

Welfare and Human Population in Austria

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Abstract: A model for a general human population dynamics is presented to approach the demographic stability problem. The main model variables are demographic variables per sexes (births, deaths, emigration, immigration and population) and the main well-being variables given by the UN: Human Development Index, Gender Development Index and Gender Empowerment Index. The model is presented in deterministic and stochastic formulations and it has been validated in the case of Austria in the 1999-2009 period.

Keywords: stochastic model; human population; well-being variables; Human Development Index; Gender Empowerment Index; Gender Development Index; demographically stable society.

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1 Introduction

An abstract stochastic mathematical model to study the evolution of human population per sexes is presented in this paper. The model involves the three United Nations (*UN*) well-being variables used by Sanz et al. (2011): *Human Development Index (HDI)*, *Gender Development Index (GDI)* and *Gender Empowerment Index (GEM)*. The main advances of the model here presented with respect to the model by Sanz et al. (2011) are the stochastic formulation, the model validation for Austria and the application case to study the demographic stability.

The authors consider in this paper that the use of *UN* well-being variables in a mathematical model is essential to study the demographic stability. There are many problems corresponding to areas like environment, economy, health, education, labor force, culture, etc., which affect human well-being and demographic stability. Nevertheless, the literature review shows that there is little research about the dynamics of human population considering both sexes, covering demographic processes such as births, deaths, migration, and using well-being variables. The papers in which the authors found the use of well-being variables are:

- Mischler et al. (2002) consider the *maturation variable* to study the stability of solutions.

- Farkas (2004) focuses on the stability of McKendrick non-linear equations, which include *fertility and mortality rates*.

- Guo and Sun (2005) study how to optimize births with a McKendrick model through a dynamics programming-based method applied to the Chinese population.

- Alho and Spencer (2005) develop a steady population model, which considers *marriages, divorces*, as well as *fertility rates*. The model is based on the Leslie method of matrices, which is applied to different economic situations.

Other papers present models in which some kind of well-being variables related to economy, education, altruism or environmental pollution are considered:

- Hritonenko et al. (2008) present an age-structured demographic model considering variables like *productivity, employment and income*. They show that current technological trends in the United States are able to compensate negative demographic trends without increasing retirement age.

- Croix and Sommacal (2009) present an economic model where they show that increasing investment in the health field implies an increasing of the life expectancy at birth.

- Zajacova et al. (2009) study the effect of education on mortality.

- Bréchet and Lambrecht (2009) develop an Overlapping Generations model where the size of generations varies over time and converges towards a steady state. These authors introduce a new variable, altruism, which can also be considered a well-being variable. They use this model to study the influence of altruism, as well as renewable resources and economic movements on the population growth.

- Dubey (2010) proposes a nonlinear mathematical model to study the effect of environmental pollution on biomass resources and on human population dynamics.

Finally, some other papers consider the *UN* well-being variables:

- Caselles et al. (2008) propose a deterministic human population dynamics model, where birth and death depend on *HDI*, although emigration and immigration rates depend on time.



- Sanz et al. (2011) present a deterministic model which is validated for Belgium in the 1997-2008 period.

- Sanz et al. (2014) present a stochastic human population-dynamics model per sexes, in which fertility and death rates depend on *GDI*. The model is validated for Spain in the 2000-2006 period. It is applied to test different strategies in different scenarios to select government interventions for the 2006-2015 period.

The benefit to use the *UN* well-being variables is their universality. In addition, they measure education, health, economy and female labor force. Thus, these variables can also help to solve problems involving the mentioned topics.

The definitions of the *UN* well-being variables are the following:

HDI measures the average achievements of a given country with three variables:

- A long and healthy life, measured by the *life expectancy at birth*.
- Education, given by the *adult literacy rate* and *combined gross enrollment ratio of primary, secondary and tertiary education*.
- A worthy standard of life, given by the Gross Domestic Product (*GDP*) per capita in Purchasing Power Parity (*PPP*) in U.S. dollars.

GDI is calculated from the same *HDI* variables, but they are differenced per sexes. In this case, *GDP* is replaced by *male* and *female income*.

GEM measures the role of women within society from:

- Political participation and decision-making is calculated from *the percentage of female and male parliament members with seats*.
- Economic participation and decision-making is measured by two variables: percentage of men and women legislators, senior officials and managers, and percentage of female and male participation occupying professional and technical posts.
- Power over economic resources which is provided by *male* and *female incomes (PPP in US. dollars)*.

The objective of the presented model is to study the demographic stability related with a well-being improving. However, improving well-being implies creating a social environment, and the human capacities must be developed in the social environment (*UNDP*, 1990-2011). Education is the main human tool to create any social environment. Note that education helps to introduce new concepts and thoughts in society and to grow its potential. Delors (1996) considers education as one of the main engines of development and, as one of its functions, that humanity can achieve fully management of its own development. Some authors such as Zajacova et al. (2009) and Mukandavire et al. (2009) reinforce the idea of Delors (1996). They say that education is essential to meet the basic needs. In short, education represents a very important role for the development of a society. Education will lead to achieve a social environment with high well-being.

On the other hand, the demographically stable society concept is here defined as the tendency towards zero of the difference between births and deaths. The literature review agrees with this objective (Gil et al., 2006; World Commission on Environment and Development, 1988; Ehrlich and Ehrlich, 1994; Brown and Mitchell, 1998; Folch, 1998; Sartori, and Mazzoleni, 2003; Diamond, 2006).

Observe in addition, that the *UN* has created a Decade of Education for a Sustainable Future (2005-2014). Its aim is to give a solution to the population growth problem. They focus this problem from the field of education. *UNESCO* is the lead agency for promotion. The suggested goal is that educators must make a commitment to all types of education, both formal (from primary school to university) and informal (museums, media ...), pay systematic attention to the situation of the world in promoting attitudes and behavior favorable to the achievement of sustainable development (Gil et al., 2006).

Also, two very confusing concepts must be disambiguated: overpopulation and population growth. Ehrlich and Ehrlich (1994) note that "from the point of view of the habitability of the Earth, over-population of the rich countries is a more serious threat than the quick population growth in poor countries."

Following this idea, the overpopulation is defined as a condition in which the population density exceeds a threshold which causes a deterioration of the environment, a decrease in well-being or a decrease in population. On the other hand, population growth is set when the number of births contrasts against the number of deaths. Europe and Africa are two continents which help explaining these concepts. Population growth in Africa is much higher than in Europe, but Europe is much more populated than Africa. Overpopulated parts of the world have a higher consumption per capita than other and they contribute to the destruction of the environment. In this sense, the World Commission on Environment and Development (1988) discusses the implications of these two phenomena: "In many parts of the world, the population rates are growing but environmental resources cannot support them. Sometimes rates are surpassing all reasonable expectations of improvement in housing, health care, food security and energy supply."

Sachs (2008) notes that "the population growth world rate has declined but the world population increases quickly in regions with less capacity to ensure the health, stability and prosperity of the people (...). The world should adopt a set of measures to help stabilize world population through voluntary decisions, instead of maintaining the current path that probably will stand at nine billion or more in 2050."

Brown and Mitchell (1998) state that "population stabilization is an essential step to stop the destruction of natural resources and ensure the satisfaction of the basic needs of all people." That is, if we want a sustainable society, we will have to build a demographically stable society, and to do it, we should reduce current growth rates.

On the other hand, our model is abstract, i.e., it is transferable to any country. Moreover, it is a stochastic dynamic model in which the forecasts in each period are obtained by means of a confidence interval given by a minimum value and a maximum value with a specific probability. This way to present results provides a measure of its reliability, unlike deterministic models (Caselles, 1992). The methodology that has been used to construct our model is the General Modeling Methodology (*GMM*), proposed by A. Caselles, which is implemented by using *SIGEM*: the Complex-Systems based Programs' Generator (Caselles, 1994, 2008). The *GMM* is a hypothetic-deductive methodology to construct complex models. This methodology has Data Mining Methods in some of its phases.

The rest of this paper is organized as follows. Section 2 presents and explains the model equations. Section 3 shows the model validation for the case of Austria in the 1999-2009 period. Section 4 suggests some strategies and scenarios to simulate the future and a way to determine the optimal strategy to achieve the proposed objective. Finally, Section 5 presents some conclusions and suggestions.



2 The model

First, all model variables are defined and listed by alphabetical order. We write the variables in this way:

XXXX Explanation variable [units of measure] (kind of variable)

Demographic variables:

DEFE Female Deaths [population] (flow variable)
DEMA Male Deaths [population] (flow variable)
DETO Total Deaths [population] (auxiliary variable)
EMIF Female Emigration [population] (flow variable)
EMIG Total Emigration [population] (auxiliary variable)
EMIM Male Emigration [population] (flow variable)
POFI Female Population at the beginning of the year [population] (constant)
POFL Female Population at the end of the year [population] (level variable)
POMI Male Population at the beginning of the year [population] (constant)
POML Male Population at the end of the year [population] (level variable)
POPI Population at the beginning of the year [population] (auxiliary variable)
POPL Population at the end of the year [population] (auxiliary variable)
PRPF Female proportion [proportion] (auxiliary variable)
PRPM Male proportion [proportion] (auxiliary variable)
RDEF Female Death Rate [%] (auxiliary variable)
RDEM Male Death Rate [%] (auxiliary variable)
REMF Female emigration rate [%] (auxiliary variable)
REMM Male emigration rate [%] (auxiliary variable)
RFEF Female Fertility rate [births/female population] (auxiliary variable)
RFEM Male Fertility rate [births/female population] (auxiliary variable)
RINF Female immigration rate [%] (auxiliary variable)
RINM Male immigration rate [%] (auxiliary variable)
TEMI Initial year [time] (constant)
TEMS Year [time] (level variable)
XACF Female Births [population] (flow variable)
XACI Total Births [population] (auxiliary variable)
XACM Male Births [population] (flow variable)
YNMF Female immigration [population] (flow variable)
YNMI Total immigration [population] (auxiliary variable)
YNMM Male immigration [population] (flow variable)

Human Development Index variables:

GDPG Gross Domestic Product [PPP US\$] (auxiliary variable)
GRRR Gross Rate Registered to level primary secondary and tertiary [%] (auxiliary variable)
LAPO Literate Adult Population [%] (auxiliary variable)
LEBI Life expectancy at birth [age] (auxiliary variable)
XDII Human development Index initial value [%] (constant)
XHDI Human development Index [%] (level variable)
YEDU Educational Index [%] (auxiliary variable)
YGDP Gross Domestic Product Index [%] (auxiliary variable)



YGRR Gross Rate Registered to level primary secondary and tertiary index [%]
(auxiliary variable)

YLAP Literacy Rate Adults [%] (auxiliary variable)

Gender Development Index variables:

GRFE Female Gross Registered to level primary, secondary and tertiary [%] (setting variable)

GRMA Male Gross Registered to level primary, secondary and tertiary [%] (setting variable)

LEBF Female life expectancy at birth [age] (setting variable)

LEBM Male life expectancy at birth [age] (setting variable)

RLAF Female literacy rate adults [%] (auxiliary variable)

RLAM Male literacy rate adults [%] (auxiliary variable)

RLIF Female literacy rate [%] (setting variable)

RLIM Male literacy rate [%] (setting variable)

YEID Equally Distributed Education Index [%] (auxiliary variable)

YEFE Female Education Index [%] (auxiliary variable)

YEMA Male Education Index [%] (auxiliary variable)

YEVD Equally Distributed life expectancy at birth index [%] (auxiliary variable)

YFEI Female income [PPP US\$] (auxiliary variable)

YFEM Female income [PPP US\$] (auxiliary variable)

YGRF Female Gross Rate Registered to level primary secondary and tertiary [%]
(auxiliary variable)

YGRM Male Gross Rate Registered to level primary secondary and tertiary [%]
(auxiliary variable)

YIIC Total income [PPP US\$] (auxiliary variable)

YIID Equally Distributed Income Index [%] (auxiliary variable)

YIFE Female income index [%] (auxiliary variable)

YIMA Male income index [%] (auxiliary variable)

YLEB Life expectancy at birth index [%] (auxiliary variable)

YLEF Female life expectancy at birth index [%] (auxiliary variable)

YLEM Male life expectancy at birth index [%] (auxiliary variable)

YMAI Male Income [PPP US\$] (auxiliary variable)

YMAL Male income [PPP US\$] (setting variable)

XGDI Gender Development Index [%] (level variable)

XGII Gender Development Index initial value [%] (constant)

Gender Empowerment Index variables:

EPIID Percentage parliamentary representation [%] (auxiliary variable)

EPIF Female Percentage parliamentary representation [%] (setting variable)

EPIIM Male Percentage parliamentary representation [%] (auxiliary variable)

PAEF Female percentage shares of positions as legislators senior officials and managers [%] (setting variable)

PAEM Male percentage shares of positions as legislators senior officials and managers [%] (auxiliary variable)

PEID Average PEID [%] (auxiliary variable)

PPPF Female percentage shares of professional and technical positions [%] (setting variable)

PPPM Male percentage shares of professional and technical positions [%] (auxiliary variable)

PSTE PEID [%] (auxiliary variable)

PPTI Professional index [%] (auxiliary variable)

XAEE PEID within index [%] (auxiliary variable)

XAEI PEID index senior [%] (auxiliary variable)

XGEM Gender Empowerment Index [%] (level variable)

XIPI Gender Empowerment Index initial value [%] (constant)

The relationships between these variables are displayed in the Forrester Diagram (Forrester, 1961) in Figure 1. This is the characteristic diagram of System Dynamics and **it** is a translation of the Causal Diagram terminology that facilitates the writing of the equations when programming the model for a computer.

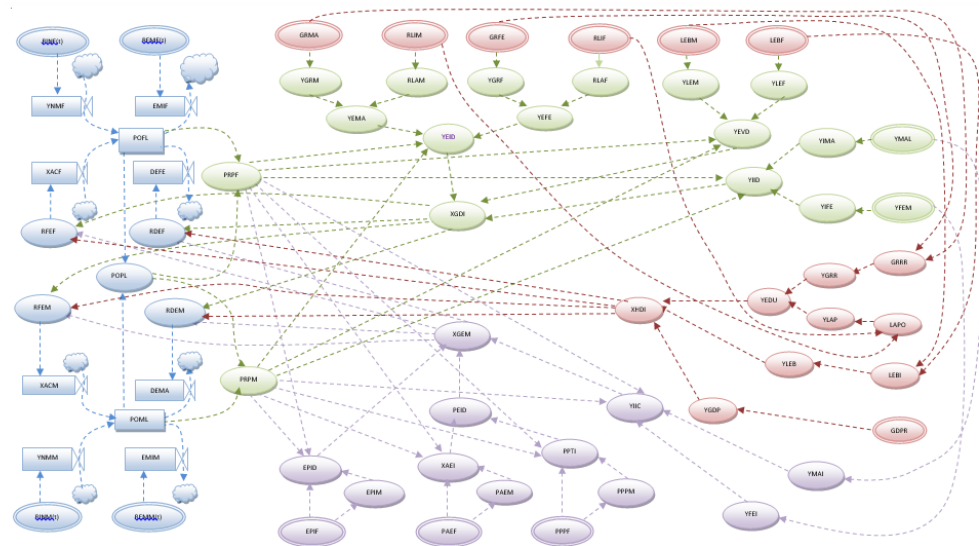


Figure 1: Forrester's Diagram (1961)

In this diagram the variables are classified into different types:

- Level variables: they require an initial value, which is an input variable, and the following values are updated. They are represented by a square or rectangle because they can be compared with tanks where a fluid is stored.
- Flow variables: they can be compared with stopcocks that regulate the flow to or from a fluid tank. They are represented by a characteristic icon.
- Auxiliary variables: intermediate variables that are used to calculate flows, or strict output variables (they aren't used in other calculations and they are usually variables to optimize). They are represented by a circle or ellipse.
- Setting variables: not controlled input variables.
- Control Variables: input variables whose value can be assigned by the decision maker or the person that uses the model.
- Constants: system parameters with known and fixed value.

All input variables and constants are represented by a double line circle or ellipse. The Sources or Sinks are represented by a cloud (they are the source of the inflows or the end of the outflows of tanks).

The model reflects the influence of fertility, mortality, emigration, immigration on the male and female population, as well as the relation between fertility and mortality rates with well-being variables (*HDI*, *GEM*, *GDI*). These rates are calculated from input variables and the resulting population.

The model equations are the same than the presented ones in Sanz et al. (2011).

2.1 Fitting Rates

In order to validate the stochastic model, all fitted functions (fertility rates, mortality rates, emigration rates and immigration rates) must maintain the specific structure described by Caselles (1992, 2008). Now we show a short summary of this.

Firstly, we calculate the mean value (represented by h in Sanz et al. (2014)) using the corresponding fitted function. Then we calculate the corresponding standard deviation (represented by s in Sanz et al. (2014)). All is made according to the method proposed by Caselles (1992, 2008). Thus, a generic variable such as Y would be obtained as: $Y = h + s \cdot \varepsilon(t)$. Being $\varepsilon(t)$ a $N(0,1)$, having h the following structure: $h = a + b_1T_1 + b_2T_2 + \dots + b_mT_m$, where a, b_1, \dots, b_m are parameters and T_1, \dots, T_m are transformed functions of the independent variables, and being s calculated by using the following formula:

$$s = s_{yx} \sqrt{1 + \frac{1}{n} + \tau' C \tau} \quad (1)$$

Where s_{yx} is the standard deviation of regression, n is the number of data points, the components of vector τ are the differences between the transformed functions T_i of the independent variables and its respective average, and C is the inverse matrix of the variance-covariance matrix corresponding to the transformed functions. For instance, in Table 1 the following equation appears:

$$RFEFHDI, GDI, GEM = \alpha_1 + \beta_1 \cdot HDI \cdot GDI \cdot GEM + \gamma_1 \cdot \cos(\mu_1 \cdot HDI \cdot GDI \cdot GEM) \quad (2)$$

In this case, the transformed functions considered are:

$$T_1 = HDI \cdot GDI \cdot GEM \quad \text{and} \quad T_2 = \cos(\mu_1 \cdot HDI \cdot GDI \cdot GEM).$$

The historical data of demographic rates have been obtained from the database of the Austria Institute of Statistics for the 1998-2008 period. The information about the independent variables of such functions (*HDI*, *GEM* and *GDI*) are extracted from the Human Development Reports (*UNDP*) for the 1997-2007 period. In order to validate them, the analysis of the residuals and the calculation of the parameter R^2 have been performed.

Next we present all the fitted functions for demographic rates: fertility, mortality, emigration and immigration rates. Tables 1 to 8 contain the needed data to write the stochastic formulation of the model and Figures 2 to 17 show the fitted functions (odd figures) and the residuals (even figures).

The mathematical structures shown in Tables 1 to 4 have been found by a trial and error process and the help of the functions finder and fitter tool *Regint* (Caselles, 1998, 2008). The work by Marchetti et al. (1996) demonstrates that fertility and death rates can be fitted by a logistic function or a sum of logistic functions. The data show a joint tendency to complete cycles (see Figures 2, 4, 6 and 8). A cycle can be formulated as an increasing logistics followed by a decreasing logistics. However, the available real data correspond to a short period, which prevents fitting the rates as a sum of logistics (too many parameters). The presented mathematical structures are the best option found to model the cycles for this



short period. Nevertheless, the fitted functions corresponding to the emigration rate and the immigration rate (Table 5 to 8) are single logistic functions. Finally the well-being indices, *HDI*, *GDI* or *GEM* are calculated with specific formulas detailed in *UNDP* (1990-2008).

$RFEFHDI, GDI, GEM = \alpha_1 + \beta_1 \cdot HDI \cdot GDI \cdot GEM + \gamma_1 \cdot \cos(\mu_1 \cdot HDI \cdot GDI \cdot GEM)$				
RFEF	α_i	β_i	μ_i	γ_i
i=1	1.1348084388e2	-3.48271345e1	40.92001	-1.247076680
Matrix C		Mean of transformed function		Estimation Error
30.79380.802930.802930.24080		T1=0.659464 T2=-0.303310		S=1.346669

Table 1: Parameters' values for RFEFHDI, GDI, GEM

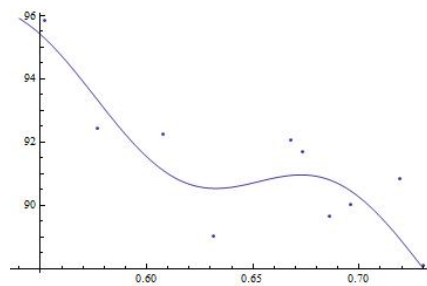


Figure 2: Fitted function (solid line) and real data (dots) of Female Birth Rate in Austria, in the 1998-2008 period. $R^2=0.733401$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F}, 11) > 0.13205$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

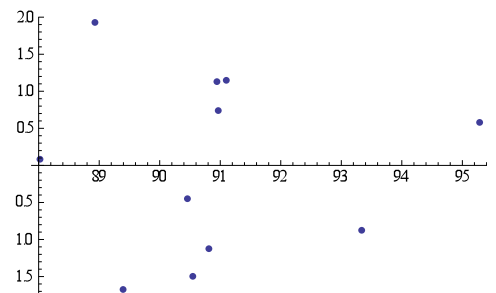


Figure 3: Residuals (ordinate) of Female Birth Rate (abscise) in Austria, from the welfare variables in the 1998-2008 period.

$RFEM_1 = \alpha_2 + \beta_2 \cdot HDI \cdot GDI \cdot GEM + \gamma_2 \cdot \cos(\mu_2 \cdot HDI \cdot GDI \cdot GEM)$				
RFEM	α_i	β_i	μ_i	γ_i
i=2	1.235915776068e2	-	36.054202	1.486311474769384
Matrix C		Mean of transformed function		Estimation Error
42.2975-2.1328-2.13280.32078		T1=0.659464 T2= 0.308286		S=1.598943

Table 2: Parameters' values for RFEMHDI, GDI, GEM

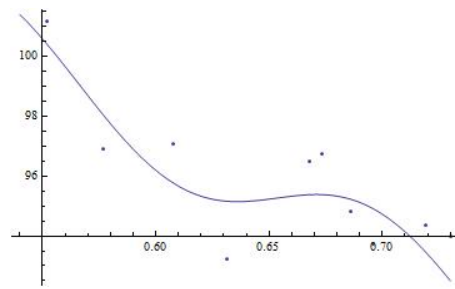


Figure 4: Fitted function (solid line) and real data (dots) of Male Birth Rate in Austria, in the 1998-2008 period. $R^2 = 0.695521$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F}, 11) > 0.160795$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

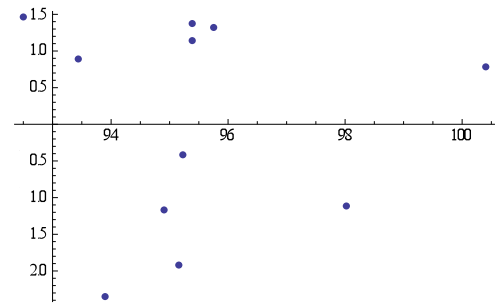


Figure 5: Residuals (ordinate) of Male Birth Rate (abscise) in Austria, from the welfare variables in the 1998-2008 period.

$RDEF_t = \alpha_3 + \beta_3 \cdot HDI \cdot GDI \cdot GEM + \gamma_3 \cdot \cos(\mu_3 \cdot HDI \cdot GDI \cdot GEM)$				
RDEF	α_i	β_i	μ_i	γ_i
i=3	1.2976652477171e2	-	74.771696	9.367690707857e-1
Matrix C		Mean of transformed function		Estimation Error
28.35650.195380.195380.15909		T1=0.659464 T2=-0.108426		S=1.962267

Table 3: Parameters' values for RDEFHDI, GDI, GEM

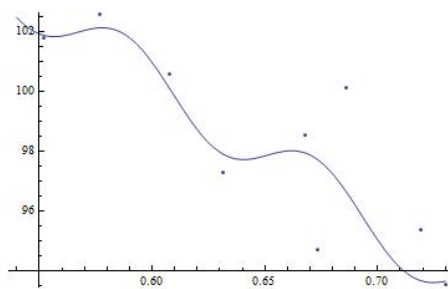


Figure 6: Fitted function (solid line) and real data (dots) of Female Death Rate in Austria, in the 1998-2008 period. $R^2 = 0.755094$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F}, 11) > 0.18025$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

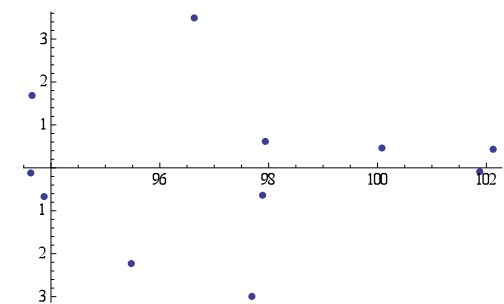


Figure 7: Residuals (ordinate) of Female Death Rate (abscise) in Austria, from the welfare variables in the 1998-2008 period.

$RDEM_t = \alpha_4 + \beta_4 \cdot HDI \cdot GDI \cdot GEM + \gamma_4 \cdot \cos(\mu_4 \cdot HDI \cdot GDI \cdot GEM)$				
RDEM	α_i	β_i	μ_i	γ_i
i=4	1.339556717394	-	2.340446	-
Matrix C		Mean of transformed function		Estimation Error
1566260674227674227290239		T1=0.659464 T2=0.026886		S=0.001134

Table 4: Parameters' values for RDEM, HDI, GDI, GEM

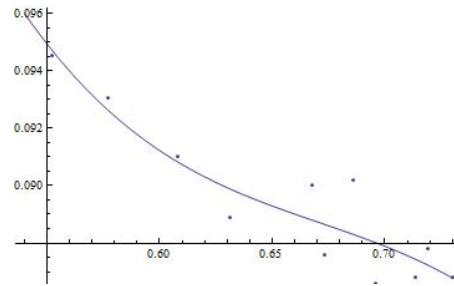


Figure 8: Fitted function (solid line) and real data (dots) of Male Death Rate in Austria, in the 1998-2008 period. $R^2 = 0.854886$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F}, 11) > 0.107544$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

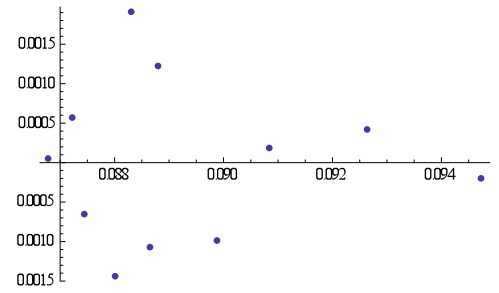


Figure 9: Residuals (ordinate) of Male Death Rate (abscise) in Austria, from the welfare variables in the 1998-2008 period.

$REMF(t) = \eta_5 + \beta_5 t + \gamma_5 \cdot \exp(\alpha_5 \cdot t) - \mu_5$					
REMF	η_i	α_i	β_i	μ_i	γ_i
i=5	$7.43002178997e-3$	1.25843	-	2000	58.1852
Matrix C			Mean of transformed function		Estimation Error
$C(0,0) = 0.715271029174$			T1=0.585007		S=0.000290

Table 5: Parameters' values for $REMF_t$

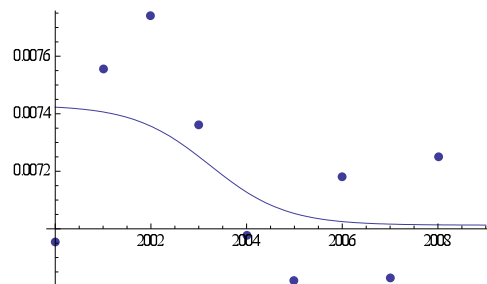


Figure 10: Fitted function (solid line) and real data (dots) of Female Emigration Rate in Austria, in the 2000-2008 period. $R^2=0.292092$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F}, 11) > 0.154679$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

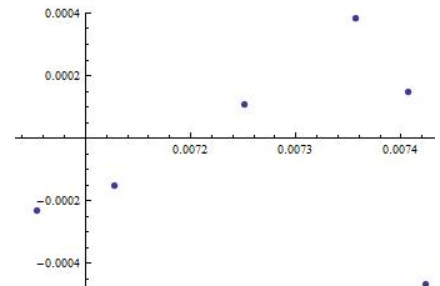


Figure 11: Residuals (ordinate) of Female Emigration Rate (abscise) in Austria, from the welfare variables in the 2000-2008 period.

$REMM(t) = \eta_6 + \beta_6 t + \gamma_6 \cdot \exp(\alpha_6 \cdot t) - \mu_6$					
REMM	η_i	α_i	β_i	μ_i	γ_i
i=6	$1.19084177191e-1$	-0.751511	-	2002	0.33799
Matrix C			Mean of transformed function		Estimation Error
$C(0,0) = 26.705210812465$			T1=0.275343		S=0.000037

Table 6: Parameters' values for $REMM_t$

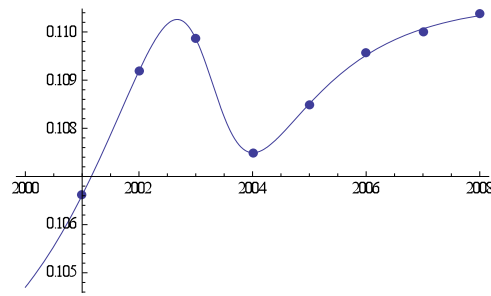


Figure 12: Fitted function (solid line) and real data (dots) of Male Emigration Rate in Austria, in the 2000-2008 period. $R^2=0.999447$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{\mu},11) > 0.235617$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

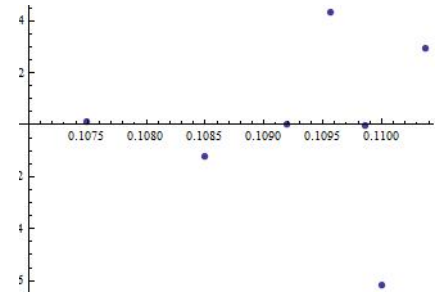


Figure 13: Residuals (ordinate) of Male Emigration Rate (abscise) in Austria, from the welfare variables in the 2000-2008 period.

$RINF(t)=\eta_7+\beta_71+\gamma_7 \cdot \exp \alpha_7 \cdot t-\mu_7$					
RINF	η_i	α_i	β_i	μ_i	γ_i
i=7	7.71178515346e-1	0.405699	1.0507450342566	2003	10.2073
Matrix C		Mean of transformed function		Estimation Error	
C(0,0) = 5.637004736678		T1=0.446858		S=0.045209	

Table 7: Parameters' values for $RINF_t$

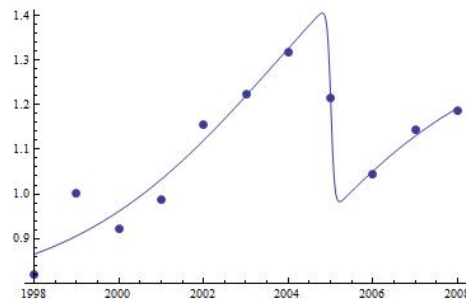


Figure 14: Fitted function (solid line) and real data (dots) of Female Immigration Rate in Austria, in the 1998-2008 period. $R^2=0.926699$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{\mu},11) > 0.186044$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

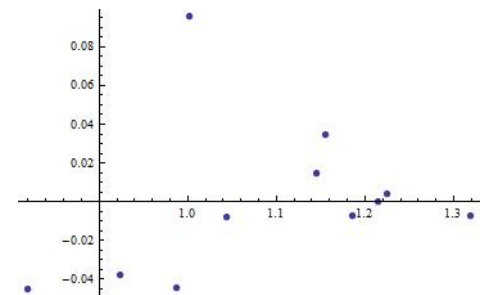


Figure 15: Residuals (ordinate) of Female Immigration Rate (abscise) in Austria, from the welfare variables in the 2000-2008 period.

$RINM(t)=\eta_8+\beta_81+\gamma_8 \cdot \exp \alpha_8 \cdot t-\mu_8$					
RINM	η_i	α_i	β_i	μ_i	γ_i
i=8	8.94551028026e-2	0.46795	1.19004999041e-1	2003	7.62838
Matrix C		Mean of transformed function		Estimation Error	
C(0,0) = 3.321784726014		T1=0.551291		S=0.005436	

Table 8: Parameters' values for $RINM_t$

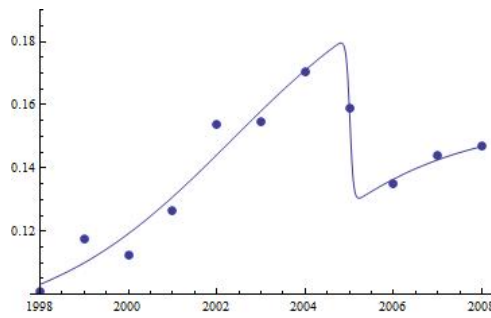


Figure 16: Fitted function (solid line) and real data (dots) of Male Immigration Rate in Austria, in the 1998-2008 period. $R^2=0.94954$. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\hat{F},11) > 0.225751$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residuals of the model.

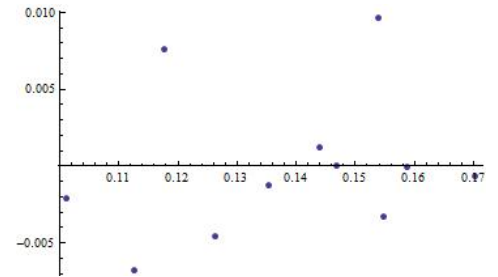


Figure 17: Residuals (ordinate) of Male Immigration Rate (abscise) in Austria, from the welfare variables in the 2000-2008 period.

3 Validation

The model validation in its two formulations, determinist and stochastic is presented in this section. The historical data used in this article to validate the model have been obtained from the Austria National Statistics Institute database in the 1999-2009 period. As mentioned in Section 1, the software tool used for model validation has been SIGEM.

3.1 Deterministic Validation

The model has been written as a set of finite difference equations and the solutions have been calculated with the Euler's approximation, following Djidjeli et al. (1998), which explain that the Euler's method is the more appropriate to solve this kind of equations. These equations have been programmed using SIGEM with the functions fitted in Section 2. Results have been obtained for the period 1999-2009.

Validation has been performed numerically, calculating the determination coefficients and testing the randomness of the residuals (Figures 18 and 19) and graphically, superposing the results obtained for each year and the historical data. The validation process has been considered successful due to three reasons:

- The graphical superposition of historical and calculated data in the first years is very good.
- The determination coefficients, R^2 , are very high. R^2 is an index useful for global fitting between data sets:

$$R^2 = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum (x_i - \mu_x)^2 \sum (y_i - \mu_y)^2}} \quad (3)$$

where (x_i, y_i) are the data to be compared, and μ_x and μ_y are the respective average values.

- The maximum relative error does not exceed 5% (really it is lower enough).

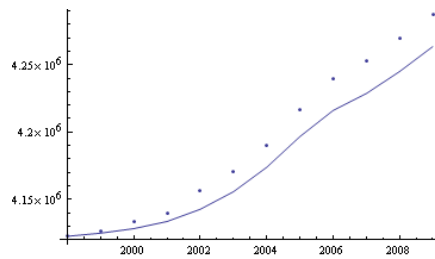


Figure 18: Forecast function (solid line) given by the model and real data (dots) for Austria's Male Population, in the 1998-2009 period, $R^2=0.935006$, with 0.586985% of maximum relative error.

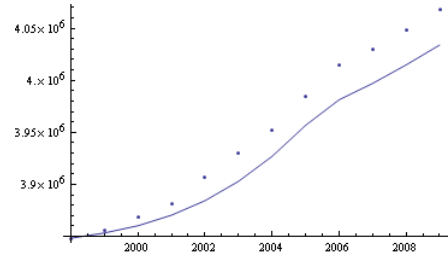


Figure 19: Forecast function (solid line) given by the model and real data (dots) for Austria's Female Population, in the 1998-2009 period, $R^2=0.920724$, with 0.836716% of maximum relative error.

Observe in Figures 18 and 19 that the model predictions show a one sided bias. This bias may be considered as negligible due to the scarce maximum relative error.

3.2 Stochastic Validation

The stochastic version of the model was the one used for simulation because it permits to determine the results' reliability (each result is obtained with its respective confidence interval or with its respective average value and standard deviation).

The procedure to verify that the stochastic formulation of the model is validated is the following:

- Observing that all the results have a normal distribution (for this purpose, *SIGEM* automatically programs a χ^2 test).
- Creating a (for instance) 95% confidence interval for each result and checking that all the historical data are within this interval.

The results corresponding to this validation type are presented in Figures 20 and 21. They confirm that the model is valid for Austria in the 1999-2009 period.

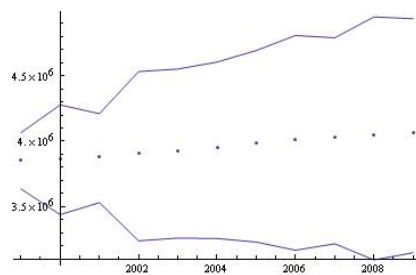


Figure 20: Austrian Male Population 1999-2009. Minimum and Maximum values (solid lines) and real values (dots).

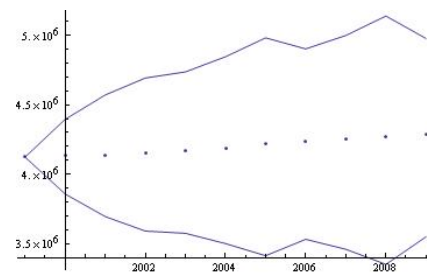


Figure 21: Austrian Female Population 1999-2009. Minimum and Maximum values (solid lines) and real values (dots).



4 Application: demographically stable society

Before making predictions with the model, we extrapolate all the input variables as a previous step to define relevant strategies and scenarios. To perform this extrapolation, firstly, the input variables are fitted with respect to time. All fitting functions are logistic sums. Logistic functions have the property that they can explain saturation of resources, and their use in demography has been proved to be very useful (Marchetti et al., 1996).

Next, these functions are extrapolated with confidence intervals using the *Extrapol* program (available in <http://www.uv.es/caselles>). The *Extrapol* program calculates the mean values (tendency values) and the limits of an interval at 95% confidence for each extrapolated year. Depending on each variable, an extrapolated value (maximum, minimum or mean) is assigned to each future scenario each year.

Figures 22 to 24 show the extrapolation of the economic variables: Gross Domestic Product, Male Income and Female Income respectively.

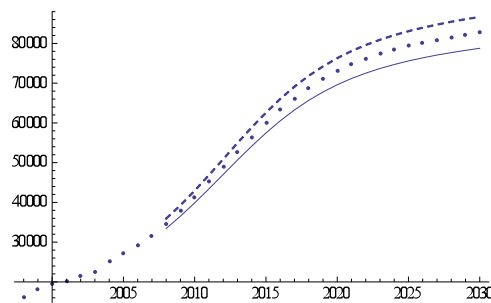


Figure 22: GDP, Gross Domestic Product

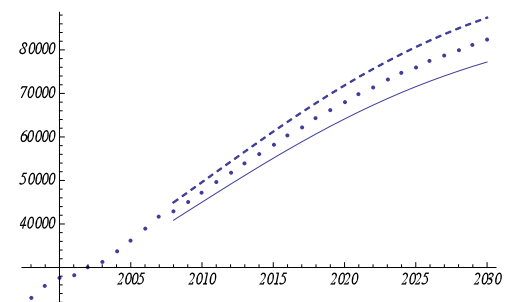


Figure 23: YMAL, Male Income.

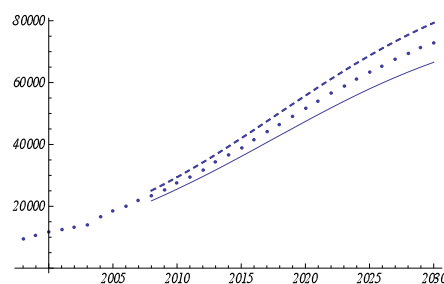


Figure 24: YFEM, Female Income.

Figures 25 and 26 show the extrapolation of the health variables: Female Life Expectancy at Birth and Male Life Expectancy at Birth, respectively.

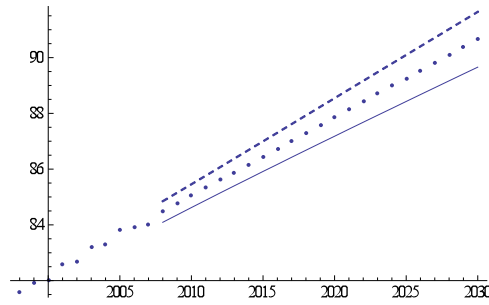


Figure 25: LEBF, Female Life Expectancy at Birth.

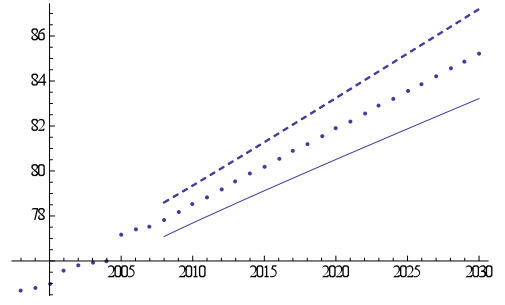


Figure 26: LEBM, Male Life Expectancy at Birth.

Figures 27 to 30 represent the extrapolation of the education variables: Male and Female Literacy Rate, Male and Female Gross Registered to level primary, secondary and tertiary, respectively.

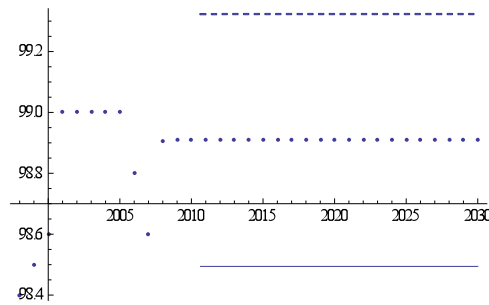


Figure 27: RLIM, Male Literacy Rate.

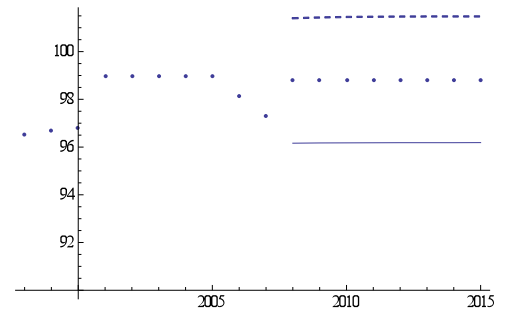


Figure 28: RLIF, Female Literacy Rate.

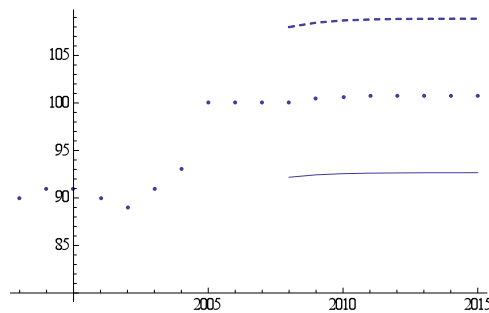


Figure 29: GRMA, Male Gross Registered to level primary, secondary and tertiary.

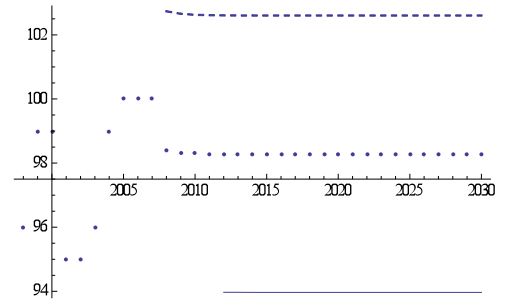


Figure 30: GRFE, Female Gross Registered to level primary, secondary and tertiary.

Finally, Figures 31 to 33 represent the extrapolation of the employment status for female's variables: Female Percentage parliamentary representation, Female percentage shares of positions as legislator senior officials and managers and Female percentage shares of professional and technical positions, respectively.

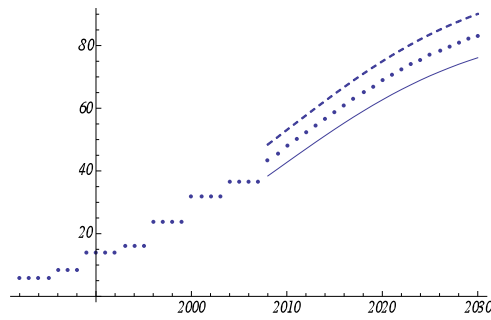


Figure 31: EPIF, Female Percentage parliamentary representation.

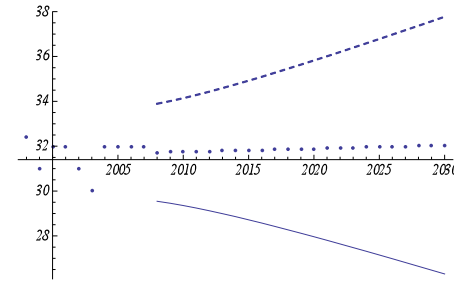


Figure 32: PAEF, Female percentage shares of positions as legislator senior officials and managers.

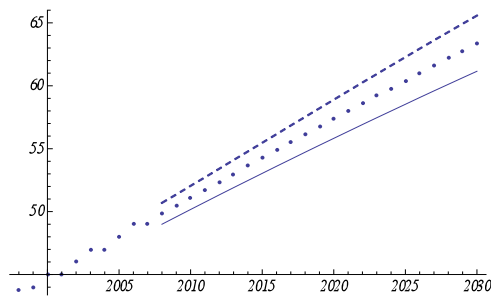


Figure 33: PPPF, Female percentage shares of professional and technical positions.

Once everything is sketched and constructed, the model can be used to simulate the behavior of the system. The proposed objective is to determine how to reach a stable population.

Although the input variables are extrapolated up to the 2030 year, the concrete stated objective is to reach a stable population in 2025 due to the forecasts of this variable in the period 2025-2030 present a chaotic behavior that prevents any correct interpretation. The corresponding objective-variable is named as $XSOS_{ij}$ where $i=1, 2, 3$ describe strategies and $j= 1, 2, 3$ describe scenarios. This variable computes the difference between births and deaths in each year of the 2010-2025 period.

In order to define some tentative strategies and scenarios, on the one hand, in the context of the model, the economic variables, $GDPR$, $YMAL$ and $YFEM$, are assumed to be scenario variables due to the possible complexity of controlling them. On the other hand, the health and education variables: Life Expectancy per gender ($LEBF$, $LEBM$), Literate Rates per gender and Gross Enrolment ratio per gender ($RLIM$, $RLIF$, $GRMA$, $GRFE$) and employment status for females ($EPIF$, $PAEF$, $PPPF$) are assumed to be control variables, because relevant authorities may be able to carry out the mechanisms to assign a value to them over time.

Strategy 1: Empowering women within the labor scope. We increase investment in health and education over the detected trend and increase the participation rate of women in the labor scope over the detected trend. To do this, the average extrapolated values of *education* and *the women position at the labor force* variables are increased.



Strategy 2: Empowering women within the family. We increase investment in health and education over the detected trend and reduce the percentage of participation of women in the labor scope below the detected trend. To do this, the average extrapolated values of *education* variables are increased, and the values of *the women position at the labor force* are decreased.

Strategy 3: Maintaining the detected trend. In this case, the values of all control variables keep the average extrapolated values.

	GDPR	YMAL	YFEM
Scenario 1	↑	↑	↑
Scenario 2	↑	↑	↓
Scenario 3	↑	↑	≈
Scenario 4	↑	↓	↑
Scenario 5	↑	≈	↑
Scenario 6	↓	↓	↓
Scenario 7	↓	↓	≈
Scenario 8	≈	↓	↓
Scenario 9	↓	≈	≈
Scenario 10	≈	↓	≈
Scenario 11	≈	≈	≈

Figure 34: Scenarios. Increasing the tendency of the economic variables: ↑. Decreasing the tendency of the economic variables: ↓. Keeping the tendency of the economic variables: ≈.

The variable to optimize is $zopt1_i$

$$zopt1_i = \sum_j X_j S_j p_j \tag{4}$$

where the index i indicates the considered strategy and where p_j are the probabilities that the experts assign to each one of the 11 considered scenarios (see Figure 34). The minimum value of this variable over time indicates which strategy is the best. In this paper we have chosen the value $p_j = 1/11$ for all j , considering tentatively that all scenarios have the same probability to occur.

The optimal strategy for achieving the proposed goal is chosen by observing the evolution of $zopt1_i$. The calculations are carried out with the simulator generated by *SIGEM* and *Mathematica 7.0*. The results from the deterministic formulation are presented in Figure 35. The results from the stochastic formulation are presented in Figure 36. However, the conclusions can be better appreciated from the numerical results of the model stochastic formulation in Table 9.

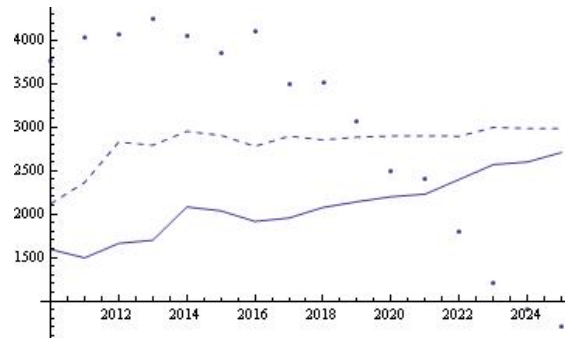


Figure 35: Deterministic formulation. Forecasted average difference between births and deaths (zopt1). Strategy 1: dots, Strategy 1: dots, Strategy 2: solid line, Strategy 3: dashed line, for period 2010-2025.

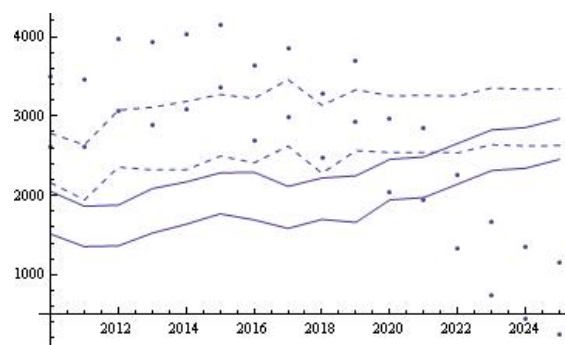


Figure 36: Stochastic formulation. Confidence intervals (95%) for the forecasted difference between births and deaths (zopt1). Strategy 1: dots, Strategy 2: solid line, Strategy 3: dashed line, for period 2010-2025.

Year	Strategy 1		Strategy 2		Strategy 3	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
2010	2618	3494	1513	2052	2163	2784
2011	2607	3459	1354	1864	1944	2639
2012	3061	3980	1365	1878	2358	3075
2013	2885	3925	1525	2086	2323	3112
2014	3092	4024	1637	2171	2325	3186
2015	3353	4156	1768	2284	2496	3273
2016	2690	3643	1692	2293	2412	3226
2017	2996	3854	1584	2114	2622	3461
2018	2485	3285	1697	2220	2280	3138
2019	2019	3704	1661	2248	2566	3334
2020	2041	2959	1944	2456	2542	3258
2021	1941	2859	1974	2486	2544	3260
2022	1341	2259	2144	2656	2538	3254
2023	741	1659	2314	2826	2640	3356
2024	441	1359	2344	2856	2627	3342
2025	441	1159	2454	2966	2629	3345

Table 9. Confidence intervals (95%) for the forecasted difference between births and deaths (zopt1).

Following the optimal strategy, i.e., Strategy 1, the conclusions are that, in Austria to achieve a stable society near 2025, Public Administrations would have to:

- Invest over the observed trend in education (as needed to achieve the desired increase in gross enrollment rate and literacy);
- Invest over the observed trend in health (as needed to achieve the desired increase in life expectancy at birth);
- Increase the percentage of participation of women in the working life.

5 Conclusion

An abstract model has been presented to study the dynamics of a human population per genders. Three well-being variables have been included in the model. These variables are submitted by the United Nations: *HDI*, *GDI* and *GEM*. Through simulations, the model allows us to obtain the possible consequences of different strategies in different scenarios, giving the results along time either with their respective confidence intervals or with their respective mean and standard deviation.

The study has been performed with two model formulations: deterministic and stochastic. The deterministic one uses new generalized functions for emigration, immigration, birth and death rates, demonstrated being valid for Austria. On the other hand, the stochastic model seems the most appropriate for the problem posed in this paper, because it takes into account the “noise” or uncertainty, calculating the reliability of the results.

An application case referred to Austria is described. The main objective in this case is to obtain a stable population about 2025. Eleven scenarios and three strategies have been tentatively assumed as possible. Strategies are: investing in education, health and female labor force with priority (Strategy 1), investing in education and health with priority (Strategy 2) and, investing in the three fields with the same proportion (Strategy 3). Scenarios are the most interesting combination between three economic variables: gross domestic product, female income and male income. The obtained best strategy is Strategy 1 (for some tentative values given to the probabilities of the scenarios).

The current model could be expanded by using another *UN* well-being variable, the Human Poverty Index (*HPI-2* for *OECD* countries and *HPI-1* for countries outside the *OECD*). It would also be interesting to see if the *HPI-1* is exchangeable with the *HPI-2* in the corresponding formulas or not.

The resolution of the demographic sustainability problem can be tackled from other points of view, for example environmental or economic. This will involve designing a model with new variables related to fertility and mortality rates. For instance:

- When tackling economic aspects, it is necessary to introduce in the model variables related to migration, investments in other countries, human poverty, job security, etc.
- When tackling environmental aspects, it is necessary to introduce variables related to energy, environmental management, demographic trends, etc.

Finally, an ambitious work would be to find a model considering several countries or regions, in order to set up the rates of migration in terms of the well-being variables of the involved countries.



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- Caselles et al. (2008). Population Growth and Social Well-being: A Dynamic Model Approach. 7th Congress of the Systems Science European Union. Lisbon (Portugal).

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