Reliability and Tolerance Comparison in Water Supply Networks[1]

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Abstract

Urban water supply is a high priority service and so looped networks are extensively used in order to considerably reduce the number of consumers affected by a failure. Looped networks may be redundant in connectivity and capacity. The concept of reliability has been introduced in an attempt to quantitatively measure the possibility of maintaining an adequate service for a given period. Numerous researchers have considered reliability as a measure of redundancy. This concept is usually implicit, but some researchers have even stated it explicitly. This paper shows why reliability cannot be considered a measure of redundancy given that branched networks can achieve high values of reliability and this would deny the fact that a looped network is more reliable than a branched network with a similar layout and size. To this end the paper discusses two quantitative indices for measuring expected network behavior: reliability and tolerance. These indices are calculated and a comparison is made between looped, branched, and mixed networks.

Keywords: water supply, reliability, tolerance, network design.

1 Introduction

Many cities in the world have very old water supply networks, and research regarding network failures, risks, and vulnerabilities has attracted the attention of researchers. As a result, many interesting papers have been recently published (Carrión et al. 2010; Christodoulou 2010; Pinto et al. 2010).

Drinking water supply networks have almost always been designed with loops so as to provide alternative paths from the source to every network node or junction. This practice is based on the need to reduce the number of affected consumers when a pipe is withdrawn from service after its failure or because of any other reason. These networks are often called closed or looped networks.

It is well known that the classical cost minimization of a looped network produces a branched network (Chiong 1985; Goulter 1993). If minimum diameters are specified to close the loops, then it is very likely that the diameters employed will not provide sufficient capacity to convey the required flows if a main pipe fails. As a result, a major service interruption may occur. Therefore, the use of loops in this way has little, if any, practical value (Martínez 2007).

Under normal operating conditions there is no need for loops – meaning that a looped network is redundant. Redundancy is the capacity of the network to distribute water to users using alternatives routes. Redundancy is only needed to maintain service, reduce deficit, and minimize the number of affected consumers when a pipe is withdrawn from service.

Redundancy is associated with additional pipes as well as increased flow capacity in those pipes. Capacity reflects the design diameter of the pipe. Due to the previously mentioned drawbacks of using minimum diameters, pipes are designed with large diameters, and so further add to redundancy. Therefore, looped networks can be redundant in connectivity and capacity.

Consequently, a distinction must be made between these two types of redundancy: connectivity redundancy and capacity redundancy. If a branched network is closed using minimum diameters, then there will be connectivity redundancy but no capacity redundancy. Redundancy therefore includes two important concepts: firstly, the connectivity necessary to provide alternative flow paths to each node; and secondly, the provision of an adequate flow capacity (diameter) for

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those paths (Martínez 2010).

Therefore, an important design factor is network layout and the comparison between branched and looped networks. The concept of redundancy appears in this comparison and has been closely related to reliability (Goulter 1992; Park & Liebman 1993; Khomsi et al. 1996). The concept of reliability was introduced to quantitatively measure the possibility of maintaining an adequate service for a given period.

As the behavior of networks is influenced by various phenomena, research has been made on assigning probabilistic values to these phenomena so as to quantify the performance reliability of a network. In this sense, several works can be consulted – including: (Lansey et al. 1989), (Bao and Mays 1990), (Goulter and Bouchart 1990), (Jacobs and Goulter 1991), (Quimpo and Shamsi 1991), (Xu and Goulter 1998).

The introduction of the reliability concept and its quantification are the consequence of the need to provide and measure an adequate level of redundancy in a network (Goulter 1993). The authors mentioned in the previous paragraph use varying definitions but, in the long run, the concept of reliability is defined as the probability that a water supply network will satisfy the design demand.

Other authors have examined the expected proportion of demand that is satisfied by a network (Park and Leibman 1993), (Xu and Goulter 1997), (Kalungi and Tanyimboh 2003).

The alternative of estimating global network reliability as the product of the non-failure probabilities of isolated pipes (Jowitt and Xu 1993) says little about the reliability of a looped network. As stated above, the classical optimization of a looped network gives the solution for a branched network by eliminating one pipe from each loop. If global reliability is calculated in this way for a branched network; the result is that the branched network is more reliable than the looped network because it has fewer pipes. This result is, of course, meaningless.

The reliability calculation in looped water supply networks – whether based on failure probabilities or expected proportion – is formulated in the literature as a function of the causes affecting consumer demand. The causes usually considered are:

1) occurrence of a demand exceeding the design demand (for instance, a fire demand),
2) growth of population served by the network,
3) pipe aging,
4) pipe failure.

Obviously, an explicit formulation of all these causes in probabilistic terms and their further integration implies considerable mathematical and algorithmic complexity. Notice, for instance, that failure (cause 4) may involve the definition of several indices if failure frequency, expected failure duration, time between failures, unsatisfied demand, etc. are considered – as in (Duan et al. 1990).

Xu and Goulter (1999; 2000) discuss causes 2), 3), and 4) in a probabilistic manner and deal with a complicated series of algorithms before presenting a solution. Other papers with a high computational burden are (Duan et al. 1990), and (Cullinae et al. 1992). Interesting works dealing with the relation between reliability and the location of valves include (Bouchart and Goulter 1991), and more recently, (Walski et al. 2006).

It was concluded by Goulter (1992) that no reliability definition had yet been proposed that was able to simultaneously fulfill the following requirements:

- explicit consideration of all causes,
- inclusion of causes in computationally feasible optimization models,
- achievable and acceptable to the professional working community.

To this it might be added that most reliability definitions and applications found in the literature are not concerned with the network layout. They can be applied to looped networks as well as to branched networks; and nothing in their numerical values makes a sharp distinction between these two types of networks. These reliability definitions cannot prevent the opening of the loops that results from optimization models (Lansey et al. 1989; Duan et al. 1990; Loganathan et al.1990).

Many researchers have considered reliability as a measure of redundancy. This concept is usually implicit, but some researchers have even stated it explicitly. This paper shows why reliability cannot be considered a measure of redundancy given that branched networks can achieve high values of reliability and this would deny the fact that a looped network is more reliable than a branched network with a similar layout and size. To this end the paper discusses two quantitative indices for measuring expected network behavior: reliability and tolerance. These indices are calculated, and a comparison is made between looped, branched, and mixed networks.

While the tolerance index definition is also unable to solve the opening of the loops when used as a constraint in the context of a design optimization, it will be seen in the example how the index definition makes a noticeable difference when applied to branched and looped networks. Other results relating tolerance with design optimization can be found in Martínez (2010).

The importance of the results presented herein is that current measures of reliability for water supply networks should no longer be considered to have a direct relation with redundancy.

2 Formulation

Following the above discussion, this paper considers a simple reliability formulation as in (Xu and Goulter 1997), (Tanyimboh et al. 2001), and (Kalungi and Tanyimboh 2003). Interesting conclusions can be drawn from such considerations, especially after comparing results with another index that we will term 'tolerance'. Before defining the tolerance concept, it is worthwhile introducing a reliability formulation that only considers pipe failures. It is assumed that a pipe temporarily withdrawn from service can be isolated and so only those consumers connected to that pipe are affected.

Only one pipe failure at a time is considered in the formulation of reliability. This is supported by the well-known fact that the probability of simultaneous failure by two or more pipes is extremely small (Morgan and Goulter 1985; Walters and Knezevic 1989; Loganathan et al. 1990; Bouchart and Goulter 1991; Park and Leibman 1993; Gupta and Bhave 1994; Khomsi et al. 1996; Xu and Goulter 1997; Xu and Goulter 1998; Xu and Goulter 1999).

Accordingly, it is accepted that the probability of simultaneous pipe failures is practically zero. In this case, the probability pf_0 of the whole network working without failure is:

$$pf_0 = 1 - \sum_{k=1}^{NT} pf_k \tag{1}$$

where:

k: pipe counter; NT: total number of pipes in the network;

pf_k: failure probability of pipe k.

The value of pf_k can be obtained from empirical formulae as a function of pipe diameter and length (Su et al. 1987; Bouchart and Goulter 1991; Cullinane et al. 1992; Gupta and Bhave 1994; Khomsi et al. 1996).

Considering an average time for the duration of pipe failure, reliability R is defined as:

$$R = \frac{1}{q^{req}} \left(q^{nf}\, pf_0 + \sum_{k=1}^{NT} q^k\, pf_k \right) \tag{2}$$

where:

q^{req}**:** total required demand by the network (the sum of all nodal demands);

q^{nf}: total flow delivered to the network when there are no failures;

q^k: total flow delivered to the network when pipe k fails.

After this definition it can be seen that reliability R represents the expected fraction of q^{req} that can be maintained for a certain time horizon, as long as the network properties used for this reliability calculation are maintained.

By calling:

$$r_0 = \frac{q^{nf}}{q^{req}} \quad \text{and} \quad r_k = \frac{q^k}{q^{req}}, \tag{3}$$

equation (2) can be written:

$$R = r_0 pf_0 + \sum_{k=1}^{NT} r_k pf_k \,. \tag{4}$$

3 Simulator

The value of q^{req} is known for an existing network, or an already dimensioned network, because this is the design demand described in a project. A computational program – usually called a simulator – is needed to calculate the value of q^{nf} and the NT different values of q^k. The simulator must calculate the actual flow delivered at nodes when the network is in a failure state, meaning that one of its pipes has failed. It is apparent that if the network performs well when there are no failures, then the value of r_0 is one, in other words $q^{nf} = q^{req}$, but this should also be checked in the simulator.

Classical water supply network simulators are called demand-driven simulators because they meet all nodal design demands and calculate the corresponding nodal pressures. Several other simulators have been presented in the literature (Jowitt and Xu 1993; Gupta and Bhave 1994; Xu and Goulter 1997; Kalungi and Tanyimboh 2003) and these simulators can calculate a network under a failure state once the head pressures at the source nodes are known. These are described as head-driven or pressure-driven simulators.

In this paper, a simulator similar to that described by Xu and Goulter (1997) is used. It applies the following pressure-driven flow equation at each node:

$$\frac{q_{real}}{D_{em}} = \left(\frac{p_{real} - p_{inf}}{p_{req} - p_{inf}} \right)^{0.5} \tag{5}$$

where:

q_{real}, D_{em}: real flow delivered and design demand at node, respectively.

$p_{real}, p_{req}, p_{inf}$: real pressure, required pressure, and inferior pressure at node, respectively.

Two conditions must be satisfied with this equation:

(i) if $p_{real} \geq p_{req}$ then $q_{real} = D_{em}$ and (ii) if $p_{real} \leq p_{inf}$ then $q_{real} = 0$.

Derivation of this equation can be found in (Tanyimboh et al. 2001).

4 Tolerance

As water supply networks should presumably behave satisfactorily under normal conditions when there are no failures ($r_0 = 1$), it is worthwhile making a separate and specific analysis of their behavior under only failure states. Accordingly, the concept of *tolerance* to failure T (Tanyimboh et al. 2001; Kalungi and Tanyimboh 2003) has been introduced by using the expression:

$$T = \frac{R - r_0 pf_0}{1 - pf_0} \tag{6}$$

In this expression, the variables are related to the whole network. However, tolerance can also be formulated for each individual node if desired.

This tolerance to failure represents the expected q^{req} fraction that the network supplies as an average when it is in a state of failure. In other words, this index answers the question of how well the network behaves, on average, when a pipe is removed from service.

Although it has not been supported theoretically by the authors in the above-mentioned papers, this index is based on equation (4). It can be seen that the first term on the right of equation (4) represents a partial reliability corresponding to the network in a non-failure state; and the second term on the right of equation (4) represents, as a whole, all failure states. In practice, the first term is considerably greater than the second term because failure probabilities are always very small, even if all pipe failure probabilities are summed. This means that the total time during which the network is in a failure state (the sum of all failure time periods) is a tiny fraction of the total time.

The numerator of equation (6) is obtained from equation (4) by solving the second term on the right:

$$R - r_0 pf_0 = \sum_{k=1}^{NT} r_k pf_k \tag{7}$$

But as the definition of tolerance involves only that period when the network is in a state of failure, the need arises to normalize that time, i.e., to consider that time fraction as 100%. To this end, tolerance T is multiplied by its probability of occurrence 1- pf_0 and equation (6) is obtained. Note that this is the same as dividing equation (7) by 1- pf_0 because, as stated before, the fraction of failure time is very small when compared to total time. From equation (7) a very important conclusion can be derived: the value of tolerance is not influenced by the value of r_0.

Kalungi and Tanyimboh (2003) showed how useful and meaningful the tolerance index is with various examples. Despite the fact that tolerance is not an explicit measure of redundancy according to the redundancy definition given above, the tolerance index seems to be a good measure of the impact of redundancy. Moreover, as stated by its authors, tolerance seems to give an adequate inverse measure of network vulnerability; or of the vulnerability of the whole system if other components are included in the calculation. It is an inverse measure, because the greater the tolerance – the lesser the vulnerability.

In short, the reliability concept, as defined above, can be regarded as simply a measure of the behavior of the network under normal conditions: meaning that reliability is not a good measure of behavior under failure situations. This is because the term $r_0\, pf_0$ from equation (4) is absolutely predominant. Moreover, tolerance T refers only to the time during which the network is in a failure state. The main reason for looping the network is to reduce the consequences of failures, and in this paper a number of examples are introduced to illustrate the use of reliability and tolerance indices when comparing the behavior of looped and branched networks.

5 Illustrative examples and results

An example network is introduced in Figure 1. The figure represents a branched network with only one source, 11 pipes, and 12 demand nodes. There are no loops at the beginning, and the dotted lines do not represent pipes. The demand unit is liters per second (L/s). Node ground elevations are given in Table 1. The required pressure in all nodes is p_{req} = 20 m and the inferior pressure is p_{inf} = 0.20 p_{req}.

Pipe length in all reaches is 400 m and the friction formula is given by Hazen-Williams with a constant coefficient of 100 for all pipes. Diameters are presented on Table 2. Total head at the source (node 1) is H_1 = 80 m.

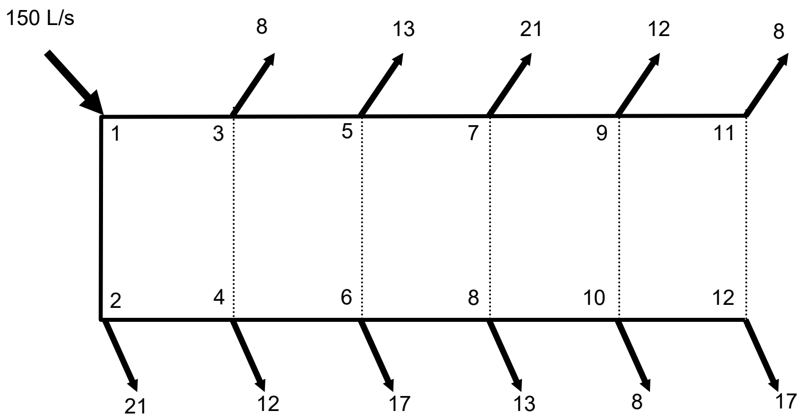


Fig. 1 Example network

Table 1 Node ground elevation

Node	1	2	3	4	5	6	7	8	9	10	11	12
Elevation (m)	40	40	42	42	44	44	46	46	48	48	50	50

Table 2 Pipe diameters

Pipe	1-2	1-3	2-4	3-5	4-6	5-7	6-8	7-9	8-10	9-11	10-12
Diam (mm)	350	400	300	300	350	300	300	250	250	250	250

Failure probability is calculated with the following expression:

$$pf_k = aL_k d_k^{-u}\, t_f / 365 \qquad (8)$$

where

L_k, d_k: length and diameter of pipe k in meters,

t_f: average failure duration = 2 days,

365: number of days per year,

a: coefficient of empirical formula = $3.50 \cdot 10^{-5}$,

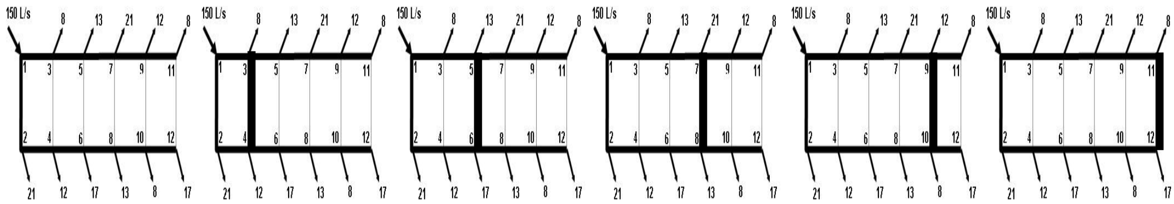
u: exponent of empirical formula = 1.27.

The following cases and groups have been defined with the aim of studying the variation in reliability and tolerance indices for different network layouts.

First group:

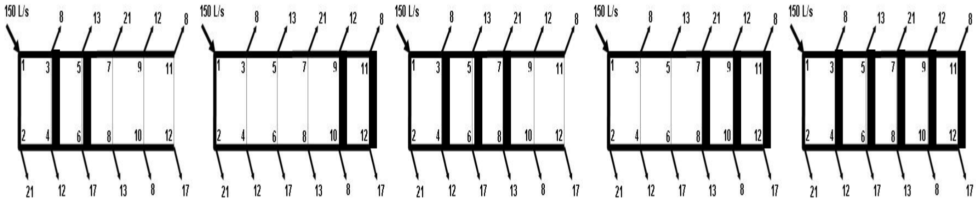
(a) network as in Figure 1 without loops
(b) adding only pipe 3 - 4 with 250 mm diameter
(c) adding only pipe 5 - 6 with 200 mm diameter
(d) adding only pipe 7 - 8 with 200 mm diameter
(e) adding only pipe 9 - 10 with 200 mm diameter
(f) adding only pipe 11- 12 with 200 mm diameter

FIRST GROUP



case	a	b	c	d	e	f
Reliability (%)	99.8931	99.9352	99.9477	99.9603	99.9647	99.9662
Tolerance (%)	73.6920	85.6454	88.7856	91.4919	92.4282	92.7430

SECOND



case	g	h	i	j	k
Reliability (%)	99.9576	99.9779	99.9777	99.9893	99.9994
Tolerance (%)	91.7070	95.7954	96.0902	98.1691	99.9107

Fig. 2 Various considered networks

All cases are run with the previously described simulator – reliability and tolerance results are shown in the upper half of Figure 2.

Case (a) is the only case of a purely branched network. Reliability is high because it has enough capacity (diameters) for the required demand. Note that reliability does not reflect the lack of connectivity redundancy: there is only one path from the source to each node. Moreover, tolerance reflects the lack of redundancy with a much lower value – indicating a vulnerable network.

Cases (b) through (e) represent mixed networks with one loop; and case (f) is a one-loop closed network. Notice that reliability and tolerance increase when the existing loop grows larger. The sharp increase in tolerance between cases (a) and (b) reflects the impact of redundancy.

Second group: (same diameters as the first group)

(g) adding only pipes 3-4 and 5-6
(h) adding only pipes 9-10 and 11-12
(i) adding only pipes 3-4, 5-6 and 7-8
(j) adding only pipes 7-8, 9-10 and 11-12
(k) adding all pipes to form 5 loops

Results can be seen on the lower half of Figure 2. Going from case (f) to case (g) shows a slight reduction in both indices because network (f) is completely closed while network (g) is mixed. Both indices then grow until case (k), which is a completely looped network that reaches very high values. In general, the second group networks are more tolerant to failure than those of the first group because they have more redundancy. It is worth emphasizing the significance of tolerance by noting that network (k) can deliver an average supply of more than 99.9% of the network design demand – even when in a failure state.

Third group:

Cases here are defined as (a1), (d1), (f1) and (k1). They are respectively similar to cases (a), (d), (f) and (k) with the only difference being that they are calculated with H_1 = 90m. Results are reported in Table 3.

Except for case (a1), the head increase at the source is reflected by an increase in reliability and an even greater increase in tolerance – when compared to their counterparts in the previous groups. Case (a1) is the exception because there is no redundancy and the head increment cannot contribute to alleviate failures. In case (a1) the values remain the same for both indices.

Table 3 Third group (and former counterparts)

case	a	d	f	k
Reliability (%)	**99.8931**	**99.9603**	**99.9662**	**99.9994**
Tolerance (%)	**73.6920**	**91.4919**	**92.7430**	**99.9107**

case	a1	d1	f1	k1
Reliability (%)	**99.8931**	**99.9730**	**99.9816**	**100.000**
Tolerance (%)	**73.6920**	**94.1981**	**96.0622**	**100.000**

Fourth group:

Cases in this group are defined as (c2), (d2), (e2) and (f2) respectively. They are similar to cases (c), (d), (e) and (f) with the difference that pipes have a smaller diameter that is equal to 100 mm. Results are shown in Table 4.

A reduction of reliability is observed when compared with their counterparts in the first group, but the reduction in tolerance is even greater. The explanation is that diameter reduction means reducing capacity redundancy; and tolerance is more sensitive to such a change. This result emphasizes the problems caused by closing loops with small diameter pipes.

Table 4 Fourth group (and former counterparts)

case	C	D	e	f
Reliability (%)	**99.9477**	**99.9603**	**99.9647**	**99.9662**
Tolerance (%)	**88.7856**	**91.4919**	**92.4282**	**92.7430**

case	c2	d2	e2	f2
Reliability (%)	**99.9080**	**99.9137**	**99.9197**	**99.9244**
Tolerance (%)	**83.2892**	**84.3274**	**85.4181**	**86.2740**

Fifth group:

Cases are defined for the fifth group as (a3), (f3), and (k3) respectively. They are similar to the cases (a), (f) and (k) with the difference that a second source is added at node 12. The head at this new source is H_{12} = 85 m, and it is assumed there are no limitations regarding the supply flow. Results are reported in Table 5.

When compared with counterparts in the first group it can be seen that reliability and tolerance increase. It can also be seen that the increase in tolerance is much greater and even occurs in the branched network in case (a3). The existence of a second source is equivalent to an increase in redundancy. In this particular case, by locating the new source at node 12, and having an unlimited supply capacity, alternative paths to numerous nodes are introduced and these reduce failure deficits. If the first source is located at node 11, then there will be alternative paths to all nodes.

Table 5 Fifth group (and former counterparts)

case	a	f	k
Reliability (%)	**99.8931**	**99.9662**	**99.9994**
Tolerance (%)	**73.6920**	**92.7430**	**99.9107**

case	a3	f3	k3
Reliability (%)	**99.9529**	**99.9996**	**100.000**
Tolerance (%)	**88.4012**	**99.9202**	**100.000**

Lastly, we will make a brief comment about cases (k1) and (k3) which show both indices at 100%. Although this may seem more theoretical than practical, the result was obtained with six zero decimal places after the decimal point – meaning that the five-loop network is very robust.

6 Conclusions

Following a thorough review of recently published papers, several current ideas and tendencies for measuring and evaluating water supply networks and their relationship with the concept of reliability are discussed.

The universal practice of looping water supply networks has been analyzed, as well as how optimization attempted with classical formulations leads to opening the network. A proposal for closing loops with minimum diameter pipes, and the argument that minimum diameters do not include the necessary redundancy, have also been addressed. The example described in the paper clearly illustrates this assertion.

The redundancy concept and how it relates to the concept of reliability is discussed. The concept of tolerance from a previously published paper is introduced and its relation with redundancy is explored. The origin of the expression for the calculation of tolerance is explained, together with how this expression reflects network behavior under failure situations.

Particularly interesting is the conclusion that the value of tolerance is independent of r_0, which means it is independent of network behavior for normal conditions. This assertion reinforces the fact that it is independent of the concept of reliability. This is precisely why tolerance is a better measure of the difference in performance between branched and looped networks.

Several evaluations of both indices are presented in the paper through an example that enables an examination of different branched, mixed, and looped network layouts. The sensitivity of the tolerance index is emphasized, as well as its effectiveness in reflecting the effect of redundancy.

Evaluations have been made for different network layouts, including variations in source head values, and the use of smaller diameters and dual sources.

It has been clearly shown why the reliability index cannot be associated with redundancy, and its weaknesses as a trustable measure of network behavior has been highlighted. The evidence provided strengthens the argument for using the tolerance index to measure the performance and vulnerability of a network under failure situations. The concept of tolerance is especially useful for looped networks.

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