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DOUBLE SAMPLING IN ATTRIBUTE CONTROL CHARTS

Miguel Angel SELLES[1], Elena PEREZ-BERNABEU[2], David JUAREZ[3], Miguel Angel PEYDRO[4]
[1]Universitat Politècnica de Valencia, maselles@dimm.upv.es
[2] Universitat Politècnica de Valencia, elenapb@eio.upv.es
[3] Universitat Politècnica de Valencia, djuarez@mcm.upv.es
[4] Universitat Politècnica de Valencia, mpeydro@mcm.upv.es

Abstract—A double sampling is applied to attribute control charts, using the methodology that Daudin developed for the X-bar charts. Daudin suggested taking two samples at the same time, and, depending on the circumstances, analyzing the second sample when necessary. This is a simple and easy method for decision making at this level, which has been shown to be effective and efficient for specific cases.

Keywords— Quality tool, attribute control charts, quality management, false alarm risk, average sample size.

I. Introduction

A control chart is widely used in industry and services to monitoring a process evolution along time. Data are gathered and plotted in time order. It allows to the process responsible to distinguish between the reasons of variation that can appear: The variability owed to common reasons that allow us to predict the state of the process or the variability owed to special reasons, which do not allow us to predict the situation of the process in a certain moment. According to the type of data, there are different kinds of control chart that can be implanted.

There are control charts for the mean, dispersion, defective items proportion and number of defects proportion or its frequencies.

When controlling the mean, we can highlight the following:

1) *Moving Averages [1]*
2) *CUSUM [2-3]*
3) *EWMA[4-5]*
4) *Sample Mean[6]*

Furthermore, it is possible to use any of these charts applying different methodologies, such as:

1) *PC (Constant Parameters).*
2) *VSS (Variable Sample Size). The sample size is not kept constant, but it is adaptable to the process, depending on its behavior.[5-7]*
3) *VSI (Variable Sample Interval). The sampling interval is not kept constant, varying it according to the behavior of the process.[8]*
4) *VSSI (Variable Sample Size and Sampling Interval). It is a combination of VSS and VSI. [6]*
5) *Daudin. This methodology uses double sampling to improve decision making aspects. [1]*

The control charts can be classified into those that are suitable for variable data and those that fit best to attribute variables. The attribute control chart can be:

1) *p chart (proportion chart)*
2) *np chart*
3) *c chart (count)*
4) *u chart.*

We are going to study the last one in depth, and implement a double sampling system to try to improve its performance and show its characteristics. We want to define a control chart using double sampling in such a way that we improve the power of the traditional *u* chart and, at the same time, without increasing the average sample size.

II. Experimental Research

The main purpose is to make a comparison between the classic u-chart and a modified u-chart (by now DS-u chart), based on J.J. Daudin's methodology [1]. Daudin modifies the Shewhart chart applying a double sampling, so that way, it improves the behavior of the chart.

In this work, it is used Daudin's strategy to the attribute control chart, concretely, to the u-chart. As the parameters seen in Table I, the modified u-chart has got a two stage scheme, with new control limits and sample size in each stage.

We have to calculate these new parameters, maintaining the most similar false alarm risk, α, and the sample size (or reducing the last one) of the classical u-chart.

The new α is calculated using (1), where RA_1 means Reject Area of the first step; AR_1 means Attention Area of the first step, and RA_2 means Reject Area of the second step. This is also represented in Fig. 1.

$$\boldsymbol{\alpha_{DS\ U} = P(u_i \in RA_1) + P(u_i \in AA_1 \ \cap \ u_i \in RA_2)} \quad (1)$$

TABLE I

PARAMETERS OF THE DS-U CHART

NAME	Definition
LCL	Lower control limit of the first stage
LAL	Lower attention limit for the first stage.
UAL	Upper attention limit for the first stage.
UCL	Upper control limit for the first stage.
LCL1	Upper control limit for the second stage.
UCL1	Upper control limit for the second stage.
n_1	Sample size for the first stage.
n_2	Sample size for the second stage.
N	Average sample size.
p_0	Proportion of defectives in the under-control process.
p_1	Proportion of defectives in the out-of-control process.

Then, the graphical representation of the DS-u chart is shown in Fig. 1.

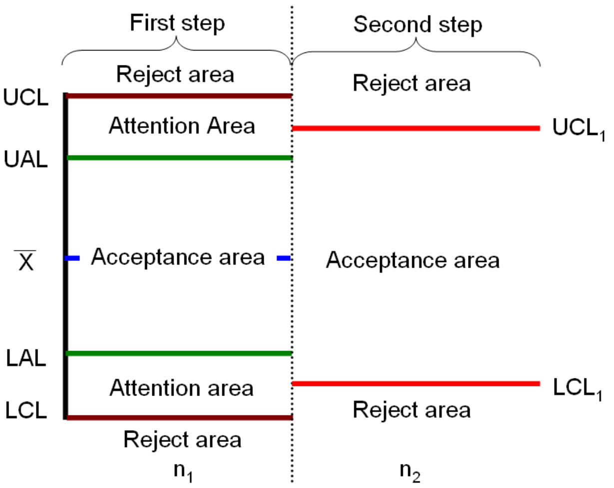

Fig. 1. DS-u Chart scheme.

III. METHODOLOGY

The method that has been chosen for calculating the new control limits of the two stages of the DS-u chart is software programming in C++.

It has been used Genetic Algorithms to get the better solutions. We used a library of free distribution, GAlib. It contains a set of C++ genetic algorithm objects. This library includes tools for using genetic algorithms to do optimization in any C++ program using any representation and genetic operators.

The parameters of the Genetic Algorithm have been selected following the rules showed in Martorell et al. [3].

As we want to know the behavior of the modified u-chart, we have selected some cases: when the defects per unit u_0 = 1, and three different false alarm risks: α= 0,027, α= 0,01, α= 0,05; and when the defects per unit u_0 = 5, and three different false alarm risks: α= 0,027, α= 0,01, α= 0,05 as seen in Table II.

TABLE II
CASES STUDIED

u_0	$\alpha_{theoretical}$	n	α_{real}
1	0.01	8	0.008566
1	0.05	8	0.047965
1	0.027	8	0.00371802
1	0.01	40	0.009132
1	0.05	40	0.047396
1	0.027	40	0.00363161
5	0.01	8	0.009131637
5	0.05	8	0.047396
5	0.027	8	0.003631613
5	0.01	40	0.009857263
5	0.05	40	0.051681
5	0.027	40	0.00269305

For analyzing data, it has also been considered calculating the maximum peak for the difference of the power between the new DS-U control chart and the classical u chart, as seen in Table III.

TABLE III
MAXIMUM PEAK VALUES FOR POT DS-U – POT U FOR U_0= 1

	n=8		n=40	
	Below u_0	Above u_0	Below u_0	Above u_0
α=0.0027	0.977	0.218	0.176	0.389
α=0.01	0.747	0.173	0.352	0.294
α=0.01	0.297	0.107	0.160	0.241

RESULTS AND CONCLUSIONS

Now, we are going to show some results obtained. We can consider that this modified u-chart is not improving its performance for all the cases, so it depends on what we need to improve.

The results obtained for the first of the cases studied, when n=8 and u_0=1, are shown in Fig. 2 and Fig. 3. It is not improved so much when u_1 is around u_0=1, and the opposite occurs when improving the power at u_1, being much lower o much greater than u_0=1. (Fig. 2). We also notice that the results are better at the time we reduce α.. In the cases that $u_1 < u_0$, the differences between the power are greater than the other case ($u_1 > u_0$), but while we improve the power difference in one value u_1, the power curve is worse than the classic u-chart for values of $u_1 > 1$.

Even we improve the power curve in u_0=1, the sample size mean is not reduced of the classic u-chart sample size. (fig. 3).

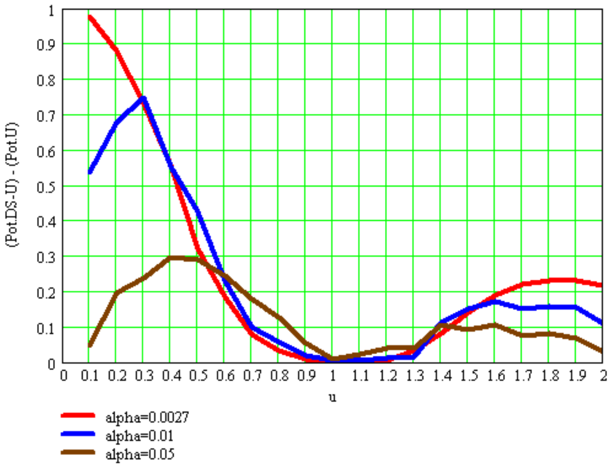

Fig. 2. Difference of power for n=8, u_0=1

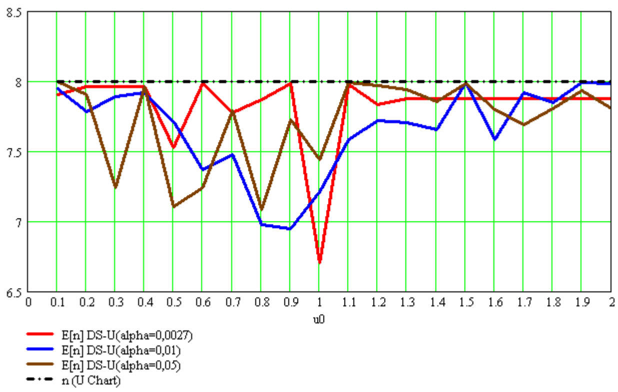

Fig. 3. Sample mean for n=8, u_0=1

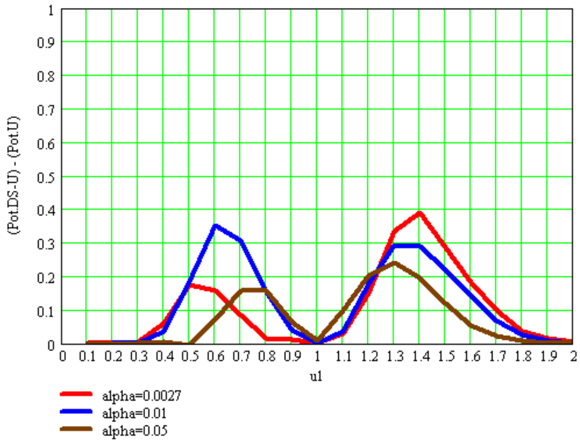

Fig. 4. Difference of power for n=40, u_0=1.

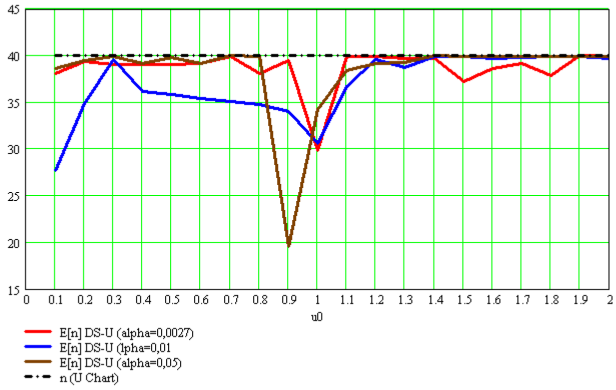

Fig. 5. Sample mean for n=40, u_0=1.

When we study the case in which n=40 and u_0=1, we obtain the results of Fig. 4 and Fig. 5. It is obtained better values of power for the new-DS-U chart, but the behavior varies in the different values of α. When α = 0,01, it looks quite symmetrical, but not for the other two values of α studied. For α=0,0027, we can consider it is very good for improving the power at u_1, being much greater than u_0=1 (Fig. 4).

For the mean sample size, the case that improves more is when α=0,01 and better for the cases in which u_1 is below u_0=1 (Fig. 5).

Now, we analyze the results obtained for the case in which n=8 and u_0=5. This case looks similar to the previous one, as we can see in Fig. 6. But in this case, the most symmetrical one happens when α = 0,05. In general the improvements are bigger when $u_1 > u_0$.

Regarding the sample size mean, as seen in Fig. 7, it doesn't follow a clear pattern, but it reduces the classical u-chart sample size when $u_1 < u_0$ and α>0,0027. For α=0,01 it also reduces the sample size when $u_1 > u_0$.

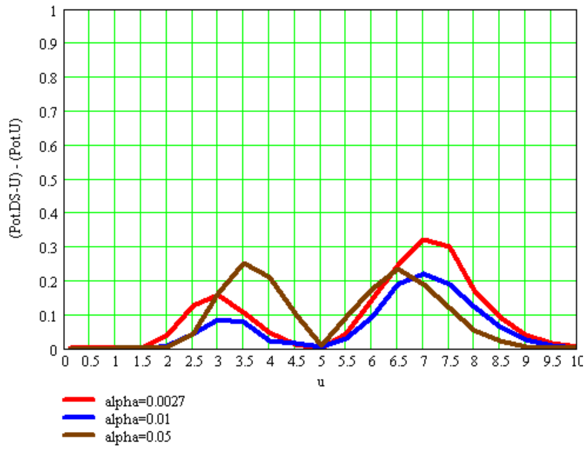

Fig. 6. Difference of power for n=8, u_0=5.

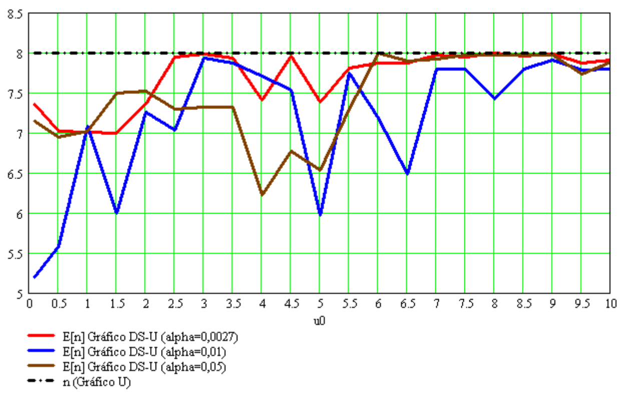

Fig. 7. Sample mean for n=8, u_0=5

If we study the case in which n=40 and u_0=5, we obtain the results of Fig. 8 and Fig. 9.

The values of power for the new-DS-U chart are not as good as in the previous cases. It just increases a little bit for values that are close to u_0=5.

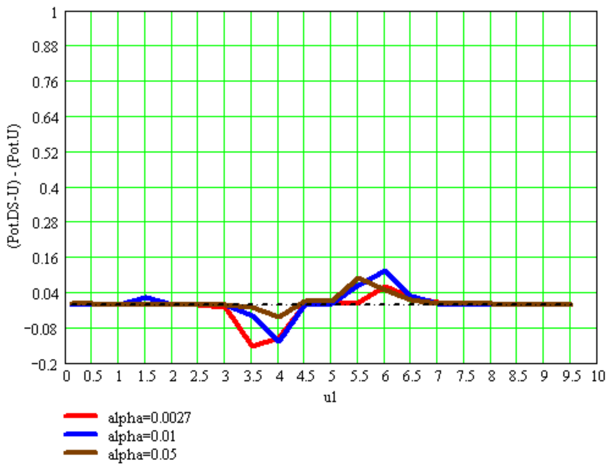

Fig. 8. Difference of power for n=40, u_0=5.

When considering the sample size mean in Fig. 9, we cannot figure out a pattern that allows us to know more about this particular case, as it looks random. But even not being a good chart for improving the power against the classical u-chart, it gets the advantage of reducing the sample size to really lower values compared to the original 40 units.

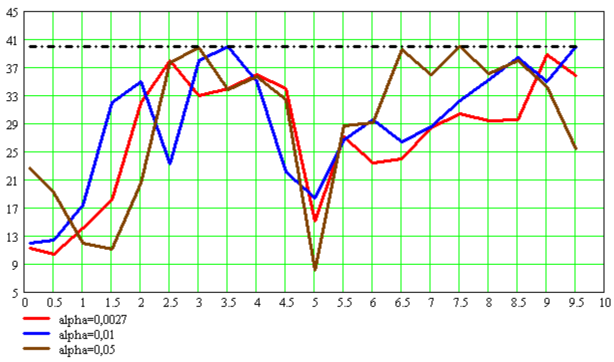

Fig. 9. Sample mean for n=40, u_0=5.

In general, we can examine two different situations in all the cases, when $u_1 < u_0$, and when $u_1 > u_0$.

As we can see in Fig. 2 to Fig.9, there is a completely different behavior according to different values of u_0, n and α. For many of the cases, it was found that the power improvement took place in a determined range of values for u_1 that were in a short distance from u_0. Regarding the sample size mean, it is possible, in general, to reduce it in many cases.

The most important cases have been analyzed to determine in which cases should be used this modified u-chart, DS-u chart.

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REFERENCES

[1] Daudin, J.J., 1992. Double Sampling X Charts,Journal of Quality Technology, 24 (2): 78-87
[2] Montgomery, D.C.,1991. Introduction to Statistical Quality Control, (Spanish edition) Grupo Editorial Iberoamérica
[3] Martorell, S. and Carlos, S. and Sánchez, A. and Serradell, V.,2000. Constrained optimization of test.
[4] Shewhart, W.A.. 1931. Economic Control of Quality. D.Van ¨Nostrand Company, Inc. Toronto. Canada.
[5] Costa, A.F.B. 1994. “ $\bar{X}$ Charts with Variable Sampling Size.” Journal of Quality Technology 26 (3): 155–163.
[6] Costa, A.F.B. 1997. “ $\bar{X}$ Chart with Variable Sampling Size and Sampling Intervals.” Journal of Quality Technology 29 (2): 197–204.
[7] Crowder, S.V. 1987. “A Simple Method for Studying Run-Length Distributions of Exponentially Weighted Moving Average Charts.” Technometrics 29:401–407.
[8] Lucas, J.M., and M.S. Saccucci. 1990. “Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements.” Technometrics 32:1–12.
[9] Page, E.S. 1954. “Continuous inspection schemes.” Biometrika 41:100–115.
[10] Prahbu, S.S., G.C. Runger, and J.B. Keats. 1993. “An Adaptive Sample Size $\bar{X}$ Chart.” International Journal of Production Research 31:2895–2909.
[11] Reynolds Jr, Marion R., Raid W. Amin, Jesse C. Arnold, and Joel A. Nachlas.1988. “ $\bar{X}$ Charts with Variable Sampling Intervals.” Technometrics 30 (2): 181–192.
[12] Roberts, S.W. 1959. “Control-Charts-Tests based on geometric moving averages.” Technometrics 1:239–250.