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Consensus over Multiplex Network To Calculate User Influence in Social Networks

M. Rebollo[1,†], E. del Val[1], C. Carrascosa[1], A. Palomares[1] and F. Pedroche[2]

[1] *Grupo Tec. Inform.-Inteligencia Artificial, Universitat Politècnica de València*

[2] *Inst. de Mat. Multidisciplinària, Universitat Politècnica de València*

Abstract. User influence determines how the information is transmitted. Most of the current methods need to consider the complete network and, if it changes, the calculations have to be repeated from the scratch. This work proposes the use of a consensus algorithm to calculate the influence of the participants in a social event through their interactions in Twitter. Retweets, mentions and replies and considered and represented in a multiplex network. The algorithm determines the influence of the users using only local knowledge.

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† **Corresponding author:** mrebollo@dsic.upv.es
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1. Introduction

User influence is a vital concept in social networks that measures the relative importance of the user inside the network referring to the diffusion of information [1, 2, 3]. In this work, the Twitter interactions associated to an event in the real world are analyzed. The engagement of the users is evaluated as the weighted sum of three measures: number of retweets, mentions and responses. Each interaction type is modeled as a layer in a multiplex network. A consensus algorithm [5] is used to compute the average reachability factor in a decentralized way, so any node in the network can determine its deviation from the mean value without any global knowledge about the network size nor its topology.

2. Methods

In networks of agents, consensus means to reach an agreement about a quantity of interest in a decentralized way [5]. Each agent only is aware of its direct neighbors and interacts just with them. Let be G an undirected graph and $x_i(0)$ the initial value of the variable for agent i. A simple algorithm to reach a consensus is defined by the expression

$$x_i(t+1) = x_i(t) + \varepsilon \sum_{j \in N_i} [x_j(t) - x_i(t)] \qquad (1)$$

where ε denotes the learning rate and $N(i)$ the neighbors of node i. The execution of this process in each node guarantees the convergence to the average value of the initial values $x_i(0)$.

The collective dynamics is given by $x(t+1) = Px(t)$, where $P = I - \varepsilon L$ is the Perron matrix and L denotes the Laplacian matrix associated to the graph. The Laplacian matrix has been used to analyze the structure of multiplex networks [4, 7]. However, the applied methods are based on spectral analysis, and therefore they need a complete view of the network. Furthermore, if the network changes, the measures must be recalculated from scratch.

3. Results

The interaction in Twitter of the users that are involved in different events has been analyzed. Events took place in the real world in 2013. They consisted in conferences, keynotes, citizen initiatives and tv programs. The analyzed events were:

- *TedX Valencia:* TED talk that took place in Valencia, June 22th.
- *SSEO4SEOS:* Professional SEO conference in Alicante, Oct 5th.
- *TAW2013:* Twitter Awards 2013, given to celebrities in different categories, 9th and 10th Oct.
- *kikodeluxe:* Reality show where a celebrity was interviewed, Oct 4th.
- *nuevosiPhone:* Apple keynote where the new iPhone 5S was presented, Sep 10th.
- *Via Catalana:* Citizen demonstration organized in Catalonian (human chain), Sep 11th.
- *Breaking Bad:* Comments during the final episode of this tv serie, Sep 29th.

User activity is measured through the engagement produced by three types of interaction: retweets, replies and mentions. Each one of these interaction types are stored in a different layer of a multiplex network. Users that make use of the hashtag but do not participate in these types of interactions are not considered in the networks. Table 1 shows the characterization of the network for each event.

Nodes are weighted (w_i) using its degree in each layer. The initial value $x_i(0)$ is the total number of interactions of each type realized during the event.

The mean user influence is calculated as the weighted, average strength of the node (see Equation (3) below). It is calculated using the weighted consensus algorithm [6], ruled by the expression

$$x_i(t+1) = x_i(t) + \frac{\varepsilon}{w_i} \sum_{j \in N_i} [x_j(t) - x_i(t)] \tag{2}$$

It can be demonstrated that, under certain conditions [6], then

$$\lim_{t \to \infty} x_i(t) = \frac{\sum_i w_i x_i(0)}{\sum_i w_i} \quad \forall i \tag{3}$$

This process is performed in a decentralized way. Each node can determine whether its activity is over o below the mean value. Furthermore, no information about the size of the network nor its topology is needed. Each user just exchanges its current mean value with its neighbors until it converges. In the experiments, the process stops when a tolerance of $1e^{-10}$ is achieved.

Table 2 shows the number of iterations needed to obtain the consensus value and the mean reachability value for each one of the events. As an example, the details of the #viaCatalana initiative are shown in Figure 1.

hashtag	tweets	n	m	γ	$\bar{d}$	l	g	c
#tedxValencia	1,803	402	1,142	-1.3318	2.87	2.77	6.0	0.63
#seo4seos	2,891	426	1,659	-1.2132	4.11	2.69	6.0	0.73
#TAW2013	12,845	2,519	7,817	-1.4650	2.92	3.46	10.0	0.67
#kikodeluxe	6,388	2,638	5,030	-1.9191	1.15	3.96	12.0	0.43
#nuevosiPhone	16,505	8,546	10,904	-2.2981	1.11	3.57	15.0	0.48
#viacatalana	135,668	47,506	129,299	-1.9608	2.20	4.45	14.0	0.28
#breakingBad	136,916	71,130	105,344	-1.9500	2.92	6.72	23.0	0.05

Table 1: Characterization of the complete networks associated to the different events; n num. of nodes, m num of links, γ power-law exponent, d mean degree, l mean path length, g network geodesic and c clustering coefficient.

hashtag	men	ret	rep	$\bar{d}_{men}$	$\bar{d}_{ret}$	$\bar{d}_{rep}$	iter	reach
#tedxValencia	1,267	650	101	8.5	4.1	0.6	556	52.64
#seo4seos	1,864	739	149	10.9	3.6	0.7	448	47.14
#TAW2013	8,739	2,902	795	6.7	2.0	0.6	10,417	83.95
#kikodeluxe	5,534	3,384	373	2.7	1.6	0.2	3,647	70.17
#nuevosiPhone	11,475	9,889	495	3.2	2.9	0.1	20,579	878.86
#viacatalana	135,434	91,339	6,523	5.0	3.3	0.2	2,528	202.84
#breakingBad	120,422	62,759	13,261	2.9	1.5	0.3	13,705	1,862.58

Table 2: For each event, the links and the mean degree of each layer, the number of iterations needed to converge and the mean reachability of the event.

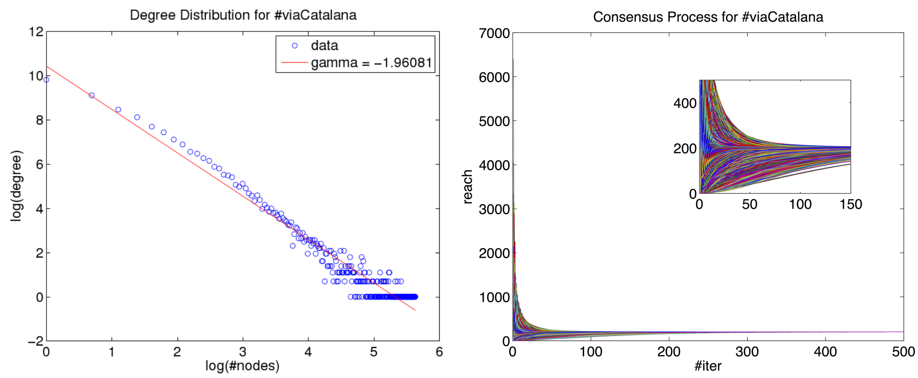

Figure 1: (Left) Degree distribution of #viaCatalana network as a power law with exponent $\gamma = -1.9608$. (Right) Consensus process to calculate the mean reachability of the event (202.84); 2,528 iterations are needed.

4. Conclusions

A consensus process has been used as a diffusion technique to perform decentralized calculus of global magnitudes in a network. Multiplex networks allow to model the behavior and the relations among the users in a social network in a natural and intuitive way. The interactions of participants in different events in the real world with a counterpart in Twitter has been studied. Using the engagement of the users, the visibility of its messages in the network related to the event can be determined. The advantage of this method is that it can be calculated locally, without any further knowledge about the size of the network nor its topology. Each node interacts with its immediate neighbors and the information is updated and spread through the network until it converges to the exact value.

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