DEMAND CATEGORISATION, FORECASTING, AND INVENTORY CONTROL FOR INTERMITTENT DEMAND ITEMS

E. Babiloni[1*], M. Cardós[2], J.M. Albarracín[3], M.E. Palmer[4]

Department of Management
Polytechnic University of Valencia, Spain
mabagri@doe.upv.es, mcardos@doe.upv.es, malbarr@doe.upv.es, marpalga@doe.upv.es

ABSTRACT

It is commonly assumed that intermittent demand appears randomly, with many periods without demand; but that when it does appear, it tends to be higher than unit sise. Basic and well-known forecasting techniques and stock policies perform very poorly with intermittent demand, making new approaches necessary. To select the appropriate inventory management policy, it is important to understand the demand pattern for the items, especially when demand is intermittent. The use of a forecasting method designed for an intermittent demand pattern, such as Croston's method, is required instead of a simpler and more common approach such as exponential smoothing. The starting point is to establish taxonomic rules to select efficiently the most appropriate forecasting and stock control policy to cope with thousands of items found in real environments. This paper contributes to the state of the art in: (i) categorisation of the demand pattern; (ii) methods to forecast intermittent demand; and (iii) stock control methods for items with intermittent demand patterns. The paper first presents a structured literature review to introduce managers to the theoretical research about how to deal with intermittent demand items in both forecasting and stock control methods, and then it points out some research gaps for future development for the three topics.

OPSOMMING

1. INTRODUCTION

In many industries, stock control has become a strategic element that determines the success of important objectives, such as fulfilling a predetermined target customer service level. A suitable inventory control policy has to take into account the characteristics of the items, especially when they present variability in both demand sise and demand occurrence. So it is necessary to characterise the type of item before selecting the best stock control policy and the best forecasting procedure.

Intermittent demand pattern appears randomly, with many periods that have no demand; but when demand does occur, it can be greater than unit sise. (See, for example, Johnston and Boylan [1]; Syntetos and Boylan [2]; Willemain et al. [3]). This situation frequently applies to spare parts and to low- and medium-movement items in many industries. In fact, it is usual to find intermittent demand patterns instead of the constant, smooth, or normal ones for which traditional models are designed.

Furthermore, appropriate models and techniques to manage those items are not used in real industrial environments, despite active research carried out on this topic in recent years. This paper contributes to the organisation of stock and forecasting models and techniques, providing background information that helps managers to select models and techniques more efficiently to manage intermittent demand items.

The paper is organised as follows: Section 2 describes and classifies the different approaches in order to categorise demand pattern, and ends by identifying the research gaps related to this topic, and the most common demand distribution function used under specific circumstances in the literature. Section 3 is dedicated to explaining methods to forecast intermittent demand patterns, and their implications in practical environments. Section 4 gathers some relevant inventory control policies applied to intermittent demand items in both periodic and continuous review policies. To end, Section 5 presents some conclusions and potential research extensions of this work.

2. CATEGORISATION OF DEMAND PATTERNS

Categorisation of demand patterns consists of defining groups or categories with the purpose of classifying the items to select the best forecasting method and inventory control policy. However, there is no standard classification, definition, or characterisation of the different demand patterns. In fact, ad hoc categorisations are the most widely used to classify real data in industrial environments (Sani and Kingsman [4]) and software packages (Boylan et al. [5]).

In this section, the different approaches related to demand pattern categorisation are classified as follows: (1) approaches based on variance partition; (2) approaches based on the accuracy of forecasting procedures; and (3) approaches based on the demand shape. The shortcomings of these approaches and the potential research gaps are highlighted.

2.1 Approaches based on variance partition

Williams [6] proposes a method based on splitting the variance of demand during a lead time (DDLT) into its causal parts. The author defines these causal parts as: (1) the number of orders arriving in successive units of time with mean n and variance *var(n)*; (2) the size of the orders, with mean **x** and variance *var(***x***)*; and (3) the lead time, with mean *L* and variance *var(L)*. Given that (1), (2) and (3) are considered to be independent and identically distributed random variables, Williams suggests dividing the variance of DDLT as follows:

$$\mathrm{var}(DDLT) = x^2 L\,\mathrm{var}(n) + nL\,\mathrm{var}(x) + n^2 x^2\,\mathrm{var}(L) \quad (1)$$

which could be expressed as the variance due to n plus the variance due to x plus the variance due to L.

By expressing (1) through its coefficients of variation (CV), we have the following expression:

$$CV_{DDLT}^2 = \frac{CV_n^2}{L} + \frac{CV_x^2}{nL} + CV_L^2 \qquad (2)$$

The size of each term of (2) is used by Williams [6] and extended by Eaves and Kingsman [7] as an indicator of the type of demand pattern. Table 1 compares both categorisations.

CV_{DDLT}^2			**Categorisation of demand**	
CV_n^2/L	CV_x^2/nL	CV_L^2	**Williams [6]**	**Eaves and Kingsman [7]**
Low	Low		Smooth	Smooth
Low	High		Smooth	Irregular
High	Low		Slow moving	Slow moving
High	High	Low	Sporadic	Mildly intermittent
High	High	High	Sporadic	Highly intermittent

Table 1: Categorisation of demand based on variance partition

According to the authors, the choice of boundaries between categories should depend on the sector and type of item, and this is essentially a management decision. Williams [6] proposes cut-off values between categories when the lead time is constant and the number of orders arriving are Poisson distributed. These limits are defined for a pure 'store' situation, which means that each product is independent, bought in, and distributed with no manufacturing activity. Eaves and Kingsman [7] deal with consumable inventories from the Royal Air Force. The authors do not state any boundaries between categories, assigning each category a percentage of the total number of the items analysed. In that particular case, the theoretical categorisations shown in Table 1 become ad hoc when boundaries are defined for each industrial environment.

2.2 Approaches based on the accuracy of the forecasting procedure

Syntetos et al. [8] suggest a categorisation scheme according to the theoretical comparison of the mean squared error (MSE), which arises from three forecasting methods: (i) Croston's method (CM), designed to forecast intermittent demand items (explained in Section 3); (ii) a modification of CM derived by Syntetos and Boylan [9] (S&B); and (iii) simple exponential smoothing (SES). To establish demand categories, the authors compare the MSE that arises from each method to identify regions of higher accuracy. These regions are delimited by the squared coefficient of variation of the demand size (CV^2) and the average inter-demand interval (p), distinguishing four demand categories: erratic, lumpy, smooth, and intermittent (see Syntetos et al. [8] for accurate definitions of each pattern). Additionally, Kostenko and Hyndman [10] suggest using the S&B forecasting procedure in the smooth pattern whenever:

$$CV^2 > 2 - (3/2)\,p \qquad (3)$$

Demand categories are shown in Figure 1, where the use of the S&B forecasting method is recommended whenever demand is categorised as erratic, lumpy, intermittent, and smooth, whereas the CM appears only for some of the smooth patterns. The SES method does not appear in the diagram, even though demand pattern is categorised as smooth.

Additionally, Boylan et al. [5] assess the use of CV^2 and p as categorisation parameters instead of the average demand used in the spare parts industry, which has an ad hoc categorisation schema for both forecast and stock control systems. According to Boylan et al. [5], the main benefit of using CV^2 in lieu of the average demand is that it allows users to distinguish between slow and lumpy items, as the average is not able to capture 'the essence of lumpiness'. In this paper, a theoretical categorisation framework is introduced as a conceptual guide to classifying non-normal demand patterns. (See Gelders and Van Looy [11]; Williams [6] for a definition of non-normal demand pattern.)

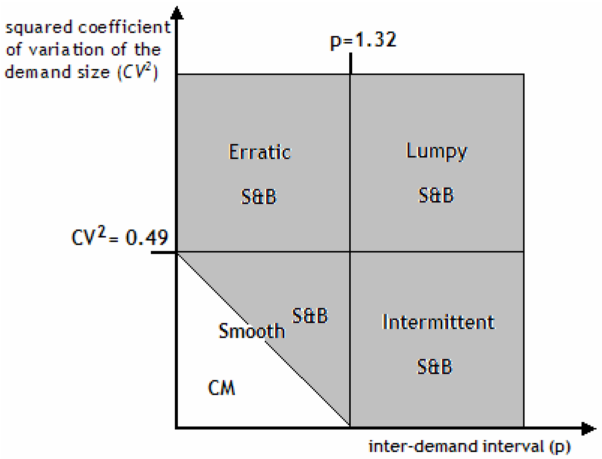

Figure 1: Categorisation of demand pattern based on the accuracy of forecasting procedures [Syntetos et al. [8] and Kostenko and Hyndman [10]]

2.3 Approaches based on the characteristics of the demand shape

Bartezzaghi et al. [12] and Zotteri [13] associate the intermittence of demand with two characteristics of the demand shape: (1) the skewness or asymmetry of the distribution, measured as the third standardised moment of the probability distribution; and (2) the existence of more than one mode (multimodality distributions). To analyse the impact of (1) and (2) on the intermittence of the demand pattern, the authors analyse six demand shapes with the same average and *CV* that differ in their asymmetry and multimodality characteristics.

The study, which is evaluated through the simulation of a stock control system, reveals that the higher the right asymmetry – that is, when the skewness is positive – the more intermittent the demand pattern; and thus higher inventory levels are required to assure a target level of service. The study points out that the existence of more than one mode also affects intermittence, but not as much as right asymmetry. However, this result is not quantified by the authors.

Although the authors suggest the importance of right asymmetry in demand pattern classification, they do not refer to any categorisation schema.

2.4 Conclusions and research gaps

The review of existing categorisation approaches points out that more research is required in this area to help managers to classify and manage inventories. Without further research, only ad hoc categorisations can be used in real industries. In those cases in which theoretical approaches are actually used, they do not seem to be accurate enough to be applied to any environment, since the boundaries between demand categories or the categories themselves are not defined.

Table 2 highlights the most important contributions of each approach and the research gaps arising from them.

A common research gap in the three approaches is to know which demand distribution function fits better with real data. Some authors recommend the use of some distribution based on the characteristics of real demand. Table 3 summarises the main suggestions on this topic, which managers can use to select the appropriate demand distribution function in each individual situation.

Approach	Categorisation factors	Contributions	Research gaps
based on variance partition Williams [6] Eaves and Kingsman [7]	• CV_n^2/L • CV_x^2/nL • CV_L^2	• States the causal parts of the variance of DDLT. • Defines demand categorisation based on the size of these causal parts.	• Only ad hoc limits are suggested between categories. • To obtain the categorisation factors, the distribution of demand has to be known.
based on the accuracy of the forecasting procedure Syntetos et al. [8]	• CV_x^2 • p	• Presents a categorisation schema based on the accuracy of three forecasting methods. • Introduces theoretical cut-off values between categories. • Suggests the best forecasting method for each demand pattern.	• Cut-off values work with forecasting procedures. However, it is not demonstrated that they work when selecting the best stock control policy. • To obtain the categorisation factors, the distribution of demand has to be known.
based on the characteristics of the demand shape Bartezzaghi et al. [12] Zotteri [13]	• Skewness • Multi-modality	• Introduces the right asymmetry of the demand shape as a factor of intermittence.	• Does not quantify the value of the skewness that causes us to consider the demand pattern as intermittent. • Does not have a categorisation schema. • To obtain the categorisation factors, the distribution of demand has to be known.

Table 2: Contributions and research gaps of demand categorisation approaches

3. FORECASTING INTERMITTENT DEMAND

Once the intermittent demand pattern has been identified, the best forecasting procedure has to be chosen to guarantee a good prediction of future demands. Time series forecasting approaches are used extensively in a routine stock control system when large number of products may be involved such as in manufacturing industries and service sectors. However, neither the simplest approaches (such as simple average [SA] and the random walk approach [Naïve forecasting]) nor more sophisticated methods (such as the autoregressive models or the exponential smoothing models generalised by the ARIMA models) are suitable when there is a large number of periods with no demand. For example, the ARIMA(0,1,1) model – that is, Simple Exponential Smoothing (SES) – focuses on the most recent data, and due to the large number of periods with no demand, it tends both to underestimate the size of the demand when it occurs, and to overestimate the long-term average demand [Croston [14]] that leads to an increase in the average stock level. Obviously, these results can be generalised to any ARIMA model, and so methods designed specifically to forecast intermittent demand patterns are required. This section presents methods and contributions related to this approach suggested by Croston [14] and its related extensions and the most widespread contributions to this topic. To end, relevant implications for industrial engineering are outlined.

Demand characteristics	Probability distribution
CV>0.5 or Right asymmetry of demand shape	• Gamma Distribution [Silver et al. [15]]
CV<0.5 and applies the central limit theorem (for example, with large lead times)	• Normal Distribution [Silver et al. [15]; [Rice [16]]
Strategic but slow-moving items	• Poisson Distribution [Silver et al. [15]] • Negative Binomial Distribution [Syntetos and Boylan [17]]
Items for which probability of no demand during a single period cannot be neglected	• Negative Binomial Distribution [Williams [6]; Syntetos and Boylan [17] • Compound Poisson Distribution [Adelson [19]; Friend [18]; Nahmias and Demmy [20]] • Compound Bernoulli Distribution [Janssen et al. [21]; Strijbosch et al. [22]]

Table 3: Probability distribution functions recommended in the literature, based on demand characteristics

3.1 Croston's method

The classic method to forecast intermittent demand was developed by Croston [14] and algebraically corrected by Rao [23]. Croston's method (CM) uses two separate exponential smoothing estimates to forecast the demand size, z_η, and the interval between non-zero demands, p_η. These estimates are updated only when demand occurs. Hence, CM is identical to conventional SES if demand occurs at each period.

Consider the model

$$y_t = x_t(\bar{z}_{\eta-1} + e_\eta), \qquad (4)$$

where

- t refers to the review interval,
- η refers to the serial number of non-zero demands,
- p is the average inter-arrival interval,
- x_t is the demand occurrence and follows a Bernoulli process with probability $1/p$,
- q_t is the inter-demand interval and is geometrically distributed with mean p,
- z_η is the non-zero observation of the demand and follows a normal distribution with mean μ and standard deviation σ.

Assuming that x_t and z_η are independent from each other and that error e_η is independent and identically normal distributed with a zero mean, then the updating procedure is as follows:

$$
\begin{aligned}
&\text{If } y_t = 0,\\
&\quad \overline{z}\eta = \overline{z}\eta{-1}\\
&\quad \overline{p}_\eta = \overline{p}_{\eta-1}\\
&\quad q_t = q_{t-1} + 1\\
&\text{If } y_t \neq 0,\\
&\quad e_\eta = y_t - \overline{z}\eta{-1}\\
&\quad \overline{z}\eta = \overline{z}\eta{-1} + \alpha\left(y_t - \overline{z}\eta{-1}\right)\\
&\quad \overline{p}_\eta = \overline{p}_{\eta-1} + \alpha\left(q_{t-1} - \overline{p}_{\eta-1}\right)\\
&\quad MAD_\eta = (1-\alpha)MAD_{\eta-1} + \alpha\left|e_\eta\right|\\
&\quad q_t = 1
\end{aligned}
\tag{5}
$$

MAD_η being the mean absolute deviation at η, and α the smoothing constant of the model.

With the forecasted demand size and interval, the estimation of the average demand per period is calculated as:

$$\overline{y}_t = \overline{z}\eta \big/ \overline{p}_\eta \tag{6}$$

the expected value of the average demand per period and the variance being as follows:

$$E\left(\overline{y}_t\right) = \frac{\mu}{p} \tag{7}$$

$$\operatorname{var}\left(\overline{y}_t\right) = \frac{\alpha}{2-\alpha}\left[\frac{(p-1)^2}{p^4}\mu^2 + \frac{\sigma^2}{p^2}\right] \tag{8}$$

Which, according to Croston [14], is lower than the variance from SES.

One of the benefits of Croston's method is that managers can easily implement it using a spreadsheet. Following Croston's work, many authors have carried out studies to analyse CM performance. Table 4 summarises the most relevant contributions.

Author	Contributions
Schultz [24]	Uses two SES estimates for demand size and interval with two different smoothing constants. He considers both distributions to be non-stationary, i.e. that the probability distribution changes over time.
Willemain et al. [3]	Compare CM and SES and conclude that some optimal degree of intermittence is needed to benefit from CM.
Johnston and Boylan [25]	Point out that the optimal degree of intermittence to apply CM should be greater than 1.25 times the review interval.
Sani and Kingsman [4]	Show very modest benefits from the application of CM to real data.
Syntetos and Boylan [2]	Find a mathematical error in Croston's estimation of the demand per period.
Snyder [26]	Proposes to estimate the variance of error σ2 and inter-

	demand interval p in a different way to comply with the stationary assumption.
Leven and Segerstedt [27]	Derive a new estimator (LS), unbiased, for the demand per period. They use the Erlang distribution to model demand size.
Syntetos and Boylan [9]	Assess the bias introduced by CM forecasts. They derive a new estimator (S&B), approximately unbiased, for the demand per period.
Shale et al. [28]	Derive the bias expected when the order arrivals follow a Poisson process and demand per period is calculated as in CM.
Boylan and Syntetos [29]	Show that the demand per period estimator introduced by Leven and Segerstedt [27] is biased.
Teunter and Sani [30]	Compare the performance of CM, S&B, and LS in terms of bias when they are used to forecast demand distributions with different degrees of intermittence.

Table 4: Main contributions following the Croston method

3.2 Extensions to the Croston method

Willemain et al. [3] compare CM and SES using artificial data created to violate Croston's assumptions, and real-world data from industrial sources. The authors argue that to obtain benefits from CM, some optimal degree of intermittence is required. Johnston and Boylan [25] quantify this degree as more than 1.25 times the review interval.

Snyder [26] identifies inconsistencies in CM since, to justify the use of SES to forecast *p* and *MAD*, it has to be assumed that *p* and the standard deviation, σ^2, change over time – that is, the process should be non-stationary. Both are supposed to be constant in CM. For this reason, the author suggests replacing them with the following expressions:

$$\overline{\sigma}^2 = \sum_{t=1}^{n} e_t^2 \Big/ \sum_{t=1}^{n} x_t \quad and \quad \overline{p} = \sum_{t=1}^{n} x_t \Big/ n \tag{9}$$

This modified version is called MCROST. In addition, the author suggests two methods: one the author called the 'LOG' method, which avoids negative demands using log-space adaptation; and the other called the 'AVAR' method, which uses variances instead of MADs.

Although CM seems to be robust and superior to other conventional methods, Sani and Kingsman [4] demonstrate that its application to real data has very few benefits. To explain this unexpected behaviour, Syntetos and Boylan [2] find a mathematical error in CM derivation, known as inversion bias [see Syntetos and Boylan [2] for further details]. The bias introduced in CM is quantified by Syntetos and Boylan [9] as

$$\frac{\alpha}{2-\alpha}\mu\frac{(p-1)}{p^2} \tag{10}$$

and therefore the authors propose an approximate unbiased estimator of average demand per period as

$$\overline{y}_t = \left(1-\frac{\alpha}{2}\right)\frac{\overline{z}_\eta}{\overline{p}_\eta} \tag{11}$$

The corrected CM, referred to in the previous section as S&B, is also known as the approximation method.

Leven and Segerstedt [27] also suggest an estimator for the average demand per period derived to avoid the inversion bias as:

$$\bar{y}_t = \alpha\left(\frac{\bar{z}_\eta}{\bar{p}_\eta}\right) + (1-\alpha)\bar{y}_{t-1} \tag{12}$$

However, Boylan and Syntetos [29] demonstrate that, in a stationary process, the expected value of (12) is biased. In a subsequent paper, Teunter and Sani [30] show that using (12) is even more biased than the Croston method.

To eliminate the bias produced by conventional forecasting methods, Shale et al. [28] derive corrector factors for the SA and the SES methods when they are used to forecast intermittent demand. When *p* and *z* are SES estimates, the corrector factor is designed to eliminate the inversion bias. Demand is assumed to follow a Poisson process, the inter-demand interval being negative exponentially distributed. Time series are treated as locally stationary. Hence, the authors suggest the following correction factor for the inversion bias:

$$1-\frac{\alpha}{2-\alpha} \tag{13}$$

Additionally, some non-Poisson arrivals are briefly considered by the authors.

3.3 Other methods and algorithms to forecast intermittent demand

Willemain et al. [31] develop a patented algorithm to forecast the cumulative distribution of intermittent demand over a constant lead time, and a new method of assessing the accuracy of these forecasts. The bootstrap method suggested by Efron [32] is adapted for use in the algorithm. The authors assume a positive correlation between zero and non-zero demand values, which is modelled using a two-stage first order Markov process. Simulation results show that the mean values of the chi-squared statistic resulting from the algorithm are smaller than those from the CM and SES, and based on this, the authors point out that the bootstrap method is more accurate than the others, despite its decreasing accuracy with the lead time.

Additionally, Hua et al. [33] propose a similar approach to forecast spare parts demand, taking into account the relationships between explanatory variables and non-zero demands. More recently, Bao et al. [34] have carried out studies on the application of a neural network algorithm, called ‘support vector machine’ or SVM, to forecast intermittent demands. Although its use seems to be promising, there are no conclusive results yet.

3.4 Conclusions and research gaps

Table 5 outlines some forecasting procedures to model and estimate demand series. Apart from the patented algorithm developed by Willemain et al. [31], the other procedures are based on CM.

An important implication for industrial engineering arising from the literature is that using traditional forecasting procedures like SA or SES is not accurate enough when dealing with intermittent demand items. For these demand patterns, separately forecasting the distribution of demand size and the distribution of the interval between positive demands entails more accurate forecasts.

Furthermore, CM produces fewer average inventory levels than SES when the inter-demand interval is above 1.25 the review interval, but guaranteeing the same, or even higher, target cycle service level. The authors have developed a simple experiment to assess the benefits of using CM, which can be sent on request.

	Demand size		Inter-demand interval	
CM Croston [14]	$N(\mu,\sigma^2)$	SES	Geometric and independent and identically distributed (i.i.d.)	SES
S&B Syntetos and Boylan [9]	$N(\mu,\sigma^2)$	SES	Geometric and i.i.d.	SES
MCROST/LOG/AVAR Snyder [26]	$N(\mu,\sigma^2)$	SES	Geometric and i.i.d.	See (9)
Corrector factor method Shale et al. [28]	Geometric	SA or SES	Negative exponential	SA or SES
Willemain Algorithm Willemain et al. [31]	Bootstrapping method		Two stage first order Markov process	

Table 5: Outline of the main characteristics of intermittent demand forecasting methods

4. INVENTORY CONTROL FOR INTERMITTENT DEMAND

It is known that traditional inventory control techniques are based on assumptions that are inappropriate for items with intermittent demand [Silver et al. [15]]. For instance, to apply continuous review policies, it is assumed that the average demand is almost invariant over time, or that undershoots at the re-order point can be neglected. To implement periodic review policies, demand between reviews must be large enough to place a replenishment order every review cycle. Obviously, these assumptions do not fit intermittent demand patterns.

Moreover, there is no agreement about what type of review policy – periodic or continuous – should be preferred to manage intermittent demand items. Sani and Kingsman [4] suggest the use of periodic review in order to minimise total inventory costs, while Yeh et al. [35] propose the use of continuous review policies to guarantee a desired service level.

This section reviews works that suggest stock control policies for intermittent demand patterns in both continuous and periodic review policies. The purpose of this section is to provide managers with enough background to control items with intermittent and slow-moving demand patterns.

The nomenclature used in this section regarding stock control policies is:

R	Review interval
S	Order up to level
s	Re-order point
(R, S)	Periodic-Review, Order-Up-To-Level policy. Every *R* units of time, a replenishment order is launched in order to raise inventory positions to level *S*.
(Q, s)	Continuous-Review, Order-Point, Order-Quantity policy. A fixed quantity *Q* is ordered whenever inventory positions drop to *s* or lower.
(R, s, Q)	Combination of *(R, S)* and *(Q, s)* policies. Every *R* units of time, the inventory position is checked. If it is at or below *s*, it is ordered to raise it to *S*.

4.1 Contributions to periodic review conditions

Schultz [24] proposes an *(R, S-1,S)* policy, delaying the placement of the replenishment order for a number of periods to achieve a reduction in holding costs. The author recommends this policy whenever the average time between demands is sufficiently large and constant-related to the lead time, and holding costs are high compared with other inventory costs. The value of the delay depends on the inter-demand interval. When demand distribution is unknown, the delay is determined using CM. Subsequently, Schultz [36] also applies the delayed model in continuous review policies.

Dunsmuir and Snyder [37] describe a model for determining the re-order point (s) to guarantee a target customer service level when the system follows the *(R,s,Q)* policy. The model is based on the method developed by Snyder [38]. The authors suggest determining the re-order point by means of the unsatisfied demand during a cycle, whereas undershoots at *s* are neglected. Janssen et al. [21] follow the same reasoning, but without neglecting undershoot at *s*. This leads to a more complex expression of the service level. In this case, demand is modeled using a compound Bernoulli process with a given probability of demand being positive. This method is called the Compound Bernoulli method (CBM).

Leven and Segerstedt [27] suggest a heuristic procedure in which demand size is modeled using the Erlang distribution in order to determine the stockout probability for the *(R, S)* policy.

Based on the work of Sani and Kingsman [4], Syntetos and Boylan [17] use an *(R,S)* policy to deal with intermittent demand items based on three constraints: (1) the specified fraction of demand satisfied directly from stock (P2); (2) the cost of shortage per unit value short (B2); and (3) the emergency delivery cost per unit value short (B3). In this case DDLT is modeled with the negative binomial distribution.

Cardós et al. [39] provide an exact method to calculate the cycle service level (CSL) when demand is modeled with a discrete and known probability distribution. The method takes the intermittent demand characteristics into account, but requires a huge computational effort, which motivates the derivation of some approximations approaches in Cardós and Babiloni [40].

4.2 Contributions to continuous review conditions

Snyder [38] proposes a heuristic model based on computing *s* to achieve a target service level. He follows the same reasoning as Dunsmuir and Snyder [37], but applies it to the *(s,Q)* policy. The model, based on Burgin [41], consists of the application of Gamma distribution and the partial expectation approach proposed by Brown [42].

Segerstedt [43] suggests an algorithm to determine when it is necessary to place a replenishment order, given a target probability for no shortage. It is assumed that the demand size, the inter-demand interval, and the length of lead time are Gamma distributed and independent from each other. The parameters of these distributions are calculated using SES estimates. Basically, the algorithm calculates two probabilities: that at least *N* orders take place during the lead time (P_A); and that the available physical inventory gives no shortage for *N* orders (P_B). Once both are calculated and given a target service level α, a replenishment order must be placed when $(P_A)\cdot(P_B)\geq(1-\alpha)$.

Haddock et al. [44] present a simple heuristic rule to determine the optimal value of *Q* that minimises the total cost associated with the production and storage of slow-moving items.

Vereecke and Verstraeten [45] propose an algorithm to be implemented in a computerised system to manage spare parts inventories that are classified as lumpy items, slow movers, and fast movers. Demand occurrence is Poisson distributed, whereas the demand size is approximated by a constant number called ‘package’. The authors called it the ‘Package Poisson’ method.

Yeh et al. [35] propose a simple graphical aid to establish an appropriate replenishment size to reach a target customer service level. The authors assume that demand size, inter-demand interval, and lead time are Gamma distributed.

Strijbosch et al. [22] use the Croston method to forecast, and the CBM suggested by Janssen et al. [21] to manage *(s,Q)* inventory policies for spare parts whose demand patterns are considered to be intermittent.

Finally, Larson et al. [46] implement a nonparametric Bayesian approach in which the previous information is characterised by a Dirichlet process.

To summarise, Table 5 identifies some relevant inventory control contributions in an intermittent demand context. This table provides information about the type of policy, the forecasting procedure, and the distribution function of demand from the inventory methods described in this section.

Ref	**Author**	**Proposed model**	**DDLT**				**Inventory Control**					
			Based on ref.	Discrete /Continuous	Type	Forecasting procedure	Review	Policy	Control parameters	Lead time	Undershoots	Criteria
1	**Snyder [38]**	Heuristic model based on Brown's partial expectation approach	-	C	Gamma	-	**C/P**	(s,Q)	SQ	C	-	P2
2	**Schultz [24]**	delays the placement of the replenishment order *k* periods	-	C/D		CM	**H**	(R,S-1,S)	SQ	Z	-	-
3	**Schultz [36]**	delays the placement of the replenishment order *k* periods	2	C	Poisson process	CM	**C**	(0,1)	-	C	-	-
4	**Dunsmuir and Snyder [37]**	Model for determining reorder levels for a target service level	1	C	Z^{+}~Gamma	-	**H**	(R,s,Q)	SQ	C	-	P2
5	**Segerstedt [43]**	Heuristic approach to calculate when a replenishment order needs to be placed		C	Z~Gamma;P~Gamma L~Gamma	SES SES SES	**C**	-	-	V	-	P2
6	**Haddock et al. [44]**	Heuristic rule to determine *Q* to minimise total costs		D	Z~Poisson; P~Poisson	-	**C**	-	SQ	C	-	-
7	**Vereecke and Verstraeten [45]**	Algorithm for managing spare parts		D	Z~C; P~Poisson 'Package Poisson'	-	**C**	(s,Q)	SQ	C		P1
8	**Yeh et al. [35]**	Simple practical aid for selecting the appropriate replenishment size	5	C	Z~Gamma;P~Gamma L~Gamma	-	**C**	-	-	V	-	P2
9	**Sani and Kingsman [4]**	Comparison between various forecasting methods and procedures to determine inventory control parameters		Depends on the control parameters determination		-	**H**	(R,s,S) (R,S)	-	C		P2
10	**Janssen et al. [21]**	Compound Bernoulli Method	4,1	D	Compound Bernoulli Process (CBP)	-	**H**	(R,s,Q)	SQ	V	√	P2
11	**Strijbosch et al. [22]**	integrate CM and CBM to manage	10,4	D	CBP	CM	**C**	(s,Q)	SQ	C	√	P2
12	**Leven and Segerstedt [27]**	rely on Croston´s method approach		C	Z~Erlang	CM	**P**	(R,S)	SQ	C	-	P2

13	**Syntetos and Boylan [17]**	study the accuracy of forecasting procedures	9	D	Negative Binomial	SES CM AM	**P**	(R,S)	SQ	C	-	P2 B2 B3
14	**Cardós et al. [39]** **Cardós and Babiloni [40]**	provide exact and approximate methods to assess the CSL		D		-	**P**	(R,S)	-	C	-	P1

Review: Continuous (C); periodic (P); hybrid (H). Control parameters: Sequential determination (SQ); simultaneous determination (SM). Lead time: Zero (Z); constant (C); variable (V). Demand distribution during the lead time (DDLT): Continuous (C); discrete (D).

Table 6: Relevant inventory control contributions in an intermittent demand context

5. CONCLUSIONS

Suitable categorisation of the demand pattern is necessary to select an appropriate forecasting and inventory control method, since identifying the demand pattern helps managers to find the best approach when dealing with items with intermittent or lumpy demand patterns. However, a standard categorisation approach is not extended in industrial environments, ad hoc classifications being most widely used. In this paper, the most relevant theoretical categorisation approaches have been classified into those based on (1) variance partition; (2) the accuracy of forecasting procedures; and (3) the characteristics of the demand shape. Yet these categorisation approaches require defining limits between demand categories before they can be applied to industrial environments. Further research should be focused on more general categorisation approaches that are applicable to any kind of inventories and industries, with an accurate definition of boundaries or cut-off values between demand categories.

Regarding forecasting methods, simple exponential smoothing and Croston's method are most commonly used to forecast intermittent demand patterns. Despite the superior performance of CM compared with SES – CM reduces average inventory levels in cycles – it assumes the demand size to be normally distributed. However, (i) normal distribution arises in practice by applying the Central Limit Theorem conditions, but these conditions cannot be recognised in an intermittent demand context; (ii) there is an unacceptable risk of forecasting negative demand; and (iii) skewness is not taken into account. Future investigations should contribute to establishing both the best probability distribution function to fit intermittent demand patterns, and the best forecasting procedure according to the demand pattern.

Finally, the most relevant inventory control methods designed for intermittent demand items were identified in Section 4. Despite the research growth in this topic in recent years, more rigorous and industrially applicable contributions are required to integrate it in practical environments.

6. ACKNOWLEDGMENTS

The authors thank the anonymous referees for their thorough reading of earlier versions of this paper, and their very constructive comments .

This research was part of the project GEMA, supported by the Ministerio de Educación y Ciencia (Ref. DPI 2007-65441).

7. REFERENCES

[1] **Johnston, F.R., Boylan, J.E.** 1996. Forecasting intermittent demand: A comparative evaluation of Croston's method. Comment, *International Journal of Forecasting*, 12 (2), pp 297-298.

[2] **Syntetos, A.A., Boylan, J.E.** 2001. On the bias of intermittent demand estimates. *International Journal of Production Economics*, 71 (1-3), pp 457-466.

[3] **Willemain, T.R., Smart, C.N., Shockor, J.H., Desautels, P.A.** 1994. Forecasting intermittent demand in manufacturing - A comparative-evaluation of Croston's method. *International Journal of Forecasting*, 10 (4), pp 529-538.
[4] **Sani, B., Kingsman, B.G.** 1997. Selecting the best periodic inventory control and demand forecasting methods for low demand items. *Journal of the Operational Research Society*, 48 (7), pp 700-713.
[5] **Boylan, J.E., Syntetos, A.A., Karakostas, G.C.** 2006. Classification for forecasting and stock control: A case study. *Journal of the Operational Research Society*, Special Issue paper, pp 1-9.
[6] **Williams, T.M.** 1984. Stock control with sporadic and slow-moving demand. *Journal of the Operational Research Society*, 35 (10), pp 939-948.
[7] **Eaves, A.H.C., Kingsman, B.G.** 2004. Forecasting for the ordering and stock-holding of spare parts, *Journal of the Operational Research Society*, 55 (4), pp 431-437.
[8] **Syntetos, A.A., Boylan, J.E., Croston, J.D.** 2005. On the categorization of demand patterns. *Journal of the Operational Research Society*, 56 (5), pp 495-503.
[9] **Syntetos, A.A., Boylan, J.E.** 2005. The accuracy of intermittent demand estimates. *International Journal of Forecasting*, 21 (2), pp 303-314.
[10] **Kostenko, A.V., Hyndman, R.J.** 2006. A note on the categorization of demand patterns. *Journal of the Operational Research Society*, 57 (10), pp 1256-1257.
[11] **Gelders, L.F., van Looy, P.M.** 1978. An inventory policy for slow and fast movers in a petrochemical plant: A case study. *Journal of the Operational Research Society*, 29 (9), pp 867-874.
[12] **Bartezzaghi, E., Verganti, R., Zotteri, G.** 1999. Measuring the impact of asymmetric demand distributions on inventories. *International Journal of Production Economics*, 60 (1), pp 395-404.
[13] **Zotteri, G.** 2000. The impact of distributions of uncertain lumpy demand on inventories. *Production Planning & Control*, 11 (1), pp 32-43.
[14] **Silver, E.A., Pyke, D.F., Peterson, R.** 1998. *Inventory management and production planning and scheduling*. 3rd edition, John Wiley & Sons.
[15] **Rice, J.A.** 1995. *Mathematical statistics and data analysis*. Duxbury Press.
[16] **Syntetos, A.A., Boylan, J.E.** 2006. On the stock control performance of intermittent demand estimators. *International Journal of Production Economics*, 103 (1), pp 36-47.
[17] **Friend, J.K.** 1960. Stock control with random opportunities for replenishment. *Operational Research Quarterly*, 11 (3), pp 130-136.
[18] **Adelson, R.M.** 1966. Compound Poisson distributions. *Operational Research Quarterly*, 17 (1), pp 73-75.
[19] **Nahmias, S., Demmy, W.S.** 1982. The Logarithmic Poisson Gamma-Distribution - A model for leadtime demand. *Naval Research Logistics*, 29 (4), pp 667-677.
[20] **Janssen, F., Heuts, R., de Kok, T.** 1998. On the (R, s, Q) inventory model when demand is modelled as a compound Bernoulli process. *European Journal of Operational Research*, 104 (3), pp 423-436.
[21] **Strijbosch, L.W.G., Heuts, R.M.J., van der Schoot, E.H.M.** 2000. A combined forecast - inventory control procedure for spare parts. *Journal of the Operational Research Society*, 51 (10), pp 1184-1192.
[22] **Croston, J.D.** 1972. Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 23 (3), pp 289-303.
[23] **Rao, A.V.** 1973. Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 24 (4), pp 639-640.
[24] **Schultz, C.R.** 1987. Forecasting and inventory control for sporadic demand under periodic review. *Journal of the Operational Research Society*, 38 (5), pp 453-458.
[25] **Johnston, F.R., Boylan, J.E.** 1996. Forecasting for items with intermittent demand. *Journal of the Operational Research Society*, 47 (1), pp 113-121.
[26] **Snyder, R.** 2002. Forecasting sales of slow and fast moving inventories. *European Journal of Operational Research*, 140 (3), pp 684-699.
[27] **Leven, E., Segerstedt, A.** 2004. Inventory control with a modified Croston procedure and Erlang distribution. *International Journal of Production Economics*, 90 (3), pp 361-367.

[28] **Shale, E.A., Boylan, J.E., Johnston, F.R.** 2006. Forecasting for intermittent demand: The estimation of an unbiased average. *Journal of the Operational Research Society*, 57 (5), pp 588-592.
[29] **Boylan, J.E., Syntetos, A.A.** 2007. The accuracy of a modified Croston procedure. *International Journal of Production Economics*, 107 (2), pp 511-517.
[30] **Teunter, R., Sani, B.** 2009. On the bias of Croston's forecasting method. *European Journal of Operational Research*, 194 (1), pp 177-183.
[31] **Willemain, T.R., Smart, C.N., Schwarz, H.F.** 2004. A new approach to forecasting intermittent demand for service parts inventories. *International Journal of Forecasting*, 20 (3), pp 375-387.
[32] **Efron, B.** 1979. Bootstrap methods - Another look at the jackknife. *Annals of Statistics*, 7 (1), pp 1-26.
[33] **Hua, Z.S., Zhang, B., Yang, J., Tan, D.S.** 2007. A new approach of forecasting intermittent demand for spare parts inventories in the process industries. *Journal of the Operational Research Society*, 58 (1), pp 52-61.
[34] **Bao, Y.K., Zou, H., Liu, Z.T.** 2006. Forecasting intermittent demand by fuzzy support vector machines. *Advances in Applied Artificial Intelligence, Proceedings*, 4031 (1080-1089.
[35] **Yeh, Q.J., Chang, T.P., Chang, H.C.** 1997. An inventory control model with gamma distribution. *Microelectronics and Reliability*, 37 (8), pp 1197-1201.
[36] **Schultz, C.R.** 1989. Replenishment delays for expensive slow-moving items. *Management Science*, 35 (12), pp 1454-1462.
[37] **Dunsmuir, W.T.M., Snyder, R.D.** 1989. Control of inventories with intermittent demand. *European Journal of Operational Research*, 40 (1), pp 16-21.
[38] **Snyder, R.D.** 1984. Inventory control with the Gamma Probability-Distribution. *European Journal of Operational Research*, 17 (3), pp 373-381.
[39] **Cardós, M., Miralles, C., Ros, L.** 2006. An exact calculation of the cycle service level in a generalized periodic review system. *Journal of the Operational Research Society*, 57 (10), pp 1252-1255.
[40] **Cardós, M., Babiloni, M.E.** 2008. Exact and approximated calculation of the cycle service level in periodic review policies. Fifteenth International Working Seminar on Production Economics, 4, pp. 39-50.
[41] **Burgin, T.A.** 1975. Gamma distribution and inventory control. *Operational Research Quarterly*, 26 (3), pp 507-525.
[42] **Brown, R.G.** 1959. *Statistical forecasting for inventory control*. McGraw Hill.
[43] **Segerstedt, A.** 1994. Inventory control with variation in lead times, especially when demand is intermittent. *International Journal of Production Economics*, 35 (1-3), pp 365-372.
[44] **Haddock, J., Iyer, N.T., Nagar, A.** 1994. A heuristic for inventory management of slow-moving items. *Production Planning & Control*, 5 (2), pp 165-174.
[45] **Vereecke, A., Verstraeten, P.** 1994. An inventory management model for an inventory consisting of lumpy items, slow movers and fast movers. *International Journal of Production Economics*, 35 (1-3), pp 379-389.
[46] **Larson, C.E., Olson, L.J., Sharma, S.** 2001. Optimal inventory policies when the demand distribution is not known. *Journal of Economic Theory*, 101 (1), pp 281-300.