, T(z_i) \rangle| \\ &\leq C \left\|(z_i)_{i=1}^n\right\|_r \delta_{p^*\sigma}((y_i^*)_{i=1}^n). \end{aligned}$$

By taking the supremum over the unit ball in $\ell_r^n(X)$ we obtain

$$\|(T^*(y_i^*))_{i=1}^n\|_{r^*} \leq C \delta_{p^*\sigma}((y_i^*)_{i=1}^n).$$

Therefore

$$\|(\langle T(x_i), y_i^*\rangle)_{i=1}^n\|_{r^*} \leq C\,\|(x_i)_{i=1}^n\|_\infty\, \delta_{p^*\sigma}((y_i^*)_{i=1}^n).$$

□

Note that if we take $\sigma = 0$, we obtain $\mathcal{D}_p^0 = \mathcal{D}_p$.

Remark 3.3. According to [16, Definition 2.2] and [30, Definition 2.2.2] we obtain

$$\mathcal{D}_p^\sigma = (\Pi_{p^*,\sigma})^{dual} = \{T \in \mathcal{L}(X,Y) : T^* \in \Pi_{p^*,\sigma}(Y^*,X^*)\} \tag{12}$$

thus $(\mathcal{D}_p^\sigma, d_p^\sigma)$ is a Banach operator ideal (see [23, Section 4.4]).

Corollary 3.4. *Consider $1 < p, q < \infty$ such that $p \leq q$. Then,*

$$\mathcal{D}_q^\sigma(X,Y) \subset \mathcal{D}_p^\sigma(X,Y)$$

Proof. It is immediate by Theorem 3.2(ii) or by $\mathcal{D}_q^\sigma = (\Pi_{q^*,\sigma})^{dual}$ and the inclusion theorem for the class $\Pi_{p,\sigma}$ (see [18, Proposition 3.3]). □

Remark 3.5. *If Y is a reflexive Banach space, then every strongly (p,σ)-continuous linear operator $T : X \to Y$ is compact.* Certainly, since $T^* : Y^* \to X^*$ is (p^*,σ)-absolutely continuous, we can conclude from the factorization theorem for the class $\Pi_{p^*,\sigma}$ [10, Theorem 3.5] and [10, Proposition 5.1] that T^* is compact, since it is completely continuous. Consequently, T is compact.

We can establish the following comparison between the classes of strongly (p,σ)-continuous linear operators and strongly p-summing linear operators.

Proposition 3.6. *Let $p > 1, 0 < \sigma < 1$. Then,*

$$\mathcal{D}_p(X,Y) \subset \mathcal{D}_p^\sigma(X,Y).$$

Moreover we have

$$d_p^\sigma(T) \leq d_p(T) \quad \text{for all} \quad T \in \mathcal{D}_p(X,Y).$$

Proof. Let $T \in \mathcal{D}_p(X,Y)$. Then its adjoint $T^* : Y^* \to X^*$ is (p^*,σ)-absolutely continuous and $\pi_{p^*,\sigma}(T^*) \leq d_p(T)$ (see [9, Theorem 2.2.2] and [18, Proposition 4.2]) and the result is obtained by (12). □

In what follows we prove more general results, also with the aim of proving that strongly p-summing and strongly (p,σ)-continuous linear operators are in fact different classes (Example 3.9). We will show first that in fact the strongly (p,σ)-continuous linear operators are the adjoints of (p,σ)-absolutely continuous linear operators. For the proof of this result we will use the following proposition.

Proposition 3.7. *If $T \in \mathcal{L}(X,Y)$. Then T is (p,σ)-absolutely continuous if and only if its second adjoint, $T^{**} \in \mathcal{L}(X^{**},Y^{**})$, is (p,σ)-absolutely continuous. In this case*

$$\pi_{p,\sigma}(T) = \pi_{p,\sigma}(T^{**}).$$

Proof. The map T^{**} extends T. By the ideal property and by the injectivity of $\Pi_{p,\sigma}$, (see [18, 16]) if $T^{**} : X^{**} \to Y^{**}$ is (p,σ)-absolutely continuous so is T, with $\pi_{p,\sigma}(T) \leq \pi_{p,\sigma}(T^{**})$.

Suppose conversely that $T \in \Pi_{p,\sigma}(X,Y)$. By the ideal property of the (p,σ)-absolutely continuous linear operators, $\Pi_{p,\sigma}(X,Y)$ may be embedded in $\Pi_{p,\sigma}(X,Y^{**})$. Then by Proposition 2.7 we can write

$$\Pi_{p,\sigma}(X,Y) \subset \Pi_{p,\sigma}(X,Y^{**}) = \left(X \widehat{\otimes}_{g_{p^*,\sigma}} Y^*\right)^*.$$

Thus we may consider T as an element of $\left(X\widehat{\otimes}_{g_{p^*,\sigma}}Y^*\right)^*$. By Theorem 2.8 T^{**} is the canonical extension of T to a bounded linear functional on $X^{**}\widehat{\otimes}_{g_{p^*,\sigma}}Y^{***}$, and the result follows from Proposition 2.7. □

Corollary 3.8. *Let $1<p<\infty$ and $0\leq\sigma<1$. Let $T\in\mathcal{L}(X,Y)$ and $T^*\in\mathcal{L}(Y^*,X^*)$ its adjoint. Then T is (p,σ)-absolutely continuous if and only if T^* is strongly (p^*,σ)-continuous.*

We show in the next example that in general $\mathcal{D}_p\neq\mathcal{D}_p^\sigma$.

Example 3.9. *Let $p>1$* and $0<\sigma<1$ *such that $p^*<\frac{p}{1-\sigma}$. Let $T\in\mathcal{L}(\ell_{p^*},\ell_{\frac{p}{1-\sigma}})$ defined by $T(e_i)=(\frac{1}{i})^{\frac{1}{p}}e_i$, where $(e_i)_{i=1}^{\infty}$ is the unit vector canonical basis of ℓ_{p^*}. The adjoint operator of T is strongly (p^*,σ)-continuous but it is not strongly p^*-summing.* In order to see this, note that by [16, Ex. 1.9] we have $T\in\Pi_{p,\sigma}(\ell_{p^*},\ell_{\frac{p}{1-\sigma}})$ and $T\notin\Pi_p\left(\ell_{p^*},\ell_{\frac{p}{1-\sigma}}\right)$. Then by [9, Theorem 2.2.2] and Corollary 3.8 we get the result.

4. The multi-ideal of Strongly (p,σ)-Continuous Multilinear Operators

In this section we extend to multilinear mappings the concept of strongly (p,σ)-continuous linear operators, for which the resulting vector space $\mathcal{D}_p^{m,\sigma}$ of the strongly (p,σ)-continuous multilinear operators is a normed (Banach) multi-ideal. We also show the Pietsch's domination theorem for such operators. We prove that $\mathcal{D}_p^{m,\sigma}$ is generated by the composition method from the operator ideal $\mathcal{D}_p^\sigma$.

Let $m\in\mathbb{N}$ and let $X,X_1,\ldots,X_m,Y$ be Banach spaces. Let $1\leq p,r<\infty$ *and* $0\leq\sigma<1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$.

Definition 4.1. *An m-linear mapping $T:X_1\times\cdots\times X_m\to Y$ is strongly (p,σ)-continuous if there is a constant $C>0$ such that for any $x_1^j,\ldots,x_n^j\in X_j$, $(1\leq j\leq m)$ and any $y_1^*,\ldots,y_n^*\in Y^*$, we have*

$$\left\|\left(\left\langle T\left(x_i^1,\ldots,x_i^m\right),y_i^*\right\rangle\right)_{i=1}^n\right\|_1\leq C\left(\sum_{i=1}^n\prod_{j=1}^m\left\|x_i^j\right\|^r\right)^{\frac{1}{r}}\delta_{p^*\sigma}((y_i^*)_{i=1}^n)\tag{13}$$

for all choices of $n\in\mathbb{N}$.

The collection of all strongly (p,σ)-continuous m-linear maps $X_1\times\cdots\times X_m\to Y$ will be denoted $\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y)$, that is readily seen to be a subspace of $\mathcal{L}(X_1,\ldots,X_m;Y)$.

The least C for which (13) holds will be written $\|.\|_{\mathcal{D}_p^{m,\sigma}}$. This is a norm for the space $\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y)$. It is easy to check that if $T\in\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y)$, then

$$\|T\|\leq\|T\|_{\mathcal{D}_p^{m,\sigma}}.$$

$\mathcal{D}_p^{m,\sigma}$ is a Banach multi-ideal with the norm $\|.\|_{\mathcal{D}_p^{m,\sigma}}$.

For $\sigma=0$, we have $\mathcal{D}_p^{m,0}(X_1,\ldots,X_m;Y)=\mathcal{D}_p^m(X_1,\ldots,X_m;Y)$, the space of Cohen strongly p-summing m-linear operators (see [4]). The next result provides a characterization of this class by means of an inequality.

Proposition 4.2. *Let $1\leq p,r<\infty$ $0\leq\sigma<1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$. The mapping $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$ is strongly (p,σ)-continuous if and only if*

$$\left\|\left(\left\langle T\left(x_i^1,\ldots,x_i^m\right),y_i^*\right\rangle\right)_{i=1}^n\right\|_1\leq C\prod_{j=1}^m\left\|(x_i^j)_{i=1}^n\right\|_{rm}\delta_{p^*\sigma}((y_i^*)_{i=1}^n)\tag{14}$$

whenever $x_1^j, \ldots, x_n^j \in X_j$, $(1 \leq j \leq m)$ *and* $y_1^*, \ldots, y_n^* \in Y^*$.

Proof. Indeed, starting from (14) we obtain (13) by replacing x_i^j by $\frac{\left(\prod_{k=1}^{m} \|x_i^k\|\right)^{\frac{1}{m}}}{\|x_i^j\|} x_i^j$. The reverse is immediate by Hölder's inequality. □

This class satisfies a Pietsch's domination theorem. For the proof we will use the full general Pietsch's domination theorem recently presented by Pellegrino et al in [25].

Theorem 4.3. *An m-linear operator $T \in \mathcal{L}(X_1, \ldots, X_m; Y)$ is strongly (p, σ)-continuous if and only if there is a constant $C > 0$ and a regular Borel probability measure μ on $B_{Y^{**}}$,(with the weak star topology) so that for all $\left(x^1, \ldots, x^m\right) \in X_1 \times \cdots \times X_m$ and for all $y^* \in Y^*$, the inequality*

$$\left|\left\langle T\left(x^1, \ldots, x^m\right), y^*\right\rangle\right| \leq C \prod_{j=1}^{m} \|x^j\| \left(\int_{B_{Y^{**}}} \left(|\varphi(y^*)|^{1-\sigma} \|y^*\|^{\sigma}\right)^{\frac{p^*}{1-\sigma}} d\mu\right)^{\frac{1-\sigma}{p^*}} \tag{15}$$

holds.

Proof. A strongly (p, σ)-continuous m-linear operator T is R_1, R_2-S-abstract $(r, \frac{p^*}{1-\sigma})$-summing (see [25, Definition 4.4]) for the parameters

$$\begin{cases} t = 2 \text{ and } k = m-1 \\ G_1 = X_m \text{ and } G_2 = Y^* \\ E_j = X_j \text{ and } j = 1, \ldots, m-1 \\ K_1 = B_{X_1^* \times \cdots \times X_m^*} \text{ and } K_2 = B_{Y^{**}} \\ \mathcal{H} = \mathcal{L}(X_1, \ldots, X_m; Y) \\ q = 1, q_1 = r \text{ and } q_2 = \frac{p^*}{1-\sigma} \\ S(T, x^1, \ldots, x^m, y^*) = \left|\left\langle T\left(x^1, \ldots, x^m\right), y^*\right\rangle\right| \\ R_1(\varphi, x^1, \ldots, x^m) = \left\|x^1\right\| \cdots \|x^m\| \\ R_2(\varphi, x^1, \ldots, x^{m-1}, y^*) = |\varphi(y^*)|^{1-\sigma} \|y^*\|^{\sigma}, \end{cases}$$

Theorem 4.6 in [25] gives the result. □

An immediate consequence of Theorem 4.3 is the following corollary.

Corollary 4.4. *Consider $1 < p, q < \infty$ and $0 \leq \sigma < 1$ such that $p \leq q$. Then*

$$\mathcal{D}_q^{m,\sigma}(X_1, \ldots, X_m; Y) \subset \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y).$$

Moreover we have $\|T\|_{\mathcal{D}_p^{m,\sigma}} \leq \|T\|_{\mathcal{D}_q^{m,\sigma}}$ *for all* $T \in \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$.

We show in what follows that the multi-ideal generated by the *composition method* from the operator ideal $\mathcal{D}_p^{\sigma}$ coincide with the space of strongly (p, σ)-continuous multilinear operators.

Proposition 4.5. *For $1 < p, r \leq \infty$ and $0 \leq \sigma < 1$ such that $\frac{1}{r} + \frac{1-\sigma}{p^*} = 1$. We have T is strongly (p, σ)-continuous m-linear operator if and only if its linearization T_L is strongly (p, σ)-continuous linear operator. In this case $\|T\|_{\mathcal{D}_p^{m,\sigma}} = d_p^{\sigma}(T_L)$.*

Proof. It is clear that $T_L \in \mathcal{D}_p^{\sigma}\left(X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_m, Y\right)$ implies $T \in \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$ and $\|T\|_{\mathcal{D}_p^{m,\sigma}} \leq d_p^{\sigma}(T_L)$.

Conversely, suppose that T is an strongly (p, σ)-continuous m-linear operator. Let $v = \sum_{i=1}^{n} x_i^1 \otimes \cdots \otimes x_i^m \in X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_m$ such that $v \neq 0$ and $y^* \in Y^*$. Then there is a regular Borel probability measure μ on $B_{Y^{**}}$, (with the weak star topology) such that

$$\begin{aligned}|\langle T_L(v), y^*\rangle| &\leq \sum_{i=1}^{n}\left|\left\langle T\left(x_i^1,\ldots,x_i^m\right), y^*\right\rangle\right| \\ &\leq \sum_{i=1}^{n}\|T\|_{\mathcal{D}_p^{m,\sigma}}\prod_{j=1}^{m}\left\|x_i^j\right\|\left(\int_{B_{Y^{**}}}(|\varphi(y^*)|^{1-\sigma}\|y^*\|^{\sigma})^{\frac{p^*}{1-\sigma}}d\mu\right)^{\frac{1-\sigma}{p^*}} \\ &= \|T\|_{\mathcal{D}_p^{m,\sigma}}\left(\sum_{i=1}^{n}\prod_{j=1}^{m}\left\|x_i^j\right\|\right)\left(\int_{B_{Y^{**}}}(|\varphi(y^*)|^{1-\sigma}\|y^*\|^{\sigma})^{\frac{p^*}{1-\sigma}}d\mu\right)^{\frac{1-\sigma}{p^*}}.\end{aligned}$$

Taking the infimum over all representations of v we get

$$|\langle T_L(v), y^*\rangle| \leq \|T\|_{\mathcal{D}_p^{m,\sigma}}\,\pi(v)\left(\int_{B_{Y^{**}}}(|\varphi(y^*)|^{1-\sigma}\|y^*\|^{\sigma})^{\frac{p^*}{1-\sigma}}d\mu\right)^{\frac{1-\sigma}{p^*}}.$$

Therefore, by Theorem 3.2, T_L is strongly (p,σ)-continuous linear operator and

$$\|T\|_{\mathcal{D}_p^{m,\sigma}} = d_p^{\sigma}(T_L).$$

□

As a consequence, we obtain the following corollary which is a straightforward consequence of the preceding proposition and Theorem 2.12.

Corollary 4.6. *The multi-ideal $\mathcal{D}_p^{m,\sigma}$ is generated by the composition method from the operator ideal $\mathcal{D}_p^{\sigma}$, i.e.,*

$$\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y) = \mathcal{D}_p^{\sigma}\circ\mathcal{L}(X_1,\ldots,X_m;Y)$$

for all Banach spaces $X_1,\ldots,X_m$ and Y.

The preceding corollary has more straightforward consequences.

Remark 4.7. *Every strongly (p,σ)-continuous m-linear operator with a reflexive range is compact.* Indeed, if $T\in\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y)$, then there is a Banach space G, an operator $u\in\mathcal{D}_p^{\sigma}(X;Y)$ and $R\in\mathcal{L}(X_1,\ldots,X_m;X)$ such that $T=u\circ R$. Since every strongly (p,σ)-continuous linear operator with a reflexive range is compact (see Remark 3.5), T is compact.

In [26] the adjoint of an m-linear operator is defined as follows. Let $X_1,\ldots,X_m,Y$ be Banach spaces. If $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$, we define the adjoint of T by

$$T^*: Y^*\to\mathcal{L}(X_1,\ldots,X_m),\ y^*\mapsto T^*(y^*): X_1\times\cdots\times X_m\to\mathbb{K}$$

with

$$T^*(y^*)(x^1,\ldots,x^m) = y^*(T(x^1,\ldots,x^m)).$$

It is easy to see that, if $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$ and $u\in\mathcal{L}(Y,Z)$ we have

$$(u\circ T)^* = T^*\circ u^*$$

A natural question is to study the connection between multilinear operators and their adjoints for different classes of summability. In the next result, we characterize the class of strongly (p,σ)-continuous m-linear operators by using the adjoint operator in a similar way as can be done in the linear case.

Theorem 4.8. *Let $1<p\leq\infty$, $0\leq\sigma<1$, $T\in\mathcal{L}(X_1,\ldots,X_m;Y)$ and T^* its adjoint. Then T is strongly (p,σ)-continuous if and only if T^* is (p^*,σ)-absolutely continuous. In this case,*

$$\|T\|_{\mathcal{D}_p^{m,\sigma}} = \pi_{p^*,\sigma}(T^*).$$

Proof. By (12) and Proposition 4.5 we have that T belongs to $D_p^{m,\sigma}(X_1,\ldots,X_m;Y)$ if and only if its linearization T_L is strongly (p,σ)-continuous and this is equivalent to $(T_L)^*: Y^* \to \big(X_1\widehat{\otimes}_\pi\cdots\widehat{\otimes}_\pi X_m\big)^*$ is (p^*,σ)-absolutely continuous. The result is obtained by the fact that $\big(X_1\widehat{\otimes}_\pi\cdots\widehat{\otimes}_\pi X_m\big)^*$ is isometrically isomorphic to $\mathcal{L}(X_1,\ldots,X_m)$. □

Corollary 4.9. *For a Banach space Y, the following assertions are equivalent.*

(i) $id_Y \in \mathcal{D}_p^\sigma(Y;Y)$.
(ii) $\mathcal{D}_p^{m,\sigma}(X_1,\ldots,X_m;Y) = \mathcal{L}(X_1,\ldots,X_m;Y)$ *for all Banach spaces* $X_1,\ldots,X_m$ *and* Y.
(iii) Y *is finite dimensional.*

Proof. For the equivalence between (i) and (ii) apply Corollary 4.6 and [6, Proposition 3.5]. For the implication (ii) $\Rightarrow$ (iii) we can define the multilinear map $T: Y\times\mathbb{R}\times\cdots\times\mathbb{R}\to Y$ given by $T(y,r_1,\ldots,r_n) = yr_1\cdots r_n$. Using the domination theorem for strongly (p,σ)-continuous multilinear operators, we obtain for all $y^*\in Y^*$,

$$\|y^*\| = \|Id_{Y^*}(y^*)\| \le C\|y^*\|^\sigma \Big(\int_{B_{Y^{**}}} |\langle y^*, y^{**}\rangle|^{p^*}\, d\mu\Big)^{\frac{1-\sigma}{p^*}},$$

and so

$$\|y^*\| \le C^{\frac{1}{1-\sigma}} \Big(\int_{B_{Y^{**}}} |\langle y^*, y^{**}\rangle|^{p^*}\, d\mu\Big)^{1/p^*}.$$

Therefore, by the Dvoretsky-Rogers Theorem Y^* is finite dimensional. To finish the proof let us show (iii) $\Rightarrow$ (i). If Y is finite dimensional, then Y^* is too, and so $(Id_Y)^* = Id_{Y^*} \in \Pi_{p^*,\sigma}$. Therefore, by duality $Id_Y \in \mathcal{D}_p^\sigma(Y;Y)$. □

5. Representation of the Multi-Ideal $\mathcal{D}_p^{m,\sigma}$ by Tensor Norms

After the results of the previous sections, we are ready to introduce a tensor norm which represents the multi-ideal $\mathcal{D}_p^{m,\sigma}$ and to show that the strongly (p,σ)-continuous m-linear operators are a dual space of an $(m+1)$-fold tensor product.

Let $X_1,\ldots,X_m,X$ be Banach spaces. We define in $X_1\otimes\cdots\otimes X_m\otimes X$ the norm

$$g_{p,\sigma}(u) = \inf \left(\sum_{i=1}^n \prod_{j=1}^m \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \delta_{p^*\sigma}((x_i)_{i=1}^n). \tag{16}$$

Where $1<p,r<\infty, 0\le\sigma<1$ such that $\frac{1}{r}+\frac{1-\sigma}{p^*}=1$ and the infimum is taken among all the representations of u as $u=\sum\limits_{i=1}^n x_i^1\otimes\cdots\otimes x_i^m\otimes x_i$, with $(x_i^j)_{i=1}^n\subset X_j, (x_i)_{i=1}^n\subset X, j=1,\ldots,m$ and $n,m\in\mathbb{N}$. Using the representations given in the proof of Proposition 4.2 we obtain a new formula for the norm $g_{p,\sigma}$,

$$g_{p,\sigma}(u) = \inf \prod_{j=1}^m \left\|(x_i^j)_{i=1}^n\right\|_{rm} \delta_{p^*\sigma}((x_i)_{i=1}^n). \tag{17}$$

Proposition 5.1. $g_{p,\sigma}$ *is a tensor norm of order* $m+1$ *(in the sense of Floret-Hunfeld, see* [14]*).*

Proof. Using classical methods it can be shown that $g_{p,\sigma}$ is a norm on $X_1\otimes\cdots\otimes X_m\otimes X$ with the metric mapping property (see the proof of Proposition 4.1 and 4.2 in [10])

Let $\phi_j \in B_{X_j^*}, \psi \in B_{X^*} (j = 1, \ldots, m)$ and let $u = \sum_{i=1}^{n} x_i^1 \otimes \cdots \otimes x_i^m \otimes x_i \in X_1 \otimes \cdots \otimes X_m \otimes X$. It follows directly from Hölder's inequality and (3) we get

$$\begin{aligned} \left|\sum_{i=1}^{n} \phi_1(x_i^1) \cdots \phi_m(x_i^m)\psi(x_i)\right| &\leq \left(\sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \|(x_i)_{i=1}^n\|_{\frac{p^*}{1-\sigma},\omega} \\ &\leq \left(\sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \delta_{p^*\sigma}((x_i)_{i=1}^n). \end{aligned}$$

Then, if ϵ is the injective norm, we have

$$\epsilon(u) = \sup_{\phi_j \in B_{X_j^*}, \psi \in B_{X^*}} \left|\sum_{i=1}^{n} \phi_1(x_i^1) \cdots \phi_m(x_i^m)\psi(x_i)\right| \leq \left(\sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \delta_{p^*\sigma}((x_i)_{i=1}^n).$$

Since is holds for every representation of u, consequently $\epsilon(u) \leq g_{p,\sigma}(u)$.
On the other hand we have

$$\begin{aligned} g_{p,\sigma}(u) &\leq \left(\sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \delta_{p^*\sigma}((x_i)_{i=1}^n) \\ &\leq \left(\sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\|^r\right)^{\frac{1}{r}} \left(\sum_{i=1}^{n} \|x_i\|^{\frac{p^*}{1-\sigma}}\right)^{\frac{1-\sigma}{p^*}}. \end{aligned}$$

By replacing in the representation of u x_i^j by $\frac{\left(\prod_{k=1}^{m} \|x_i^k\| \|x_i\|\right)^{\frac{1}{rm}}}{\|x_i^j\|} x_i^j$ and x_i by $\frac{\left(\prod_{k=1}^{m} \|x_i^k\| \|x_i\|\right)^{\frac{1-\sigma}{p^*}}}{\|x_i\|} x_i$, a simple calculation gives

$$g_{p,\sigma}(u) \leq \sum_{i=1}^{n}\prod_{j=1}^{m} \left\|x_i^j\right\| \|x_i\|.$$

Taking the infimum over all representations of u, we find $g_{p,\sigma}(u) \leq \pi(u)$. □

As in the basic two-spaces tensor product case, it is clear by the definition that these tensor norms are finitely generated. The main result of this section is the following

Theorem 5.2. *Let $X_1, \ldots, X_m, Y$ be Banach spaces and let $1 < p, r < \infty, 0 \leq \sigma < 1$ such that $\frac{1}{r} + \frac{1-\sigma}{p^*} = 1$. The space $(\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y), \|.\|_{\mathcal{D}_p^{m,\sigma}})$ is isometrically isomorphic to $(X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*$.*

Proof. It is easy to see that the correspondence

$$\Psi : (\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y), \|.\|_{\mathcal{D}_p^{m,\sigma}}) \to (X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*$$

defined by

$$\Psi(T)(x^1 \otimes \cdots \otimes x^m \otimes y^*) = \left\langle T(x^1, \ldots, x^m), y^* \right\rangle,$$

for every $T \in (\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$, $x^j \in X_j$ $(j = 1, \ldots, m)$ and $y^* \in Y^*$, is linear. It is clearly also injective. It remains to show the surjectivity and that

$$\|\Psi(T)\|_{(X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*} = \|T\|_{\mathcal{D}_p^{m,\sigma}}.$$

Take $T \in (\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$, and let

$$u = \sum_{i=1}^{n} x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^* \in X_1 \otimes \cdots \otimes X_m \otimes Y^*,$$

where $(x_i^j)_{i=1}^n \subset X_j (j = 1, \ldots, m), (y_i^*)_{i=1}^n \subset Y^*$. We have

$$\begin{aligned} |\Psi(T)(u)| &= \left| \sum_{i=1}^n \left\langle T(x_i^1, \ldots, x_i^m), y_i^* \right\rangle \right| \\ &\leq \|T\|_{\mathcal{D}_p^{m,\sigma}} \left(\sum_{i=1}^n \prod_{j=1}^m \left\| x_i^j \right\|^r \right)^{\frac{1}{r}} \delta_{p^*\sigma}((y_i^*)_{i=1}^n). \end{aligned}$$

Since it holds for every representation of u we get $|\Psi(T)(u)| \leq \|T\|_{\mathcal{D}_p^{m,\sigma}} g_{p,\sigma}(u)$. It follows that

$$\|\Psi(T)\|_{(X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*} \leq \|T\|_{\mathcal{D}_p^{m,\sigma}}.$$

In order to establish the reverse inequality, let $\phi \in (X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*$ define the m-linear mapping $T \in \mathcal{L}(X_1, \ldots, X_m; Y)$ by

$$\left\langle T(x^1, \ldots, x^m), y^* \right\rangle = \phi(x^1 \otimes \cdots \otimes x^m \otimes y^*).$$

Let $(x_i^j)_{i=1}^n \subset X_j (j = 1, \ldots, m)$ and $(y_i^*)_{i=1}^n \subset Y^*$, for $\lambda_1, \ldots, \lambda_n \geq 0$ we can write

$$\begin{aligned} & \left| \sum_{i=1}^n \lambda_i \left\langle T(x_i^1, \ldots, x_i^m), y_i^* \right\rangle \right| \\ = & \left| \sum_{i=1_i}^n \lambda_i \phi\left(x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^*\right) \right| \\ \leq & \|\phi\| \, g_{p,\sigma} \left(\sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^* \right) \\ \leq & \|\phi\| \, \|(\lambda_i)_{i=1}^n\|_\infty \left(\sum_{i=1}^n \prod_{j=1}^m \left\| x_i^j \right\|^r \right)^{\frac{1}{r}} \delta_{p^*\sigma} \left((y_i^*)_{i=1}^n \right). \end{aligned}$$

By taking the supremum over all $\lambda_1, \ldots, \lambda_n \geq 0$ with $\|(\lambda_i)_{i=1}^n\|_\infty \leq 1$ and using the equality (1) we get that $T \in \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$ and

$$\|T\|_{\mathcal{D}_p^{m,\sigma}} \leq \|\phi\| = \|\Psi(T)\|_{(X_1 \otimes \cdots \otimes X_m \otimes Y^*, g_{p,\sigma})^*}.$$

□

6. The Factorization Theorem

Let $X_1, \ldots, X_m, Y$ be Banach spaces, $1 \leq p, r < \infty$, $0 \leq \sigma < 1$ such that $\frac{1}{r} + \frac{1-\sigma}{p^*} = 1$ and a regular Borel probability measure η on $B_{Y^{**}}$, (with the weak star topology). We denote by e the isometric embedding $Y^* \to C(B_{Y^{**}})$ given by $e(y^*) = \langle y^*, \cdot \rangle$. For $f \in e(Y^*)$ consider the seminorm $\|f\|_{p,\sigma} = \inf \sum_{k=1}^n \|f_k\|_{e(Y^*)}^\sigma \left(\int_{B_{Y^{**}}} |f_k|^p \, d\eta \right)^{\frac{1-\sigma}{p}}$, the infimum computed over all decompositions of f as $f = \sum_{k=1}^n f_k$ in $e(Y^*)$. Following [10, Section 3.2], let $L_{p,\sigma}(\eta)$ be the completion of the quotient normed space $e(B_{Y^*})/\| \cdot \|_{p,\sigma}^{-1}(0)$ of all classes of functions as $\langle y^*, \cdot \rangle \in e(B_{Y^*}) \subset C(B_{Y^{**}})$, $y^* \in Y^*$, with the quotient norm $\|\cdot\|_{p,\sigma}$. Let us call $J_{p,\sigma} : e(Y^*) \to L_{p,\sigma}(\eta)$ the projection on the quotient. Although in [10, Section 3.2] the factorization is done through a subspace $X_{p,\sigma}$ of $L_{p,\sigma}(\eta)$, a quick look to the proof shows that in fact $X_{p,\sigma}$ coincides with the whole space $L_{p,\sigma}(\eta)$. For the sake of clarity, notice that with this notation in the case $\sigma = 0$ the space $L_{p,0}(\eta)$ do not coincide with $L_p(\eta)$ but with the subspace of $L_p(\eta)$ that allows the factorization theorem for p-summing operators.

Theorem 6.1. *(See Theorem 3.5 in* [10]*). For every operator $T : X \to Y$, the following statements are equivalent.*

(i) *T is (p,σ)-absolutely continuous.*

(ii) *There exist a regular Borel probability measure η on B_{X^*} (with the weak star topology) and an operator $\widetilde{T} \in \mathcal{L}(L_{p,\sigma}(\eta), Y)$ such that the following diagram commutes*

$$\begin{array}{ccc} X & \xrightarrow{\quad T \quad} & Y \\ {\scriptstyle e}\downarrow & & \uparrow{\scriptstyle \widetilde{T}} \\ e(X) & \xrightarrow{\quad J_{p,\sigma} \quad} & L_{p,\sigma}(\eta) \end{array}$$

We define the m-linear mapping $K : X_1 \times \cdots \times X_m \to \mathcal{L}(X_1, \dots, X_m)^*$ by

$$K(x^1, \dots, x^m)(\phi) := \phi(x^1, \dots, x^m) \text{ for all } \phi \in \mathcal{L}(X_1, \dots, X_m).$$

It is easy to see that K is continuous and $\|K\| = 1$. On the other hand, if $T \in \mathcal{L}(X_1, \dots, X_m; Y)$ and $k_Y : Y \hookrightarrow Y^{**}$ is the natural embedding. Then the following diagram commutes

$$\begin{array}{ccc} X_1 \times \cdots \times X_m & \xrightarrow{\quad K \quad} & \mathcal{L}(X_1, \dots, X_m)^* \\ {\scriptstyle T}\downarrow & & \downarrow{\scriptstyle T^{**}} \\ Y & \xrightarrow{\quad k_Y \quad} & Y^{**} \end{array}$$

i.e., $k_Y \circ T = T^{**} \circ K$.

Using Theorem 4.8 and [10, Theorem 3.5], we present the following factorization theorem concerning the class of strongly (p,σ)-continuous multilinear operators.

Theorem 6.2. *For every multilinear operator $T : X_1 \times \cdots \times X_m \to Y$, the following statements are equivalent.*

(i) T is strongly (p,σ)-continuous.

(ii) There exist a regular Borel probability measure μ on $B_{Y^{**}}$ and a continuous m-linear mapping $u_* : X_1 \times \cdots \times X_m \to (L_{p^*,\sigma}(\mu))^*$ such that

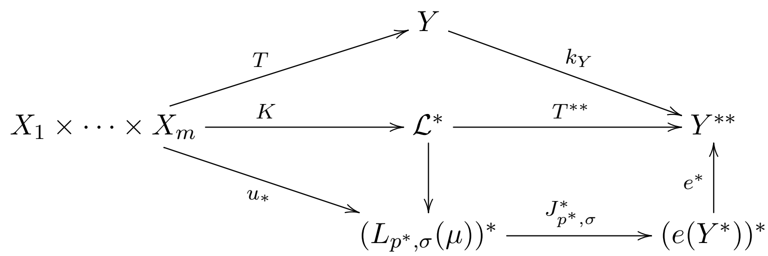
$$k_Y \circ T = e^* \circ J^*_{p^*,\sigma} \circ u_*.$$

Proof. To simplify the notation, let us write $\mathcal{L}^*$ instead of $\mathcal{L}(X_1, \dots, X_m)^*$.

(i) $\Rightarrow$ *(ii)* Assume that T is strongly (p,σ)-continuous. By Theorem 4.8 we have that $T^* : Y^* \to \mathcal{L}(X_1, \dots, X_m)$ is (p^*,σ)-absolutely continuous with $\|T\|_{\mathcal{D}_p^{m,\sigma}} = \pi_{p^*,\sigma}(T^*)$. By Theorem 6.1, there exist a regular Borel probability measure μ on $B_{Y^{**}}$ and a bounded linear operator u such that the following diagram commutes,

$$\begin{array}{ccc} Y^* & \xrightarrow{\quad T^* \quad} & \mathcal{L}(X_1, \dots, X_m) \\ {\scriptstyle e}\downarrow & & \uparrow{\scriptstyle u} \\ e(Y^*) & \xrightarrow{\quad J_{p^*,\sigma} \quad} & L_{p^*,\sigma}(\mu) \\ \big\downarrow & & \\ C(B_{Y^{**}}) & & \end{array}$$

since $L_{p^*,\sigma}(\mu)$ is the closure of $(J_{p^*,\sigma} \circ e)(Y^*)$. By transposing the diagram above, we obtain the following diagram, which commutes



where u_* is the m-linear mapping defined by

$$\begin{aligned} \left\langle u_*(x^1,\ldots,x^m), J_{p^*,\sigma}\circ e(y^*)\right\rangle &:= \left\langle u^*\circ K(x^1,\ldots,x^m), J_{p^*,\sigma}\circ e(y^*)\right\rangle \\ &= u\left(J_{p^*,\sigma}\circ e(y^*)\right)(x^1,\ldots,x^m) \end{aligned}$$

for all $y^* \in Y^*, x^j \in X_j (j = 1, \ldots, m)$. It is clear that u_* is well-defined. On the other hand we have

$$\begin{aligned} \left|\left\langle u_*(x^1,\ldots,x^m), J_{p^*,\sigma}\circ e(y^*)\right\rangle\right| &\leq \left\|u(J_{p^*,\sigma}\circ e(y^*))\right\|_{L_{p^*,\sigma}(\mu)} \prod_{j=1}^m \left\|x^j\right\| \\ &\leq \|u\| \left\|J_{p^*,\sigma}\circ e(y^*)\right\|_{L_{p^*,\sigma}(\mu)} \prod_{j=1}^m \left\|x^j\right\|. \end{aligned}$$

We take the supremum over all $J_{p^*,\sigma} \circ e(y^*)$ with $\|J_{p^*,\sigma} \circ e(y^*)\|_{L_{p,\sigma}(\mu)} \leq 1$ in order to obtain

$$\left\|u_*(x^1,\ldots,x^m)\right\| \leq \prod_{j=1}^m \|u\| \left\|x^j\right\|.$$

Therefore, u_* is continuous with norm $\leq \|u\|$.

$(ii) \Rightarrow (i)$ Assume that $k_Y \circ T = e^* \circ J^*_{p^*,\sigma} \circ u_*$. The natural inclusion/quotient map $J_{p^*,\sigma}$ is (p^*, σ)-absolutely continuous [10, Lemma 3.4]. Then $J^*_{p^*,\sigma}$ is strongly (p, σ)-continuous by Corollary 3.8. Consequently, the m-linear mapping is $J^*_{p,\sigma} \circ u_*$ is strongly (p, σ)-continuous by Corollary 4.6 and so $k_Y \circ T \in \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y^{**})$ by the ideal property.

It remains to show that the mapping T is in $\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$. By Theorem 4.8 we have

$$(k_Y \circ T)^* = (e^* \circ J^*_{p^*,\sigma} \circ u_*)^* = T^* \circ k_Y^* \in \mathcal{P}_{p^*,\sigma}(Y^{***}, \mathcal{L}(X_1, \ldots, X_m)).$$

The fact that $k_Y^* \circ k_{Y^*} = id_{Y^*}$ implies

$$T^* = (e^* \circ J^*_{p^*,\sigma} \circ u_*)^* \circ k_{Y^*} \in \mathcal{P}_{p^*,\sigma}(Y^*, \mathcal{L}(X_1, \ldots, X_m)).$$

Thus $T \in \mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$. □

A direct consequence of the previous theorem is the following

Corollary 6.3. *A multilinear operator $T : X_1 \times \cdots \times X_m \to Y$ belongs to $\mathcal{D}_p^{m,\sigma}(X_1, \ldots, X_m; Y)$ if and only if $T^{**} \in D_p^\sigma\big(\mathcal{L}(X_1, \ldots, X_m)^*, Y^{**}\big)$.*

For the case $m = 1$ we obtain the factorization theorem for the linear case, which as we said in the Introduction is also new. Let us write it separately.

Theorem 6.4. *For every linear operator $T : X \to Y$, the following statements are equivalent.*
(i) T is strongly (p, σ)-continuous.

(ii) There exist a regular Borel probability measure μ on $B_{Y^{**}}$ and a continuous linear mapping $u_* : X \to (L_{p^*,\sigma}(\mu))^*$ such that

$$k_Y \circ T = e^* \circ J^*_{p^*,\sigma} \circ u_*.$$

Let us finish the paper by writing the factorization theorem for Cohen strongly p-summing multilinear operators. The linear case (i.e. for $m = 1$) is also new. Putting $\sigma = 0$ in Theorem 6.2, we obtain the following

Theorem 6.5. *For every multilinear operator $T : X_1 \times \cdots \times X_m \to Y$, the following assertions are equivalent.*
(i) T is Cohen strongly p-summing.
(ii) There exist a regular Borel probability measure μ on $B_{Y^{**}}$, a subspace S of the Lebesgue space $L_{p^*}(\mu)$ and a continuous m-linear mapping $u_* : X_1 \times \cdots \times X_m \to S^*$ such that

$$k_Y \circ T = e^* \circ J^*_{p^*} \circ u_*.$$

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