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Nonlinear ultrasound simulations including complex frequency dependent attenuation

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Abstract

Including non-squared frequency dependent attenuation is essential to obtain accurate acoustic predictions in biological media when wide-band signals and/or nonlinear effects are taken into account. Thus, a time domain nonlinear acoustics model is presented, where the dispersion and attenuation are included by means of relaxation processes. In this way, an efficient implementation by finite differences avoiding convolutional operators is developed. By optimizing a pair of relaxing parameters, the model exhibits an attenuation frequency response that fits a power law experimental data for most biological tissues. In this way, it is possible to obtain arbitrary frequency dependent attenuation and dispersion in order to model biological media. Furthermore, due to the generalized formulation, typical relaxation processes can be modeled as those observed in air or seawater, and used to model the losses of longitudinal waves observed in other complex heterogeneous media, such as soil or porous rock.

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1. Introduction

When accounting for finite amplitude acoustic perturbations in biological media the inclusion of the correct frequency dependent attenuation and dispersion is critical to accurately model nonlinear effects in the propagation. Thus, the dynamics of the microscopic-heterogeneous structure of the biological media for wavelengths bigger than the microstructure can be modeled by a macroscopic-homogeneous attenuation and dispersion. In this way, the attenuation presents a complex frequency dependence that in most cases can be approximated by a frequency power-

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law dependence as $\alpha(f)=\alpha_0 f^{\gamma}$ (Duck, 1990, Hill et al., 2004) where the exponent, γ, is close to unity. In order to include the observed losses in the acoustic equations, there exist numerous phenomenological approaches (Hill et al., 2004, Wismer et al., 1995). On the other hand, it is common in literature to describe the losses in soft tissues as multiple-relaxation processes (Pierce, 1989, Nachman et al., 1990, Hill et al., 2004), where the relaxation frequencies can be associated to either a tissue specific physical mechanism or empirically optimized to fit the observed tissue attenuation (Cleveland et al., 2009, Pinton et al., 2009). Moreover, fractional partial differential operators have demonstrated the ability to describe frequency power law attenuation (Szabo, 1994, Prieur et al., 2011). These operators can be included in the modeling by means of time (Szabo, 1994), space (Chen et al., 2004) or combined time-space fractional derivatives (Caputo 1967, Wismer, 2006). In order to solve these models, many time-domain numerical methods have been developed. Attenuation modeled by relaxation processes can be solved by means of finite-differences in time-domain (FDTD) solvers in linear regime (Yuan et al., 2004) and in nonlinear regime applied to augmented Burger's equation (Cleveland et al., 2009), Khokhlov-Zabolotskaya-Kuznetsov (Yuan et al., 2005) and Westervelt (Pinton et al., 2009) nonlinear wave equations. On the other hand, time-dependent fractional derivatives can be solved in nonlinear regime (Liebler et al., 2004) by convolutional operators. This approach requires the memory storage of certain time history, and although the memory can be strongly reduced compared to direct convolutions, this algorithm employs up to ten auxiliary fields and a memory buffer of three time steps. In order to overcome this limitation, time-space fractional derivatives or fractional Laplacian can be used to model frequency power laws without time-domain convolutional operators (Treeby et al., 2010, Chen at al., 2004). Recently *k*-space and pseudo-spectral methods have been applied in order to solve fractional Laplacian operators efficiently in nonlinear regimes.

The aim of this work is to summarize a numerical method that solves the complete set of equations (continuity, momentum and state equations) including nonlinear propagation and frequency power law attenuation based on multiple relaxation processes.

2. Physical model

The main constitutive relations for nonlinear acoustic waves for a viscous fluid can be expressed as (Naugolnykh and Ostrovsky, 1998)

$$\frac{\partial \rho}{\partial t}=-\nabla\cdot\left(\rho\mathbf{v}\right) \ , \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\eta\nabla^2\mathbf{v}+\left(\zeta+\frac{\eta}{3}\right)\nabla\left(\nabla\cdot\mathbf{v}\right), \tag{2}$$

where ρ is the total density field, $\mathbf{v}$ is the particle velocity vector, p is the pressure, η and ζ are the coefficients of shear and the bulk viscosity respectively. The acoustic waves described by this model exhibit viscous losses with squared power law dependence on frequency. In order to include a power law frequency dependence on the attenuation, a multiple relaxation model will be added into the time domain equations.

The basic mechanism for energy loss in a relaxing media is the appearance of a phase shift between the pressure and density fields. This behavior is commonly modeled as a time dependent connection at the fluid state equation, that for a fluid retaining the nonlinear effects up to second order an be expressed as (Naugolnykh and Ostrovsky, 1998, Rudenko et al., 1977)

$$p=c_0^2\rho'+\frac{c_0^2}{\rho_0}\frac{B}{2A}\rho'^2+\int_{-\infty}^{t}G(t-t')\frac{\partial\rho'}{\partial t}dt, \tag{3}$$

where $\rho'=\rho-\rho_0$ is the density perturbation over the stationary density ρ_0, B/A is the nonlinear parameter, c_0 is the small amplitude sound speed, and $G(t)$ is the kernel associated with the relaxation mechanism. The first two terms describe the instantaneous response of the medium and the convolutional third term accounts for the "memory time" of the relaxing media. Thus, by choosing an adequate time function for the kernel $G(t)$ the model can present an attenuation and dispersion response that fits the experimental data of the heterogeneous media. If a sum of N exponential forms of the kernel $G(t)$ is taken into account, the integral form of the state eq. (3) leads to

$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \sum_{n=1}^{N} S_n. \tag{4}$$

Here the "frozen" sound speed for N mechanisms is defined as

$$c_\infty^2 = c_0^2 \left(1 + \sum_{n=1}^{N} \eta_n \right), \tag{5}$$

and each state variable S_n obeys

$$\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n} S_n + \frac{\eta_n c_0^2}{\tau_n} \rho' \tag{6}$$

for each relaxation process. Model equations (1-2, 4, 6) are solved by finite differences in time domain method (FDTD). Thus, a space-time staggered discretization is employed and central finite differences operators are applied for solving both spatial and time differential operators.

3. Validation

3.1. Single relaxation

In order to validate the frequency dependent attenuation and dispersion of the numerical method a simulation was done in linear regime including a single relaxation process. A plane wave front traveling in $+z$ direction was considered. Thus, at $z = 0$ the media was excited with a negative second derivative of a Gaussian function:

$$p(t, z_0) = p_0 \left(1 - 2 (\pi f_0 t)^2 \right) e^{-(\pi f_0 t)^2}. \tag{7}$$

Here, the central frequency of the broadband signal was set to $f_0 = 10$ MHz, and the amplitude was set to $p_0 = 1$ μPa, small enough to neglect the nonlinear propagation terms. As the wave propagates, a single relaxation process changes the complex amplitude as $\exp(-az)$, where a can be expressed as (Naugolnykh and Ostrovsky, 1998):

$$a = \frac{\omega \eta_1 (i\omega^2 \tau_1^2 - \omega \tau_1)}{2c_0 (1 + \omega^2 \tau_1^2)}. \tag{8}$$

The results, plotted in Fig. 1, show excellent agreement between the attenuation and dispersion obtained by numerical solution and those predicted by the theory for a single relaxation process, demonstrating that the inclusion of relaxation processes in nonlinear equations by means of the proposed model exhibit attenuation and dispersion in correctly. However, typical FDTD numerical dispersion can be observed in the high frequency limit ($f > 20$ MHz), where the cumulative phase error leads to phase velocity mismatching. This error can be mitigated in the present numerical algorithm by increasing the number of elements per wavelength.

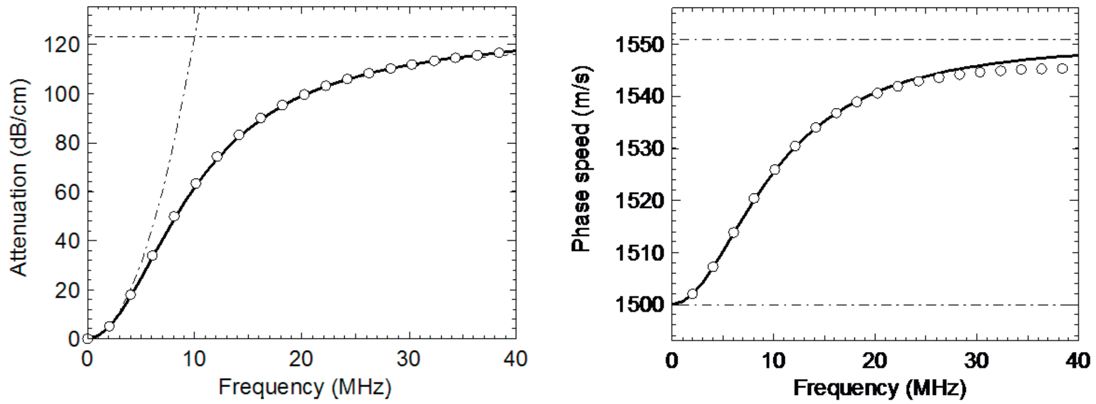

Fig. 1 Left: Attenuation retrieved by the numerical method (circles), theoretical (continuous line), low frequency limit (ω^2) and high frequency limit ω^0 (dashed line). Right: Dispersion retrieved by the numerical method (circles), theoretical (continuous line), low frequency limit (c_0) and high frequency limit (c_∞).

3.2. Modelling frequency power law attenuation

An optimization algorithm has been used to fit the numerical attenuation response due to multiple relaxation to a frequency power law $\alpha(f) = \alpha_0 f^\gamma$. In order to find the proper relaxation coefficients, this algorithm uses the *fmincon* function in the optimization toolbox in MATLAB v7.13. Thus, an optimization of the relaxation times τ_n and relaxation modulus η_n values has been done in order to minimize the relative error between the target power law and the computed attenuation. Only two independent relaxation processes were employed in this section to obtain the target frequency power laws that are plotted in Fig. 2 for different frequency power laws covering the range of that observed in tissues $\gamma = [1, 1.3, 1.6, 2]$ over the typical frequency range for medical ultrasound applications, i. e. 1 to 20 MHz.

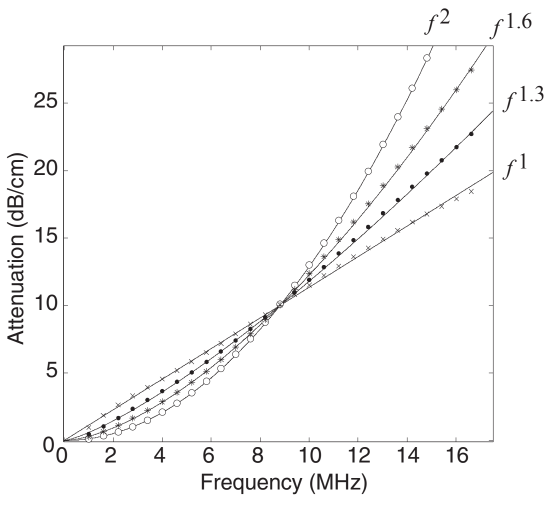

Fig. 2. Numerical frequency power law attenuation for γ=2 (white circles), γ=1.6 (asterisk), γ=1.3 (black circles), γ=1 (cross).

3.3. Modelling relaxation processes in air and seawater

Due to the general formulation of the relaxation in the present model, a description of other complex frequency dependent attenuation can be obtained if correct values for the relaxation parameters are chosen. Thus, Fig. 3 shows the frequency dependence of the attenuation for classical relaxation processes of oxygen and nitrogen in air and boric acid and magnesium sulfate in seawater.

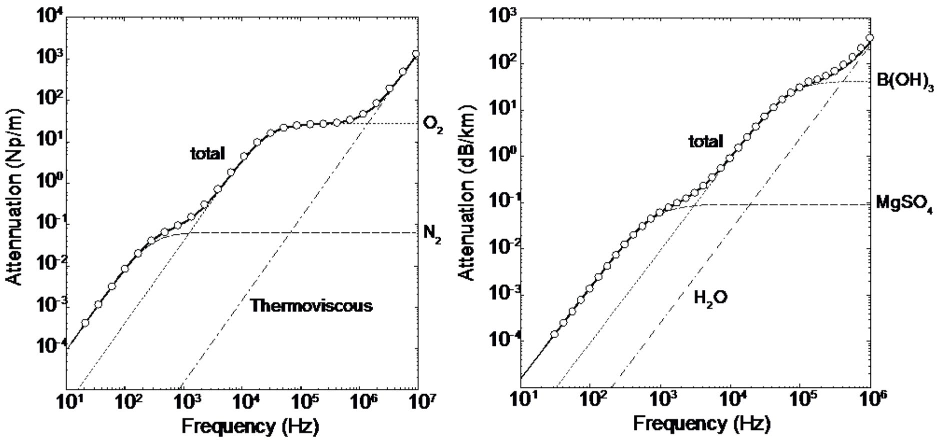

Fig. 3. Attenuation retrieved by the numerical method for the relaxation processes of oxygen and nitrogen in air (left), and magnesium sulfate and boric acid in seawater (right)

3.4. Modelling nonlinear effects

In order to validate the method in the nonlinear regime a full-wave simulation was developed in a monorelaxing medium. Thus, the analytical (inverted) solution for the steady solution with $p = -p_0$ for $\tau = -\infty$, $p = +p_0$ for $\tau = \infty$ and $p = 0$ for $\tau = 0$, for the retarded time $\tau = t - x/c_0$ reads

$$\tau = \tau_n \ln \frac{\left(1 + p/p_0\right)^{D-1}}{\left(1 - p/p_0\right)^{D+1}} \tag{9}$$

Where $D = \eta_n \rho_0 c_0^2 / 2\beta p_0$ measures the ratio of relaxation effects to nonlinear effects. Numerical and analytical solutions of Eq. (9) are shown in Fig. 4, where good agreement is achieved.

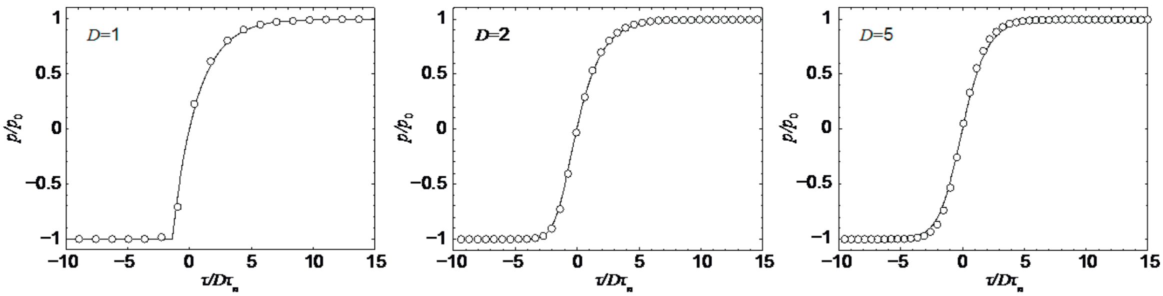

Fig. 4 Numerical (circles) and analytical (solid line) solutions of a step waveform in a monorelaxing media for D= 1, 2 and 5.

4. Conclusions

A generalized formulation of finite amplitude waves in multiple relaxation media has been developed for the constitutive relations of nonlinear acoustics. Thus, a numerical method based on finite differences in time domain has been developed and validated, where excellent agreement between numerical and analytical solutions has been observed for multiple relaxing media (relaxation processes of air and seawater), frequency power-law attenuation media (soft-tissue like) and monorelaxing media in nonlinear regime.

Due to the model being developed from the constitutive relation for nonlinear acoustics, most wave phenomena is captured. As a difference from the one-way models the proposed model implicitly includes multiple wave direction, and, due to the Lagrangian density is computed exactly, multiple scattering and resonance effects are accurately described (unlike Westervelt-type nonlinear-wave equations). Moreover, unlike Augmented Burgers, KZK and other parabolic approximations, the proposed model captures the diffraction exactly, so for simulation of acoustic beams the field is not approximated only on the beam axis, but also in the near field, far from the beam axis and thus high focusing devices can be simulated. The main counterpart of the proposed model is the FDTD solver, where the numerical dispersion can lead to huge errors in the higher harmonics, even with fine meshes if the propagation distance increases, and due the description of shock waves (numerical discontinuities) is limited to viscous media.

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