Comparative Study with New Accuracy Metrics for Target Volume Contouring in PET Image Guided Radiation Therapy.

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Abstract—The impact of PET on radiation therapy is held back by poor methods of defining functional volumes of interest. Many new software tools are being proposed for contouring target volumes but the different approaches are not adequately compared and their accuracy is poorly evaluated due to the ill-definition of ground truth. This paper compares the largest cohort to date of established, emerging and proposed PET contouring methods, in terms of accuracy and variability. We emphasise spatial accuracy and present a new metric that addresses the methods, in terms of accuracy and variability. We emphasise a custom-built PET phantom representing typical problems in institutions to contour functional VOIs in clinical PET/CT and a custom-built PET phantom representing typical problems in image guided radiotherapy. Contouring methods are grouped according to algorithmic type, level of interactivity and how they exploit structural information in hybrid images. Experiments reveal benefits of high levels of user interaction, as well as simultaneous visualisation of CT images and PET gradients to guide interactive procedures. Method-wise evaluation identifies the danger of over-automation and the value of prior knowledge built into an algorithm.

I. INTRODUCTION

Positron emission tomography (PET) with the metabolic tracer $^{18}$F-FDG is in routine use for cancer diagnosis and treatment planning. Target volume contouring for PET image-guided radiotherapy has received much attention in recent years, driven by the combination of PET with CT for treatment planning [1], unprecedented accuracy of intensity modulated radiation therapy (IMRT) [2] and on-going debates [3], [4] over the ability of the standardised uptake value (SUV) to define functional volumes of interest (VOIs) by simple thresholding. Many new methods are still threshold-based, but either automate the choice of SUV threshold specific to an image [5], [6] or apply thresholds to a combination (eg ratio) of SUV and an image-specific background value [7], [8]. More segmentation algorithms are entering PET oncology from the field of computer vision [9] including the use of image gradients [10], deformable contour models [11], [12], mutual information in hybrid images [13], [14] and histogram mixture models for heterogeneous regions [15], [16]. The explosion of new PET contouring algorithms calls for constraint in order to steer research in the right direction and avoid so-called vapetism (Yet Another PET Image Segmentation Method) [17]. For this purpose, we identify different approaches and compare their performance.

Previous works to compare contouring methods in PET oncology [18], [19], [20] do not reflect the wide range of proposed and potential algorithms and fall short of measuring spatial accuracy. [18] compare 3 threshold-based methods used on PET images of non-small cell lung cancer in terms of the absolute volume of the VOIs, ignoring spatial accuracy of the VOI surface that is important to treatment planning. Greco et al. [19] compare one manual and 3 threshold-based segmentation schemes performed on PET images of head-and-neck cancer. This comparison also ignores spatial accuracy, being based on absolute volume of the VOI obtained by manual delineation of complementary CT and MRI. Vees et al. [20] compare one manual, 4 threshold-based, one gradient-based and one region-growing method in segmenting PET gliomas and introduce spatial accuracy, measured by volumetric overlap with respect to manual segmentation of complimentary MRI. However, a single manual segmentation can not be considered the unique truth as manual delineation is prone to variability [21], [22].

Outside PET oncology, the society for Medical Image Computing and Computer Assisted Intervention (MICCAI) has run a ‘challenge’ in recent years to compare emerging methods in a range of application areas. Each challenge takes the form of a double-blind experiment, whereby different methods are applied by their developers on common test-data and the results analysed together objectively. In 2008, two examples of
pathological segmentation involved multiple sclerosis lesions in MRI [23] and liver tumours in CT [24]. These tests involved 9 and 10 segmentation algorithms respectively, and evaluated their accuracy using a combination of the Dice similarity coefficient [25] and Hausdorff distance [26] with respect to a single manual delineation of each VOI. In 2009 and 2010, the challenges were to segment the prostate in MRI [27] and parotid in CT [28]. These compared 2 and 10 segmentation methods respectively, each using a combination of various overlap and distance measures to evaluate accuracy with respect to a single manual ground truth per VOI. The MICCAI challenges have had a major impact on segmentation research in their respective application areas, but this type of large-scale, double-blind study has not previously been applied to PET target volume delineation for therapeutic radiation oncology, and the examples above are limited by their dependence upon a single manual delineation to define ground truth of each VOI.

This paper reports on the design and results of a large-scale, multi-centre, double-blind experiment to compare the accuracy of 30 established and emerging methods of VOI contouring in PET oncology. The study uses a new, probabilistic accuracy metric [29] that removes the assumption of unique ground truth, along with standard metrics of Dice similarity coefficient, Hausdorff distance and composite metrics. We use both a new tumour phantom [29] and patient images of head-and-neck cancer imaged by hybrid PET/CT. Experiments first validate the new tumour phantom and accuracy metric, then compare conceptual approaches to PET contouring by grouping methods according to how they exploit CT information in hybrid images, the level of user interaction and 10 distinct algorithm types. This grouping leads to conclusions about general approaches to segmentation, also relevant to other tools not tested here. Regarding the role of CT, conflicting reports in the literature further motivate the present experiments: while some authors found that PET tumour discrimination improves when incorporating CT visually [30] or numerically [31], others report on the detrimental effect of visualising CT on accuracy [32] and inter/intra-observer variability [21], [22]. Further experiments directly evaluate each method in terms of accuracy and, where available, inter-/intra operator variability. Due to the large number of contouring methods, full details of their individual accuracies and all statistically significant differences are provided in the supplementary material and summarised in this paper.

The rest of this paper is organised as follows. Section II describes all contouring algorithms and their groupings. Section III presents the new accuracy metric and describes phantom and patient images and VOIs. Experiments in section IV evaluate the phantom and accuracy metric and compare segmentation methods as grouped and individually. Section V discusses specific findings about manual practices and the types of automation and prior knowledge built into contouring and section VI gives conclusions and recommendations for future research in PET-based contouring methodology for image-guided radiation therapy.

II. CONTOURING METHODS

Thirteen contouring ‘teams’ took part in the experiment. We identify 30 distinct ‘methods’, where each is a unique combination of team and algorithm. Table I presents the methods along with labels (first column) used to identify them hereafter. Some teams used more than one contouring algorithm and some well-established algorithms such as thresholding were used by more than one team, with different definitions of the quantity and its threshold. Methods are grouped according to algorithm type and distinguished by their level of dependence upon the user (section II-B) and CT data (section II-C) in the case of patient images. Contouring by methods MD<sup>b</sup>, RG<sup>a</sup> and RG<sup>b</sup> was repeated by two users in the respective teams, denoted by subscripts 1 and 2, and the corresponding segmentations are treated separately in our experiments.

Some of the methods are well known for PET segmentation while others are recently proposed. Of the recently proposed methods, some were developed specifically for PET segmentation (e.g. GR, T2<sup>d</sup> and PL<sup>c</sup>) while some were adapted and optimised for PET tumour contouring for the purpose of this study. The study actively sought new methods, developed or newly adapted for PET tumours, as their strengths and weaknesses will inform current research that aims to refine or replace state of the art tools, whether those tools are included.

<table>
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<tr>
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<th>team</th>
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<th>interactivity</th>
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here or not. Many of the algorithms considered operate on standardised uptake values (SUVs), whereby PET voxel intensity $I$ is rescaled as $\text{SUV} = I \times (\beta/a_n)$ to standardise with respect to initial activity $a_n$ of the tracer in Bq ml$^{-1}$ and patient mass $\beta$ in grams [33]. The SUV transformation only affects segmentation by fixed thresholding while methods that normalise with respect to a reference value in the image or apply thresholds at a percentage of the maximum value are invariant to the SUV transformation.

### A. Method types and descriptions

**Manual delineation methods (MD)** use a computer mouse to delineate a VOI slice-by-slice, and differ by the modes of visualisation such as overlaying structural or gradient images and intensity windowing. MD$^a$ is performed by a board-certified radiation oncologist and nuclear medicine physician, who has over a decade of research and clinical experience in PET-based radiotherapy planning. MD$^b$ is performed by two independent, experienced physicians viewing only PET image data. For each dataset, the grey-value window and level were manually adjusted. MD$^c$ performed on the PET images by a nuclear medicine physicist who used visual aids derived from the original PET: intensity thresholds, both for the PET and the PET image-gradient, were set interactively for the purpose of visual guidance.

**Thresholding methods (T1 - T4)** are divided into 4 types according to whether the threshold is applied to signal (T1 & T2) or a combination of signal and background intensity (T3 & T4) and whether the threshold value is chosen a priori, based on recommendations in the literature or the team’s own experience (T1 & T3) or chosen for each image, either automatically according to spatial criteria or visually by the user’s judgement (T2 & T4). Without loss of generalisation the threshold value may be absolute or percentage (e.g., of peak) intensity or SUV. T1$^a$ & T1$^b$ employ the widely used cut-off values of 2.5 SUV and 40% of the maximum in the VOI, as used for lung tumour segmentation in [34] and [35] respectively. Method T1$^c$ is the only method of all in table I that is directly affected by the conversion from raw PET intensity to SUVs. The maximum SUV used by method T1$^b$ was taken from inside the VOI defined by T1$^a$. To calculate SUV for the phantom image, where patient weight $\beta$ is unavailable, all voxel values were re-scaled with respect to a value of unity at one end of the phantom where intensity is near uniform, causing method T1$^c$ to fail for phantom scan 2 as the maximum was below 2.5 for both VOIs. T1$^c$ applies a threshold at 50% of the maximum SUV. Method T2$^a$ is the thresholding scheme of [6], which automatically finds the optimum relative threshold level (RTL) based on an estimate of the absolute volume of the VOI in the image. The RTL is relative to background intensity, where background voxels are first labelled automatically by clustering. An initial VOI is estimated by a threshold of 40% RTL, and its maximum diameter is determined. The RTL is then adjusted iteratively until the absolute volume of the VOI matches that of a sphere of the same diameter, convolved with the point-spread function (PSF) of the imaging device, estimated automatically from the image. Methods T2$^b$ & T2$^c$ automatically define thresholds according to different criteria. They both use the results of method T1$^a$ as an initial VOI, and define local background voxels by dilution. Method T2$^a$ uses two successive dilations and labels the voxels in the second dilation as background. The auto-threshold is then defined as 3 standard deviations above the mean intensity in this background sample. Method T2$^b$ uses a single dilation to define the background and finds the threshold that minimises the within-class variance between VOI and background using the optimization technique in [36]. Finally, method T2$^c$ applies a closing operation to eliminate any holes within the VOI, which may also have the effect of smoothing the boundary. Method T2$^d$ finds the RTL using the method of [6] in common with method T2$^a$ but with different parameters and initialisation. Method T2$^d$ assumes a PSF of 7 mm full width at maximum (FWHM) rather than estimating this value from the image. The RTL was initialized with background defined by a manual bounding box rather than clustering and foreground defined by method T3$^c$ with a 50% threshold rather than 40% RTL. Adaptive thresholding method T2$^e$ starts with a manually defined bounding box then defines the VOI by the iso-contour at a percentage of the maximum value within the bounding box. Methods T3$^a$ & T3$^b$ are similar to T1$^c$, but incorporate local background intensity calculated by a method equivalent to that Daise et al. [37]. A threshold value is then 41% and 50% of the maximum plus background value, respectively. Method T4$^a$ is an automatic SUV-thresholding method implemented in the ‘Rover’ software [38]. After defining a search area that encloses the VOI, the user provides an initial threshold which is adjusted in two steps of an iterative process. The first step estimates background intensity $I_b$ from the average intensity over those voxels that are below the threshold and within a minimum distance of the VOI (above the threshold). The second step re-defines the VOI by a new threshold at 39% of the difference $I_{\text{max}} - I_b$, where $I_{\text{max}}$ is the maximum intensity in the VOI. Methods T4$^b$ & T4$^c$ use the source-to-background algorithm in [8]. The user first defines a background region specific to the given image, then uses parameters $a$ and $b$ to define the threshold $t = a I_{\text{VOL}} + b I_{\text{BG}}$, where $I_{\text{VOL}}$ and $I_{\text{BG}}$ are the mean SUV in the VOI and background respectively. The parameters are found in a calibration procedure by scanning spherical phantom VOIs of known volume. As this calibration was not performed for the particular scanner used in the present experiments (GE Discovery), methods T4$^b$ and T4$^c$ use parameters previously obtained for Gemini and Biograph PET systems respectively.

**Region growing methods (RG)** use variants of the classical algorithm in [39], which begins at a ‘seed’ voxel in the VOI and agglomerates connected voxels until no more satisfy criteria based on intensity. In RG$^a$, the user defines a bounding sphere centred on the VOI, defining both the seed at the centre of the sphere and a hard constraint at the sphere surface to avoid leakage into other structures. The acceptance criterion is an interactively adjustable threshold and the final VOI is manually modified in individual slices if needed. Methods RG$^b$ & RG$^c$ use the region growing tool in Mirada XD (Mirada Medical, Oxford, UK) with seed point location and...
acceptance threshold defined by the user. In RG\(^b\) only, the
results are manually post-edited using the 'adaptive brush' tool
available in Mirada XD. This 3D painting tool adapts in shape
to the underlying image. Also in method RG\(^b\) only, CT images
were fused with PET for visualisation and the information used
to modify the regions to exclude airways and unaffected bone.

Watershed methods (WS) use variants of the classical
algorithm in [40]. The common analogy pictures a gradient-
filtered image as a 'relief map' and defines a VOI as one or
more pools, created and merged by flooding a region with
water. Method WS\(^b\), adapted from the algorithm in [41] for
segmenting natural colour images and remote-sensing images,
makes use of the content as well as the location of user-
defined markers. A single marker for each VOI (3 × 3 or
5 × 5 pixels depending on VOI size) is used along with a
background region to train a fuzzy classification procedure
where each voxel is described by a texture feature vector.
Classification maps are combined with image gradient and the
familiar 'flooding' procedure is adapted for the case of multi-
ple surfaces. Neither the method nor the user were specialized
in medical imaging. Method WS\(^b\), similar way to that in [42],
uses two procedures to overcome problems associated with
local minima in image gradient. First, viscosity is added to
the watershed, which closes gaps in the edge-map. Second, a
set of internal and external markers are identified, indicating
the VOI and background. After initial markers are identified
in one slice by the user, markers are placed automatically in
successive slices, terminating when the next slice is deemed
no longer to contain the VOI according to a large drop in
the 'energy', governed by area and intensity, of the segmented
cross section. If necessary, the user interactively overrides the
automatic marker placement.

Pipeline methods (PL) are more complex, multi-step algo-
rithms that combine elements of thresholding, region growing,
watershed, morphological operations and techniques in [43],
[44], [15]. Method PL\(^a\) is a deformable contour model adapted
from white matter lesion segmentation in brain MRI. The main
steps use a region-scalable fitting model [45] and a global
standard convex scheme [46] in energy minimization based on
the 'Split Bregman' technique in [43]. Methods PL\(^b\) – PL\(^d\) are
variants of the 'Smart Opening' algorithm, adapted for PET
from the tool in [44] for segmenting lung nodules in CT data.
In contrast to CT lung lesions, the threshold used in region
growing can not be set \textit{a priori} and is instead obtained from
the image interactively. Method PL\(^b\) was used by an operator
with limited PET experience. The user of method PL\(^c\) had
more PET experience and, to aid selection of boundary points
close to steep PET gradients, also viewed an overlay of local
maxima in the edge-map of the PET image. Finally, method
PL\(^d\) took the results of method PL\(^c\) and performed extra pro-
cessing by dilation, identification of local gradient maxima in
the dilated region, and thresholding the gradient at the median
of these local maxima. Methods PL\(^c\) & PL\(^d\) use the so-called
'poly-segmentation' algorithm without and with post editing
respectively. PL\(^c\) is based on a multi-resolution approach,
which segments small lesions using recursive thresholding
and combines 3 segmentation algorithms for larger lesions.
First, the watershed transform provides an initial segmentation.
Second, an iterative procedure improves the segmentation by
adaptive thresholding that uses the image statistics. Third, a
region growing method based on regional statistics is used.
The interactive variant (PL\(^g\)) uses a fast interactive tool for
watershed-based sub-region merging. This intervention is only
necessary in at most two slices per VOI. Method PL\(^b\) is a
new fuzzy segmentation technique for noisy and low resolution
oncological PET images. PET images are first smoothed using
a nonlinear anisotropic diffusion filter and added as a second
input to the fuzzy C-means (FCM) algorithm to incorporate
spatial information. Thereafter, the algorithm integrates the à trous wavelet transform in the standard FCM algorithm to
handle heterogeneous tracer uptake in lesions [15].

The Gradient based method (GR) is the novel
edge-finding method in [10], designed to overcome the low
signal-to-noise ratio and poor spatial resolution of PET im-
ages. As resolution blur distorts image features such as iso-
contours and gradient intensity peaks, the method combines
dge restoration methods with subsequent edge detection.
Edge restoration goes through two successive steps, namely
dge-preserving denoising and deblurring with a deconvolu-
tion algorithm that takes into account the resolution of a
given PET device. Edge-preserving denoising is achieved by
bilateral filtering and a variance-stabilizing transform [47].
Segmentation is finally performed by the watershed transform
applied after computation of the gradient magnitude. Over-
segmentation is addressed with a hierarchical clustering of
the watersheds, according to their average tracer uptake. This
produces a dendrogram (or tree-diagram) in which the user
selects the branch corresponding to the tumour or target.
User intervention is usually straightforward, unless the uptake
difference between the target and the background is very low.

The Hybrid method (HB) is the multi-spectral algorithm in
[14], adapted for PET/CT. This graph-based algorithm exploits
the superior contrast of PET and the superior spatial resolution
of CT. The algorithm is formulated as a Markov Random
Field (MRF) optimization problem [48]. This incorporates an
energy term in the objective function that penalizes the spatial
difference between PET and CT segmentation.

B. Level of interactivity

Levels of interactivity are defined on an ordinal scale of
'max', 'high', 'mid', 'low' and 'none', where 'max' and 'none'
refer to fully manual and fully automatic methods respectively.
Methods with a 'high' level involve user initialisation, which
locates the VOI and/or representative voxels, as well as run-
time parameter adjustment and post-editing of the contours.
'Mid'-level interactions involve user-initialisation and either
run-time parameter adjustment or other run-time information
such as wrongly included/excluded voxels. 'Low'-level inter-
action refers to initialisation or minimal procedures to re-
start an algorithm with new information such as an additional
mouse-click in the VOI.

C. Level of CT use

We define the levels at which contouring methods exploit
CT information in hybrid patient images as 'high', 'low' or
'none', where 'high' refers to numerical use of CT together with PET in calculations. The 'low' group makes visual use of CT images to guide manual delineation, post-editing or other interactions in semi-automatic methods. The 'none' group refers to cases where CT is not used, or is viewed incidentally but has no influence on contouring as the algorithm is fully automatic. None of the methods operated on CT images alone.

III. EXPERIMENTAL METHODS

A. Images

We use two images of a new tumour phantom [29], manufactured for this study and two clinical PET images of different head-and-neck cancer patients. The test images are available on-line [49], along with ground truth sets described in section III-C. All imaging used the metabolic tracer $^{18}$F-Fluorodeoxyglucose (FDG) and a hybrid PET/CT scanner (GE Discovery), but CT images from phantom scans were omitted from the test set. Table II gives more details of each image type. The tumour phantom contains glass compartments of irregular shapes shown in figure 1 (top row), mimicking real radiotherapy target volumes. The tumour compartment

![Fig. 1](image1.png)  
(a) tumour and (b) nodal chain VOIs of the phantom. *Top*: Digital photographs of glass compartments. *Middle*: PET images from scan 1 (sagittal view). *Bottom*: 3D surface view from an arbitrary threshold of simultaneous CT, lying within the glass wall.

(b) has branches to recreate the more complex topology of some tumours. This and the nodal chain compartment (b) are based on cancer of the oral cavity and lymph node metastasis respectively, manually segmented from PET images of two head and neck cancer patients and formed by glass blowing. The phantom compartments and surrounding container were filled with low concentrations of FDG and scanned by a hybrid device (1, middle and bottom rows). Four phantom VOIs result from scans 1 and 2, with increasing signal to background ratio achieved by increasing FDG concentration in the VOIs. Details of the 4 phantom VOIs are given in the first 4 rows of table III. Figure 2 shows the phantom VOIs from scan 1, confirming qualitatively the spatial and radiometric agreement between phantom and patient VOIs.

![Fig. 2](image2.png)  
Fig. 2: Axial PET images of phantom and real tumour (top) and lymph node (bottom) VOIs with profile lines traversing each VOI. Plots on the right show the image intensity profiles sampled from each image pair.

For patient images, head and neck cancer was chosen as it poses particular challenges to PET-based treatment planning due to the many nearby organs at risk (placing extra demand on GTV contouring accuracy), the heterogeneity of tumour tissue and the common occurrence of lymph node metastasis. A large tumour of the oral cavity and a small tumour of the larynx were selected from two different patients, along with a metastatic lymph node in the first patient (figure 3). These target volumes were chosen as they were histologically proven and have a range of sizes, anatomical locations/surroundings and target types (tumour and metastasis). Details of the 3 patient VOIs are given in the last 3 rows of table III.

B. Contouring

With the exception of the hybrid method (HB) that does not apply to the PET-only phantom data, all methods contoured all 7 VOIs. In the case of patient VOIs, participants had the option of using CT as well as PET, and were instructed to contour the gross tumour volume (GTV) and metastatic tissue of tumours and lymph node respectively. All contouring
TABLE II: Details of phantom and patient PET/CT images.

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<td>width/height (pixels)</td>
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<td>256</td>
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<tr>
<td>patient</td>
<td>3.0</td>
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Fig. 3: Axial neck slices of 18F-FDG PET images overlain on simultaneous CT. (a) & (b) Oral cavity tumour & lymph node metastasis in patient 1 (c) Laryngeal tumour in patient 2.

methods were used at the sites of the respective teams using their own software and workstations. Screen-shots of each VOI were provided in axial, sagittal and coronal views, with approximate centres indicated by cross-hairs and their voxel coordinates provided to remove any ambiguity regarding the ordering of axes and direction of increasing indices. No other form of ground truth was provided. Teams were free to refine their algorithms and practice segmentation before accepting final contours. This practicing stage was done without any knowledge of ground truth and is considered normal practice.

Any contouring results with sub-voxel precision were down-sampled to the resolution of the PET image grid and any results in mm were converted to voxel indices. Finally, all contouring results were duplicated to represent VOIs first by the voxels on their surface, and second by masks of the solid VOI including the surface voxels. These two representations were used in surface-based and volume-based contour evaluation respectively.

C. Contouring evaluation

Accuracy measurement generally compares the contour being evaluated, which we denote C, with some notion of ground truth, denoted GT. We use a new probabilistic metric [29] denoted AUC', as well as a variant of the Hausdorff distance [26] denoted HD' and the standard metric of Dice similarity coefficient [25] (DSC). AUC' and HD' are standardised to the range 0 . . . 1 so that they can be easily compared or combined with DSC and other accuracy metrics occupying this range [50], [51], [52]. Treated separately, AUC', HD' and DSC allow performance evaluation with and without the assumption of unique ground truth, and in terms of both volumetric agreement (AUC' and DSC) and surface-displacement (HD') with respect to ground truth.

AUC' is a probabilistic metric based on receiver operating characteristic (ROC) analysis, in a scheme we call inverse-ROC (I-ROC). The I-ROC method removes the assumption of unique ground truth, instead using a set of p arbitrary ground truth definitions \{GT_i\}, i \in \{1 . . . p\} for each VOI. While uniquely correct ground truth in the space of the PET image would allow deterministic and arguably superior accuracy evaluation, the I-ROC method is proposed for the case here, and perhaps all cases except numerical phantoms, where such truth is not attainable. The theoretical background of I-ROC is given in Appendix A and shows that the area under the curve (AUC) gives a probabilistic measure of accuracy provided that the arbitrary set can be ordered by increasing volume and share the topology and general form of the (unknown) true surface. The power of AUC' as an accuracy metric also relies on the ability to incorporate the best available knowledge of ground truth into the arbitrary set. This is done for phantom and patient VOIs as follows.

For phantom VOIs, the ground truth set is obtained by incrementing a threshold of Hounsfield units (HU) in the CT data from hybrid imaging (figure 4). Masks acquired for all CT slices in the following steps:

(i) reconstruct/down-sample the CT image to the same pixel grid as the PET image
(ii) define a bounding box in the CT image that completely encloses the glass VOI as well as C
(iii) threshold the CT image at a value HU_i
(iv) treat all pixels below this value as being 'liquid' and all above it as 'glass'

Fig. 4: (a) 3D visualisation of phantom VOI from CT thresholded at a density near the internal glass surface. (b) Arbitrary ground truth masks of the axial cross section in (a), from 50 thresholds of HU.
(v) label all 'liquid' pixels that are inside the VOI as positive, but ignore pixels outside the VOI.
(vi) repeat for \( p \) thresholds \( HU_i, i \in \{1 \ldots p\} \) between natural limits \( HU_{\text{min}} \) and \( HU_{\text{max}} \).

This ground truth set is guaranteed to pass through the internal surface of the glass compartment and exploits the inherent uncertainty due to partial volume effects in CT. It follows from derivations in Appendix A.2-3 that AUC is equal to the probability that a voxel drawn at random from below the unknown CT threshold at the internal glass surface, lies inside the contour \( C \) being evaluated.

For patient VOIs, the ground truth set is the union of an increasing number of expert manual delineations. Experts contoured GTV and node metastasis on PET visualised with co-registered CT. In the absence of histological resection, we assume that the best source of ground truth information is manual PET segmentation by human experts at the imaging site, who have experience of imaging the particular tumour-type and access to extra information such as tumour stage, treatment follow-up and biopsy where available. However, we take the view that no single manual segmentation provides the unique ground truth, which therefore remains unknown. In total, 3 delineated each VOI on 2 occasions (denoted \( N_{\text{experts}} = 3 \) and \( N_{\text{occasions}} = 2 \)) with at least a week in between. The resulting set of \( p = N_{\text{experts}} \times N_{\text{occasions}} \) ground truth estimates were acquired to satisfy the requirements in Appendix A.3 as follows:

(i) define a bounding box in the CT image that completely encloses all \( N_{\text{experts}} \times N_{\text{occasions}} \) manual segmentations \( \{G{T}_i\} \) and the contour \( C \) being evaluated
(ii) order the segmentations \( \{G{T}_i\} \) by absolute volume in \( \text{cm}^3 \)
(iii) use the smallest segmentation as \( G{T}_2 \)
(iv) form a new VOI from the union of the smallest and the next largest VOI in the set and use this as \( G{T}_3 \)
(v) repeat until the largest VOI in the set has been used in the union of all \( N_{\text{experts}} \times N_{\text{occasions}} \) VOIs
(vi) create homogeneous masks for \( G{T}_1 \) and \( G{T}_p \), having all negative and all positive respectively.

The patient ground truth set encodes uncertainty from inter-/intra-expert variability in manual delineation and AUC is the probability that a voxel drawn at random from the unknown manual contour at the true VOI surface, lies inside the contour \( C \) being evaluated. Finally, we rescale AUC to the range \( \{0 \ldots 1\} \) by

\[
AUC' = \frac{AUC - 0.5}{0.5}, \quad 0 \leq AUC' \leq 1 = \text{maximum accuracy.}
\]

Reference surfaces that profess to give the unique ground truth are required to measure the Hausdorff distance and Dice similarity. We obtain the 'best guess' of the unique ground truth, denoted \( G{T}^* \) from the sets of ground truth definitions introduced above. For each phantom VOI we select the CT threshold having the closest internal volume in \( \text{cm}^3 \) to an independent estimate. This estimate is the mean of three repeated measurements of the volume of liquid contained by each glass compartment. For patient VOIs, \( G{T}^* \) is the union of the population, which is irrespective of multiple comparison effects. The rank-based metric is also immune to the multiple comparison effect and we use the median rather than mean rank to avoid misleading results for a method that ranks highly in only one of the metrics AUC', DSC, HD and A*, considered an outlier.
Intra-operator variability was measured by the raw Hausdorff distance in mm between the first and second segmentation result from repeated contouring (no ground truth necessary). However, this was only done for some contouring methods. For fully automatic methods, variability is zero by design and was not explicitly measured. Of the remaining semi-automatic and manual methods, 11 were used twice by the same operator: MD$^a_c$, MD$^b_2$, RG$^a$, HB, WS$^b$, RG$^b_1$, RG$^b_2$, RG$^c_1$, RG$^c_2$, GR and MD$^c$ and for these we measure the intra-operator variability which allows extra, direct comparisons in section IV-E.

IV. Experiments

This section motivates the use of the new phantom and accuracy metric (IV-A), then investigates contouring accuracy by comparing the pooled accuracy of methods grouped according to their use of CT data (section IV-B), level of user interactivity (section IV-C) and algorithm type (section IV-D). Section IV-E evaluates methods individually, using condensed accuracy metrics in table IV. With the inclusion of repeated contouring by methods MD$^b$, RG$^b$ and RG$^c$ by a second operator, there are a total of $n = 33$ segmentations of each VOI, with the exception of phantom VOIs where $n = 32$ by the exclusion of method HB. Also, method T1 failed to recover phantom VOIs in scan 1 as no voxels were above the predefined threshold. In this case a value of zero accuracy is recorded for two out of 4 phantom VOIs.

A. Phantom and AUC$'$

This experiment investigates the ability of the phantom to pose a realistic challenge to PET contouring, by testing the null-hypothesis that both phantom and patient VOIs lead to the same distribution of contouring accuracy across all methods used on both image types. First, we take the mean accuracy over the 4 phantom VOIs as a single score for each contouring method. Next, we measure the accuracy of the same methods used in patient images and take the mean over the 3 patient VOIs as a single score for each method. Finally, a paired-samples t-test is used for the difference of means between accuracy scores in each image type, with significant difference defined at a confidence level of $p \leq 0.05$. Figure 5 shows the results separately for accuracy defined by AUC$'$, DSC and HD$'$. There is no significant difference between accuracy in phantom and patient images measured by AUC$'$ or DSC. A significant difference is seen for HD$'$, which reflects the sensitivity of HD$'$ to small differences between VOI surfaces. In this case the phantom VOIs are even more difficult to contour accurately than the patient images, which could be explained by the absence of anatomical context in these images, used by operators of manual and semi-automatic contouring methods. A similar experiment found no significant difference between phantom and patient VOIs in terms of intra-operator variability. On the whole we accept the null-hypothesis meaning that the phantom and patient images pose the same challenge to contouring methods in terms of accuracy and variability.

![Figure 5: Contouring accuracy in phantom and patient images](image)

Figure 5 also supports the use of the new metric AUC$'$. Although values are generally higher than DSC and HD, which may be explained by the involvement of multiple ground truth definitions increasing the likelihood that a contour agrees with any one in the set, the variance of accuracy scores is greater for AUC$'$ than the other metrics (table V), which indicates higher sensitivity to small differences in accuracy between any two methods.

<table>
<thead>
<tr>
<th>Table V: Variance of AUC$'$ and standard accuracy metrics calculated for all 7 VOIs (second column), and for the 4 and 3 VOIs in phantom and patient images respectively.</th>
</tr>
</thead>
<tbody>
<tr>
<td>metric</td>
</tr>
<tr>
<td>AUC$'$</td>
</tr>
<tr>
<td>DSC</td>
</tr>
<tr>
<td>HD$'$</td>
</tr>
</tbody>
</table>

B. Role of CT in PET/CT contouring

For contouring in patient images only, we test the benefit of exploiting CT information in contouring (phantom VOIs are omitted from this experiments as the CT was used for ground truth definitions and not made available during contouring). This information is in the form of anatomical structure in the case of visual CT-guidance ('low' CT use) and higher-level, image texture information in the case of method HB with 'high' CT use. The null-hypothesis is that contouring accuracy is not affected by the level of use of CT information.

We compare each pair of groups $i$ and $j$ that differ by CT use, using a t-test for unequal sample sizes $n_i$ and $n_j$, where the corresponding samples have mean accuracy $\mu_i$ and $\mu_j$ and standard deviation $\sigma_i$ and $\sigma_j$. For the $i$th group containing $n_{\text{methods}}$ contouring methods, each segmenting $n_{\text{VOIs}}$ targets, the sample size $n_i = n_{\text{methods}} \times n_{\text{VOIs}}$ and $\mu_i$ and $\sigma_i$ are calculated over all $n_{\text{methods}} \times n_{\text{VOIs}}$ accuracy scores. We calculate the significance level from the t-value using the number of degrees of freedom given by the Welch-Satterthwaite formula for unequal sample sizes and sample standard deviations. Significant
differences between groups are defined by confidence interval of $p \leq 0.05$. For patient images only, $n_{\text{VOIs}} = 3$ and for the grouping according to CT use in table I, $n_{\text{methods}} = 1, 6$ and 26 for the groups with levels of CT use 'high', 'low' and 'none' respectively (methods RG$^b$ in the 'low' group and MD$^b$ & RG$^c$ in the 'none' group were used twice by different operators in the same team). We repeat for 4 accuracy metrics AUC', DSC, HD' and their weighted sum $A^\ast$, Figure 6 shows the results for all groups ordered by level of CT use, in terms of each accuracy metric in turn.

![Figure 6: Effect of CT use on contouring accuracy in patient images](image)

With the exception of AUC' the use of CT as a visual guidance ('low'), out-performed the 'high' and 'none' groups consistently but without significant difference. The fact that the 'high' group (method HB only) significantly out-performed the lower groups in terms of AUC' alone indicates that the method had good spatial agreement with one of the union-of-experts masks for any given VOI, but this union mask did not have absolute volume most closely matching the independent estimates used in calculations of DSC and HD'. We conclude that the use of CT images as visual reference ('low' use) generally improves accuracy, as supported by the consistent improvement in 3 out of 4 metrics. This is in agreement with experiments in [30] and [31], which found the benefits of adding CT visually and computationally, in manual and automatic tumour delineation and classification respectively.

**C. Role of user interaction**

This experiment investigates the effect of user-interactivity on contouring performance. The null hypothesis is that contouring accuracy is not affected by the level of interactivity in a contouring method. We compare each pair of groups $i$ and $j$ that differ by level of interactivity, using a t-test for unequal sample sizes as above. For the grouping according to level of interactivity in table I, groups with interactivity level 'max', 'high', 'mid', 'low' and 'none' have $n_{\text{methods}} = 4, 3, 7, 13$ (12 for phantom images by removal of method HB) and 6 respectively (methods MD$^b$, RGMD$^b$ and RGMD$^c$ in the 'max', 'high' and 'mid' groups respectively were used twice by different operators in the same team). We repeat for patient images ($n_{\text{VOIs}} = 3$), phantom images ($n_{\text{VOIs}} = 4$) and the combined set ($n_{\text{VOIs}} = 7$) and, as above, for each of the 4 accuracy metrics. Figure 7 shows all results for all groups ordered by level of interactivity.

The trends for each of phantom, patient and all VOIs are consistent over all metrics. The most accurate methods were those in the 'high' and 'max' groups for phantom and patient images respectively. For patient images, the 'max' group is significantly more accurate than any other and this trend carries over to the pooled accuracies in both image types despite having less patient VOIs ($n = 3$) than phantom VOIs ($n = 4$). For phantom VOIs, with the exception of HD', there are no significant differences between 'high' and 'max' groups and these both significantly out-perform the 'low' and 'none' groups in all metrics. For HD' alone, fully manual delineation is significantly less accurate than semi-automatic methods with 'high' levels of interaction. This may reflect the lack of anatomical reference in the phantom images, which is present for patient VOIs and guides manual delineation. As high levels of interaction still appear most accurate, the reduced accuracy of fully manual methods is not considered likely to be caused by a bias of manual delineations toward manual ground truth, given the levels of inter-user variability. Overall, we conclude that manual delineation is more accurate than semi- or fully-automatic methods, and that the accuracy of semi-automatic methods improves with the level of interaction built in.

**D. Accuracy of algorithm types**

This experiment compares the accuracy of different algorithm types, defined in section II-A. The null hypothesis is that contouring accuracy is the same for manual or any numerical method regardless of the general approach they take. We compare each pair of groups $i$ and $j$ that differ by algorithm type, using a t-test for unequal sample sizes as above. For the grouping according to algorithm type in table I, $n_{\text{methods}} = 4, 3, 5, 2, 3, 5, 2, 1, 1$ (0 for phantom images by removal of method HB) and 7 for algorithm-types MD, T1, T2, T3, T4, RG, WS, GR, HB and PL respectively (methods MD$^b$ in the MD, and RG$^b$ & RG$^c$ in the RG group were used twice by different operators in the same team). As above, we repeat for patient images ($n_{\text{VOIs}} = 3$), phantom images ($n_{\text{VOIs}} = 4$) and the combined set ($n_{\text{VOIs}} = 7$), and for each of the 4 accuracy metrics. Figure 8 shows the results separately for all image sets and accuracy metrics.

Plot (b) reproduces the same anomalous success of the hybrid method (HB) in terms of AUC' alone, as explained above. Manual delineation exhibits higher accuracy than other algorithm types, ranking in the top 3 for any accuracy metric.
Fig. 7: Effect of user interaction on contouring accuracy measured by top row: AUC' for (a) phantom, (b) patient and (c) both VOI types, second row: DSC for (d) phantom (e) patient and (f) both image types, third row: HD' for (g) phantom, (h) patient and (i) both image types, and bottom row: A* for (j) phantom, (k) patient and (l) both VOI types. Significant differences between any two levels of user interaction are indicated by '.cp'.
Fig. 8: Contouring accuracy of all algorithm types measured by top row: \( AUC' \) for (a) phantom, (b) patient and (c) both VOI types, second row: \( DSC \) for (d) phantom (e) patient and (f) both image types, third row: \( HD' \) for (g) phantom, (h) patient and (i) both image types and bottom row: \( A^* \) for (j) phantom, (k) patient and (l) both VOI types. Significant differences between any two algorithm types are indicated by '\( \uparrow \uparrow \uparrow \)'. 
in phantom images and the top two for any metric in patient images. The pooled results over all images reveal manual delineation as the most accurate in terms of all 4 metrics. With the exception of T4 in terms of HD’ (patient and combined image sets), the improvement of manual delineation over any of the thresholding variants T1 - T4 is significant, despite these being the most widely used (semi-)automatic methods. A promising semi-automatic approach is the gradient-based (GR) group (one method), which has the second highest accuracy by all metrics for the combined image set and significant difference from manual delineation. Conversely, the watershed group of methods that also rely on image gradients exhibit consistently low accuracy. This emphasized the problem of poorly-defined edges and noise-induced false edges typical of PET gradient filtering, which in turn suggests that edge-preserving noise reduction by the bi-lateral filter plays a large part in the success of method GR.

E. Accuracy of individual methods

The final experiments directly compare the accuracy of all methods. Where two algorithms have arguably minor difference, as in the case of PLc and PLd which differ by an extra processing step applied by PLd, these are treated as separate methods because the change in contouring results is notable and can be attributed to the addition of the processing step, which is informative. Repeated segmentations by two different users in the cases of methods MDb1,2, RGb1,2 and RGc1,2 are counted as two individual results so there are a total of n = 32 ‘methods’, or n = 33 for patient VOIs in PET/CT only by inclusion of hybrid method HB. The null hypothesis is that all n cases are equally accurate. We compare each pair of methods i and j that differ by method, using a t-test for equal sample sizes ni = nj = nVOIs, where mean accuracy µi and µj and standard deviation σi and σj are calculated over all VOIs and there are 2nVOIs – 2 degrees of freedom.

As above, we repeat for all image sets and accuracy metrics. Figure 9 shows the results separately for phantom, patient and combined image sets in terms of A* only. Full results for all metrics and significant differences between methods are given in the supplementary material.

The generally low values of A* in figure 9 and other metrics in the supplementary material highlight the problem facing accurate PET contouring. These results also reiterate the general finding that manual practices can be more accurate than semi- or fully-automatic contouring. For patient images, and the combined set, the most accurate contours are manually delineated by method MDc. Also for these image sets the second and third most accurate are another manual method (MDb2) and the ‘smart opening’ algorithm (PLb) with mid-level interactivity.

For phantom VOIs only, methods RGb and T1b, with high- and low-level interactivity, out-perform manual method MDc with no significant difference. Method RGb is based on SRG with post-editing by the adaptive brush and showed low accuracy for patient VOIs with RGb2 being significantly less accurate than the manual method MDc (see supplementary material). Method T1b is based on thresholding and showed low accuracy for patient VOIs, being significantly less accurate than the manual methods MDc and MDb2 (see supplementary material). Their high accuracy in phantom images alone could be explained by methods T1b and RGc being particularly suited to the relative homogeneity of the phantom VOIs.

Methods WSb, Ti c and T3b have the 3 lowest accuracies by mean A* across all 3 image sets. The poor performance of method WSb could be explained by its origins (colour photography and remote-sensing) and user having no roots or specialism in medical imaging. Threshold methods Ti c and T3b give iso-contours at 50% of the local peak intensity without and with adjustment for background intensity respectively. Their poor performance in all image types highlights the limitations of thresholding.

Table VI presents the composite metrics explained in section III-C along with intra-operator variability where available (last two columns), measured by the Hausdorff distance in mm between two segmentations of the same VOI, averaged over the 3 patient or 4 phantom VOIs. This definition of intra-operator variability gives an anomalously high value if the two segmentations resulting from repeated contouring of the same VOI do not have the same topology, as caused by an internal hole in the first contouring by method RG1. Notably, we find no correlation between intra-operator variability and the level of interactivity of the corresponding methods. The same is true for inter-operator variability (not shown) calculated by the Hausdorff distance between segmentations by different users of the same method (applicable to methods MDc, RGc and RGb). This finding contradicts the general belief that user input should be minimised to reduce variability. Table VI reaffirms the finding that manual delineation is the most accurate method type, with examples MDc and MDb1,2 scoring highly in all metrics. The most consistently accurate non-manual methods are the semi- and fully-automatic methods PLb and PLc. More detailed method-wise comparisons are made in the next section.

V. DISCUSSION

We have evaluated and compared 30 implementations of PET segmentation methods ranging from fully manual to fully automatic and representing the range from well established to never-before tested on PET data. Region growing and watershed algorithms are well established in other areas of medical image processing, while their use for PET target volume delineation is relatively new. Even more novel approaches are found in the ‘pipeline’ group and the two distinct algorithms of gradient-based and hybrid segmentation. The gradient-based method [10] has already had an impact in the radiation oncology community and the HB method [14] is one of few in the literature to make numerical use of the structural information in fused PET/CT. The multispectral approach is in common with classification experiments in [13] that showed favourable results over PET alone.

A. Manual delineation

Free-hand segmentation produced among the most accurate results, which may be counter-intuitive. One explanation
comes from the incorporation of prior knowledge regarding the likely form and extent of pathology. In the case of the patient images alone, bias toward MD may be suspected as the ground-truth set is also built up from manual delineations. However, this does not explain the success of manual methods as they performed better still for phantom VOIs where the ground truth comes from CT thresholds. The use of multiple ground truth estimates by I-ROC may falsely favour manual delineation due to its inherent variability. However, this too does not explain the success of manual methods as they also perform well in terms of DSC and HD’ that use a unique, 'best-guess' of ground truth (at least one MD is among the 5 highest DSC and HD for each of the patient phantom VOI sets). These observations challenge the intuition, that manual delineation is less accurate. Although many (semi-)automatic methods out-perform free-hand delineation in the literature, the inherent bias toward positive results among published work makes this an unfair basis for intuition.

Of the 4 manual delineations (MD\textsuperscript{a}, MD\textsuperscript{b,1}, MD\textsuperscript{b,2} and MD\textsuperscript{c}), method MD\textsuperscript{c} out-performed the rest in all of n(n.s.d), n(>\mu+\sigma), median rank and intra-operator variability where known, with significant improvement over MD\textsuperscript{b,1,2} in terms of AUC' for patient VOIs (although the multiple comparison effect can mean that one or more of these differences are falsely detected as significant). The obvious difference between these 4 is the user. It is interesting, and indicative of no bias in terms of user group, that the delineator of MD\textsuperscript{c} was a nuclear medicine physicist while the other users, in common with the experts providing ground truth estimates, were experienced physicians. However, while users of MD\textsuperscript{a} and MD\textsuperscript{b,1,2} only viewed the PET images during delineation, the physicist using MD\textsuperscript{c} also viewed an overlay of the PET gradient magnitude and, in the case of patient images, simultaneous CT. These modes of visual guidance could in part compensate for the relative lack of clinical experience, although no concrete conclusion can be made as clinical sites may disagree on the correct segmentation.

B. Automation vs. user guidance

Two method comparisons provide evidence that too much automation in a semi-automatic algorithm is detrimental to contouring accuracy. First, we compare the accuracy of methods PL\textsuperscript{c} and PL\textsuperscript{d}. Method PL\textsuperscript{d} starts with the same segmentation achieved by PL\textsuperscript{c}, then performs extra steps in the automatic pipeline intended to improve on the results. However, these extra steps reduce the final accuracy. Second, we compare the accuracy of methods RG\textsuperscript{b,1,2} and RG\textsuperscript{c,1,2}. These differ in that RG\textsuperscript{b,1,2} also employs post-editing by the adaptive brush tool. While the adaptive brush may improve accuracy for phantom VOIs, accuracy is reduced for patient VOIs indicated by n(n.s.d) and median rank. This suggests
TABLE VI: Summarised accuracy and variability of phantom (ph.) and patient (pt.) contouring by all methods ordered as in table I and using ranked and other composite accuracy metrics in section III-C. Data are not available (n/a) for method HB in phantom results and most methods in variability results.

<table>
<thead>
<tr>
<th>method</th>
<th>n(n.s.d)</th>
<th>n(&gt;ε+σ)</th>
<th>median rank</th>
<th>intra-operator HD (mm)</th>
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<tbody>
<tr>
<td>PL&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>0 0</td>
<td>17 19</td>
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<td>1 1</td>
<td>23.5 27</td>
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</tr>
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<td>0 0</td>
<td>4 7</td>
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<td>2 0</td>
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<td>3 0</td>
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<td>0 2</td>
<td>0 0</td>
<td>7 7.5</td>
<td>n/a</td>
</tr>
<tr>
<td>T2&lt;sup&gt;d&lt;/sup&gt;</td>
<td>4 3</td>
<td>0 1</td>
<td>18.5 26</td>
<td>n/a</td>
</tr>
<tr>
<td>PL&lt;sup&gt;j&lt;/sup&gt;</td>
<td>4 3</td>
<td>0 3</td>
<td>8.5 29.5</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Interactive post-editing by user-defined watershed markers and sub-regional merging. Method PL<sup>j</sup> is consistently more accurate than PL<sup>f</sup> over all 12 combinations of accuracy metric and image type. A second example comes from comparing 5 thresholding schemes used at the same institution (team 13). Methods T1<sup>c</sup>, T3<sup>a</sup> and T3<sup>b</sup> use intensity thresholds of 50% maximum and 41% & 50% of maximum-plus-background, while T2<sup>d</sup> and T2<sup>e</sup> use thresholds chosen to match an estimate of the VOI’s absolute volume and the user’s visual judgement of VOI extent respectively. Of these, T2<sup>e</sup> is most highly influenced by the user and ranks consistently higher than the other 4 in all 12 combinations of accuracy metric and image set, significantly out-performing T1<sup>c</sup> once, T3<sup>b</sup> twice and T3<sup>a</sup> three times (notwithstanding the possibility of false significance by the multiple comparison effect).

Fully automated contouring has the potential to reduce the user-time involved, whereas contouring speed is not included in the present evaluation strategy. This study focuses on accuracy, given that even fully automatic results can in principle be edited by medical professionals, who ultimately decide how much time is justified for a given treatment plan as well as just where the final contours should lie. The CPU-time of the more computationally expensive algorithms could be quantified as the subject of further work, but its relevance is debatable given that CPUs have different speeds and large data sets can be processed off-line, allowing the medical professional to work on other parts of a treatment in parallel.

C. Building prior knowledge into contouring

As already seen from figure 9 method WS<sup>a</sup> consistently gave the lowest accuracy. This method was adapted from an algorithm designed for segmenting remote sensing imagery and its user declared no expertise in medical image analysis. Conversely, two methods were adapted for the application of PET oncology, from other areas of medical image segmentation. Method PL<sup>a</sup> has origins in white matter lesion segmentation in brain MRI and method PL<sup>b</sup> is adapted from segmentation of lung nodules in CT images. These two examples far out-perform method WS<sup>a</sup>, with method PL<sup>b</sup> having the joint second highest median ranking for patient images and no significant difference from the most accurate methods in terms of any metric for any image set.

Some methods were designed for PET oncology, incorporating numerical methods to overcome known challenges. Examples are method GR that overcomes poorly defined gradients around small volumes due in part to partial volume effects, and method PL<sup>j</sup> allows for regional heterogeneity that is known to confound PET tumour segmentation. These methods rank reasonably highly, in patient images, ranking similarly to all manual delineations and the semi-automatic ‘smart opening’ algorithm (PL<sup>f</sup>), despite neither GR nor PL<sup>j</sup> having any user intervention or making any use of simultaneous CT. Method PL<sup>j</sup> performs relatively poorly in phantom images, where the problem of tissue heterogeneity is not reproduced.

The benefits of prior knowledge are also revealed by comparing 3 thresholding schemes T4<sup>a</sup>, T4<sup>b</sup> and T4<sup>c</sup> used by the same institution (team 04). Of these, method T4<sup>a</sup>
was considerably less accurate in terms of both n(n.s.d) and median rank. Methods T4\(^b\) and T4\(^c\) were calibrated using phantom data to build in prior knowledge of the imaging device. Even though the two devices used to calibrate T4\(^b\) and T4\(^c\) are from different vendors (Siemens and Biograph devices) than the one that acquired the test images (GE Discovery), they are consistently more accurate than method T4\(^d\) implemented at the same site, which does not learn from scanner characteristics but instead has an arbitrary parameter (39%). Methods T4\(^b\) and T4\(^c\) also out-perform the majority of the other low-interactivity thresholding schemes, suggesting that the calibration is beneficial and generalises across imaging devices. This apparent generalisation is further evidenced by no significant differences between methods T4\(^b\) and T4\(^c\) in any individual metric for patient or phantom VOIs.

Finally, the low accuracy of methods T4\(^a\) and T4\(^d\) may be due to erroneous prior knowledge. These two implementations of the same algorithm [6] inherently approximate the volume of interest as a sphere. Both perform poorly, with median ranking from 4 - 7 over all 4 metrics in contouring both phantom and patient VOIs. These low accuracies are likely to arise from the spherical assumption rather than the initialisation of the method, as the low accuracies are similar despite different methods of initialisation described in section II.

D. Accuracy evaluation

Accuracy measurement is fundamentally flawed in many medical image segmentation tasks due to the ill-definition of the true surface of the VOI. It is most common to estimate the ground truth by manual delineation performed by a single expert (e.g. [53], [19], [54]). However, even among experts, inter- and intra-operator variability are inevitable and well documented in PET oncology [21], [22]. The new metric AUC\(^g\) exploits this variability in a probabilistic framework, and we have also defined a single 'best guess' ground truth, for use with traditional metrics of DSC and HD, from the union of a sub-set of expert contours. For patient VOIs, the I-ROC scheme incorporates knowledge and experience of multiple experts as well as structural and clinical information into accuracy measurement and rewards the ability of an algorithm to derive the same information from image data. The I-ROC method considers all ground truth estimates to be equally valid a priori, and any one estimate can become the operating point on the I-ROC curve built for a given contour under evaluation. This is in common with the Simultaneous Truth and Performance Level Estimation (STAPLE) algorithm by Warfield et al. [55]. Theirs is also a probabilistic method, which uses maximum likelihood estimation to infer both the accuracy of the segmentation method under investigation and an estimate of the unique ground truth built from the initial set.

Other authors have evaluated segmentation accuracy using phantoms. The most common phantoms used in PET imaging contain simple compartments such as spherical VOIs, attempting to mimic tumours and metastases in head and neck cancer [10], [12], lung nodules [56] and gliomas [20] and cylindrical VOIs, attempting to mimic tumours [37]. The ground truth surface of such VOIs is precisely known due to their geometric form, but many segmentation algorithms are confounded by irregular surfaces and more complex topology such as branching seen in clinical cases and in the new phantom presented here. Another limitation of phantom images including those used here is the difficulty of mimicking heterogeneous or multi-focal tumours as seen in some clinical data.

Digital images of histological resection can in some cases provide unique ground truth, removing the need to combine multiple estimates. A recent example demonstrates this for PET imaging of prostate cancer [57]. While this approach could provide the standard for accuracy evaluation where available, histology-based accuracy measurement is currently limited as described in [58], with errors introduced by deformation of the organ and co-registration of digital images (co-registration in [57] required first registering manually to an intermediate CT image). Furthermore, tumour excision is only appropriate for some applications. For head-and-neck cancer, the location of the disease often calls for non-invasive, in vitro treatment by radiotherapy and in such cases the proposed use of multiple ground truth estimates may provide a new standard.

Neither deterministic metrics with flawed, unique ground truth (DSC and HD) nor probabilistic methods like I-ROC or STAPLE, measure absolute accuracy. However, the relative accuracy of methods or method groups is of interest to our aim of guiding algorithm development. For this purpose, a large and varied cohort of segmentation methods is desirable, and the composite metrics based on method ranking, distributions of accuracy scores n>(µ+σ) and the frequency of having no significant reduction in accuracy with respect to the most accurate n(n.s.d) become more reliable as the number of contouring tools increases. However, without a simultaneous increase in the number of VOIs, significance tests of the difference in accuracy of any one pair of methods becomes less reliable due to multiple comparison effects.

VI. Conclusions

The multi-centre, double-blind comparison of segmentation methods presented here is the largest of its kind completed for VOI contouring in PET oncology. This application has an urgent need for improved software given the demands of modern treatment planning. The number and variety of contouring methods used in this paper alone confirms the need for constraint, if the research is to converge on a small number of contouring solutions for clinical use.

We found that structural images in hybrid PET/CT, now commonly available for treatment planning, should be used for visual reference during semi-automatic contouring while the benefits of high-level CT use by multispectral calculations are revealed only by the new accuracy metric. We also concluded that higher levels of user interaction improves contouring accuracy without increasing intra- or inter-operator variability. Indeed, manual delineation overall out-performed all semi-or fully-automatic methods. However, two methods (T2\(^b\) and PL\(^J\)) with a low-level of interactivity and two automatic methods (PL\(^a\) and PL\(^b\)) are characterized by accuracy scores that are frequently not significantly different from those of the best...
manual method. Contouring research should pursue a semi-automatic method that achieves the same level of accuracy as expert manual delineation, but must strike a balance between (i) guiding manual practices to reduce levels of variability and (ii) not over-influencing the expert or overriding his or her knowledge. To strike this balance, techniques that show promise are (i) visual guidance by both CT and PET-gradient images, (ii) model-based handling of heterogeneity and blurred edges that characterise oncological VOIs in PET and (iii) departure from the reliance on the SUV transformation and iso-contours of this parameter or another scalar multiple of PET intensity, given its dependence on the imaging time window and countless other confounding factors.

These results go a long way towards constraining subsequent development of PET contouring methods, by identifying and comparing the distinct components and individual methods used or proposed in research and the clinic. In addition, we provided detailed results and statistical analyses in supplementary material for use by others in retrospective comparisons according to criteria or method groups not attempted here, as well as access to the test images and ground truth sets [49] that can be used to evaluate other contouring methods in the future. While our tests focused on head-and-neck oncology, only the fixed threshold method T1 made any assumptions about the tracer or tumour site so results for the remaining methods tested here provide a benchmark for future comparisons.

Recently proposed methods in [11], [12] and [59] would be of particular interest to test. However, if the number of tested methods increases without increasing the number of VOIs, the chance of falsely finding significant differences between a pair of methods increases due to the multiple comparison effect so the composite metrics are favoured over pair-wise comparisons for such a benchmark.

Future work using the data from the present study should categorise the 30 methods in terms of user-group and compare segmentation methods in more head and neck VOIs. Future work with a larger set of test data (images and VOIs) is expected to provide more statistically significant findings and should repeat for VOIs outside the realm of FDG in head-and-neck cancer and for images of different signal/background quality. For this purpose the experimental design including phantom, accuracy metrics and the grouping of contemporary segmentation methods, will generalise for other tumour types and PET tracers.

VII. ACKNOWLEDGEMENTS

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APPENDIX

In order to derive the new accuracy metric and explain its probabilistic nature, we recall the necessary components of conventional receiver operating characteristic (ROC) analysis, then demonstrate the principles of inverse-ROC (I-ROC) for a simple data classification problem and explain the extension to topological ground truth for contour evaluation.

A.1 Conventional ROC: multiple decision makers

Receiver operating characteristic (ROC) analysis is well established in medical imaging as a means of evaluating region- and voxel-wise data classification [60]. Data comes in the form of $N = N_+ + N_-$ measurements, comprising $N_+$ 'positive' data with truth labels +1 and $N_-$ 'negative' data with labels −1. A binary classifier divides all $N$ data into positive and negative sets, and has at least one internal parameter that affects this division. ROC analysis is performed by varying an internal parameter in $p$ increments. In threshold classification, the threshold is the internal parameter and data above the threshold are counted as either true positive (TP) or false positive (FP) according to agreement or otherwise, with the ground truth labels. Similarly, true negative (TN) or false negative (FN) classifications are counted below the threshold.

The counts $N_{TP}$, $N_{FP}$, $N_{TN}$ and $N_{FN}$ of true/false positives and negatives yield the true positive ratio $TPR_i$ and false positive ratio $FPR_i$ for the $i$th threshold and the pair $\{TPR_i, FPR_i\}$ becomes a single point on a ROC curve. The whole curve is generated by varying the internal parameter between natural limits. For the threshold classifier in figure 10, the limits are the minimum and maximum value in all N data. The fixed ground truth in figure 10 are drawn from Gaussian distributions with $\mu_+ = 3.0$, $\mu_- = −3.0$ and $\sigma_+ = \sigma_- = 2.5$.

The ROC curve occupies the range $\{0 \dots 1\}$ in both $TPR$ and $FPR$ and has two limiting cases. The first limit is the diagonal line (--- in figure 10) which has an area under the curve (AUC) of 0.5 and indicates failure to classify data better than random assignment of labels ±1. The second limiting case (--- in figure 10) has AUC = 1 and indicates perfect classification. As a result, AUC is commonly used as a measure of classifier accuracy. ROC analysis simultaneously yields the operating point of the classifier, defined as the internal parameter setting (e.g. threshold) that minimises the combined cost of false positives and false negatives.

If positive and negative ground truth are normally distributed, the ROC curve has exponential form and AUC can be calculated by fitting an analytic function and integrating between the limits 0 to 1. In this case, AUC is a monotonic function $z^{-1}$ of the distance between the means $\mu_+, \mu_-$ of the true distributions, scaled by their standard deviations $\sigma_+, \sigma_-$, where

$$z(AUC) = \frac{\mu_+ - \mu_-}{\sqrt{\sigma_+^2 + \sigma_-^2}}$$

and AUC is equal to the Gaussian probability that a measurement drawn at random from the positive set will be correctly classified. If the assumption of normally distributed data is relaxed the probabilistic interpretation still holds, where the probability is that sought by a Wilcoxon signed ranks test and AUC is evaluated using the trapezium rule [61].

In summary, AUC is a probabilistic measure regardless of the underlying distributions and ROC analysis can be used as a metric combining sensitivity and specificity.
A.2 I-ROC: multiple ground truth representations

The new ROC technique is referred to as inverse as, rather than unique ground truth labelling and various arbitrary decision makers, it assumes a single classification and varies the definition of ground truth. Figure 11 demonstrates this for the example of threshold classification. In common with figure 10, data being classified in figure 11 are a mixture of Gaussians with means $\mu_-$ and $\mu_+$ separated by 6 units and standard deviations $\sigma_- = \sigma_+ = 2.5$, and the total number of data is fixed at $N_+ + N_- = 2 \times 10^4$. To simulate a change in ground truth labelling for the same underlying data, the means of the positive and negative distributions are shifted by $\delta_i$ so that $\mu_- = -3.0 + \delta_i$ and $\mu_+ = 3.0 + \delta_i$, where $\delta_i$ increases from an arbitrary (negative) minimum to an arbitrary (positive) maximum in $p = 19$ increments, and the proportion $\rho$ of data in the positive set decreases as $\rho = 1 - i/p$. To classify data that has the $i^{th}$ ground truth labelling, we fix the threshold at $T=0$ for all $i \in \{1 \ldots p\}$. In line with the requirements of conventional ROC, the multiple ground truth definitions are

A.2(i) ordered by monotonically (in-) decreasing $N_-$,
A.2(ii) obtained by independent means, not the threshold classifier being evaluated,
A.2(iii) incorporate the best knowledge of the unique (unknown) ground truth, and
A.2(iv) ‘pass through’ the unique (unknown) ground truth as closely as possible.

Requirement A.2(iii) is realised by fixing the difference of means $\mu_+ - \mu_-$ and having $\rho$ increase with $\mu_+$. Requirement A.2(iv) means that there exist labellings $\{GT_i\}$ and $\{GT_{i+1}\}$ with $\mu_+$ and $N_+$ (similarly $\mu_-$ and $N_-$) either side of the operating point.

The shape of the ROC curve in figure 11, the operating point and, within the accuracy of the trapezium integration, the AUC are the same for the I-ROC as for the equivalent analysis in figure 10 by virtue of the choice of parameters, which merely serves to illustrate the ability to perform equivalent ROC analyses by shifting decision maker (ROC) or ground truth labelling (I-ROC).

A.3 I-ROC with topographic ground truth

In the context of VOI contouring, the notion of ‘positives’ refers to voxels inside a contour, which is a spatial distinction
and may or may not correspond to voxel values above a threshold. Truth labels in turn are separated by a surface in image space, and stored as a binary mask of \( \{ \pm 1 \} \). We refer to \( \{ \mathcal{G}_i \} \) as a contour or mask interchangeably. The I-ROC method evaluates the accuracy of a fixed result of a contouring algorithm denoted \( C \), using a set of arbitrary ground truth masks \( \{ \mathcal{G}_i \}, i \in \{ 1 \ldots p \} \). The term ‘arbitrary’ refers to the fact that no single mask in the set is the closest \( a \)-priori to the unknown, unique ground truth and does not mean that their shapes are arbitrary. Following from the requirements for the shifting threshold in A.2, the natural limits \( \mathcal{G}_1 \) and \( \mathcal{G}_p \) contain none and all of the image voxels (inside a bounding box) respectively and the set \( \{ \mathcal{G}_i \}, i \in \{ 1 \ldots p \} \).

A.3(i) is ordered monotonically by volume where \( \mathcal{G}_i \) completely encloses \( \mathcal{G}_{i-1} \).

A.3(ii) is obtained independently of the contouring algorithm under evaluation.

A.3(iii) incorporates the best available knowledge of ground truth, and

A.3(iv) ‘passes through’ the unknown, unique ground truth surface as closely as possible.

Requirement A.3(i) can always be met by defining each \( \mathcal{G}_i \) as the union of contours from an original set. Requirements A.3(ii) and (iii) can also always be met, whereby suggested sources of independent information are complementary imaging or clinical information unseen to the tool under evaluation.

Requirement A.3(iv) means that topology and general shape are conserved within the set as in the analogy of inflating a novelty balloon, and can also always be met by the procedure used to obtain all \( \mathcal{G}_i \), such as the suggested use of union masks.

If the general shape common to all \( \{ \mathcal{G}_i \} \) is representative of the unknown ground truth then AUC is higher when the contour under evaluation shares this shape. Figure 12 demonstrates this for the case where the ground truth set has a different (a) and the same (c) shape as a circular contour \( C \) under evaluation. Using a square ground truth set (a) gives AUC < 1, equivalent to the case of overlapping histograms in figure 11, although the similar form of the curve and value of AUC = 0.98 are only due to the parameters and shapes used for illustration. A circular set, chosen for its agreement with \( C \) to illustrate the possibility of achieving AUC = 1, indicates perfect contouring accuracy. More generally, AUC approaches 1 as the contour \( C \) approaches any contour in the set \( \{ \mathcal{G}_i \} \) and this indicates perfect agreement with the general form of the unknown, unique ground truth all in the set \( \{ \mathcal{G}_i \} \) share this form. It follows that AUC is equal to the probability that a voxel drawn at random from inside the optimal \( \mathcal{G}_i \), which is not known \( a \)-priori, lies inside the contour \( C \) being evaluated.

Formally, the I-ROC method will generalise for any shape of ground truth set or contour under evaluation if

\[
N(c \mathcal{G}_j) = \sum_{i=1}^{j} N(\mathcal{G}_{i} \cup \mathcal{G}_j) \quad \text{and} \quad (a)
\]

\[
N(c \mathcal{G}_j) + N(\mathcal{G}_j) = \text{constant} \quad \forall j \quad (b)
\]

where \( N(c \mathcal{G}_j) \) and \( N(\mathcal{G}_j) \) denote the number of voxels inside and outside the \( j \)-th ground truth definition. Equation 6(a) holds if requirement A.3(i) is met and 6(b) is satisfied by the fixed bounding box enclosing the set \( \{ \mathcal{G}_i \} \).

REFERENCES


Fig. 12: Inverse-ROC analysis of a fixed contour (red circle) performed by varying ground truth contours as squares (a) or circles (c) of increasing size. ROC curves in (b) and (d) are built from the corresponding true and false counts that lie inside or outside the $i$th ground truth contour. Operating points are shown in green.