Adding a new station and a road link to a road-rail network in the presence of modal competition

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Abstract

In this paper we study the problem of locating a new station on an existing rail corridor and a new junction on an existing road network, and connecting them with a new road segment under a budget constraint. We consider three objective functions and the corresponding optimization problems, which are modeled by means of mixed integer non-linear programs. For small instances, the models can be solved directly by a standard solver. For large instances, an enumerative algorithm based on a discretization of the problem is proposed. Computational experiments show that the latter approach yields high quality solutions within short computing times.

Keywords: Station location, road-rail network design, infrastructure planning, non-linear programming.

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1. Introduction

Railway systems offer many advantages with respect to other transportation modes. Among others they provide safety, speed, stable travel times, and non-dependency on petrol. For these and other reasons, large investments have recently been devoted to the construction or improvement of railway networks. When designing such networks it is not sufficient to take connectivity and efficiency issues into account, but one must also consider competition and interconnectivity with other transportation modes.

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1.1. Literature review

The literature of optimal location of stations on a railway network dates back to the beginning of the 20^{th} century (see Vuchic and Newell [16] for a review). The problems considered in the early papers dealt with the determination of the optimal interstation spacing by minimizing the total travel time of the passengers commuting to the city center along a railway line. Several papers have since been devoted to this problem, and a variety of criteria have been considered.

Starting with passenger travel time criteria, we mention Vuchic and Newell [16], who examine the problem of determining a number of stations and the interstations spacings to minimize the total travel time. The model is limited to the people commuting to the central part of the city and takes into account several realistic aspects such as access speed, dwell times, kinematics of trains, modal competition and passenger population along the line. These authors solved the problem through a set of second order difference equations.

A related criterion is the maximization of travel time reduction when introducing new stations. This problem was shown to be NP-hard by Hamacher et al. [4] when the travel time includes both accessing and in-vehicle time. A similar problem regarding a high-speed line was treated by Repolho et al. [10]. These authors present a mixed-integer program which is applied to the location of stations on several corridors of a planned high-speed line in Portugal, which competes with other transportation modes.

The minimization of the additional travel time induced by the stops of the trains at the new stations while covering all the demand sites has been studied by Schöbel [11] and Schöbel et al. [13] in the context of urban public transport. The problem was proven to be NP-hard. The same problem has been studied in Carrizosa et al. [1], but considering this time the kinematics of the trains between stops.

Since constructing stations is costly and such decisions are strategic due to their permanent character, one of the most commonly used criteria in real situations is to maximize coverage. This problem was studied in a paper by Laporte et al. [7] where stations for a rapid transit line with lower and upper bounds on the interstations spacings are located on a predefined alignment. To solve this problem for a finite candidate set, the authors made use of a graph representation and applied a longest path algorithm. The continuous version of this problem, where the number of new stations is fixed, was shown to be NP-hard by Kranakis et al. [6]. Schöbel [12] considered the biciteria problem of maximizing coverage while minimizing the number of new stations. This problem was solved in quadratic time for the particular case of a polygonal line with an additional assumption.

The maximization of ridership, which is a more realistic criterion, was introduced by Vuchic [14] and further studied by Laporte et al. [8]. The context of the problem dealt with by Vuchic is similar to that of Vuchic and Newell [16], while in Laporte et al. [8] a bounded-length line and their stations are simultaneously located. Körner et al. [5] used the same criterion for the location of two new stations on segments and on tree-like networks in a mixed planar network environment. The authors have provided a polynomial time algorithm. Gross et al. [3] have dealt with the maximum accessibility problem where a fixed number of stations is to be located and the sum of distances to demand points is to be minimized. The problem was shown to be NP-hard for two different environments: in a plane and on a street network. Finally, the problem of minimizing the cost of constructing the new stations or the simpler one of minimizing the number of stations, while covering all sites was proved to be NP-hard by Hamacher et al. [4].

1.2. Linking stations to the road network

Building train stations relatively far from city centers and linking these stations to the road network is becoming a trend in certain high-speed railway networks for several reasons. At a macroscopic engineering level, the main reason for constructing intermediate stations away from city centers is because this allows high-speed lines to have a smoother shape, also allowing higher speed between the end cities of the line (these end cities usually provide a larger traffic than intermediate stations). Cost minimization is a common reason since avoiding city centers often means building fewer stations, and bringing railway lines into city centers tends to be expensive. Another reason is political: when two nearby cities want a railway station, it is often politically expedient to build one half-way between them in order to avoid favoring one city at the expense of the other. As case in point, 30% of the stations of the Spanish high-speed railway network are now located outside cities. As an example, the Camp de Tarragona station on the Madrid-Barcelona line lies 12 km from Tarragona, the nearest large city with more than 100000 inhabitants, (see Figure 1). In such cases it becomes desirable or even necessary to build new links between the road network and the out-of-town station. Another example is the case of new stations in commuter systems. Line 5 of the commuter system of Seville (Spain) was established

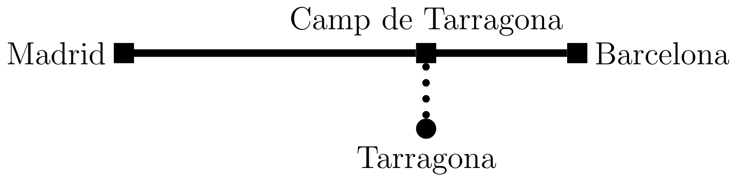


Figure 1: The Madrid-Barcelona line (solid line). The Camp de Tarragona station is 12 km away from the nearest large city, Tarragona. A road link (dotted line) joins these two points.

by partially using a regional railway. One of the stations of this line is the Valencina-Santiponce station which needed a new link to be connected with the road connecting both towns.

Some of the earlier station location models (see for instance Hamacher et al. [4], Laporte et al. [7], Schöbel [12]) did not take into account competition with other transportation modes, whereas some others did, like Vuchic [14] and more recently Repolho et al. [10]. Körner et al. [5] introduced and solved a station-location problem consisting of locating two stations on an existing tree-like railway network. The problem presented in this paper also considers static modal competition but differs from that introduced in Körner et al. [5] because we allow connections with any point of the road network, not just with a node.

The remainder of the paper is structured as follows. In Section 2 we introduce the input data for our problem. Section 3 details the variables and constraints needed to build the presented mixed integer non-linear programming models. In Section 4, an iterative process to solve the model in Section 3 and a heuristic procedure are introduced, which are tested and compared with each other in Section 5 via a computational experience. In Section 6 we illustrate our procedures on a realistic instance. Conclusions follow in Section 7.

2. Formal description of the problem

Our problem takes as input data a *road-rail* connected network composed of both road and rail links, as well as cities, train stations and junctions. Formally, we consider an undirected road-rail network $G = (R \cup T, E_R \cup E_T)$, where $R \cup T$ is the node set and $E_R \cup E_T$ is the edge set. The set R is a set of cities and junctions without a train station, T is the set of cities and junctions with a train station, E_R is the set of road links joining the nodes of R among themselves or one node of R with one node of T, and E_T is the set of rail links joining the nodes of T. As an example consider Figure 2, where the set R is represented by circles, the set T is represented by squares, the edges of E_R are represented by dotted lines, and the edges in E_T by solid lines.

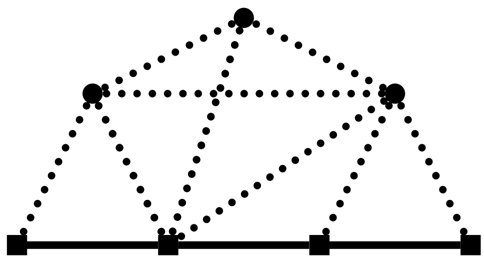


Figure 2: Input data to the problem. A set of cities (filled circles) and a set of train stations (filled squares) are joined by rail tracks (solid edges). Cities are linked among themselves and with the stations by means of road links (dotted edges).

The aim is to build a new station on the rail network and a new junction on the road network, and to build a road link between the new stations and the new junctions, as shown in Figure 3. To illustrate the problem, consider

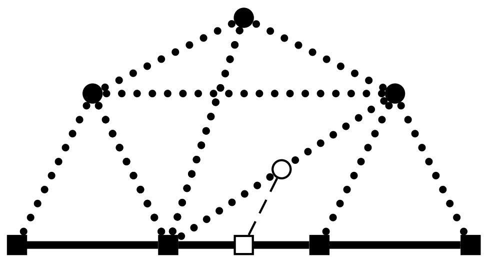
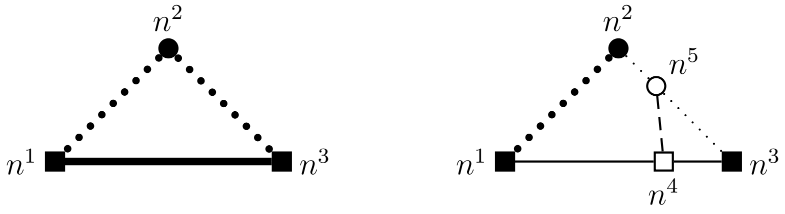


Figure 3: A possible solution to the problem depicted in Figure 2. The empty square represents the new station and the empty circle represents the new junction to be connected with the new station, by means of a new road link represented by dashed lines.

the following example.

Example 2.1. *There are three cities n^1, n^2, n^3, a rail line between cities n^1 and n^3, and a road network made up of edges (n^1, n^2) and (n^2, n^3). All these* *elements form a road-rail network (see Figure 4a). The aim is to build a station at node n^4 on the rail line between cities n^1 and n^3, a junction n^5 on the road network, and a link between this junction and the new station so that a certain objective function is optimized and a budget constraint is satisfied. One possible solution to this example is shown in Figure 4b.*



a) Existing network.   b) Network with an added road-rail link.

Figure 4: Figure 4a shows the existing road-rail network. In Figure 4b, the new station is located at n^4, the new junction is located at n^5 on the road edge (n^2, n^3), and the dashed edge represents the new road edge between the new station and the new junction. Note that (n^1, n^2) is the only edge remaining from the original network in Figure 4a, the other five are new (they are obtained after adding the new station and the junction).

The problem will be modeled by means of the following notation:

- We are given two train stations $T = \{n^1, n^k\} \subset \mathbb{R}^2$, a set of cities and junctions $R = \{n^2, ..., n^{k-1}\} \subset \mathbb{R}^2$, one rail link joining the two stations in T (set E_T), and a set of road links joining elements of R or one element of T with one element of R (set E_R). We assume that all road and rail edges are linear. Let $G = (R \cup T, E_R \cup E_T)$ be the resulting undirected road-rail network. In the example of Figure 4a, $R = \{n^2\}, T = \{n^1, n^3\}, E_R = \{(n^1, n^2), (n^2, n^3)\}, E_T = \{(n^1, n^3)\}$. For the sake of simplicity, we identify the nodes of $R \cup T$ by their indices whenever this creates no confusion, and we denote the edge connecting n^i and n^j by (i, j) although their elements are not ordered. We assume that n^1 is located at $(0, 0)$ and n^k is located at $(b, 0)$, $b > 0$. Let (n^i_1, n^i_2) be the coordinates of node $n^i \in R \cup T$.

- In order to compute traffic flows, we will need the set of (directed) arcs associated with the edges in E_R and E_T. Let $A(E_R)$ and $A(E_T)$ denote these arc sets. Note that $E_R = \{(i,j) \in A(E_R) : i < j\}, E_T = \{(i,j) \in A(E_T) : i < j\}$. In the rest of the paper we will use the same notation, so if E is a set of (undirected) edges, we let $A(E)$ denote its associated set of (directed) arcs.

- With each arc $(i,j) \in A(E_R) \cup A(E_T)$ we associate a travel time t_{ij}. We assume that the travel time on an arc is proportional to its Euclidean length. We also assume that the train is faster than the road mode. That is, if d_{ij} denotes the Euclidean length of arc (i,j), then $t_{ij} = \alpha_1 d_{ij}$ if $(i,j) \in A(E_R)$ and $t_{ij} = \alpha_2 d_{ij}$ if $(i,j) \in A(E_T)$, with $\alpha_1 > \alpha_2 > 0$. The reader may note that when adding a new junction on the road network the number of interactions among cars flowing in different directions goes up, which increases the probability of congestion. In order to take into account this extra congestion in the road network, we will consider that if the new station is linked with road edge $(i,j) \in E_R$, then $t_{ij} = \alpha_1 d_{ij} + \psi$, where $\psi \geq 0$ is a congestion parameter.

- We are given a set of origin/destination (O/D) pairs $W \subseteq (R \cup T) \times (R \cup T)$. For each $(p,q) \in W$, g_{pq} denotes the number of potential trips of this O/D pair, and u_{pq}^{ROAD} denotes the traveling time using the road network only.

- We assume that if passengers travel from n^1 to n^k (or from n^k to n^1) using the train during their trip, they will incur a stop time β at the new station.

- The construction costs of the new station and of the new junction are assumed to be equal to c_1 and c_2, respectively. The construction cost of the road edge linking the new station and the junction is assumed to be equal to τ times its Euclidean length. A construction budget equal to $C_{\max}$ is available.

The problem consists of choosing a location for the new station x on E_T at node $k+1$, a location for a new junction y on E_R at node $k+2$, and building a road segment linking nodes $k+1$ and $k+2$, so that a certain objective function is optimized, without violating the budget constraint.

Note that if the new junction y is located on edge $(i^*, j^*) \in E_R$, the road-rail network then has two new nodes (the new station $k+1$ and the new junction $k+2$), two new rail edges $((1,k+1),(k,k+1))$ instead of $(1,k)$, two new road edges $(i^*, k+2), (j^*, k+2)$ instead of (i^*, j^*), and another new road edge which is the connection between the new station and the new junction $(k+1, k+2)$. Thus, the set of edges in the new road-rail network, denoted by $E_{(i^*,j^*)}$, is

$$(E_R \setminus \{(i^*, j^*\}) \cup \{(1, k+1), (k, k+1), (i^*, k+2), (j^*, k+2), (k+1, k+2)\}.$$

The new road-rail network we will work with is therefore

$$(R \cup T \cup \{n^{k+1}, n^{k+2}\}, E_{(i^*,j^*)}).$$

For example, in Figure 4b, road edge $(2, 3)$ does not exist and is replaced with $(2, 5)$ and $(3, 5)$; rail edge $(1, 3)$ does not exist and is replaced with $(1, 4)$ and $(3, 4)$; and there is also a new road edge $(4, 5)$. Therefore, the new node set is $R \cup T \cup \{n^4, n^5\}$, and the new edge set is

$$E_{(2,3)} = \{(1, 2), (1, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}.$$

The reader may note that both the locations of the two new nodes and the lengths of the five new edges are variables.

3. Mathematical models

In this section we will model our problem assuming the new junction is located on a predefined road link by means of mixed integer non-linear programming (MINLP) formulations. We will consider three different objectives, namely minimizing the total travel time, maximizing the number of travelers who will use the rail corridor (ridership), and maximizing the number of users who are positively affected by the construction of the new station (winners). We will therefore end up with three different mathematical programming models, one for each objective. Note that although the second and third objectives are different, they both maximize functions estimating the expected number of passenger trips (not necessarily leading to the same solutions). According to Vuchic [15] (page 186), maximizing passenger attraction is the most appropriate objective to consider when planning transit systems.

3.1. Variables

Assuming that the new junction is to be located on edge $(i^*, j^*) \in E_R$, we first have to update $t_{i^*j^*} = \alpha_1 d_{i^*j^*} + \psi$ so that the extra congestion due to the inclusion of the new junction and link to the road network is considered in the model. In order to build the mathematical programming program, we need the following variables:

1. $x_1 \in [0, b]$ is the first coordinate of the location of the new station, located between the two already existing stations located in $(0, 0)$ and $(b, 0)$. Actually, in realistic settings the domain of this variable is reduced in order to avoid locating the new station too close to the existing ones. Moreover, there are possibly forbidden regions for locating the stations (for example tunnels). Note that here we assume that the rail corridor is a straight line segment.
2. $\lambda \in [0, 1]$ determines the convex combination of the endnodes of edge $(i^*, j^*) \in E_R$ where the new junction on the road network is to be located. Note that here we assume that the road network is composed of straight edges.
3. The variables y_1 and y_2 denote the first and second coordinates of the location of the new junction that will be linked with the new station.
4. The continuous variables δ_{ij} define the travel times of the new arcs.
5. The binary variable f_{ij}^{pq} indicates whether or not O/D pair $(p, q) \in W$ uses arc $(i, j) \in A(E_{(i^*,j^*)})$.
6. The binary variable v_{pq} indicates whether or not O/D pair (p, q) incurs a stop time at the new station.
7. u_{pq} is the travel time associated with O/D pair (p, q).

Some of these variables are now explicitly defined:

$$
\begin{gathered}
y_1 = \lambda n_1^{i^*} + (1-\lambda) n_1^{j^*}, \\
y_2 = \lambda n_2^{i^*} + (1-\lambda) n_2^{j^*}, \\
\delta_{i^*,k+2} = (1-\lambda) t_{i^*j^*}, \delta_{j^*,k+2} = \lambda t_{i^*j^*}, \\
\delta_{1,k+1} = \alpha_2 x_1,\ \delta_{k,k+1} = \alpha_2 (b - x_1), \\
\delta_{k+1,k+2} = \alpha_1 \sqrt{(x_1 - y_1)^2 + (0 - y_2)^2}, \\
\delta_{ij} = \delta_{ji},\ \forall\ (i,j) \in A(E_{(i^*,j^*)} \setminus E_R) : i > j, \\
u_{pq} = \sum_{(i,j) \in A(E_R \setminus (i^*,j^*))} t_{ij} f_{ij}^{pq} + \sum_{(i,j) \in A(E_{(i^*,j^*)} \setminus E_R)} \delta_{ij} f_{ij}^{pq} + \beta v_{pq},\ \forall\ (p,q) \in W.
\end{gathered}
$$

Note that the definition of u_{pq} contains the quadratic terms $t_{ij}f_{ij}^{pq}$ which can easily be linearized. Unfortunately, the non-linearity in the definition of $\delta_{k+1,k+2}$ cannot be removed, which makes our model non-linear.

3.2. The constraints

The following constraints are common to all three models:

$$c_1 + c_2 + \tau\sqrt{(x_1 - y_1)^2 + (0 - y_2)^2} \le C_{max} \tag{1}$$

$$\sum_{i:(i,p)\in A(E_{(i^*,j^*)})} f_{ip}^{pq} = 0, \ \ (p,q) \in W \tag{2}$$

$$\sum_{j:(p,j)\in A(E_{(i^*,j^*)})} f_{pj}^{pq} = 1, \ \ (p,q) \in W \tag{3}$$

$$\sum_{i:(i,q)\in A(E_{(i^*,j^*)})} f_{iq}^{pq} = 1, \ \ (p,q) \in W \tag{4}$$

$$\sum_{j:(q,j)\in A(E_{(i^*,j^*)})} f_{ij}^{pq} = 0, \ \ (p,q) \in W \tag{5}$$

$$\sum_{i:(i,r)\in A(E_{(i^*,j^*)})} f_{ir}^{pq} - \sum_{j:(r,j)\in A(E_{(i^*,j^*)})} f_{rj}^{pq} = 0, \ \ (p,q) \in W, \ \ r \in (R \cup T) \setminus \{p,q\} \tag{6}$$

$$f_{1,k+1}^{pq} + f_{k+1,k}^{pq} + f_{k,k+1}^{pq} + f_{k+1,k}^{pq} - 1 \le v_{pq}, \ \ (p,q) \in W \tag{7}$$

$$x_1 \in [0,b], \lambda \in [0,1], f_{ij}^{pq}, v_{pq} \in \{0,1\}. \tag{8}$$

Constraint (1) ensures that the cost of building the new station c_1 plus the cost of building the new junction c_2 plus the cost of building the new road link does not exceed the available budget. Constraints (2) to (6) are flow conservation constraints on the f variables. Note that, even if more than one of the binary f variables can take the value 1, these constraints ensure that $f_{ij}^{pq} = 1$ if and only if arc (i,j) belongs to the path used by O/D pair (p,q). Constraints (7) imply that $v_{pq} = 1$ if two rail arcs are used by the O/D pair (p,q), meaning that this O/D pair will incur a stop time at the new station n^{k+1}.

3.3. Objectives

The three considered objectives can be formulated as follows:

1. Minimizing the total travel time of the road-rail network:

$$\text{minimize } z_{TTT} := \sum_{(p,q)\in W} g_{pq} u_{pq}, \tag{9}$$

Minimizing (9) subject to constraints (1) to (7) yields model $TTT_{(i^*,j^*)}$ (locating station-junction on (i^*, j^*) minimizing the total travel time).

2. Maximizing ridership, i.e., the number of travelers who will use the rail corridor. This number will be estimated by a logit function, that is, we assume that the proportion of travelers in (p, q) who will use the rail corridor is given by

$$\psi(u_{pq}^{ROAD} - u_{pq}) = \frac{1}{1 + \gamma_1 e^{-\gamma_2(u_{pq}^{ROAD} - u_{pq})}},$$

where $\gamma_1, \gamma_2 > 0$ are two parameters to be calibrated depending on the instance. Note that, if $u_{pq}^{ROAD} - u_{pq}$ is sufficiently large, then $\psi(u_{pq}^{ROAD} - u_{pq})$ tends to one. Conversely, if $u_{pq}^{ROAD} - u_{pq}$ is sufficiently small, then $\psi(u_{pq}^{ROAD} - u_{pq})$ tends to zero. The reader may note that ψ is monotone increasing. Therefore, in this case the objective is

$$\text{maximize } z_{RID} := \sum_{(p,q)\in W} g_{pq} \frac{1}{1 + \gamma_1 e^{-\gamma_2(u_{pq}^{ROAD} - u_{pq})}}. \tag{10}$$

Unfortunately, this function is neither convex nor concave, which makes the problem difficult to solve. Maximizing (10) subject to constraints (1) to (7) yields model $RID_{(i^*,j^*)}$ (locating station-junction on (i^*, j^*) maximizing ridership).

3. Maximizing the number of users who are positively affected by the construction of the new station and the new road link (winners). An O/D pair (p, q) is considered a winner if $u_{pq} < u_{pq}^{ROAD}$. To this end, we define the following set of variables:

   - $s_{pq} = 1$ if the travel time for O/D pair (p, q) is shorter using the rail corridor than using the road network only, and zero otherwise.

   These binary variables were used in Laporte et al. [9] to estimate the number of trips attracted by a rail network. The objective is modeled as follows:

$$\text{maximize } z_{WIN} := \sum_{(p,q)\in W} g_{pq} s_{pq}. \tag{11}$$

Note that in this case we have to add the following constraints in order to ensure that variables s_{pq} are zero whenever the travel times using the rail corridor are greater than or equal to the travel times without using this corridor:

$$u_{pq} - u_{pq}^{ROAD} + \varepsilon \leq (1 - s_{pq}),\ (p, q) \in W, \tag{12}$$

where ε is a small positive number ensuring that if $u_{pq} = u_{pq}^{ROAD}$ then $s_{pq} = 0$. Maximizing (11) subject to (12) and constraints (1) to (7) yields model $WIN_{(i^*,j^*)}$ (locating station-junction on (i^*, j^*) maximizing the number of winners).

4. Algorithms

We now present two algorithms for the problem. The first one makes use of a MINLP solver (Section 4.1). Since the problem is non-linear and non-convex, global optimality is not guaranteed. The second one (Section 4.2) uses a discretization strategy which is exact if the feasible solution set is discrete.

4.1. Mathematical programming-based algorithm

Once we have defined the MINLP models that optimally locate a new station on the rail corridor and a junction on a given edge $(i^*, j^*) \in E_R$, it is easy to allow the junction to be located on any edge in E_R (or a subset of it). To this end, we only have to solve the previous models for each such edge. A pseudocode of the algorithm to minimize the total travel time (called TTT, locating station-junction minimizing the total travel time) is given in Algorithm 1. The algorithms used to maximize ridership or maximize number of winners are analogous.

4.2. Enumerative algorithm

In this section we propose an iterative process based on an enumeration of the feasible locations for both the new station and the new junction, which will be tested one at a time. Let $\bar{x}$ and $\bar{y}$ be the (known) locations of the new station and the junction. In this case, the computation of any of the three objectives ($z_{TTT},\ z_{RID},\ z_{WIN}$) is straightforward since it reduces to computing shortest paths between O/D pairs in the resulting road-rail network. We

Data: A road-rail network with two train stations.
Set $z^*_{TTT} = \infty$;
for $(i, j) \in E_R$ **do**
  Set $t_{ij} = \alpha_1 d_{ij} + \psi$.
  Solve $TTT_{(i,j)}$ by means of a MINLP solver.
  Let $x^{(i,j)}, y^{(i,j)}$ be the resulting optimal locations for the new station and the new junction, and let $z_{TTT}(i, j)$ be the total travel time of the corresponding network;
  if $z_{TTT}(i, j) < z^*_{TTT}$ **then**
    $(x^*, y^*) = (x^{(i,j)}, y^{(i,j)}), z^*_{TTT} = z_{TTT}(i, j)$
  end
  Set $t_{ij} = \alpha_1 d_{ij}$.
end
Result: Locations for the new station and the junction, (x^*, y^*), yielding a locally minimal total travel time z^*_{TTT}.

Algorithm 1: Local optimal algorithm for the location of a new station on the rail corridor and a junction on the road network minimizing the total travel time.

now detail the enumerative algorithm to solve the station-junction location problem minimizing the total travel time. The enumerative algorithms for the other two objectives are analogous.

Denote by F^x the discrete set of possible locations for the new station, and by $F^y(i, j)$ the set of possible locations for the new junction on edge $(i, j) \in E_R$. For $(\bar{x}, \bar{y}) \in F^x \times F^y(i, j)$, let $z_{TTT}(\bar{x}, \bar{y})$ be the total travel time of the network after adding a new station at $\bar{x}$, a new junction at $\bar{y}$, and a road edge between the two. A pseudocode of the enumerative algorithm when the objective is to minimize the total travel time is provided in Algorithm 2.

5. Computational results

All computational experiments were run on a desktop PC, 3.20 GHz processor, 4MB of RAM Memory, on Windows 7 64 bits, and the software was GAMS 22.9. The MINLP solver was LINDOglobal which, in all instances, would stop processing when the relative gap was lower than 5%. We be-

Data: A road-rail network with two train stations.
Set $z^*_{TTT} = \infty$;
for $(i,j) \in E_R$ **do**
  Set $t_{ij} = \alpha_1 d_{ij} + \psi$.
  Set $z_{TTT}(i,j) = \infty$;
  for *feasible* $\bar{x} \in F^x$ *and* $\bar{y} \in F^y(i,j)$ **do**
    Compute $z_{TTT}(\bar{x},\bar{y})$ of the corresponding network;
    if $z_{TTT}(\bar{x},\bar{y}) < z_{TTT}(i,j)$ **then**
      $(\bar{x}^{(i,j)}, \bar{y}^{(i,j)}) = (\bar{x},\bar{y}), z_{TTT}(i,j) = z_{TTT}(\bar{x},\bar{y})$
    end
  end
  if $z_{TTT}(i,j) < z^*_{TTT}$ **then**
    $(\bar{x}^*, \bar{y}^*) = (\bar{x}^{(i,j)}, \bar{y}^{(i,j)}), z^*_{TTT} = z_{TTT}(i,j)$
  end
  Set $t_{ij} = \alpha_1 d_{ij}$.
end
Result: Locations for the new station and the new junction $(\bar{x}^*, \bar{y}^*)$ yielding a total travel time equal to z^*_{TTT}.

Algorithm 2: Enumerative algorithm for the station-junction location problem minimizing the total travel time.

gin in Section 5.1 by showing how the random instances were generated. We then provide in Section 5.2 a comparison between the three mathematical programming based algorithms presented. We conclude in Section 5.3 by comparing the performance of the mathematical programming algorithm minimizing the total travel time with that of the enumerative algorithm minimizing the same objective.

5.1. Instance generation

We have randomly generated a number instances for four different configurations. In each configuration, the existing road-rail network consists of k nodes, $k = 3, ..., 6$. The train stations are $T = \{n^1 = (0,0), n^k = (2,0)\}$, and the only rail edge is $E_T = \{(n^1, n^k)\}$. If $\mathcal{U}(a_1, a_2)$ denotes a uniform distribution on the interval $[a_1, a_2] \subset \mathbb{R}$, the location of the nodes in R and the configuration of the road network E_R are:

- Configuration 1 ($k = 3$): $R = \{n^2 = (\mathcal{U}(0.5, 1.5), 1)\}$, $E_R = \{(1,2),$

$(2,3)\}$.

- Configuration 2 ($k=4$): $R = \{n^2 = (\mathcal{U}(0.5, 1.5), 1), n^3 = (\mathcal{U}(0.5, 1.5), -1)\}$, $E_R = \{(1,2), (1,3), (2,4), (3,4)\}$.
- Configuration 3 ($k=5$): $R = \{n^2 = (\mathcal{U}(0.25, 0.75), 1), n^3 = (\mathcal{U}(0.5, 1.5), -1)$, $n^4 = (\mathcal{U}(1.25, 1.75), 1)\}$, $E_R = \{(1,2), (1,3), (2,4), (3,5), (4,5)\}$.
- Configuration 4 ($k=6$): $R = \{n^2 = (\mathcal{U}(0.25, 0.75), 1), n^3 = (\mathcal{U}(0.25, 0.75), -1), n^4 = (\mathcal{U}(1.25, 1.75), 1), n^5 = (\mathcal{U}(1.25, 1.75), -1)\}$, $E_R = \{(1,2), (1,3), (2,4), (3,5), (4,6), (5,6)\}$.

Graphical descriptions of these configurations can be seen in Figure 5.

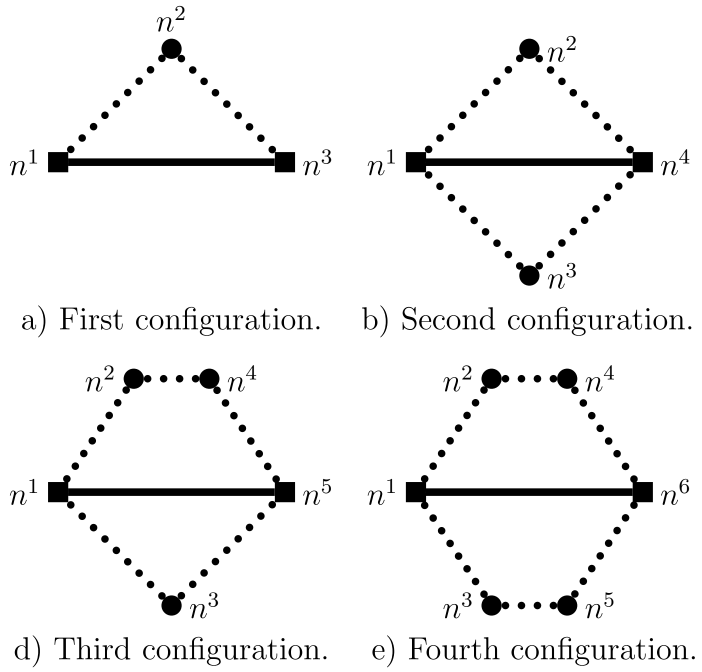


Figure 5: Possible instances for each of the four configurations used in the experiments.

In the four cases, $W = E_R \cup E_T$. For each $(p,q) \in W$, the expected flow g_{pq} is randomly generated from a gravitational model:

$$g_{pq} = \sqrt{\frac{w_p w_q}{d_{pq}}}, \tag{13}$$

where $w_p \sim \mathcal{U}(5, 15)$ is a uniform random variable simulating the node's population and d_{pq} is the Euclidean distance between n^p and n^q. We also impose $x_1 \in [0.5, 1.5]$ in order to avoid locating the new station too close to the existing ones. The travel times on road edges were set equal to their Euclidean lengths (i.e, $\alpha_1 = 1$), and the travel times on rail edges were set equal to $\alpha_2 = 0.25$ times their Euclidean lengths. We assumed $c_1 = 1.5, c_2 = 0.5$, and $c_{ij} = d_{ij}$. The maximum budget was limited to $C_{\max} = 2.8$.

5.2. Comparison between mathematical programming based algorithms

We have performed initial comparisons between the mathematical programming based algorithms corresponding with the three objectives presented: locating station-junction minimizing the total travel time (TTT), locating station-junction maximizing the ridership (RID), and locating station-junction maximizing the number of winners (WIN). To this end, we have solved a set of 20 randomly generated instances. For each configuration in Section 5.1, five instances were generated by using different seeds in the pseudo-random number generator. Because the aim of these experiments is to show that one of these three models outperforms the other two, the logit parameters (needed to calculate the ridership) were set to $\gamma_1 = \gamma_2 = 1$ without previous calibration. Table 1 summarizes the results, complete results are shown in the Appendix, Table 5.

	z_{TTT}	z_{RID}	z_{WIN}	Seconds
TTT	74.8	30.9	28.5	236.0
RID	75.1	30.8	24.8	1162.2
WIN	83.8	28.9	33.3	1275.2

Table 1: Average value for each objective and CPU time (columns) obtained by the MINLP-based algorithms (rows).

From Table 1, we can draw the following conclusions:

- The total travel time (column z_{TTT}) yielded by model TTT is on average better than that obtained by the other two models. Only in one instance was the total travel time yielded by this model improved by that yielded by model RID (due to the non-guarantee of global optimality or to the 5% maximum gap allowed).

- The ridership (column z_{RID}) yielded by model TTT is on average better than the ridership yielded by the other two models and, in all instances but one, model TTT produced the maximum ridership. Note that, although model RID was created specially for maximizing ridership, model TTT obtains higher average ridership. This is due to the fact that model RID incorporates the logit function, which is non-convex, and therefore model RID is more complex than model TTT. Besides, the algorithms for the non-linear programs only guarantee local optimality.

- The number of winners (column z_{WIN}) yielded by model WIN is on average better than that yielded by the other two models.

- The CPU times (column Seconds) are significantly lower for model TTT than for the other two models (four to five times less).

We therefore decided to pursue our experiments using model TTT only since it is computationally simpler than the other models and yields satisfactory results for all three objectives. Nevertheless, all three models become intractable for large instances.

5.3. Comparison between the mathematical programming based algorithm and the enumerative algorithm

For the experiments described in this section we randomly generated 50 instances for each of the four configurations proposed. Each of these 200 instances was solved by means of the following algorithms:

- The MINLP-based algorithm minimizing the total travel time (Algorithm 1) by means of LINDOglobal, allowing a maximum 5% GAP.

- The enumerative algorithm presented in Algorithm 2 assuming the potential locations for the new station are from (0.5,0) to (1.5,0) with step size ρ, and the potential locations for the new junction are between the end nodes of the edge in E_R with the same step size, for $\rho = 0.1, 0.05, 0.025$. In our instances, assuming that the two existing stations are located 200 km away from each other (one length unit equals 100 km), these steps mean that potential locations for the new station are 20, 10, and 5 km far from each other. The same could be stated for the new junction. This algorithm is called ENUM$_\rho$. For each feasible pair of locations for the new station and the link on the road network, the total travel time of the corresponding network ($z_{TTT}(\bar{x}, \bar{y})$) was calculated using Dijkstra's algorithm Dijkstra [2].

The Appendix shows the results obtained for the four configurations tested (Tables 6 to 9). The column headings are:

- *Seed* is the seed used by GAMS to pseudo-randomly generate the locations of the nodes in R, and the weight given to any such node which was used to estimate the flows between O/D pairs by means of the gravitational model (see Equation (13)).

- z_{TTT} represents the total travel time obtained in the solution of each of the four algorithms tested (Algorithm 1, and Algorithm 2 with step sizes 0.1, 0.05 and 0.025, respectively).

- *Sec* represents the computational time in seconds needed to obtain the solution by each of the four algorithms tested.

- *%gap* represents, for each of the three enumerative algorithms tested, the percent relative difference in total travel time between the solution obtained by the corresponding enumerative algorithm and the solution obtained by the MINLP-based algorithm, computed through the following formula:

$$\%gap = 100\frac{z_{TTT}(ENUM) - z_{TTT}(MINLP)}{z_{TTT}(MINLP)},$$

  where $z_{TTT}(MINLP)$ and $z_{TTT}(ENUM)$ are the total travel time of the road-rail network yielded by the MINLP-based algorithm and the enumerative algorithm, respectively.

Table 2 summarizes the results presented in the Appendix. From these results, we draw the following conclusions:

- The enumerative algorithms (Algorithm 2) with any of the three tested step sizes produced better average total travel times than the MINLP-based approach (Algorithm 1).

- The average CPU time in seconds is clearly shorter for the enumerative algorithm than for the mathematical programming based algorithm. As expected, shorter step sizes in the enumerative algorithm increase the CPU time (since more evaluations need to be done).

Algorithm	MINLP	$\text{ENUM}_{0.1}$	$\text{ENUM}_{0.05}$	$\text{ENUM}_{0.025}$
Avg. TTT	77.21	76.36	76.25	76.21
Avg. CPU time (sec.)	149.20	0.20	0.74	3.01
Avg. %gap	–	−0.5	−0.68	−0.75

Table 2: Average results when comparing the MINLP-based algorithm and the enumerative algorithms.

- The average %gap shows that the enumerative algorithms with a step size equal to 0.1 is on average 0.5 % better than the MINLP-based algorithm. When the step size decreases to 0.05 the enumerative algorithm produces solutions on average 0.68% better than the MINLP-based enumerative procedure. When the step size decreases to 0.025 the enumerative algorithm is on average 0.75 % better than the MINLP-based algorithm.

From these results we first remark that computational times are much shorter for the enumerative algorithms than for the MINLP-based algorithm and, within the enumerative algorithms, reducing the step size significantly increases computational times. We have also noted that the quality of the solution given by the enumerative algorithms in terms of total travel time is basically the same for the three step sizes tested. When compared with the MINLP-based algorithm, the enumerative algorithm yields better solutions on average.

6. Case study

We now present the results obtained on a realistic instance: the high speed corridor between Madrid and Valladolid (Spain), which has 179.3 km of length. Originally, this corridor linked Madrid with Valladolid without any intermediate stop. In 2008, a new station was added close to the city of Segovia. We will see in this section that our algorithms suggest a similar location for the new station.

For this case study, we have considered cities that are close to the original high-speed track and have a significant potential demand. There are many small towns which, although they are close enough to the railway line, do not have a large enough population. Data about the cities considered such as a numeric identifier, population, and geodesic coordinates, can be found in

Table 3. The last two columns are the coordinates obtained after rotations and translations over the original ones so that Madrid is located at (0,0) and Valladolid is located at (179.3,0). Other geometric operations to rectify the high-speed line were also performed. A graphical description of our input data after such geometric operations is given in Figure 6.

City	Id	Population	Latitude	Longitude	x	y
Madrid	1	5098322*	40.42	−3.70	0.0	0.0
Valladolid	2	309714	41.66	−4.73	179.3	0.0
Colmenar Viejo	3	46955	40.66	−3.77	25.5	5.8
Collado Villalba - Galapagar	4	95207	40.63	−4.01	40.0	29.3
Segovia	5	54309	40.95	−4.12	75.5	−2.1
Laguna de Duero	6	22590	41.58	−4.72	172.9	−1.0
Miraflores de la Sierra	7	5907	40.81	−3.77	38.8	−4.7
Garcillán	8	477	40.98	−4.27	92.9	1.4
Santa María la Real de Nieva	9	993	41.07	−4.40	106.1	1.3
Olmedo	10	3776	41.29	−4.68	144.9	−0.3
Matapozuelos	11	1032	41.41	−4.79	163.4	−0.0
Cuéllar	12	9861	41.40	−4.31	128.5	−17.4

Table 3: Cities considered in the case study. *The population of Madrid includes its metropolitan area.

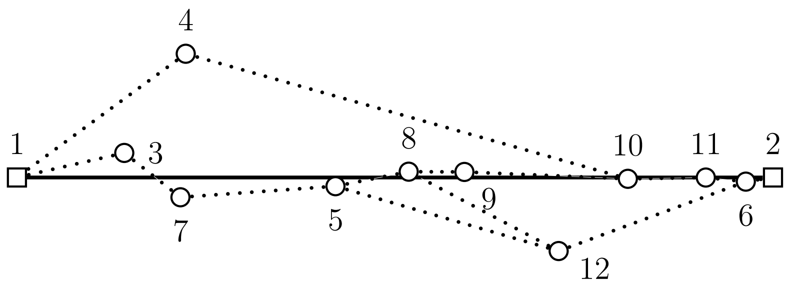


Figure 6: Case study network. A high speed train corridor between cities 1 and 2, represented by a solid line. Ten other cities considered, represented by empty dots. Dotted lines represent the road network.

Data about travel times using the road network were obtained using *Google maps*, and data about travel times using the train network were obtained from the operator's website, www.renfe.com. In order to keep a minimum interstation space, we assume that the new station should be located at least 30 km away from the nearest station. We have considered the tunnels along this corridor, which means that a new station cannot be located between km 30 and km 66 either. We also obtained data about passenger flows between stations directly from the operator, but we were asked to treat them confidentially and therefore these are not provided here. Passenger flows using the road network were estimated using the gravity model presented in Section 5.3.

Here we run the MINLP based approach minimizing total travel time and the heuristic algorithm with step size of one km and the same objective. A time limit of 30 minutes was set for every iteration of the MINLP-based approach (that is, the MINLP solver had 30 minutes to find the best station-connection pair assuming the connection point must be on a given road link). The results obtained were rather similar for both methods, see Table 4. In this table, and for each procedure, column "TTT" gives the total travel time of the corresponding network. Column "Station" gives the first coordinate of the location of the new station. Columns "Connection" give the coordinates of the point in the road network that will be linked to the new station. Finally, "Seconds" gives the CPU time in seconds needed by each procedure.

	TTT	Station	Connection		Seconds
Heuristic	27963495.69	75	75.65334116	−2.102538069	201.54
MINLP	28042844.56	74.7828814	75.54361877	−2.070662348	19244.40

Table 4: Solutions to the case study applying the heuristic procedure and the MINLP-based approach.

Note that the solutions obtained using the two algorithms are rather similar. The heuristic locates the new station at km 75 of the railway corridor, and joins it with a point of the road network very close to Segovia. The MINLP-based approach minimizing TTT locates the new station at km 74.78 and joins it directly with Segovia. Both solutions are rather close to the current location of the Segovia-Guiomar station, built in 2008 and 4 km away from Segovia.

It is also interesting to note that the total travel time of the new network obtained using the heuristic procedure is slightly shorter than that obtained by the MINLP-based approach. This may be due to the lack of global optimality guarantee of MINLP solvers or to the imposed CPU time limit. Regarding CPU times, we note that the heuristic needed two orders of magnitude less CPU time than the MINLP-based approach.

7. Conclusions

We have introduced, modeled, and solved the problem of locating a new station on a rail corridor and a new junction on a road network, and of building a new road segment between the two, respecting a budget constraint. The optimization was carried out under three different criteria: total travel time, ridership, and number of benefited travelers. Preliminary experiments showed that, in terms of computing times and of quality of solution, minimizing the total travel time yields the best results.

Minimizing the total travel time also yields shorter computing times than the models with the other two objectives. Nevertheless, the resulting MINLP-based algorithm is too slow, even for small instances. We have therefore proposed a faster enumerative algorithm. This algorithm is based on the evaluation of a number of feasible solutions, and is exact when the sets of possible locations for the new station and the connection with the road network are both discrete. When such sets include a continuous range of locations, the proposed enumerative algorithm discretizes the continuous sets and is only approximate.

In order to test the quality of the solution returned by the enumerative algorithm in the continuous case, we have performed some computational experiments over a set of small instances randomly built over four different configurations. We have observed that the quality of the solution returned by the enumerative algorithms in terms of total travel time is slightly better than the MINLP-based algorithm, whereas the computational times are significantly lower.

We have also tested our procedures over a medium-size case study. For this instance, both the MINLP-based algorithm and the heuristic algorithm yield very similar solutions under a travel time minimization objective, whereas computational times are two order of magnitude lower for the heuristic than for the MINLP-based algorithm. It is also interesting to highlight that the solutions yielded by these algorithms are very close to the location of the current intermediate station, which was decided in 2008 independently of this research.

Future research efforts on this topic will focus on solving the problem of locating more than one station and more than one junction. Unfortunately our mathematical programming approach cannot easily be extended to this case. A major complication is that it would be necessary to consider all possible intersections of the new road links, and the new links that such intersections would generate.

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Appendix: detailed results

		TTT				RID				WIN			
Conf.	Seed	z_{TTT}	z_{RID}	z_{WIN}	Sec	z_{TTT}	z_{RID}	z_{WIN}	Sec	z_{TTT}	z_{RID}	z_{WIN}	Sec
1	1	16.93	8.91	8.36	1.39	16.93	8.91	8.36	3.71	20.84	8.10	15.08	4.89
1	2	18.15	9.61	16.34	1.19	18.15	9.61	16.34	3.69	23.29	8.51	16.34	5.16
1	3	25.56	14.21	23.45	1.85	25.62	14.20	14.15	3.47	34.57	12.28	23.45	5.10
1	4	27.14	15.12	25.02	1.32	27.20	15.10	14.42	3.39	36.60	13.09	25.02	4.99
1	5	28.64	15.93	14.58	1.12	28.64	15.93	14.58	2.58	32.14	15.28	26.48	5.51
2	1	51.28	21.43	16.70	32.23	51.28	21.43	16.70	579.15	64.29	18.52	25.72	818.22
2	2	53.37	22.10	21.91	42.48	53.37	22.10	21.91	366.79	59.68	20.73	29.62	276.83
2	3	53.75	22.38	31.35	21.18	53.75	22.38	31.35	464.48	61.71	20.67	31.35	529.16
2	4	55.78	22.17	17.86	58.42	55.63	22.21	25.82	343.83	68.70	19.26	25.82	902.00
2	5	56.23	21.99	12.76	33.59	56.29	21.98	18.52	340.22	61.23	20.92	26.26	1130.31
3	1	82.42	31.68	34.14	1097.11	82.65	31.62	20.99	3003.05	85.53	31.00	34.14	3069.73
3	2	93.50	37.26	25.23	1072.57	93.50	37.26	25.23	2008.37	98.06	36.26	38.56	2418.84
3	3	81.92	34.63	33.84	85.52	81.99	34.61	25.28	2011.34	93.53	32.01	33.84	1748.32
3	4	92.97	39.41	22.74	87.85	92.97	39.41	22.74	2011.68	104.96	36.69	40.86	2225.02
3	5	84.93	34.78	28.70	59.34	85.29	34.71	24.76	3003.79	93.33	32.93	33.90	2153.37
4	1	133.69	53.27	50.19	254.82	136.16	52.66	35.41	2015.16	147.63	50.05	50.19	2458.64
4	2	150.08	58.88	51.89	147.06	152.23	58.34	37.38	1026.72	172.47	53.65	51.89	1700.10
4	3	117.64	46.34	40.33	332.16	117.64	46.34	40.33	2028.13	126.87	44.21	42.11	2485.53
4	4	132.42	51.17	44.64	1130.09	132.42	51.17	44.64	2015.00	140.95	49.24	44.64	2372.91
4	5	139.77	56.55	50.86	258.29	139.90	56.51	36.86	2010.03	148.79	54.54	50.86	1188.38
Average		74.81	30.89	28.54	235.98	75.08	30.82	24.79	1162.23	83.76	28.90	33.31	1275.15

Table 5: Comparison between the three MINLP-based algorithms. The first row denotes the model. The second row denotes the configuration and seed of the instance, and (for each of the three models) we show the yielded total travel time, ridership, number of winners, and CPU time required to compute the solution. The last row shows average results.

	MINLP		ENUM$_{0.1}$			ENUM$_{0.05}$			ENUM$_{0.025}$		
Seed	z_{TTT}	Sec	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap
1	16.93	1.40	17.08	0.05	0.87	16.86	0.15	−0.40	16.86	0.64	−0.43
2	18.27	1.31	18.41	0.04	0.77	18.18	0.15	−0.51	18.15	0.64	−0.67
3	25.62	1.26	25.81	0.05	0.74	25.58	0.17	−0.14	25.56	0.64	−0.22
4	27.20	1.28	27.38	0.04	0.66	27.29	0.15	0.35	27.14	0.64	−0.20
5	28.64	1.11	28.83	0.04	0.68	28.73	0.15	0.30	28.64	0.64	0.00
6	13.95	0.95	14.07	0.04	0.88	13.99	0.15	0.35	13.95	0.63	0.07
7	15.80	1.27	15.93	0.04	0.79	15.85	0.15	0.30	15.81	0.82	0.06
8	17.48	1.27	17.60	0.05	0.71	17.52	0.15	0.26	17.48	0.67	0.05
9	18.99	1.36	19.12	0.06	0.65	19.04	0.17	0.23	19.00	0.62	0.04
10	19.98	1.06	20.12	0.04	0.69	20.04	0.15	0.26	19.99	0.61	0.05
11	21.06	1.27	21.23	0.05	0.81	21.12	0.16	0.29	21.07	0.61	0.05
12	16.72	1.04	16.84	0.04	0.71	16.76	0.16	0.26	16.73	0.62	0.05
13	17.82	1.05	17.96	0.04	0.83	17.87	0.17	0.32	17.83	0.63	0.06
14	26.22	1.13	26.24	0.04	0.10	26.24	0.15	0.10	26.23	0.63	0.05
15	27.71	1.16	27.66	0.05	−0.17	27.66	0.15	−0.17	27.66	0.64	−0.17
16	18.66	1.90	18.59	0.04	−0.33	18.59	0.16	−0.33	18.59	0.62	−0.33
17	20.30	1.38	20.24	0.04	−0.29	20.24	0.15	−0.29	20.24	0.62	−0.29
18	16.96	1.39	17.06	0.04	0.57	16.85	0.15	−0.64	16.83	0.62	−0.75
19	18.52	1.22	18.45	0.04	−0.36	18.41	0.17	−0.57	18.40	0.62	−0.60
20	19.90	1.87	19.84	0.04	−0.31	19.82	0.15	−0.38	19.82	0.61	−0.38
21	21.10	0.89	21.10	0.04	0.00	21.10	0.16	0.00	21.10	0.63	0.00
22	22.22	0.88	22.25	0.05	0.14	22.25	0.16	0.14	22.24	0.63	0.07
23	17.03	0.92	17.03	0.04	−0.01	17.03	0.15	−0.01	17.03	0.62	−0.01
24	18.30	0.88	18.48	0.04	1.02	18.30	0.16	0.00	18.30	0.63	0.00
25	26.76	1.02	26.82	0.04	0.21	26.76	0.16	0.01	26.76	0.62	0.01
26	18.17	1.38	18.18	0.04	0.06	18.17	0.17	0.00	18.17	0.63	0.00
27	19.98	1.46	19.98	0.04	0.00	19.98	0.15	0.00	19.98	0.62	0.00
28	21.65	1.41	21.65	0.05	0.02	21.65	0.16	0.02	21.65	0.62	0.00
29	17.43	1.11	17.43	0.04	−0.02	17.43	0.15	−0.02	17.43	0.63	−0.02
30	19.14	1.19	19.42	0.04	1.48	19.14	0.16	0.00	19.14	0.63	0.00
31	20.69	1.38	20.69	0.04	0.03	20.69	0.16	0.02	20.69	0.62	0.00
32	22.02	0.94	22.02	0.04	0.00	22.02	0.15	0.00	22.02	0.63	0.00
33	23.07	1.30	23.08	0.05	0.06	23.07	0.15	−0.01	23.07	0.63	−0.01
34	23.97	1.67	24.02	0.04	0.21	23.97	0.15	0.00	23.97	0.62	0.00
35	18.41	0.83	18.67	0.04	1.40	18.41	0.15	0.00	18.41	0.63	0.00
36	17.27	1.93	17.38	0.04	0.62	17.31	0.15	0.24	17.28	0.63	0.04
37	19.16	1.26	19.17	0.05	0.06	19.17	0.16	0.06	19.16	0.62	0.03
38	20.82	1.44	20.82	0.04	0.00	20.82	0.15	0.00	20.82	0.63	0.00
39	22.33	1.54	22.34	0.04	0.06	22.33	0.15	0.00	22.33	0.63	0.00
40	23.73	1.29	23.77	0.05	0.18	23.73	0.16	0.00	23.73	0.66	0.00
41	19.14	0.86	19.32	0.05	0.91	19.14	0.17	0.00	19.14	0.67	0.00
42	20.59	0.89	20.59	0.05	0.00	20.59	0.17	0.00	20.59	0.70	0.00
43	21.87	1.01	21.88	0.05	0.01	21.88	0.16	0.01	21.87	0.69	−0.01
44	23.00	1.20	23.03	0.05	0.13	23.03	0.16	0.13	23.01	0.68	0.03
45	24.28	1.18	24.06	0.05	−0.89	23.99	0.16	−1.17	23.99	0.66	−1.17
46	15.61	1.19	15.77	0.04	1.02	15.64	0.17	0.20	15.62	0.67	0.08
47	17.60	0.97	17.79	0.05	1.06	17.60	0.16	0.00	17.60	0.65	0.00
48	19.44	1.21	19.59	0.04	0.76	19.41	0.16	−0.16	19.40	0.64	−0.22
49	21.11	1.26	21.24	0.05	0.63	21.18	0.16	0.34	21.05	0.63	−0.27
50	22.61	1.16	22.76	0.04	0.64	22.68	0.15	0.30	22.59	0.62	−0.09
Avg.	20.50	1.23	20.58	0.04	0.38	20.50	0.16	−0.01	20.48	0.64	−0.10

Table 6: Results obtained for the instances corresponding with the first configuration.

Seed	MINLP		ENUM$_{0.1}$			ENUM$_{0.05}$			ENUM$_{0.025}$		
	z_{TTT}	Sec	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap
1	51.28	55.49	51.36	0.15	0.15	51.32	0.56	0.06	51.31	2.11	0.06
2	53.37	31.06	53.51	0.13	0.25	53.42	0.53	0.08	53.40	2.14	0.05
3	53.95	142.15	54.11	0.13	0.29	53.83	0.51	−0.22	53.74	2.07	−0.38
4	55.78	163.54	55.92	0.13	0.25	55.84	0.51	0.11	55.63	2.10	−0.27
5	56.23	29.61	56.43	0.14	0.35	56.32	0.53	0.16	56.21	2.13	−0.03
6	42.07	56.65	42.10	0.14	0.06	42.10	0.52	0.06	42.10	2.09	0.06
7	45.01	52.96	45.05	0.14	0.10	45.05	0.53	0.10	45.02	2.10	0.02
8	47.33	25.02	47.35	0.14	0.04	47.28	0.52	−0.09	47.28	2.08	−0.09
9	48.72	17.46	48.92	0.15	0.41	48.79	0.51	0.15	48.73	2.09	0.03
10	49.56	39.26	49.66	0.14	0.21	49.58	0.52	0.04	49.58	2.08	0.04
11	50.38	39.24	50.47	0.14	0.17	50.43	0.51	0.10	50.39	2.09	0.02
12	37.17	22.81	37.17	0.14	0.00	37.17	0.52	0.00	37.17	2.11	0.00
13	47.98	45.73	48.13	0.14	0.33	48.04	0.52	0.13	47.99	2.12	0.03
14	49.77	39.97	49.85	0.14	0.16	49.85	0.52	0.16	49.79	2.08	0.04
15	51.92	56.78	51.92	0.14	0.00	51.92	0.52	0.00	51.92	2.09	0.00
16	42.42	26.59	42.48	0.14	0.13	42.48	0.52	0.13	42.45	2.08	0.07
17	60.29	11.12	60.62	0.14	0.54	60.41	0.52	0.19	60.31	2.08	0.03
18	46.28	13.38	46.50	0.13	0.48	46.36	0.52	0.19	46.29	2.10	0.03
19	48.37	17.34	48.44	0.14	0.15	48.44	0.52	0.15	48.38	2.09	0.02
20	62.29	13.13	62.29	0.13	0.00	62.29	0.51	0.00	62.29	2.11	0.00
21	64.11	23.63	64.24	0.13	0.20	64.15	0.54	0.07	64.12	2.10	0.01
22	65.50	20.91	65.53	0.15	0.05	65.53	0.55	0.05	65.53	2.16	0.05
23	48.91	42.02	48.91	0.14	0.00	48.91	0.54	0.00	48.91	2.16	0.00
24	51.23	41.49	51.44	0.14	0.41	51.29	0.54	0.12	51.25	2.14	0.04
25	53.37	26.95	53.55	0.15	0.35	53.44	0.57	0.14	53.38	2.10	0.03
26	43.88	22.00	44.05	0.14	0.39	43.96	0.51	0.18	43.89	2.10	0.03
27	57.42	9.49	57.71	0.14	0.50	57.53	0.52	0.19	57.44	2.09	0.03
28	59.67	8.96	59.95	0.14	0.48	59.78	0.54	0.18	59.68	2.15	0.03
29	45.60	8.66	45.79	0.14	0.41	45.66	0.52	0.15	45.61	2.10	0.02
30	48.09	24.52	48.14	0.13	0.10	48.09	0.52	0.00	48.09	2.10	0.00
31	49.65	23.66	49.68	0.14	0.08	49.65	0.52	0.01	49.65	2.11	0.01
32	50.38	36.25	50.45	0.14	0.13	50.45	0.52	0.13	50.39	2.11	0.02
33	62.43	25.29	62.49	0.14	0.09	62.49	0.53	0.09	62.45	2.12	0.02
34	63.16	26.74	63.36	0.14	0.32	63.25	0.53	0.15	63.17	2.11	0.02
35	47.52	39.30	47.70	0.14	0.38	47.58	0.52	0.13	47.53	2.08	0.02
36	40.30	33.52	40.30	0.14	0.01	40.30	0.51	0.01	40.30	2.08	0.01
37	42.21	22.69	42.23	0.13	0.04	42.23	0.52	0.04	42.23	2.09	0.04
38	43.48	22.96	43.51	0.14	0.06	43.51	0.52	0.06	43.49	2.12	0.01
39	44.20	9.57	44.24	0.14	0.09	44.19	0.53	−0.01	44.19	2.10	−0.01
40	55.55	15.41	55.69	0.14	0.26	55.58	0.53	0.05	55.58	2.12	0.05
41	58.73	13.58	59.00	0.13	0.47	58.82	0.54	0.16	58.74	2.12	0.02
42	61.37	9.77	61.60	0.14	0.37	61.49	0.52	0.18	61.39	2.11	0.03
43	63.40	8.95	63.55	0.14	0.23	63.53	0.53	0.21	63.43	2.09	0.04
44	64.32	15.11	64.57	0.14	0.39	64.46	0.52	0.22	64.34	2.09	0.04
45	64.35	23.21	64.55	0.13	0.32	64.50	0.52	0.25	64.38	2.11	0.05
46	37.93	44.92	38.05	0.14	0.31	37.98	0.52	0.11	37.94	2.09	0.02
47	51.81	49.06	51.93	0.14	0.24	51.87	0.51	0.11	51.70	2.10	−0.21
48	54.33	26.47	54.53	0.14	0.37	54.39	0.53	0.11	54.35	2.30	0.03
49	56.60	29.19	56.61	0.14	0.02	56.61	0.51	0.02	56.61	2.10	0.02
50	58.44	32.14	58.47	0.14	0.05	58.47	0.54	0.05	58.36	2.16	−0.14
Avg.	52.16	32.71	52.28	0.14	0.22	52.21	0.52	0.09	52.16	2.11	0.00

Table 7: Results obtained for the instances corresponding with the second configuration.

Seed	MINLP		ENUM$_{0.1}$			ENUM$_{0.05}$			ENUM$_{0.025}$		
	z_{TTT}	Sec	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap
1	82.42	254.67	82.41	0.26	0.00	82.41	0.96	0.00	82.41	3.71	0.00
2	93.68	173.856	93.61	0.25	−0.07	93.61	0.93	−0.07	93.52	3.66	−0.17
3	85.99	245.037	82.36	0.24	−4.22	81.92	0.89	−4.73	81.92	3.67	−4.73
4	94.10	165.082	93.30	0.24	−0.85	92.84	0.89	−1.34	92.84	3.61	−1.34
5	87.78	281.076	85.31	0.23	−2.82	84.93	0.88	−3.25	84.93	3.64	−3.25
6	85.58	365.321	85.58	0.25	0.00	85.58	0.95	0.00	85.58	3.87	0.00
7	98.67	1165.37	98.75	0.26	0.08	98.68	0.93	0.01	98.68	3.84	0.01
8	108.67	1215.46	108.73	0.25	0.05	108.67	0.95	0.00	108.67	3.81	0.00
9	81.96	154.539	82.05	0.25	0.11	82.05	0.93	0.11	82.01	3.82	0.06
10	88.09	227.228	88.12	0.23	0.03	88.09	0.87	0.00	88.09	3.73	0.00
11	74.79	77.657	74.81	0.23	0.02	74.81	0.87	0.02	74.79	3.55	0.00
12	69.74	66.066	69.74	0.23	0.01	69.74	0.86	0.01	69.74	3.50	0.00
13	83.15	64.576	83.44	0.23	0.35	83.26	0.87	0.13	83.17	3.53	0.02
14	74.03	93.544	74.28	0.23	0.34	74.12	0.86	0.12	74.05	3.47	0.02
15	84.71	83.966	85.04	0.24	0.40	84.94	0.85	0.27	84.71	3.73	0.00
16	97.78	87.109	97.88	0.25	0.11	97.78	0.93	0.01	97.78	3.66	0.01
17	92.57	74.509	92.87	0.25	0.32	92.68	0.90	0.11	92.59	3.65	0.02
18	89.67	1293.86	87.81	0.24	−2.07	87.81	0.90	−2.07	87.81	3.62	−2.07
19	98.39	199.251	96.64	0.24	−1.79	96.53	0.88	−1.89	96.48	3.62	−1.94
20	109.04	216.497	109.04	0.24	0.01	109.04	0.89	0.01	109.04	3.59	0.00
21	103.86	82.42	104.44	0.24	0.56	103.87	0.87	0.01	103.86	3.58	0.00
22	91.57	85.418	91.64	0.23	0.08	91.57	0.87	0.00	91.57	3.55	0.00
23	86.08	57.096	86.08	0.23	0.00	86.08	0.86	0.00	86.08	3.53	0.00
24	95.12	89.586	95.39	0.23	0.28	95.21	0.86	0.10	95.14	3.51	0.02
25	71.62	80.517	71.84	0.22	0.30	71.70	0.86	0.11	71.63	3.48	0.02
26	84.83	162.316	84.89	0.24	0.07	84.83	0.93	0.00	84.83	3.70	0.00
27	118.56	95.685	118.57	0.24	0.01	118.57	0.91	0.01	118.55	3.65	0.00
28	89.92	1064.16	90.11	0.24	0.21	90.04	0.91	0.13	89.94	3.66	0.02
29	87.94	321.837	86.31	0.24	−1.85	85.87	0.90	−2.36	85.86	3.65	−2.37
30	95.95	230.683	96.06	0.25	0.12	96.03	0.91	0.08	95.96	3.66	0.01
31	85.90	141.186	85.87	0.23	−0.03	85.87	0.88	−0.03	85.87	3.67	−0.03
32	80.75	276.916	78.74	0.24	−2.50	78.74	0.88	−2.50	78.74	3.56	−2.50
33	87.34	57.817	87.89	0.24	0.63	87.36	0.88	0.02	87.34	3.58	0.00
34	98.36	70.351	98.36	0.23	0.00	98.36	0.87	0.00	98.35	3.51	−0.01
35	90.93	70.696	91.22	0.23	0.32	91.04	0.87	0.12	90.95	3.51	0.02
36	71.98	259.336	72.13	0.23	0.21	71.99	0.92	0.00	71.91	3.73	−0.11
37	83.16	127.972	83.56	0.24	0.48	83.18	0.91	0.02	83.16	3.70	0.00
38	91.50	231.209	91.74	0.25	0.25	91.73	0.93	0.25	91.50	3.72	0.00
39	82.08	209.866	80.13	0.24	−2.37	80.13	0.91	−2.37	80.12	3.71	−2.38
40	94.78	207.172	94.84	0.24	0.06	94.84	0.92	0.06	94.80	3.69	0.02
41	91.97	483.556	89.85	0.24	−2.30	89.67	0.91	−2.50	89.67	3.67	−2.50
42	102.74	258.571	102.75	0.24	0.01	102.75	0.89	0.01	102.74	3.65	0.00
43	112.81	78.975	112.81	0.24	0.00	112.81	0.89	0.00	112.81	3.63	0.00
44	84.70	1077.52	84.70	0.24	0.00	84.70	0.89	0.00	84.70	3.60	0.00
45	95.03	121.431	95.10	0.24	0.08	95.10	0.88	0.08	95.05	3.59	0.02
46	90.06	260.171	90.11	0.25	0.05	90.06	0.95	0.00	90.06	3.93	0.00
47	102.95	1194.55	103.05	0.24	0.10	102.97	0.94	0.02	102.97	3.82	0.02
48	100.02	124.844	100.06	0.25	0.04	100.02	0.92	0.00	100.02	3.79	0.00
49	111.43	116.062	111.45	0.25	0.02	111.43	0.93	0.00	111.43	3.75	0.00
50	100.46	249.531	100.90	0.26	0.43	100.51	0.94	0.05	100.48	3.74	0.01
Avg.	91.30	287.84	91.05	0.24	−0.29	90.93	0.90	−0.42	90.90	3.66	−0.46

Table 8: Results obtained for the instances corresponding with the third configuration.

	MINLP		ENUM$_{0.1}$			ENUM$_{0.05}$			ENUM$_{0.025}$		
Seed	z_{TTT}	Sec	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap	z_{TTT}	Sec	%gap
1	137.98	246.75	133.68	0.39	−3.12	133.68	1.42	−3.12	133.68	5.73	−3.12
2	154.33	128.82	150.08	0.38	−2.75	150.08	1.39	−2.75	150.08	5.54	−2.75
3	121.15	247.58	117.65	0.40	−2.89	117.65	1.45	−2.89	117.64	5.93	−2.90
4	136.32	245.46	132.87	0.37	−2.54	132.42	1.42	−2.86	132.42	5.72	−2.86
5	144.09	245.89	139.77	0.39	−3.00	139.77	1.37	−3.00	139.77	5.61	−3.00
6	140.75	2394.20	140.93	0.42	0.13	140.78	1.54	0.02	140.78	6.27	0.02
7	161.05	2160.76	161.07	0.42	0.02	161.05	1.52	0.00	161.05	6.14	0.00
8	175.27	1647.11	175.29	0.39	0.01	175.28	1.46	0.00	175.27	5.96	0.00
9	111.04	447.47	111.05	0.40	0.01	111.05	1.42	0.01	111.04	5.80	0.00
10	124.79	45.68	122.49	0.36	−1.84	122.34	1.29	−1.96	122.28	5.24	−2.01
11	130.46	28.80	128.57	0.34	−1.45	128.43	1.25	−1.56	128.37	5.07	−1.60
12	126.15	1029.91	123.13	0.38	−2.39	123.05	1.34	−2.45	123.05	5.39	−2.45
13	141.13	36.04	138.07	0.37	−2.17	137.89	1.29	−2.30	137.73	5.27	−2.41
14	136.36	132.55	132.81	0.37	−2.60	132.81	1.34	−2.60	132.80	5.52	−2.60
15	127.59	127.06	124.08	0.36	−2.75	123.64	1.31	−3.10	123.64	5.38	−3.10
16	147.13	245.86	141.93	0.38	−3.53	141.93	1.38	−3.53	141.92	5.62	−3.54
17	148.39	129.29	145.28	0.37	−2.10	144.72	1.36	−2.47	144.71	5.49	−2.48
18	146.94	256.02	143.41	0.39	−2.41	143.19	1.41	−2.55	143.01	5.75	−2.68
19	161.58	15.31	157.81	0.38	−2.34	157.59	1.38	−2.47	157.47	5.64	−2.54
20	177.51	10.43	174.17	0.36	−1.88	173.95	1.33	−2.01	173.85	5.43	−2.06
21	139.66	8.81	137.55	0.35	−1.51	137.39	1.30	−1.63	137.33	5.34	−1.67
22	128.74	8.70	126.93	0.36	−1.40	126.76	1.28	−1.54	126.70	5.16	−1.59
23	150.94	127.52	144.92	0.37	−3.99	144.92	1.33	−3.99	144.92	5.46	−3.99
24	158.12	128.33	155.15	0.36	−1.88	154.70	1.31	−2.16	154.64	5.34	−2.20
25	128.86	132.99	125.25	0.37	−2.80	125.23	1.37	−2.82	125.23	5.61	−2.82
26	149.90	248.81	144.95	0.40	−3.30	144.93	1.46	−3.32	144.93	5.85	−3.32
27	166.63	246.10	162.21	0.37	−2.65	161.58	1.43	−3.03	161.58	5.71	−3.03
28	126.33	130.36	124.13	0.39	−1.75	123.99	1.38	−1.85	123.88	5.56	−1.95
29	149.39	129.59	143.91	0.39	−3.67	143.91	1.43	−3.67	143.91	5.83	−3.67
30	158.83	132.48	155.68	0.40	−1.99	155.50	1.40	−2.10	155.41	5.83	−2.16
31	148.55	10.85	145.94	0.37	−1.76	145.77	1.36	−1.87	145.70	5.60	−1.91
32	134.01	8.44	131.78	0.37	−1.66	131.59	1.33	−1.81	131.52	5.40	−1.86
33	125.07	10.98	122.33	0.36	−2.20	122.16	1.28	−2.33	122.08	5.25	−2.39
34	141.51	8.70	138.79	0.38	−1.92	138.62	1.27	−2.04	138.54	5.19	−2.10
35	160.18	127.29	155.86	0.37	−2.70	155.14	1.32	−3.14	155.14	5.41	−3.14
36	129.85	261.07	126.16	0.41	−2.84	126.14	1.50	−2.86	126.14	6.08	−2.86
37	148.04	251.41	143.89	0.40	−2.80	143.89	1.53	−2.80	143.88	5.90	−2.81
38	160.84	19.52	156.56	0.39	−2.66	156.56	1.41	−2.66	156.56	5.76	−2.67
39	123.86	19.66	121.13	0.40	−2.21	120.97	1.40	−2.33	120.89	5.61	−2.40
40	137.16	11.51	133.60	0.37	−2.60	133.60	1.36	−2.60	133.50	5.51	−2.68
41	124.32	137.95	121.32	0.41	−2.42	120.95	1.40	−2.71	120.95	5.78	−2.71
42	165.28	13.14	161.31	0.38	−2.40	161.31	1.39	−2.40	161.31	5.61	−2.40
43	181.30	10.86	177.33	0.39	−2.19	177.33	1.34	−2.19	177.33	5.48	−2.19
44	115.50	127.64	112.38	0.38	−2.71	112.38	1.31	−2.71	112.38	5.37	−2.71
45	133.05	127.98	129.97	0.37	−2.32	129.97	1.27	−2.32	129.97	5.22	−2.32
46	166.92	483.60	161.08	0.42	−3.50	161.08	1.54	−3.50	161.00	6.28	−3.55
47	150.83	482.38	146.59	0.43	−2.81	146.02	1.54	−3.19	146.02	6.30	−3.19
48	165.07	252.01	159.93	0.42	−3.11	159.93	1.53	−3.11	159.92	6.24	−3.12
49	182.34	132.41	177.81	0.40	−2.49	177.23	1.49	−2.80	177.22	6.11	−2.81
50	141.80	138.69	138.17	0.41	−2.56	137.71	1.53	−2.89	137.70	6.20	−2.89
Avg.	144.86	275.02	141.54	0.38	−2.29	141.37	1.39	−2.40	141.34	5.65	−2.42

Table 9: Results obtained for the instances corresponding with the fourth configuration.