Highly Efficient Heterogeneous Modal Superposition Method for the Full-Wave Analysis of Arbitrarily Shaped H Plane Structures Fed through Rectangular Waveguides

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Abstract

A new fully modal characterisation procedure is proposed to reduce the number of unknowns needed to characterise the ports of an arbitrary H plane device in rectangular waveguide, so that no equivalent currents will be involved in the characterisation of the ports. Moreover, since no currents are needed to characterise the ports, the weights of the scattered modes can be unknowns of the resulting system of equations. This is an important fact, since, to find the wanted modal weights, a subsequent projection step is not necessary anymore. The new method is then advantageous when compared to other hybrid alternatives in the literature, not only because the ports are solved using fewer unknowns, which is the main advantage of the proposal, but also because the scattering parameters can be computed directly, once the system of equations is posed.

I. Introduction

The use of hybrid techniques is not a new idea. In certain situations, for example when the problem is big compared to the wavelength of the excitation, it is possible to apply approximate expressions of Maxwell's equations. These approximate expressions are easier to solve; this is the case of the geometrical theory of diffraction or the unified theory of diffraction. The first attempts of combining several solving techniques include one of these asymptotic formulations and another more rigorous numerical technique [1], [2]. Recently, some works still treat this topic because it is necessary to employ these asymptotic approximations of Maxwell's equations when the problem is really large [3]–[5].

One of the most common techniques employed in developing hybrid formulations is mode-matching, which is a very fast but not a general approach. A very popular choice to combine with mode-matching is then the method of moments, which is very general. For example, in [6], a hybrid mode-matching and method of moments technique was recently published. This technique can be applied to solve arbitrarily-shaped problems fed through one or more canonical waveguides, i.e. a waveguide whose modes are known analytically. In [6], the ports of the problem are characterised by means of a single current formulation; in other formulations [7] the ports are solved using both electric and magnetic currents. Thanks to this simplification, the weights of the scattered modes become unknowns of the resulting system of equations; this fact is exploited to achieve certain efficiency improvement.

In this paper a fully modal characterisation of the ports is proposed, in opposition to the traditional two currents based approach of [7] or the single current based approach of [6]. This fully modal characterisation of the ports produces smaller systems of equations, so an additional efficiency improvement is achieved.

II. Problem formulation

In this section an arbitrarily shaped H plane closed cavity fed through several canonical waveguides (see Fig. 1) is solved using a fully modal characterisation of the ports.

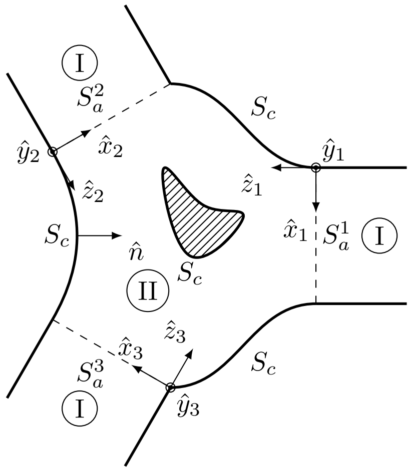


Fig. 1. Base problem.

Obviously, the total reflected field towards medium I can be expressed in terms of the regressive modes of the rectangular waveguides that feed the ports

$$\vec{E}^{T,I}_{(x_i,y_i,z_i)} = \sum_{n=1}^{N_i} b_{ni} \vec{e}_{ni}^{\,T,I,-}(x_i,y_i) e^{\jmath\beta^I_{ni} z_i} \tag{1}$$

$$\vec{H}^{T,I}_{(x_i,y_i,z_i)} = \sum_{n=1}^{N_i} b_{ni} \vec{h}_{ni}^{T,I,-}(x_i,y_i) e^{\jmath\beta^I_{ni} z_i} \tag{2}$$

and the same is valid for the transversal fields, that is

$$\vec{E}^{I}_{(x_i,y_i,z_i)} = \overline{\overline{T}}_i \vec{E}^{T,I}_{(x_i,y_i,z_i)} = \sum_{n=1}^{N_i} b_{ni} \vec{e}_{ni}^{\,I}(x_i,y_i) e^{\jmath\beta^I_{ni} z_i} \tag{3}$$

$$\vec{H}^{I}_{(x_i,y_i,z_i)} = \overline{\overline{T}}_i \vec{H}^{T,I}_{(x_i,y_i,z_i)} = -\sum_{n=1}^{N_i} b_{ni} \vec{h}_{ni}^{I}(x_i,y_i) e^{\jmath\beta^I_{ni} z_i} \tag{4}$$

where

- N_i is the number of modes considered for the i-th port.
- b_{ni} is the weight associated to the n-th mode emergent through the i-th port.
- $\vec{e}/\vec{h}_{ni}^{T,I,-}(x_i,y_i)$ are the regressive auto-vectors for the modes belonging to the waveguide of the i-th port for the total electric/magnetic field.
- $\overline{\overline{T}}_i$ is the projection matrix over the transversal coordinates of the i-th port.

- $\vec{e}/\vec{h}^{I}_{ni}(x_i, y_i)$ are the auto-vectors for the modes belonging to the waveguide of the i-th port for the transversal electric/magnetic field.

If the modes of the cavity were known, the same could be done for the fields of medium II. The cavity that defines the medium II, however, is arbitrarily shaped, so the modes are not known. Fortunately, it is not necessary to know the modes of the cavity to modally characterise the ports.

In fact, if superposition is applied, it can be concluded that several modal sets can be simultaneously used to characterise the problem. Every modal set, of course, must accomplish by itself the Maxwell's equations inside the cavity.

To see that this is possible the cavity contour has to be broken into pieces and a different modal set has to be independently assigned to each piece. Every chosen modal set has to be valid at every point inside the cavity; it has also to be able to force the accomplishment of arbitrary boundary conditions along its own contour piece.

When all of these modal sets are defined, arbitrary boundary conditions can be forced along the whole contour of the cavity. If the uniqueness theorem [8] is applied, it means that any solution of the problem could be synthesised using a proper linear superposition of these modal sets.

Clearly, the most difficult part of this procedure is finding an appropriate division of the cavity contour and modal sets associated to each considered contour piece.

A good starting point could be to treat separately each port, i.e., to break the cavity contour in port and non-port parts, S_a and S_c, respectively (see Fig. 1). Then, S_a can be broken again, and every piece of the contour belonging to each port can be separately treated. In other words, S_a can be broken into S_a^1, $\ldots$ S_a^A, where A is the number of ports feeding the cavity (see again Fig. 1).

The ports are always plane, so the most appropriate modal set for these ports is a plane wave spectrum. The plane wave spectrum, which emanates from a plane, is physically valid in only one of the half-spaces that this plane defines [9]. This means that the position of a given port has to be chosen so that the entire cavity belongs to the half-space where the plane spectrum associated to this port remains physically valid; fortunately, this is always possible.

In order to construct a fully valid plane wave spectrum for the i-th port, with coordinates x_i, y_i and z_i, two different sets of plane waves are necessary, a TM^y plane wave set plus a TE^y plane wave set [9]. Fortunately, thanks to the symmetry of a rectangular waveguide H plane problem, in this case, only a TM^y plane wave will be necessary.

The TM^y plane wave has the following expression,

$$\begin{aligned} \vec{e}^{\,T,II,TM} &= \left(a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) \\ &\left(a_y^{(TM+)} e^{-j\beta_y y_i} + a_y^{(TM-)} e^{j\beta_y y_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{y}_i \end{aligned} \tag{5}$$

$$\begin{aligned} \vec{h}^{T,II,TM} &= \frac{-\beta_z}{k\eta} \left(a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) \\ &\left(a_y^{(TM+)} e^{-j\beta_y y_i} + a_y^{(TM-)} e^{j\beta_y y_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{x}_i \\ &+ \frac{-\beta_x}{k\eta} \left(-a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) \\ &\left(a_y^{(TM+)} e^{-j\beta_y y_i} + a_y^{(TM-)} e^{j\beta_y y_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{z}_i \end{aligned} \tag{6}$$

where

- $a_{x,y,z}^{(TM\pm)}$:are arbitrary weights.
- $\beta_x^2 + \beta_y^2 + \beta_z^2 = k^2$
- These modes have physical sense in the cavity since the waves are attenuated when z_i grows, once β_x and β_y are fixed, and a propagating solution is not obtained (β_z pure imaginary).

In addition, if the excitation is constant along y, and the only non-zero component of its electric field is E_y, the scattered field generated by the arbitrarily shaped cavity of Fig. 1 will show the same properties than the excitation. This happens, for example, when the cavity is excited by the fundamental mode of the feeding rectangular waveguides. In this case, β_y must be equal to zero, and the appropriate plane wave spectrum to characterise the problem is then

$$\vec{e}^{\;T,II,TM} = \left(a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{y}_i \tag{7}$$

$$\begin{aligned}\vec{h}^{T,II,TM} = & \frac{-\beta_z}{k\eta} \left(a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{x}_i \\ & + \frac{-\beta_x}{k\eta} \left(-a_x^{(TM+)} e^{-j\beta_x x_i} + a_x^{(TM-)} e^{j\beta_x x_i}\right) a_z^{(TM+)} e^{-j\beta_z z_i} \hat{z}_i\end{aligned} \tag{8}$$

These plane waves can synthesise arbitrary tangential fields at every port so that a given boundary condition can be forced. The spectrum derived from the plane waves of equations (5) and (6) is continuous, however a discrete spectrum is needed. A discrete spectrum is obtained when the field over the whole plane containing the port is periodic. Fortunately, it is necessary to synthesise only an arbitrary field in the region of the plane belonging to the port, while the fields outside the port surface are not important, so a discrete spectrum can be used to force an arbitrary contour condition at the port.

To obtain the expressions for the modes, let us analyse a simple case: synthesising an arbitrary E_{y_i} field using the TM^y wave of (5). Since the port is the region of interest, and a periodic solution is possible, a Fourier series in x_i can be used to synthesise the field. Moreover, since the port is of size a in x_i, the most attractive possibility is to choose this size, a, as the base period of the Fourier series.

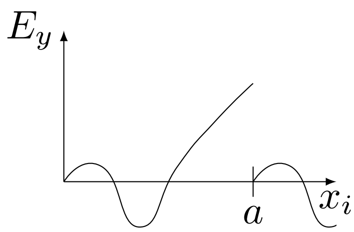


Fig. 2. i-th port initial period.

A Fourier series with $\beta_x^{(m)} = 2m\pi/a$ will allow us to synthesise an arbitrary function over the port surface. This Fourier expansion has a high risk, however, of showing an undesirable Gibbs' effect; this is because it is not guaranteed that the synthesised periodic function is continuous.

To ensure continuity, rather than considering the period of Fig. 2, a bigger period, specifically, a period of size $2a$, will be used (see Fig. 3).

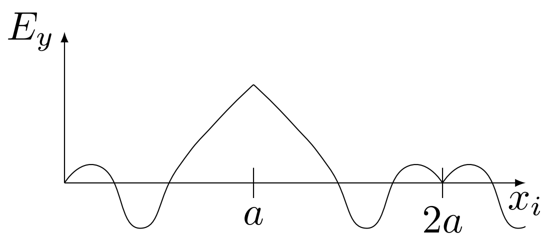


Fig. 3. i-th port extended rectangular period.

In order to synthesise such an extended period a Fourier series with $\beta_x^{(m)} = m\pi/a$ ($m = -\infty, \ldots, -1, 0, 1, \ldots \infty$) has to be used. The shape of the period outside the wanted region of Fig. 3, i.e. the interval between zero and a is not important; for instance, a period showing even symmetry (see Fig. 3) can be therefore chosen. This even symmetry ensures a continuous periodic field over the plane which contains the port and the Gibbs' effect will be almost non-existent. If the Fourier weights for such a period are computed, then $a_{x,m}^{(TM+)} = a_{x,m}^{(TM-)}$, where $a_{x,m}^{(TM+)}$ and $a_{x,m}^{(TM-)}$ are the weights of (7) and (8) when $\beta_x^{(m)} = m\pi/a$. The equality between the positive exponential and negative exponential weights notably simplifies the discrete TM^y plane wave modes (see (7) and (8))

$$\begin{aligned} \vec{e}_{mi}^{\,T,II} &= a_m^{(TM)} \cos\left(\frac{m\pi}{a} x_i\right) e^{-j\beta_z z_i} \hat{y}_i \\ \vec{h}_{mi}^{T,II} &= a_m^{(TM)} \frac{-\beta_z}{k\eta} \cos\left(\frac{m\pi}{a} x_i\right) e^{-j\beta_z z_i} \hat{x}_i \\ &\quad + a_m^{(TM)} \frac{m\pi}{jk\eta a} \sin\left(\frac{m\pi}{a} x_i\right) e^{-j\beta_z z_i} \hat{z}_i \end{aligned} \tag{9}$$

where all the constants of (7) and (8) have been grouped into the single plane wave mode weight, $a_m^{(TM)}$. Finally, since the weight has to be considered separately in the following expressions, the modes can be written definitely as

$$\begin{aligned}
\vec{e}_{mi}^{\,T,II} &= \cos\left(\frac{m\pi}{a}x_i\right)e^{-j\beta_z z_i}\hat{y}_i \\
\vec{h}_{mi}^{T,II} &= \frac{-\beta_z}{k\eta}\cos\left(\frac{m\pi}{a}x_i\right)e^{-j\beta_z z_i}\hat{x}_i \\
&\quad + \frac{m\pi}{jk\eta a}\sin\left(\frac{m\pi}{a}x_i\right)e^{-j\beta_z z_i}\hat{z}_i
\end{aligned} \tag{10}$$

Using the modes of (10) an arbitrary tangential electric or magnetic field can be synthesised at the port; it is known from uniqueness [8] that this will be enough to successfully accomplish an arbitrary boundary condition at the port.

To be more specific, the field at every point of medium II, (x, y, z), generated by these plane wave modes associated to the ports of the cavity can be calculated as

$$\begin{aligned}
\vec{E}_{(x,y,z)}^{T,II} &= \sum_{m=1}^{M} d_m \vec{e}_m^{\,T,II}(x, y, z) \\
\vec{H}_{(x,y,z)}^{T,II} &= \sum_{m=1}^{M} d_m \vec{h}_m^{T,II}(x, y, z)
\end{aligned} \tag{11}$$

where

- M is the total number of modes considered at all ports.
- $\vec{e}/\vec{h}_m^{T,II}$ is the electric/magnetic part of the m-th TM plane wave mode.
- d_m is the weight associated to the m-th mode.

Until now, the ports, S_a, have been characterised, but the conducting part of the cavity, S_c, have not yet been solved. Unfortunately, the shape of this part of the contour it is not known a priori (it was known that every S_a^i was plane). Therefore, another strategy has to be used to force the boundary condition over S_c, for example, an unknown surface current, $\vec{J}_c$, over S_c. Finally, in order to complete the discretisation of the equations, the current, $\vec{J}_c$, has to be expanded as sum of certain basis functions; that is, the well-known method of moments [10] is going to be applied.

$$\vec{J}_c = \sum_{q=1}^{Q} I_q \vec{J}_q \tag{12}$$

To pose the problem, the corresponding boundary conditions over the cavity limits have to be enforced, that is, continuity of electric and magnetic field at the ports and null tangential electric field over S_c

$$\begin{aligned} &\vec{E}^{I}_{inc,S^i_a} + \sum_{n=1}^{N_i} b_{ni} \vec{e}_{ni}^{\,I}(S^i_a) = \\ &\qquad \overline{\overline{T}}_i \sum_{m=1}^{M} d_m \vec{e}_m^{\,T,II}(S^i_a) + \sum_{q=1}^{Q} I_q \vec{E}^{II}_{S^i_a}(\vec{J}_q) \\ &\qquad i = 1, 2, \ldots A \end{aligned} \tag{13}$$

$$\begin{aligned} &\vec{H}^{I}_{inc,S^i_a} - \sum_{n=1}^{N_i} b_{ni} \vec{h}^{I}_{ni}(S^i_a) = \\ &\qquad \overline{\overline{T}}_i \sum_{m=1}^{M} d_m \vec{h}^{T,II}_m(S^i_a) + \sum_{q=1}^{Q} I_q \vec{H}^{II}_{S^i_a}(\vec{J}_q) \\ &\qquad i = 1, 2, \ldots A \end{aligned} \tag{14}$$

$$\hat{n}_{S_c} \times \left\{ \sum_{m=1}^{M} d_m \vec{e}_m^{\,T,II}(S_c) + \sum_{q=1}^{Q} I_q \vec{E}^{T,II}_{S_c}(\vec{J}_q) \right\} = 0 \tag{15}$$

where $\vec{E}^{T,II}_S(\vec{J})/\vec{H}^{T,II}_S(\vec{J})$ is the total electric/magnetic field over S generated by an electric current $\vec{J}$ in the medium II and $\vec{E}^{II}_S(\vec{J})/\vec{H}^{II}_S(\vec{J})$ is the electric/magnetic field tangential to S generated by an electric current $\vec{J}$ in the medium II.

There are, fortunately, some shapes for the conducting contour that can be solved modally. For instance, if a certain piece of the conducting contour was plane, and if the whole cavity belonged to the physically valid half-space of an hypothetically plane wave spectrum over such plane part of the contour, the same modes presented before for every S^i_a could be used to characterise this conducting and plane part of the contour. This situation is not as uncommon as one would think. There are many structures with such plane conducting surfaces limiting the cavity, as can be seen in the results. Of course, these special parts of the contour have been used to improve the efficiency of the technique presented in this paper.

III. Testing the equations

To determine the modal coefficients and the weights for the currents, equations (13)-(15) must be discretised. They will be projected over a well chosen set of test functions distributed along $S_0 \equiv S_a \cup S_c$.

Considering that the electric current, $\vec{J}$, has been expanded as a weighted sum of Q different basis functions, equation (15) should be tested using also Q test functions, $\vec{\omega}_r$, $r = 1, 2, \ldots R$, $R = Q$, placed along S_c.

The rest of test functions will be placed over S_a. For the i-th port N_i modes have been used to expand the field in the medium I and M_i modes to expand the field in the medium II. Equation (13) will be therefore projected over a certain set of test functions $\vec{u}_{si}$, $s = 1, 2, \ldots S_i$ and equation (14) over a different set of test functions $\vec{v}_{ti}$, $t = 1, 2, \ldots T_i$, where $S_i = T_i = N_i = M_i$. The same process will be repeated for every port of the problem.

To simplify the notation, it is more convenient to reorder the port dependent parameters, b_{ni}, $\vec{e}^{\,I}_{ni}$, $\vec{h}^{I}_{ni}$, $\vec{u}_{si}$ and $\vec{v}_{ti}$ in terms of a single index. Then the index i, which refers to the port number in all the port-dependent parameters, can be eliminated. For example, if this reordering is applied to the coefficients of the emergent modes, b_{ni}, the column vector $\overline{b}$ of $N = \sum_{i=1}^{A} N_i$ elements can be defined.

$$\overline{b}^T = [b_{11}, \cdots, b_{N_1 1}, \cdots, b_{1A}, \cdots, b_{N_A A}] \tag{16}$$

By applying the reordering to the other port dependent variables, the column vectors $\overline{e}^I$ and $\overline{h}^I$ of N elements and the vectors $\overline{u}$ of $S = \sum_{i=1}^{A} S_i$ elements, and $\overline{v}$ of $T = \sum_{i=1}^{A} T_i$ elements can be defined ($N = S = T$).

Finally, when the equations of (13)-(15) are projected over these sets of test functions, a system of equations is obtained. The construction of this matrix system is detailed in the appendix. The solution of this system provides b_n, d_m and I_q.

$$\begin{pmatrix} \overline{\overline{Z}}_{11} & \overline{\overline{Z}}_{12} & \overline{\overline{Z}}_{13} \\ \overline{\overline{X}}_{21} & \overline{\overline{X}}_{22} & \overline{\overline{X}}_{23} \\ \overline{\overline{Z}}_{31}=0 & \overline{\overline{Z}}_{32} & \overline{\overline{Z}}_{33} \end{pmatrix} \begin{pmatrix} \overline{b} \\ \overline{d} \\ \overline{I}^c \end{pmatrix} = \begin{pmatrix} \overline{E}_i \\ \overline{H}_i \\ 0 \end{pmatrix} \tag{17}$$

IV. Scattering parameters computation

In (17), the first set of unknowns are the weights b_{ni} for the scattered modes through the ports towards medium I. Then, in order to compute the scattering parameters, only a port, for example, port j, has to be excited with a single mode, for example the m-th mode, with unitary amplitude ($a_{mj} = 1$),

$$[\vec{E}/\vec{H}]^I_{inc,S_a} = \begin{cases} [\vec{e}/\vec{h}]_{mk}(S^k_a) & \text{if } k = j \\ 0 & \text{rest} \end{cases} \tag{18}$$

then $s^{ij}_{nm} = b_{ni}$

A. *Direct solution*

That can be done if the following matrices are defined,

$$\overline{\overline{Z}}_{ac} = -\overline{\overline{Z}}_{33}^{-1}\overline{\overline{Z}}_{32} \tag{19}$$

$$\begin{aligned} \overline{\overline{Z}}_G &= \overline{\overline{Z}}_{11}^{-1}(\overline{\overline{Z}}_{12} + \overline{\overline{Z}}_{13}\overline{\overline{Z}}_{ac}) \\ \overline{\overline{X}}_G &= \overline{\overline{X}}_{21}^{-1}(\overline{\overline{X}}_{22} + \overline{\overline{X}}_{23}\overline{\overline{Z}}_{ac}) \end{aligned} \tag{20}$$

and it is taken into account that $\overline{E}_i = -\overline{\overline{Z}}_{11}\overline{a}$ and $\overline{H}_i = \overline{\overline{X}}_{21}\overline{a}$, where $\overline{a}$ is a vector which contains the weights for the incident modes. Then the generalised scattering matrix is

$$\overline{\overline{S}} = -\left\{\overline{\overline{I}} + 2\overline{\overline{Z}}_G\left[\overline{\overline{X}}_G - \overline{\overline{Z}}_G\right]^{-1}\right\} \tag{21}$$

V. Comparison against the reference

To compare the technique of [6] and the technique of this paper, the same test functions, $\vec{u}_s$, $\vec{v}_t$ and $\vec{\omega}_r$ have to be used; the same basis functions, $\vec{J}_q$, have to be used to expand the current over the conducting limits of the cavity in both methods, as well; so that $Q_1 = Q_2$ in Tab. I.

TABLE I
COST COMPARISON.

	[6]	New method
Num. of elem.	$4N_1^2 + 3Q_1N_1 + Q_1^2$	$4N_2^2 + 3Q_2N_2 + Q_2^2$
Cost of prod.	$Q_1^2N_1 + 2Q_1N_1^2 + 3N_1^3$	$Q_2^2N_2 + 2Q_2N_2^2 + 3N_2^3$
Cost of inv.	$O\left(Q_1^3 + \left(1 + \frac{2}{A^2}\right) N_1^3\right)$	$O\left(Q_2^3 + \left(1 + \frac{2}{A^2}\right) N_2^3\right)$

Due to the fact that the characterisation of the ports is very different in both methods, $N_1 \neq N_2$; the comparison requires a more exhaustive analysis.

If simple basis functions, such as pulses, are used to discretise the currents of [6], more unknowns will be needed to adequately characterise the ports than using plane wave modes, that is $N_2 < N_1$. The cost of filling the matrices and computing the matrix products and inversions will be notably smaller for the method of this paper (see Tab. I).

If full domain basis functions are used to characterise the ports, for example the $\cos()$ part of (10), N_1 can be reduced, and forced to be almost equal to N_2. In this case, the cost of the matrix products and inversions is the same for the techniques compared here. However, the cost of computing the matrix elements related to the ports, $\overline{\overline{Z}}_{12}$, $\overline{\overline{X}}_{22}$ and $\overline{\overline{Z}}_{32}$, will be nearly prohibitive in the technique of [6]. This is because full domain basis functions have been used and that will imply the solution of large and difficult (singular at many evaluation points) numerical quadrature rules to compute the port-related matrix elements. In comparison, in the technique presented in this paper, a single evaluation of a function will be enough to compute the same elements. In short, in this case, the technique of [6] will be also slower and, furthermore, very difficult to implement.

VI. RESULTS

At this point several simple problems will be solved using the technique proposed in the previous sections. Specifically, a 90° bend, several beveled bends, a 135° bend, a charged T-junction and two filters of coupled cavities will be analysed (all H-plane problems inside a rectangular waveguide); the solutions will be compared with some results found in the literature [6], [11].

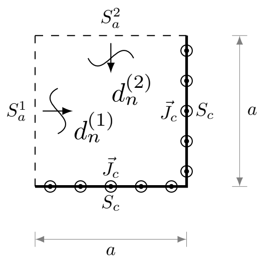
As test and basis functions, a combination of basis and test functions that facilitate the computation of the coefficients matrix elements is chosen.

The basis functions will be $\vec{J}_q = P_q \hat{y}$, where P_q are rectangular pulses uniformly distributed along S_c.

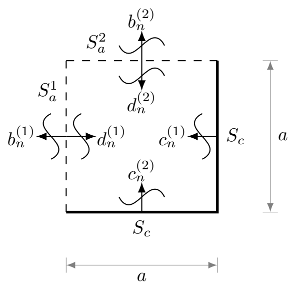
The test functions, $\vec{\omega}_r$, will be $\vec{\omega}_r = W_r \hat{y}$, with W_r equal to Dirac's delta functions uniformly distributed along S_c. The test functions, $\vec{u}_{si}$ and $\vec{v}_{ti}$, will be $\vec{u}_{si} = U_{si}\hat{y}$ and $\vec{v}_{ti} = V_{si}(\hat{n} \times \hat{y})$, with U_{si} and V_{ti} also Dirac's delta functions uniformly distributed along S_a, so that the method has been reduced to a typical point-matching.

The rectangular waveguide modes and the solution of a two-dimensional scattering problem using the method of moments (point-matching) are well documented in the literature [10], [12], [13], so they will not be reproduced here.

As it was mentioned before, to increase the efficiency of the technique, the straight conducting limits of the cavity will be modally characterised, as the ports are. This implies, for example, that for the 90° bend, the discretisation of Fig. 4(a) will be replaced by the discretisation of Fig. 4(b). Both alternatives are valid, but the modal one (Fig. 4(b)) is more efficient.



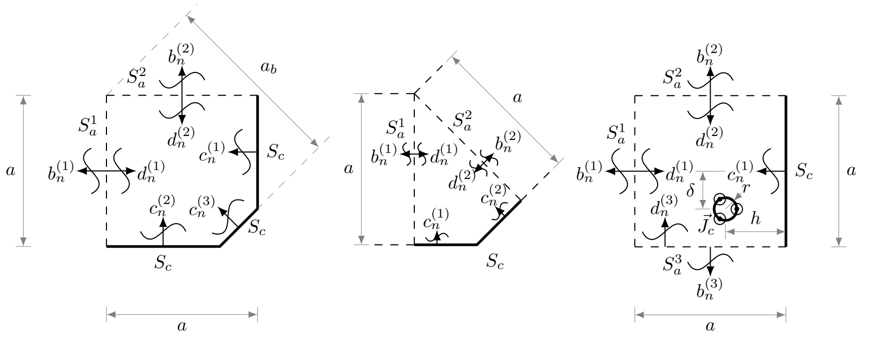
(a) With currents



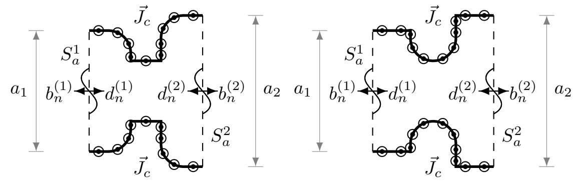
(b) With modes

Fig. 4. Discretisation for the 90° bend when the straight conducting segments are and are not modally characterised.

To find the solution of the analysed problems, a basis functions density equal to 100 pulses per wavelength has been used; in ports $N_i = M_i = 20$ modes have proven to be enough to provide accurate results. Or in other words, the scattering parameters of $N_i = 20$ different modes will be found at each port after the method is applied.



(a) Beveled bend (b) 135° bend (c) Charged T-junction



(d) Coupling window 1 (e) Coupling window 2

Fig. 5. Discretisation proposed for the devices analysed in this paper.

For the beveled bends of Fig. 5(a) and the 135° bend of Fig. 5(b), the waveguides will have a width equal to $a = 15.799$ mm. Three beveled bends will be analysed, with $a_b = a/\sqrt{2}$, $a_b = 0.98a$ and $a_b = a\sqrt{2}$ (the 90° bend of Fig. 4(b)). On the other hand, for the charged T-junction of Fig. 5(c), the width of the waveguides will be $a = 22.86$ mm; the values for δ and h will be, $\delta = 0.1$ mm and $h = 8.8$ mm. Finally, the coupling windows of Figs. 5(d) and 5(e) will be used to analyse two filters of coupled cavities. To improve the efficiency of the solution, every coupling window will be analysed separately. Then, all of the coupling windows will be linked using only 11 of the 20 modes (for efficiency) calculated at each port and the generalised scattering matrix of the whole filter will be computed using the procedure of [14]. In this particular case, the scattering matrices computed for each coupling window must be unavoidably multimodal, since the cascade cannot be accurately done using monomodal scattering matrices. First, a filter with rounded ($r_c = 3$ mm) corners in the coupled cavities (Fig. 5(d)) will be considered . Second, a filter with rounded corners ($r_w = 2$ mm) in the coupling windows (Fig. 5(e)) will be analysed. The dimensions of the filters are tabulated in Tabs. II and

III.

TABLE II

DIMENSIONS FOR THE FILTER WITH ROUNDED CORNERS IN THE CAVITIES ($r_c = 3$ MM).

$l_1 = l_7 = 9.284$ mm	$L_1 = L_6 = 9.77$ mm	$a_{in} = 19.05$ mm
$l_2 = l_6 = 6.344$ mm	$L_2 = L_5 = 11.118$ mm	$a_{out} = 19.05$ mm
$l_3 = l_5 = 5.814$ mm	$L_3 = L_4 = 11.273$ mm	$a = 22$mm
$l_4 = 5.738$ mm		

TABLE III

DIMENSIONS FOR THE FILTER WITH ROUNDED CORNERS IN THE COUPLING WINDOWS ($r_w = 2$ MM).

$l_1 = l_7 = 8.768$ mm	$L_1 = L_6 = 9.159$ mm	$a_{in} = 19.05$ mm
$l_2 = l_6 = 5.837$ mm	$L_2 = L_5 = 10.581$ mm	$a_{out} = 19.05$ mm
$l_3 = l_5 = 4.311$ mm	$L_3 = L_4 = 10.752$ mm	$a = 22$ mm
$l_4 = 5.235$ mm		

The meaning of the parameters shown in these tables can be seen in Fig. 6

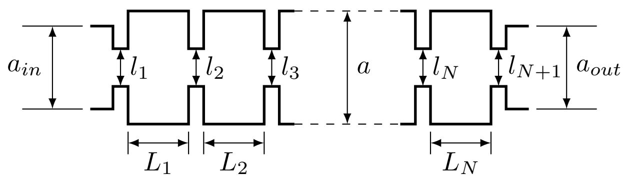
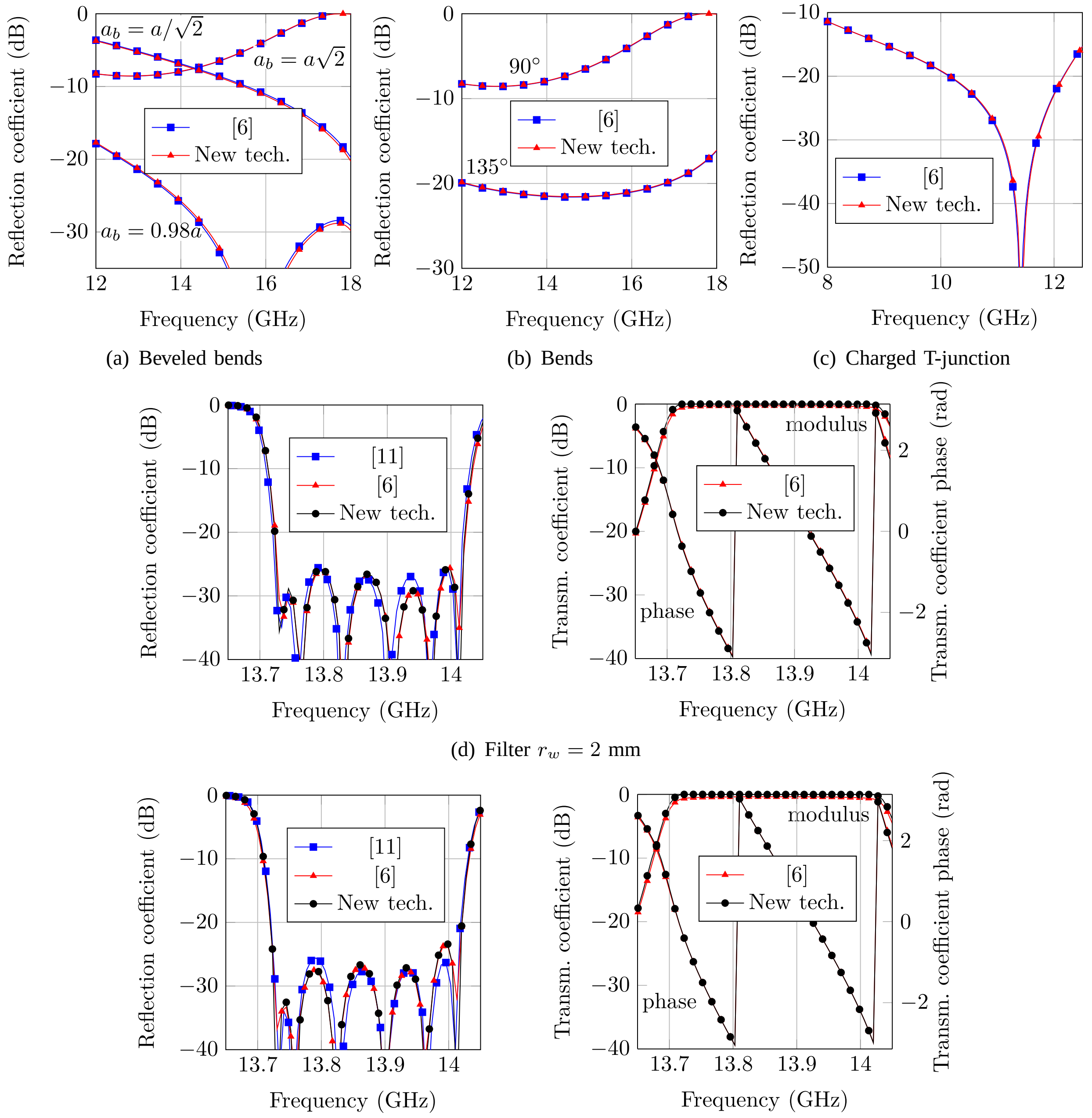


Fig. 6. Filter of coupled cavities

In Fig. 7, one can see the comparison between the results obtained after applying the technique developed in the present paper and several results taken from the literature. A good agreement for both responses can be observed.

In Tab. IV, a summary of solving temporal costs is shown. There, one can see that the cost of solving these problems is actually smaller than the cost of applying the technique of [6], as it was advanced in section V.

Not only the generalised scattering matrix of the different structures has been computed, but also the field scattered in response to the incidence against the port 1 of the fundamental mode.



(a) Beveled bends (b) Bends (c) Charged T-junction

(d) Filter $r_w = 2$ mm

(e) Filter $r_c = 3$ mm

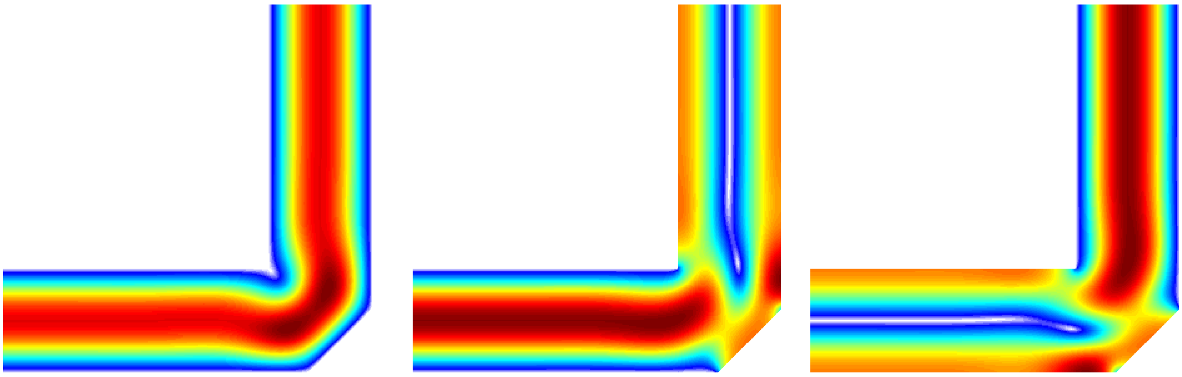
Fig. 7. Comparison of the results with the references.

The graphical representation of the fields for the beveled bend with $a_b = 0.98a$ at $f = 15.8$ GHz can be seen in Fig. 8.

TABLE IV

SUMMARY OF TEMPORAL COSTS AND NUMBER OF UNKNOWNS CONSIDERED FOR EVERY DEVICE USING AN INTEL CORE2DUO (3.00 GHZ) PROCESSOR WITH 4 GB OF RAM.

	[6]		New method	
	Unknowns	S parameters sec/freq. point	Unknowns	S parameters sec/freq. point
90° bend	294	0.06	120	0.005
135° bend	262	0.04	96	0.004
Bev. bend ($a_b = a/\sqrt{2}$)	238	0.03	108	0.004
Bev. bend ($a_b = 0.98a$)	260	0.04	113	0.004
Filter ($r_w = 2$ mm)	2896	0.64	2004	0.26
Filter ($r_c = 3$ mm)	2756	0.56	2198	0.34



(a) E_{y_1} (b) H_{x_1} (c) H_{z_1}

Fig. 8. Field inside a beveled bend with $a = 15.799$ mm and $a_b = 0.98a$, when the incidence of fundamental mode against the port 1 is considered for a frequency equal to $f = 15.8$ GHz. a) Transversal E, b) Transversal H and c) Longitudinal H.

VII. CONCLUSION

A new formulation for a hybrid mode-matching and method of moments technique which modally characterises every port of an arbitrarily shaped cavity (see Fig. 1) has been developed.

When the ports are characterised using a planar modal expansion, the number of unknowns in most situations is smaller than in [6]. If more elaborate basis functions are used, the number of unknowns can be comparable, but at a high cost; this is because it is necessary to evaluate an important number of numerical integrals in order to fill the matrices while, in the technique presented in this paper, this numerical integration does not appear. This can be seen in section

VI, where the efficiency of the new method is compared with the method of [6]. From this comparison one can conclude that, for a general device, the new method performs, at least, twice faster, even ten times faster for particular geometries.

Finally, it is necessary to remark that the technique presented in this paper has been particularised to analyse arbitrary H plane problems in rectangular waveguides, but this technique can be used to solve full three-dimensional problems after applying the appropriate generalisations. The work presented in this paper for the two-dimensional case can be a good starting point to extend the technique to three-dimensional problems. The solution and the analysis of the results of several three-dimensional devices will show if this formulation is able to provide efficiency improvements comparable to those obtained for the two-dimensional devices analysed in this paper.

VIII. Acknowledgment

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Appendix

To complete the calculation of the system of equations resulting from the discretisation of (13)-(15), the projection operator which will allow to discretise the equations needs to be defined

$$< \vec{\kappa}, \vec{v}(x,y,z) >= \iint_S \left[\vec{\kappa} \cdot \vec{v}(x,y,z)\right] dS \tag{22}$$

where $\vec{\kappa} \cdot \vec{v}$ is the scalar product between the vectors $\vec{\kappa}$ and $\vec{v}$, S can be S_a or S_c; $\vec{\kappa}$ is any of the vectorial test functions that has been defined in section III.

Next,the expression for the resulting system of equations after applying the method of moments to (13)-(15) is presented. In order to do that, first the unknowns are organised in a column vector $\overline{x}$

$$\overline{x}^T = (b_1, b_2, \cdots, b_N, d_1, d_2, \cdots, d_M, I_1, I_2, \cdots, I_Q) \tag{23}$$

the excitation is stored in another column vector, $\overline{c}^{inc}$ of size $B\times 1$, $B = N+M+Q = S+T+R$

$$\overline{c}^{inc} = \begin{pmatrix} < \vec{u}_1, \vec{E}^I_{inc,S_a} > \\ < \vec{u}_2, \vec{E}^I_{inc,S_a} > \\ \vdots \\ < \vec{u}_S, \vec{E}^I_{inc,S_a} > \\ < \vec{v}_1, \vec{H}^I_{inc,S_a} > \\ < \vec{v}_2, \vec{H}^I_{inc,S_a} > \\ \vdots \\ < \vec{v}_T, \vec{H}^I_{inc,S_a} > \\ \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} R \end{pmatrix} \tag{24}$$

and finally, a $B \times B$ matrix, $\overline{\overline{Z}}$, is defined, whose elements can be grouped in the following sub-matrices

$$\overline{\overline{Z}} = \begin{pmatrix} \overline{\overline{Z}}_{11(S\times N)} & \overline{\overline{Z}}_{12(S\times M)} & \overline{\overline{Z}}_{13(S\times Q)} \\ \overline{\overline{X}}_{21(T\times N)} & \overline{\overline{X}}_{22(T\times M)} & \overline{\overline{X}}_{23(T\times Q)} \\ \overline{\overline{Z}}_{31(R\times N)} & \overline{\overline{Z}}_{32(R\times M)} & \overline{\overline{Z}}_{33(R\times Q)} \end{pmatrix} \tag{25}$$

- $\overline{\overline{Z}}_{11(S\times N)}$ also can be divided into blocks:

$$\begin{pmatrix} \overline{\overline{E}}_{11(S_1\times N_1)} & \cdots & \overline{\overline{E}}_{1A(S_1\times N_A)} \\ \vdots & \ddots & \vdots \\ \overline{\overline{E}}_{A1(S_A\times N_1)} & \cdots & \overline{\overline{E}}_{AA(S_A\times N_A)} \end{pmatrix} \tag{26}$$

  and only the diagonal blocks will be non-zero. The elements of a diagonal block, $\overline{\overline{E}}_{ii}$, can be computed

$$e^{(ii)}_{sn} = - < \vec{u}_{si}, \vec{e}^{\,I}_{ni}(S^i_a) > \tag{27}$$

  with $s = 1, 2, \ldots S_i$, $n = 1, 2, \ldots N_i$.

- $\overline{\overline{Z}}_{12(S\times M)}$

$$z_{sm}^{(12)} =< \vec{u}_s, \overline{\overline{T}}_i \vec{e}_m^{\,T,II}(S_a^i) > \tag{28}$$

$s = 1,2,\ldots S$, $m = 1,2,\ldots M$ and i is the port which s belongs to.

- $\overline{\overline{Z}}_{13(S\times Q)}$

$$z_{sq}^{(13)} =< \vec{u}_s, \vec{E}_{S_a^i}^{II}(\vec{J}_q) > \tag{29}$$

$s = 1,2,\ldots S$ and $q = 1,2,\ldots Q$.

- $\overline{\overline{X}}_{21(T\times N)}$ whose elements can also be grouped

$$\begin{pmatrix} \overline{\overline{H}}_{11(T_1\times N_1)} & \cdots & \overline{\overline{H}}_{1A(T_1\times N_A)} \\ \vdots & \ddots & \vdots \\ \overline{\overline{H}}_{A1(T_A\times N_1)} & \cdots & \overline{\overline{H}}_{AA(T_A\times N_A)} \end{pmatrix} \tag{30}$$

and a block-diagonal matrix is obtained. The elements of a block in the main diagonal, $\overline{\overline{H}}_{ii}$, can be computed

$$h_{tn}^{(ii)} =< \vec{v}_{ti}, \vec{h}_{ni}^{I}(S_a^i) > \tag{31}$$

with $t = 1,2,\ldots T_i$, $n = 1,2,\ldots N_i$.

- $\overline{\overline{X}}_{22(T\times M)}$

$$x_{tm}^{(22)} =< \vec{v}_t, \overline{\overline{T}}_i \vec{h}_m^{T,II}(S_a^i) > \tag{32}$$

$t = 1,2,\ldots T$, $m = 1,2,\ldots M$ and i is the port which t belongs to.

- $\overline{\overline{X}}_{23(T\times Q)}$

$$x_{tq}^{(23)} =< \vec{v}_t, \vec{H}_{S_a^i}^{II}(\vec{J}_q) > \tag{33}$$

$t = 1,2,\ldots T$ and $q = 1,2,\ldots Q$.

- $\overline{\overline{Z}}_{31(R\times N)}$=0, because (15) do not depend on b_n.
- $\overline{\overline{Z}}_{32(R\times M)}$

$$z_{rm}^{(32)} =< \vec{\omega}_r, \hat{n}_{S_c} \times \vec{e}_m^{\,T,II}(S_c) > \tag{34}$$

$r = 1, 2, \ldots R$ and $m = 1, 2, \ldots M$.

- $\overline{\overline{Z}}_{33(R\times Q)}$

$$z_{rq}^{(33)} = < \vec{\omega}_r, \hat{n}_{S_c} \times \vec{E}_{S_c}^{T,II}(\vec{J}_q) > \quad (35)$$

$r = 1, 2, \ldots R$ and $q = 1, 2, \ldots Q$.

Finally, the following system of equations can be constructed

$$\overline{\overline{Z}}\overline{x} = \overline{c}^{inc} \quad (36)$$

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