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Estimating returns and conditional volatility: a comparison between the ARMA-GARCH-M Models and the Backpropagation Neural Network

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Abstract. Econometric models have usually estimated both returns and conditional volatility in financial assets. This paper is intended in the comparison of this traditional approach with the more recent Backpropagation neural network. When applied to the Spanish Ibex-35 stock market index, we find that the neural network achieved significantly better performance in predicting conditional volatility, but similar results when predicting financial returns.

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1. Introduction

Quantitative research tries to identify investment opportunities that maximize returns while assessing any risks involved by measuring the volatility of returns, an aspect to which investors give great importance. Modelling has thus become a primary research field (Bollerslev [1], Bollerslev and Mikkelsen [2];, Deo *et al* [3], among others), using the capacity of econometric models to estimate the returns on investments, stock market volatility, and the relationship between these two variables (Ghahramani and Thavaneswaran [5], Lundbergh and Teräsvirta [6], Schepper and Goovaerts [8]).

This paper describes a comparison of one of the econometric models most widely used in risk simulation, the ARMA-GARCH-M, with a model based on artificial intelligence, the Backpropagation neural network.

The rest of the paper is laid out as follows: the following section deals with a brief description of the methods used to estimate returns on investments and conditional volatility by econometric models and neural networks. In Section 3 the performance of ARMA-GARCH-M and the neural network Backpropagation are assessed by processing a historical series of the Spanish Ibex-35 closing prices and the results are compared by means of different error statistics. The main conclusions drawn from the work are presented in the final section.

2. The GARCH econometric model vs. the Backpropagation neural network model

One of the variants of the GARCH econometric models proposed by Bollerslev [1] and the ARCH-M proposed by Engle et al. [4] is the GARCH-M or GARCH-in-Mean. This model proposes incorporating conditional variance into the returns equation; in other words, the expected returns will also depend on their conditional variance. The analytical expression of the GARCH-M model is given in the equations below, in which (1) expresses the conditional variance equation and (2) expresses the returns equation.

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} = \alpha_0 + A(L)\,\varepsilon_t^2 + B(L)\,h_t \tag{1}$$

$$R_t = \delta + \gamma h_t + h_t^{1/2} \varepsilon_t \tag{2}$$

The influence of conditional volatility on performance can be expressed in different ways: as variance h_t, as the logarithm of variance $\log h_t$ or standard deviation $h_t^{1/2}$. The last option is the one most widely used in empirical studies and appears in expression (2).

Our proposal is to compare this model with the Backpropagation neural network (Rumelhart *et al.* [7]), which uses delta rule-based supervised learning, or *error backpropagation*. In the case of this network, the learning algorithm is generalized so that it can be used with networks of more than two layers. Operations are carried out in two phases. The information initially enters the first-layer neurons and generates an association of input-output data pairs. In the second phase, the information is propagated to the rest of the neurons in the rest of the layers and the different neuron outputs are compared to the desired output, after which the learning error is calculated.

The errors from each neuron are then transmitted backwards from the output neurons in order to determine the contribution of each neuron to the total error. With this new information the weights of the neurons are varied until a certain error threshold is reached.

Applying the Backpropagation algorithm requires the neurons to have a continuous and differentiable activation function, usually sigmoidal in type.

3. An application to the estimation of stock returns and conditional volatility of the Ibex-35 Index.

The series of Ibex-35 daily closing prices chosen for the comparative study ranged from 3 January 2000 until 14 July 2010 and contained a total of 2,658 observations. The series included periods of both rising and falling price trends and high and low volatility.

When designing the GARCH model, we must also find the ARMA model that better fit the sample. The best election was the ARMA(1,1).

The GARCH model estimation was performed considering different delays, as an explanatory variable in the model in three different ways: (1) incorporating volume in the variance equation, (2) incorporating lagged volume, (3) including the lagged logarithmic form.

The model was chosen using the Schwarz and Hannan-Quinn criteria. For the chosen sample, both criteria select the same model: ARMA(1,1)-GARCH-M(2,1). The values of both criteria are shown in Table 3, modelling the conditional variance equation in its three possible forms: variance (GARCH), logarithm of variance (LN GARCH) and standard deviation (DESV GARCH). Also considered was the possibility of including the logarithmic form of the lagged volume.

According to the results given in Table 1, the model with the best scores for both criteria is the one that expresses the conditional variance equation in the form of standard deviation and includes delayed volume in its logarithmic form (Table 1, last column).

Table 1: Selection of the ARMA(1,1)-GARCH-M(2,1) model

Returns Equation	GARCH		LN GARCH		DESV GARCH	
Variance Equation	Vol(-1)	Ln Vol(-1)	Vol(-1)	Ln Vol(-1)	Vol(-1)	Ln Vol(-1)
Schwarz Criterion	-5.9188	-5.9185	-5.9211	-5.9205	-5.9204	-5.9175
Hannan-Quin Criterion	-5.9303	-5.9299	-5.9326	-5.9320	-5.9319	-5.9290

After designing the definitive model, its coefficients were estimated from the 2,658 observations in the sample. Table 2 gives different error statistics for the returns and conditional variance equations: MAPE (Mean Absolute

Percentage Error), MAE (Mean Absolute Error), MSE (Mean Squared Error), AMPE (Absolute Mean Percentage Error) and RMSE (Root Mean Squared Error).

Table 2: Error statistics for the returns and conditional volatility equations in the ARMA-GARCH-M model

	Returns Equation	Conditional Volatility Equation
MAPE	1.0983	2.0463
MAE	0.0108	0.0002
MSE	0.0002	0.0000
AMPE	0.9715	1.8903
RMSE	0.0154	0.0003

From the different neural network configurations we chose the Backpropagation for its capacity to adapt neuron weights from the errors made during the learning process.

The inputs established for the network learning process were: index returns with one time lag $(t-1)$, conditional variance with one $(t-1)$ and two time lags $(t-2)$, and volume with one time lag $(t-1)$. The outputs were financial returns and conditional variance at time t.

The same variables were chosen for both systems in order to make it possible to compare the performance of the neural network with the econometric model. On one hand, network training indicates possible relationships between returns and their time lag (ARMA (1,1)). Conditional variance with one and two time lags establishes the relationship between returns and conditional variance (GARCH-M). Finally, the relationship between delayed volume and conditional volatility is also included.

Table 3 gives the error statistics of the three networks considered that minimize: the average absolute error, the mean squared error, and the root mean squared error. It can be seen that there is little difference between the results of the three networks when estimating the returns equation, while the differences are in general somewhat higher when it comes to estimating conditional volatility.

If these results are compared with those from the econometric model, we again find little difference as to the returns equation, but more significant ones in estimating conditional volatility: in this case, the results of the Backpropagation neural network are a considerable improvement on those of the ARMA-GARCH-M econometric model.

Table 3: Error statistics for the returns equation and conditional volatility in the Backpropagation Neural Network Model

	Returns equation			Volatility equation		
	Average absolute error	Mean squared error	Root Mean squared error	Average absolute error	Mean squared error	Root Mean squared error
MAPE	1.1420	1.0711	1.1489	0.1888	0.1483	0.1754
MAE	0.0108	0.0108	0.0108	0.0000	0.0000	0.0000
MSE	0.0002	0.0002	0.0002	0.0000	0.0000	0.0000
AMPE	0.9915	0.9980	1.0247	0.1046	0.0060	0.0898
RMSE	0.0154	0.0154	0.0154	0.0001	0.0001	0.0001

4. Conclusions

This paper presents a comparison of the performance of the GARCH family of econometric models and neural networks in estimating the returns and conditional variance of the Ibex-35 Spanish Stock Exchange Index. As a fairly long period (11) of daily closing prices was analysed, the sample contained a significant number of observations (2,658) with stages of both rising and falling stock prices, as well as high and low volatility.

From a comparison of the results of both models it can be concluded that there are no significant differences in their explanations of the returns equation, so that one model cannot be said to be better than the other in this respect.

However, significant differences were found in favour of the neural network for its explanation of conditional variance in each of the three networks estimated with different optimized error criteria. It can therefore be concluded that the Backpropagation neural network is better able to explain index volatility than the ARMA-GARCH-M econometric model.

References

[1] T. Bollerslev, Generalized Autorregressive Conditional Heteroskedasticity, *Journal of Econometrics* **31**, 307-327 (1986).

[2] T. Bollerslev and H. Mikkelsen, Modeling and Pricing long memory in Stock Market Volatility, Journal of Econometrics **73**, 151-184 (1996).

[3] M. Deo, K. Srinivasan K. and Devanadhen, The empirical relationship between stock returns, trading volume, and volatility: Evidence from select Asia–Pacific stock market, *European Journal of Economics, Finance and Administrative Sciences* **12**, 58-68 (2008).

[4] R.F. Engle, D.M. Lilien and R.P. Robins, Estimating Time-Varying Risk Premia in the Term Structure: the ARCH-M Model, *Econometrica* **55**, 391-407 (1987).

[5] M. Ghahramani and A. Thavaneswaran, A Note on GARCH Model Identification, *Computers and Mathematics with Applications* **55**, 2469-2475 (2008).

[6] S. Lundbergh and T. Teräsvirta, Evaluating GARCH models. *Journal of Econometrics* **110**, 417-435 (2002).

[7] D. Rumelhart, G. Hinton and R. Williams, Learning representations by back-propagating errors. *Nature* **323**, 533–536 (1986).

[8] A.D. Schepper and M.J. Goovaerts, The GARCH(1,1)-M model: results of densities of the variance and the mean. *Insurance: Mathematics and Economics* **24**, 83-94 (1999).