



**UNIVERSITAT POLITÈCNICA DE VALÈNCIA
DEPARTAMENT DE CIÈNCIA ANIMAL**

MODELOS DE CRECIMIENTO DE LA DORADA (*Sparus aurata*, L.) EN CONDICIONES DE PRODUCCIÓN EN LA COSTA MEDITERRÁNEA.

TESIS PRESENTADA POR:

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Para la obtención *del título de Doctor por la UPV*

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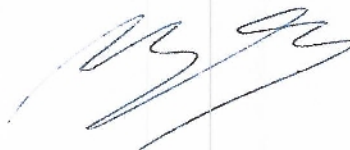
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Resumen

La presente tesis doctoral aborda el estudio de diversos modelos matemáticos de crecimiento para la dorada (*Sparus aurata*) en condiciones de producción y desarrolla un nuevo modelo que permite establecer la evolución de los pesos en función de la temperatura del agua y de la distribución inicial del peso de los alevines proporcionados por la hatchery. Para llegar al modelo final partimos, en un primer artículo, de la selección de modelos adecuados para describir la evolución de los valores medios del peso de la población de peces en una jaula marina. En un segundo artículo se aplica la regresión cuantil en base al modelo seleccionado (modelo *TGC*) y se observan dos comportamientos, que permiten clasificar los lotes según dos patrones de crecimiento: crecimiento homogéneo y crecimiento heterogéneo. También se identifican las variables que influyen en la existencia de dichos patrones. En el tercer artículo, tras detectar un cambio en la dinámica del crecimiento de la dorada a lo largo de su ciclo de crecimiento que permite establecer dos periodos de crecimiento diferenciado; se desarrolla un modelo que se ajusta a dicho comportamiento.

Resum

La present tesi doctoral aborda l'estudi de diversos models matemàtics de creixement per a l'orada (*Sparus aurata*) en condicions de producció i desenvolupa un nou model que permet establir l'evolució dels pesos en funció de la temperatura de l'aigua i de la distribució inicial del pes dels alevins proporcionats per l'hatchery. Para arribar al model final partim, en un primer article, de la selecció de models adequats per descriure l'evolució dels valors mitjans del pes de la població de peixos en una gàbia marina. En un segon article s'aplica la regressió quantil d'acord al model seleccionat (model *TGC*) i s'observen dos comportaments, que permeten classificar els lots en dos patrons de creixement: creixement homogeni i creixement heterogeni. També s'identifiquen les variables que influeixen en l'existència dels patrons. En el tercer article, després de detectar un canvi en la dinàmica del creixement de l'orada al llarg del seu cicle que permet establir dos períodes de creixement diferenciat; es desenvolupa un model s'ajusta al comportament.

Abstract

In this Thesis, the study of several mathematical models of growth for the gilthead sea bream (*sparus aurata*) under conditions of production, and a new model that established the evolution of the weights has been developed.

In a first article, the growth of gilthead sea bream has been studied considering five multiple exponential regression models.

In a second article, quantile regression techniques were used to describe the evolution of weight distribution. Different Thermal-unit Growth Coefficients (TGC) were obtained for several quantiles, and two growth patterns were observed. Finally, discriminant analysis helped to identify variables that affected the classification of batches.

In a third article, two stages for the growth of gilthead sea bream were detected by means of TGC seasonal models using the continuous temperature curves.

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1. INTRODUCCIÓN

1.1 SITUACIÓN ACTUAL DE LA ACUICULTURA

Según los informes del “El Estado Mundial de la Pesca y la Acuicultura” publicados por la FAO en 2012 y 2014, en las tres últimas décadas (1980-2010), la producción mundial de peces comestibles procedentes de la acuicultura se ha multiplicado por 12. La producción acuícola mundial ha seguido creciendo, aunque a un ritmo más lento que en las décadas de 1980 y 1990. La tasa media anual de producción de peces comestibles durante este periodo fue del 10,8%. En la década de 1990-2000 disminuyó al valor de 9,5% y en el periodo de 2000 al 2012 se redujo al 6,2%. Esta disminución en la tasa media de producción no supone una menor producción de peces, no obstante desde el 2000 hasta el 2012 se duplica la producción acuícola mundial de peces comestibles, pasando de las 32,4 millones de toneladas en el 2000 hasta las 66,6 millones de toneladas en el 2012.

Según las últimas estadísticas publicadas por la FAO, en 2012 se alcanzó un máximo histórico de 66,6 millones de toneladas (excluidas las plantas acuáticas y los productos no alimentarios), un valor total estimado de 137.700 millones de dólares americanos.

Los datos más recientes de la FAO estiman que en 2013 la producción mundial de peces comestibles aumentó un 5,8%, alcanzando los 70,5 millones de toneladas y siendo China la responsable del aporte del 62%.

La importancia que adquiere la acuicultura en el suministro total de pescado se ha mantenido de forma ininterrumpida. Los peces comestibles producidos por la acuicultura han supuesto un aporte del 42,2% del total de 158 millones de toneladas de pescado producido por la pesca de captura (incluido el que no está destinado a usos alimentarios), y más del 50% del total de peces comestibles.

El crecimiento de la producción acuícola sigue siendo muy importante, no obstante continúa el descenso en los países industrializados como Japón, Estados Unidos de América, España, Francia, Italia y la República de Corea. Estos países presentan un denominador común: el descenso de la producción de peces de escama.

Las principales especies de peces producidas en el mundo son las carpas, trucha y salmón, pero en entorno del Mediterráneo las especies marinas como lubina y dorada son las más importantes.

1.2 SITUACIÓN ACTUAL DE LA PRODUCCIÓN DE LA DORADA (*Sparus aurata* L.)

En el panorama estatal y atendiendo al último informe de “Asociación Empresarial de Productores de Cultivos Marinos de España” (APROMAR), la dorada (*Sparus aurata*), la lubina (*Dicentrarchus labrax*), el rodaballo (*Psetta máxima*) y la corvina (*Argyrosomus regius*) son las especies más relevantes producidas en los países europeos meridionales.

La mayor parte de los países ribereños del Mediterráneo producen dorada y lubina mediante acuicultura, coexistiendo en muchas ocasiones ambas especies en las mismas granjas. Grecia es, con diferencia, el principal país productor. Muchos de estos mismos países producen además corvina, compartiendo granja con dorada y lubina. Por otra parte, la crianza del rodaballo se realiza en la vertiente atlántica europea, principalmente en España, siendo reducido el número de países productores.

Según estadísticas de la “Federación Europea de Productores de Acuicultura” (FEAP) y de APROMAR, la producción acuícola total de dorada (*Sparus aurata*) en Europa y el resto del mundo en 2014 se estima en 173.024 toneladas. Esta cifra es un 4.3% inferior a la del 2013 con (179.924 t) que supuso un repunte importante y un máximo histórico en la producción dorada (figura 1).

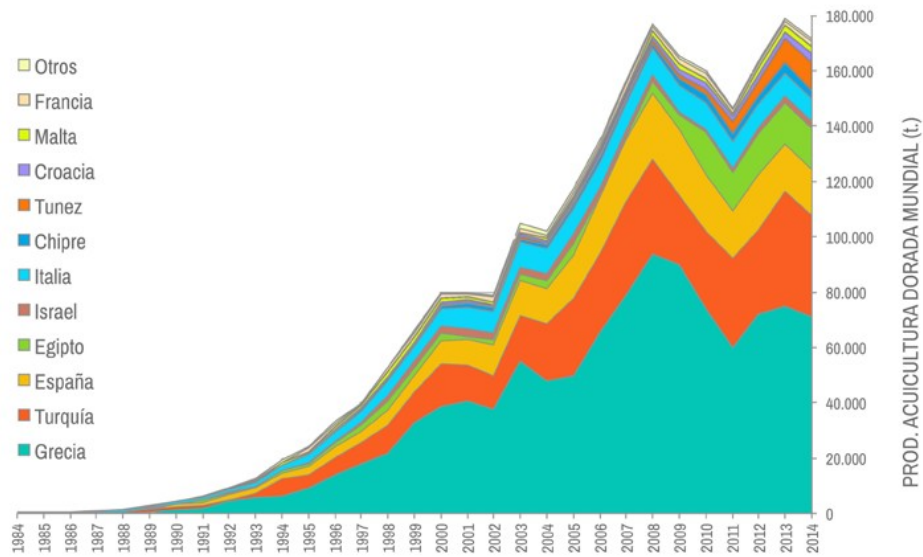


Figura 1. Evolución de la producción de acuicultura de dorada en el área mediterránea y el resto del mundo en el periodo 1984-2014. APROMAR 2015.

España es uno de los principales productores de dorada con 16.230 toneladas (9,3%) en 2014. Con esta producción, España se sitúa en tercer lugar del mundo por detrás de la producción de Grecia (41%) y Turquía (21,4%).

La producción de la dorada en España fue en 2014 un 3,4% menos que en 2013. Este dato supone una nueva ligera caída añadida a la del 2013 (13,6%).

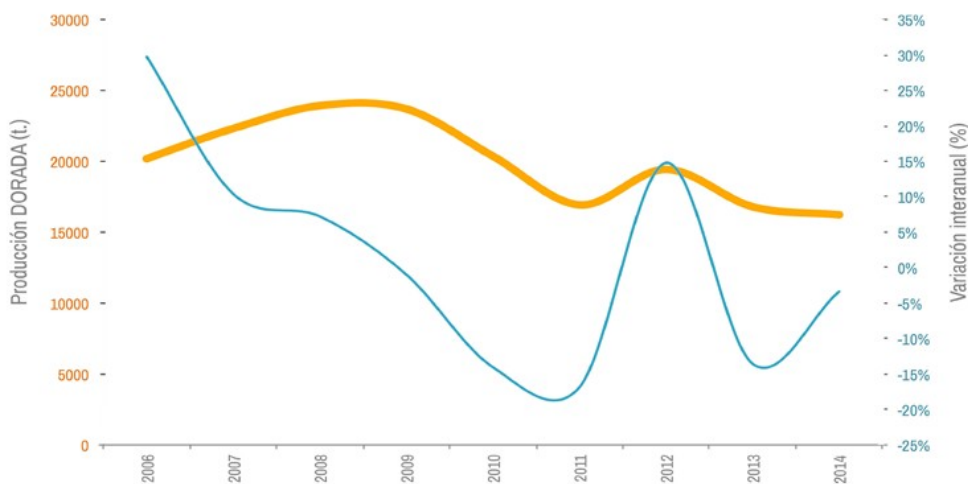


Figura 2: Evolución de la producción acuícola de dorada (*Sparus aurata*) en España (2006-2014). Se muestra la tasa de variación interanual. APROMAR 2015.

La Comunidad Valenciana sigue encabezando la mayor producción de dorada de acuicultura en España (con el 53,4% del total), seguida por Murcia (24%), Canarias (9,8%), Andalucía (7%) y Cataluña (5,9%). Como es lógico, el descenso en la producción ha conllevado la disminución del número de granjas.

En esta notable disminución del número de granjas, desde el punto de vista de APROMAR, “subyace una maduración sectorial asociada a una reestructuración del tejido empresarial que racionaliza los tamaños de granja viables, corrigiendo un intervencionismo desde las administraciones públicas vía tamaño de las concesiones, que conllevó la creación, en su momento, de un sector excesivamente atomizado en algunas regiones”.

Aunque siguen existiendo en España granjas de acuicultura marina perfectamente eficientes de tamaños diversos, adaptadas a sus objetivos de mercado, la dimensión media de las granjas, así como el número de granjas por cada empresa, ha ido creciendo con el paso de los años.

La rentabilidad de la producción está condicionada por dos aspectos diferenciados, por una parte, el precio de venta, que depende del mercado y escapa en ocasiones a las decisiones empresariales, y por otra, por los costes de producción, que en gran medida dependen de la propia gestión de la empresa acuícola.

El precio medio de primera venta de dorada de acuicultura producida en España en 2014 ha sido de 5,45 euros/kg (APROMAR 2015), lo que significa un incremento del 13,8 % respecto a 2013. Este precio supone mayor normalización si se compara con el de ejercicios anteriores, en los que la rentabilidad de las empresas productoras se vio comprometida, dado que los costes de producción medios pueden estimarse en una horquilla entre 3,80 y 4,30 euros/kg.

El precio medio de venta al público (PVP) ha sido de 7,48 euros/kg, con un consumo medio por familia de 600gr anuales.

APROMAR estima en su último informe que la producción nacional cubre aproximadamente un 60% del mercado nacional.

Aún persiste una excesiva volatilidad de los precios, causada por el elevado número de operadores y comercializadores, extranjeros y españoles, que fraccionan en exceso la oferta en contraste con la notable concentración de la demanda. Dicha volatilidad debe ser, y lo es en aquellas empresas que aún perduran, el motor para ajustar costes de producción para los productos acuícolas.

Durante 2014 y 2015, los precios de venta se han incrementado como consecuencia de la crisis sectorial ocurrida en Grecia, que ha provocado una reducción de la oferta.

En cambio, los costes de producción dependen de la gestión de la propia empresa acuícola, y son susceptibles de optimizar mediante adecuadas decisiones técnicas. Los principales costes de producción están relacionados con la compra de pienso (40%) y alevines (25%) y, aunque su precio depende de las empresas fabricantes y de las granjas de reproducción, es posible reducir el coste mediante una adecuada elección de ambas y mediante un manejo adecuado, que mejore la eficiencia y la supervivencia respectivamente.

Por otra parte, la gestión de la biomasa de peces en las granjas marinas es fundamental para planificar la producción y organizar las diferentes operaciones de producción, principalmente, la alimentación diaria, pero también los desdobles, las clasificaciones, las previsiones de venta, etc, y para ello es fundamental disponer de modelos de crecimiento adecuados para las diferentes especies de interés comercial.

1.3 MODELOS PARA LA ESTIMACIÓN DEL CRECIMIENTO DE PECES

1.3.1 INTRODUCCIÓN.

El problema de establecer modelos para el crecimiento de peces ha sido abordado por diversos autores. La mayoría de los estudios trabajan sobre poblaciones naturales y, por lo general, los resultados presentan un interés relativo en el contexto de la acuicultura intensiva. Los modelos deterministas clásicos pueden clasificarse en dos categorías:

1. Modelos formales o descriptivos, que pretenden ajustar, de la mejor manera posible, los valores a obtener mediante el modelo a los datos observados. Estos modelos no suelen basarse en hipótesis de carácter biológico.
2. Modelos lógicos o interpretativos, que se fundamentan sobre hipótesis de carácter biológico.

El más conocido de los modelos interpretativos es el modelo general de Von Bertalanffy (1938), basado en una serie de consideraciones fisiológicas sobre el anabolismo y el catabolismo. Este modelo conduce a la siguiente ecuación

$$W(t) = W_{\infty} \cdot (1 - e^{-kt})^b \quad (1)$$

Donde $W(t)$ es el peso del pez en el instante t y, en general para biología de pesca, $b=3$; W_{∞} es el peso asintótico (a largo plazo); k es el coeficiente instantáneo de crecimiento lineal, aceptándose la hipótesis teórica de que $W(0) = 0$. En su forma usual, el modelo de Von Bertalanffy resulta de la resolución de la ecuación diferencial

$$\frac{dW}{dt} = H \cdot W^d - K \cdot W^m \quad (2)$$

donde $H \cdot W^d$ representa la síntesis o anabolismo y $K \cdot W^m$ representa la degradación o catabolismo de la sustancia corporal. La ecuación (2) fue integrada por Beverton y Hold,

para el caso particular $d = \frac{2}{3}$ y $m=1$.

$$\frac{dW}{dt} = H \cdot W^{\frac{2}{3}} - K \cdot W,$$

cuya solución exacta es

$$W(t) = \left(\frac{H}{K} + e^{\frac{1}{3}Kt} \cdot C_1 \right) \quad (3)$$

Si en (3) se supone que $K > 0$, $\lim_{t \rightarrow +\infty} W(t) = W_{\infty}$ y $W(0)=0$, entonces se llega a que

$$W_{\infty} = \left(\frac{H}{K} \right)^3 \text{ y } C_1 = \left(\frac{H}{K} \right);$$

y se obtiene

$$W(t) = \left(\frac{H}{K} \right)^3 (1 - e^{-kt})^3$$

es decir,

$$W(t) = W_{\infty} \cdot (1 - e^{-kt})^3 \quad (4)$$

o, lo que es lo mismo, se ha obtenido la ecuación (1) para $b=3$ y $k=\frac{K}{3}$. Este caso particular se basa en la aceptación de que el catabolismo en peces es proporcional al tamaño y en asumir que el consumo de oxígeno debe ser proporcional a $W^{\frac{2}{3}}$ (regla 2/3 del metabolismo). Un detalle interesante es que la curva que describe la ecuación (4) tiene un punto de inflexión cuando la tasa de crecimiento es máxima, lo cual sucede cuando el peso es $W = 0.296 \cdot W_{\infty}$.

El modelo de Von Bertalanffy es un caso particular de la denominada familia de los modelos de crecimiento de Richards, la cual se define a partir de la ecuación diferencial

$$\frac{dW}{dt} = \frac{k \cdot W}{1-m} \cdot \left(\left(\frac{W_{\infty}}{W} \right)^{1-m} - 1 \right) \quad (5)$$

La ecuación diferencial (5) describe un modelo de crecimiento para diferentes patrones de crecimiento orgánico, dependiendo del valor del parámetro m , siendo W_{∞} el peso máximo admisible, y k una constante de proporcionalidad. La solución analítica general a la ecuación (5) es

$$W(t) = W_{\infty}^{\frac{1}{1-m}} \cdot (1 - A \cdot e^{-kt})$$

Casos particulares, de especial interés, de la familia de Richards son:

- El modelo de Von Bertalanffy: $m=2/3$ en (5), descrito anteriormente.
- El modelo Monomolecular: $m=0$ en (5) o, lo que es lo mismo,

$$\frac{dW}{dt} = -k \cdot W \cdot \left(\left(\frac{W_\infty}{W} \right)^{-1} - 1 \right) = k \cdot W \cdot \left(1 - \frac{W}{W_\infty} \right)$$

cuya solución general es

$$W(t) = W_\infty^{-1} \cdot (1 - A \cdot e^{-kt})$$

Esta curva de peso presenta un punto de inflexión cuando el peso es $W(t) = \frac{W_\infty}{2}$

El modelo de Von Bertalanffy y el resto de los pertenecientes a la familia de Richards pueden ser adecuados para el estudio de poblaciones naturales, pero se utilizan muy poco para el estudio del crecimiento en acuicultura intensiva, ya que las condiciones de producción son, en general, diferentes al medio natural y normalmente sólo interesa el peso, hasta el momento en que se alcanzan tamaños comerciales.

1.3.2 MODELOS DE CRECIMIENTO EN ACUICULTURA INTENSIVA

Algunos modelos clásicos.

La evolución temporal del peso en acuicultura suele seguir un típico perfil sigmoideal, susceptible de ser representado por diversidad funciones matemáticas, aunque numerosos autores consideran que, básicamente, para cortos periodos de tiempo, se puede considerar el crecimiento según un modelo exponencial,

$$W(t) = C \cdot e^{kt} \quad (6)$$

donde C y k son constantes. No obstante, esta consideración exige que, para abarcar periodos largos de tiempo, tenga que recurrirse a ciertos artificios como, por ejemplo, definir el crecimiento a tramos, recurriendo a distintas funciones exponenciales.

Parker y Larkin (1959), formularon la hipótesis de que, en cada fase del crecimiento, la tasa de crecimiento instantánea del peso, $\frac{dW}{dt}$, es proporcional a una potencia, b , del peso instantáneo

$$\frac{dW}{dt} = K \cdot W^b \quad (7)$$

que lleva a la solución exacta

$$\frac{1}{K \cdot (b-1)} \cdot (W(t))^{1-b} + t = C_1$$

Si se tiene la condición inicial de que $W(0) = W_0$; se llega a que

$$\frac{1}{K \cdot (b-1)} \cdot W_0^{1-b} = C_1$$

por lo tanto,

$$(W(t))^{1-b} = W_0^{1-b} - K \cdot (b-1) \cdot t \quad (8)$$

El modelo (8) responde a la relación general

$$(W(t))^a = W_0^a + C \cdot t \quad (9)$$

donde C es la pendiente del crecimiento (dependiente de la temperatura, ración, factores ambientales, etc.) y t es el tiempo, en días. El modelo dado por (9) asume la hipótesis de que hay una potencia del peso que sigue un crecimiento lineal.

Otro modelo interesante es el propuesto por Cho (1992), que plantea una predicción del crecimiento usando un índice denominado Thermal Growth Coefficient (*TGC*) o “coeficiente de crecimiento térmico”. Según dicho modelo

$$(W(t))^{\frac{1}{3}} = W_0^{\frac{1}{3}} + TGC \cdot \sum_{j=1}^t T_j \quad (10)$$

lo que es lo mismo,

$$W(t) = \left(W_0^{\frac{1}{3}} + TGC \cdot \sum_{j=1}^t T_j \right)^3 \quad (11)$$

donde t son días, W_0 es el peso en el instante inicial ($t=0$), y T_k es la temperatura, en grados centígrados en el k -ésimo día.

Modelo general.

Una generalización, en forma diferencial, del modelo dado en (7) es el que proporciona la formulación con condición inicial

$$\frac{dW}{dt} = f(t) \cdot W^a \quad (12)$$

$$W(t_0) = W_0 \quad (13)$$

donde $f(t)$ es una función continua del tiempo. La solución a (12) - (13), es

$$W(t) = \left(W_0^{1-a} + (1-a) \cdot \int_{t_0}^t f(t) \cdot dt \right)^{\frac{1}{1-a}} \quad (14)$$

Si se considera el caso particular en que $f(t)$ es una función de la temperatura, que a su vez es una función continua del tiempo, $f(t) = F(T(t))$ se tiene que

$$W(t) = \left(W_0^{1-a} + (1-a) \cdot \int_{t_0}^t F(T(t)) \cdot dt \right)^{\frac{1}{1-a}} \quad (15)$$

Se observa que de (15), para el caso particular $F(T(t)) = \frac{k}{1-a} \cdot T(t)$ y $\frac{1}{1-a} = 3$, ($a=2/3$)

se obtiene

$$W(t) = \left(W_0^{\frac{1}{3}} + k \int_{t_0}^t T(t) \cdot dt \right)^3$$

si, además, $[t_0; t] = [t_0; t_1] \cup [t_1; t_2] \cup \dots \cup [t_{n-1}; t_n]$; $t_n = t$ y la temperatura es constante en cada subintervalo, es decir, $T(x) = T_1$; para todo $x \in [t_0; t_1]$ y $T(x) = T_j$; para todo $x \in [t_{j-1}; t_j]$; $j=2, 3, \dots, n$, entonces

$$W(t) = \left(W_0^{\frac{1}{3}} + k \sum_{j=1}^n T_j (t_j - t_{j-1}) \right)^3 \quad (16)$$

En particular, si en (16), $t_j = t_0 + j$; $j=1, 2, \dots, n$ (los intervalos temporales son de longitud 1 día), entonces

$$W(t_0 + n) = \left(W_0^{\frac{1}{3}} + k \sum_{j=1}^n T_j \right)^3 \quad (17)$$

es decir, se obtiene el modelo de Cho (10) con $TGC=k$.

Por último, cabe mencionar algunos trabajos que han estudiado el crecimiento de la dorada. Petridis y Rogdakis (1996) consideran la igualdad,

$$SGR=0.000103 \times W_i^{0.523} \times T^{3.74} \quad (18)$$

El modelo descrito en (18) es adecuado para calcular el peso final utilizando el *Specific Growth Rate (SGR)* pero solamente es recomendable para periodos cortos de tiempo.

Por otra parte, Lupatsch and Kissil (1998) y Lupatsch et al. (2003) propusieron modelos de regresión múltiple para calcular la ganancia de peso diaria (*WG*) considerando el efecto de la temperatura (*T*), descritos por las ecuaciones (19) y (20):

$$WG=0.0167 \times W_i^{0.621} \times \exp(0.055 \times T), \quad (19)$$

$$WG=0.024 \times W_i^{0.514} \times \exp(0.060 \times T), \quad (20)$$

respectivamente.

1.3.3 MODELOS ESTACIONALES A PARTIR DE MODELOS GENERALES

Modelo de crecimiento tradicional

Muchas fórmulas de crecimiento se obtienen como soluciones de ecuaciones diferenciales. Estas se expresan generalmente por una expansión como la siguiente

$$\frac{dW}{dt} = g(W) \quad (21)$$

donde *W* es el peso (o la longitud), y *t* es el tiempo.

A partir de (21) se obtiene, dado que se trata de una ecuación diferencial de variables separables, la expresión diferencial equivalente

$$\frac{dW}{g(W)} = dt \quad (22)$$

La integración de ambos miembros de (22) lleva a

$$G(W) = t + C \quad (23)$$

donde c es la constante de integración, y $G(W)$ viene dada por

$$G(W) = \int \frac{dW}{g(W)} \quad (24)$$

La solución general de la ecuación (21) es

$$W(t) = G^{-1}(t + C) \quad (25)$$

donde con G^{-1} se denota la función inversa de G .

En matemáticas, la ecuación diferencial (22) y su solución integral (25) se consideran idénticas. Sin embargo, la expresión (25) es más útil a efectos prácticos.

La ecuación (21) no suele ser suficiente para describir con cierto detalle el crecimiento de los peces. Una ecuación diferencial más completa es

$$\frac{dW}{dt} = h(W, t) \quad (26)$$

Sin embargo, para (26) es más difícil obtener una solución general. En lugar de la ecuación (26), se considera la ecuación (27), que es un caso particular:

$$\frac{dW}{dt} = G(W) \cdot f(t) \quad (27)$$

Puesto que (27) es de variables separables, se puede resolver como sigue.

Sea

$$T = F(t) = \int f(t) dt \quad (28)$$

Diferenciando ambos miembros de (28) se obtiene

$$dT = f(t) dt \quad (29)$$

Teniendo en cuenta las ecuaciones (27), (29) se tiene que

$$\frac{dW}{g(W)} = f(t) \cdot dt = dT \quad (30)$$

Análogamente a como se ha razonado con las ecuaciones de la (22) a la (25), la solución general de (30) es

$$W = G^{-1}(T + C) = G^{-1}(F(t) + C) \quad (31)$$

Si consideramos que $f(t)$ es la temperatura del agua en función del tiempo t , entonces $F(t)$ sería la temperatura acumulada del agua. Por lo tanto, se tiene una metodología para obtener un modelo matemático, dado en (31), por el método de sustituir la variable tiempo, t , en (25) por la temperatura acumulada $F(t)$. Aunque se puede utilizar cualquier tipo de función temperatura, $f(t)$, a la hora de diseñar el modelo en biología suele recurrirse a una función periódica ajustada a las variaciones estacionales.

Los modelos de crecimiento obtenidos de este modo se denominan modelos de crecimiento estacional. También es posible llegar a la expresión (27) partiendo de la ecuación diferencial (32)

$$\frac{dW}{dT} = G(W) \quad (32)$$

teniendo en cuenta que

$$\frac{dT}{dt} = f(t), \quad (33)$$

puesto que

$$\frac{dW}{dt} = \frac{dW}{dT} \frac{dT}{dt} = G(W) \cdot f(t) \quad (34)$$

2. JUSTIFICACIÓN Y OBJETIVOS

2.1 JUSTIFICACIÓN

Aunque existe una extensa bibliografía que describe el crecimiento de la especie objeto de estudio en el presente trabajo de Tesis Doctoral, la dorada (*Sparus aurata*), gran parte de estos trabajos estudian a escala de laboratorio los requerimientos nutricionales, las fuentes proteicas alternativas a las harinas de sangre o de pescado, los requerimientos de aminoácidos o el perfil adecuado de lípidos, y un sinnúmero de estrategias que intentan obtener el mejor crecimiento reduciendo los costes económicos y ambientales, pero sin llegar a modelar dicho crecimiento. Otros trabajos proponen modelos que incluyen variables y parámetros muy diversos y quizá muy necesarios para una descripción del crecimiento muy ajustada a unas condiciones particulares pero que, en general, no son útiles para prever crecimientos de otros lotes. Además, no existen demasiadas investigaciones que aborden el problema de describir el crecimiento bajo condiciones reales de producción.

Para diseñar modelos útiles para describir el crecimiento de la dorada, atendiendo a condiciones reales de producción, habría que establecer variables susceptibles de poder ser controladas en una granja marina (origen de los lotes, fecha de introducción, alimentación, manejo, etc) que pueden llegar a explicar dicho crecimiento, aunque deben tenerse en cuenta criterios de sencillez y de practicidad, ya que los condicionantes operativos pueden dificultar el seguimiento y/o control de las variables. Una vez seleccionadas las variables más adecuadas, significativas y controlables en una instalación para la descripción del crecimiento, se iniciaría el proceso de modelización matemática.

Tras obtener el “mejor” modelo teórico en base a datos conocidos, es necesario contrastar el mismo con datos reales de producción procedentes de otras granjas para validar el modelo y, por consiguiente, dar una mayor robustez a la descripción del crecimiento.

A partir del modelo se podría describir el crecimiento futuro de los peces o, lo que es lo mismo, establecer predicciones. Las poblaciones de peces que se estabulan en jaulas marinas tienen una característica común en el crecimiento: la heterocedasticidad, la cual cuestiona como modelos útiles para la gestión aquellos que se basan simplemente en un ajuste mínimo-cuadrático de valores promedio del peso mediante modelos de

regresión simple o múltiple. Dichos modelos, protagonistas no obstante de la mayoría de la bibliografía, presentan muchas limitaciones para la descripción completa de la evolución del peso para una población de peces en una granja marina. Un modelo de regresión proporciona una descripción correcta de la evolución de valores promedio, pero la gestión de una granja exige saber algo más, fundamentalmente conocer cómo evoluciona en conjunto el peso de la población de peces que proporciona inicialmente la granja de reproducción. Téngase en cuenta que tras dos evoluciones semejantes de valores promedio para el peso, pueden esconderse en realidad, para cada instante, poblaciones caracterizadas por pesos con muy distinto apuntamiento y diferente simetría, es decir, dos poblaciones con una evolución de la distribución del peso muy diferente.

Para tener una descripción mucho más completa y fiel del crecimiento, se debe construir el modelo utilizando la máxima información posible, obtenida ésta a partir del mayor número posible de datos reales.

La regresión cuantil, como contraposición a la regresión mínimo-cuadrática clásica, permite una mejor aproximación al problema de describir la evolución de la distribución de una variable dependiente aleatoria, en función de una variable independiente determinista. En la regresión clásica, un único valor medio obtenido en una muestra (200 ejemplares simplifica una población de entre 100.000 individuos hasta casi un millón), mientras que la regresión cuantil permite considerar todos los datos de la muestra en su conjunto, proporcionando una distribución muestral como aproximación a la distribución poblacional, obteniendo diversas curvas de crecimiento de los peces, en función de sus características genéticas o adaptativas. A modo de ejemplo, con la regresión cuantil se puede detectar la evolución del crecimiento de las llamadas “colas” y las “cabezas” de la población, pudiéndose caracterizar posibles fenómenos de competencia intraespecífica o de canibalismos, e incluso, considerando la información remitida por la granja de reproducción, detectar lotes iniciales formados a partir de las colas de varias puestas.

Una vez seleccionados, por una parte el modelo más adecuado y por otra la técnica que más información aporta, se dispondrá de una herramienta adecuada que facilitará la gestión de la producción.

2.2 OBJETIVOS

Por todo lo comentado, el objetivo principal de la presente Tesis Doctoral se centra en desarrollar un modelo matemático de crecimiento de la dorada (*Sparus aurata*) en condiciones reales de producción en jaulas o corrales marinos, que permita predecir la evolución del peso de los peces en función de la temperatura y distribución inicial de pesos, para optimizar la gestión de la producción de la empresa acuícola.

Asimismo, los objetivos parciales serían:

1. Seleccionar el modelo descriptivo que, de una forma sencilla, mejor describa la evolución de los valores medios de peso para los individuos de la población en condiciones reales de producción.
2. Detectar las posibles variables que pueden caracterizar patrones de crecimiento diferente en las distintas poblaciones.
3. Desarrollar un modelo de regresión cuantil asociado al modelo descriptivo, que permita estimar el crecimiento de los diferentes grupos de peces de un mismo lote, con diferentes velocidades de crecimiento.
4. Profundizar en la descripción del modelo, detectando la existencia de patrones de crecimiento asociados al peso de la dorada.

3. MATERIAL Y MÉTODOS

3.1 DESCRIPCIÓN DE LOS DATOS.

Para desarrollar los modelos de la presente Tesis, se han procesado los datos de peso correspondientes a 20 lotes de dorada (*Sparus aurata*), producidos en la Granja Marina Safor entre los años 1998 y 2000. Dichos lotes se disponían en jaulas flotantes de 16 metros de diámetro, en una concesión de explotación marina ubicada a 2 millas de la costa de Gandía, Valencia.

La temperatura del agua en la que se localiza la granja se registró a lo largo del periodo de estudio, oscilando entre 13.2°C en febrero y 26.5°C en septiembre.

Cada jaula contenía entre 70.000 y 230.000 doradas, que fueron introducidas con pesos medios iniciales en un rango entre 6 y 44 gr. Los pesos medios finales oscilaron entre 310 y 552 gramos. La densidad mantuvo entre los 15 y los 25 kg/m³.

Las doradas fueron alimentadas dos veces al día, seis días por semana, con dos dietas comerciales, de 43/21 y 44/17, en porcentaje de proteína y lípidos respectivamente.

A lo largo de todo el periodo de producción, se extraían muestras de 300 peces de cada lote, en cada muestreo, midiéndose el peso y la longitud de cada dorada. Para medir el peso se utilizaron dinamómetros de 50gr, 100gr, 200gr y 500gr. Dichos muestreos se realizaron a intervalos entre 30 y 60 días. Los datos de longitud y, fundamentalmente, de temperatura y peso de los 20 lotes son los que han sido utilizados para desarrollar los estudios que se presentan en esta memoria.

También se obtuvieron datos de temperatura y peso correspondientes a 6 lotes producidos entre 2001 a 2004, que se reservaron para validar los análisis y modelos desarrollados. Los 6 lotes indicados fueron producidos en jaulas de 25 metros de diámetro, con unos pesos medios iniciales situados entre los 10 y los 85gramos y unos pesos finales entre 371 y 452 gramos.

En el primer artículo *Predicting the growth of gilthead sea bream (Sparus aurata L.) farmed in marine cages under real production conditions*, se consideraron 5 modelos que estimaban el peso final en función del peso inicial (kg), el tiempo (días) y la temperatura del agua (°C), o la suma de las temperaturas efectivas (°C). Los parámetros para cada modelo se obtuvieron mediante ajuste minimocuadrático del promedio de los

pesos en cada muestreo, respecto a los pesos iniciales correspondientes al muestreo anterior, considerando como variable independiente, en unos casos, la temperatura media y en otros, la temperatura efectiva acumulada del periodo entre muestreos, utilizando como metodología de ajuste la de Marquard en la herramienta de regresión no lineal del software estadístico Statgraphics Plus, versión 5.1.

Para asegurar que los resultados finales no se ven afectados significativamente por errores sistemáticos, el azar u otros factores externos debido a la naturaleza de los datos o a las condiciones reales de producción, para el análisis de validación se consideraron los datos de peso de los seis lotes de dorada producidos de 2001 a 2004 descritos anteriormente. Las diferencias entre los errores de predicción se analizaron mediante ANOVA simple, utilizando el test *t* de Student de Newman Keuls y el mismo software estadístico.

El segundo artículo *Use of quantile regression and discriminant analysis to describe growth patterns in farmed gilthead sea bream (Sparus aurata)* se estructura en tres partes. En primer lugar se describe el crecimiento de los 20 lotes de referencia, tomando como base el modelo *TGC* (Cho y Bureau, 1998), mediante regresión cuantil (Koenker y Bassett, 1978). La obtención de los parámetros que caracterizan el modelo para cada uno de los cuantiles considerados son obtenidos a partir del procesamiento de los datos de peso en función de las temperaturas efectivas acumuladas, utilizando la rutina *quantreg* específica para regresión cuantil lineal, que proporcionar el software RDCT (2005). En segundo lugar, se procede a realizar una clasificación de los lotes estudiados en homogéneos y heterogéneos, usando una técnica ANOVA (Koenker y Bassett, 1982), usando también el software RDCT (2005), considerando siempre un nivel de confianza del 95% para establecer la existencia de diferencias estadísticamente significativas entre las pendientes de las rectas, o, lo que es lo mismo diferencias estadísticamente significativas para los valores de *TGC*, correspondientes a los diferentes cuantiles. Aceptar la hipótesis nula de igualdad de valores *TGC* para los diferentes cuantiles significará clasificar el lote como homogéneo, lo cual quiere decir que la distribución del peso se mantiene a lo largo del ciclo de crecimiento. Si alguno de los valores de *TGC*, correspondiente a alguno de los cuantiles, es significativamente distinto, desde la perspectiva estadística, a los demás valores de *TGC* que corresponden a otros cuantiles, entonces el lote es clasificado como heterogéneo, lo cual significa que la distribución del peso varía y cambia a lo largo del ciclo de crecimiento. La

clasificación obtenida supone establecer la existencia de distintos patrones de crecimiento resumidos en dos: el patrón según el cual la distribución de los valores de peso evoluciona regularmente a partir de la distribución inicial (lotes homogéneos) y el patrón según el cual la distribución evoluciona irregularmente con dinámicas de evolución singulares para alguno de los cuantiles (lotes heterogéneos). Finalmente, tras la clasificación, se utiliza la técnica del análisis discriminante, considerando, para cada muestreo y lote, diversas variables (medidas estadísticas asociadas la distribución inicial del peso de los peces y algunas variables ambientales obtenidas a partir de la evolución de la temperatura del agua durante todo el período de producción). Con el análisis discriminante se pretende valorar la relación de dicha clasificación con las variables influyentes en la misma, es decir, para detectar las variables que más contribuirían a dar sentido a dicha clasificación. En el análisis discriminante, para evitar la influencia de las diferencias entre las varianzas de las variables considerada, se utilizan datos tipificados y los cálculos se realizan utilizando la opción específica del software estadístico STATGRAPHICS Plus 5.1.

En el tercer artículo *A two-stage growth model for gilthead sea bream (Sparus aurata) based on the thermal growth coefficient* se justifica la evidencia de que el modelo *TGC* clásico (Cho y Bureau, 1998), cuya base teórica sería modelar la evolución del peso tras una linealización consistente en una transformación potencial (elevando el peso a $1/3$), debe ser generalizado, estableciéndose dos etapas en el crecimiento para las cuales se debe considerar dos linealizaciones de los datos mediante su transformación en base a dos potencias distintas ($1/3$ y $2/3$), que dan lugar a los modelos $TGC_{1/3}$ (el clásico) y el modelo $TGC_{2/3}$, respectivamente, para obtener un mejor ajuste matemático a la realidad de la evolución de los pesos medios en los distintos lotes. En los modelos analizados se incluyó una función senoidal, periódica de periodo 365 días, que modeliza la evolución de la temperatura del agua en función del tiempo (días). Los parámetros que definen la función temperatura se obtuvieron mediante ajuste mínimo-cuadrático no lineal, utilizando el método de Levenberg–Marquardt disponible en el software de cálculo MATLAB v. 5.3. Se estableció un valor crítico de referencia para el peso medio (117 g), donde se produce un cambio claro en la dinámica de la evolución del peso medio de los peces. Dicho cambio lleva a desarrollar un tercer modelo, modelo *TGC* mixto, en base a la hipótesis de que debe producirse una transición suave desde el modelo $TGC_{1/3}$ al modelo $TGC_{2/3}$. Para la obtención de los parámetros de los modelos

$TGC_{1/3}$ y $TGC_{2/3}$ (y por lo tanto del mixto), se ha recurrido a la regresión lineal simple (previa linealización de los datos) disponible en el software Statgraphics plus 5.1.

Finalmente, se compararon los valores reales, correspondientes a los seis lotes reservados para la validación, y los valores estimados por los tres modelos, contrastando la igualdad de las medias de los errores absolutos mediante un ANOVA simple y también valorando el comportamiento de las predicciones respecto a los datos reales mediante contrastes de hipótesis para datos apareados de pesos medios, reales y estimados. Para estos últimos análisis también se ha recurrido a las opciones específicas disponibles en el software Statgraphics plus 5.1.

4. ARTÍCULO 1

“Predicting the growth of gilthead sea bream (*sparus aurata L.*) farmed in marine cages under real production conditions using temperature and time-dependent models”. Adaptación para la tesis.

Predicting the growth of gilthead sea bream (*Sparus aurata* L.) farmed in marine cages under real production conditions using temperature- and time-dependent models

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Abstract

The growth of gilthead sea bream (*Sparus aurata*) has been studied considering five multiple exponential regression models using data from 20 lots of gilthead sea bream growing in 20 marine cages from a Mediterranean commercial fish farm. The final weight (W_f) of fish was predicted in relation to the initial weight (W_i), time (t) and temperature (T), or the sum of effective temperatures ($\sum T_{ef}$). The estimated weight results from the simulation using the five models have been compared with the real final weight values using the mean of the absolute values of the prediction errors in short and long term (the precision value). All models presented a high determination coefficient, above 96%, and good prediction values in the short term. Regression models were tested using data from six new cages. The best models for predicting the growth of sea bream long term were the ones where final weight is expressed in relation to the initial weight and the sum of effective temperature, $W_f = 3.528 W_i^{0.783} e^{0.00042\sum T_{ef}}$ and $W_f = 1.516 W_i^{0.796} \sum T_{ef}^{0.161}$, obtaining long-term prediction errors 12.9% and 10.7% respectively.

Keywords: regression model, management of fish farms

Introduction

Fish growth prediction is essential in aquaculture to organize the production planning during the design stage of new fish farms, as well as for planning and

carrying out several management operations, such as classification and distribution of fish, unloading of new batches, harvesting schedule and also for calculating the daily feeding rates.

Traditionally, fish growth has been expressed through the Von Bertalanffy (1938) equations, which are appropriate for studying natural populations, where fish reach adult size, but not in aquaculture because the commercial weight of fish is lower, reached before the natural cycle of growth could be completed, and also because these models do not consider the effect of water temperature along the year.

The growth of fish in farms is influenced by several factors: density, dietary level of nutrients, feeding rate, environmental conditions, origin of fish and mainly water temperature. However, the most frequently used index for evaluating fish growth in commercial marine farms and in research trials is the 'specific growth rate' (SGR), which only provides an approximation of a measure of the weight increase in relation to time (t), expressed in days [Eq (1)].

$$SGR = \frac{\ln(W_f) - \ln(W_i)}{t} \quad (1)$$

If experiments are conducted under the same conditions (similar initial and final weight, temperature, etc.), then the SGR could be a good index for comparing the growth of several groups of fish in order to study different variables (diets, density, etc.). For predicting fish growth, the final weight can be estimated from the previous SGR values [Eq (2)].

$$W_f = W_i \times \exp(SGR \times t) \quad (2)$$

The problem of using the SGR model is that the SGR value varies according to initial weight of fish (W_i) and temperature (T), as reported by Petridis and Rogdakis (1996) [Eq (3)]. Therefore, when the SGR growth model is used for gilthead sea bream, it is possible to estimate the SGR value in relation to initial weight and temperature, and afterwards calculate the final weight using Eq (2) for short periods of time.

$$\text{SGR} = 0.000103 \times W_i^{0.523} \times T^{3.74} \quad (3)$$

In recent years, Lupatsch and Kissil (1998) and Lupatsch, Kissil and Sklan (2003) proposed some exponential multiple regression models for evaluating the daily weight gain, WG (g fish day⁻¹) considering the effect of temperature (T) [Eqs (4) and (5) respectively].

$$\text{WG} = 0.0167 \times W_i^{0.621} \times \exp(0.055 \times T) \quad (4)$$

$$\text{WG} = 0.024 \times W_i^{0.514} \times \exp(0.060 \times T) \quad (5)$$

Likewise, Cho and Bureau (1998), for salmonids, and Kaushik (1998), for other species, proposed a new model based on the 'Thermal-Unit Growth Coefficient' (TGC), which is calculated from final and initial weights and the summation of daily effective temperature ($\sum T_{\text{ef}}$) [Eq (6)].

$$\text{TGC} = \frac{W_f^{\frac{1}{3}} - W_i^{\frac{1}{3}}}{\sum T_{\text{ef}}} \quad (6)$$

The main advantage of considering the TGC model is that its value is theoretically independent from the initial body weight. If the TGC value is available from previous studies, by solving W_f in Eq (6), a growth prediction can be obtained from the expected daily effective temperatures in the period [Eq (7)].

$$W_f = \left(W_i^{\frac{1}{3}} + \text{TGC} \times \sum T_{\text{ef}} \right)^3 \quad (7)$$

Nevertheless, Dumas, de Lange, France, Hua and Bureau (2006) have recently reported that the TGC is not independent of initial fish weight, presenting three different values for rainbow trout considering three growth periods: the first from 0.2 to 20 g, the second from 20 to 500 g and the third > 500 g.

The intensive production of sea bream (*Sparus aurata*) is the most widespread in the Mediterranean Sea, but the sale price has decreased over the past few years, reducing the profitability of marine farms. The development of a suitable growth prediction model, adjusted to the real conditions of intensive production, could be an important tool for reducing the production costs by optimizing the daily food

ration, the organization of management operations and the production plan.

Most of the previously cited models have been developed from laboratory growth data, under experimental and controlled conditions, or from real production growth data in the sea, but considering a small number of lots in small cages, it is necessary to check and calibrate these models for their adaptation to the real production conditions.

The aim of this work is to obtain an accurate and simple prediction model for gilthead sea bream growing in marine cages in the Mediterranean Sea, taking into account previous models cited in the bibliography and other regression models developed from data obtained in a Spanish marine fish farm, under real production conditions.

Materials and methods

Weight data from 20 batches of sea bream, with origins in different hatcheries, growing in floating cages along the year, were sampled in the marine farm, 'Granja Marina Safor', located in the Western Mediterranean opposite Gandia, Valencia, Spain, 2 miles off the coast and with a water depth of around 20 m. The marine cages had a 16 m diameter and a 10 m depth.

Water temperature was recorded every day and varied from a lowest of 13.2 °C in February, to the highest of 26.5 °C in September. Each cage contained 69 500–236 000 fish. Considering complete growth cycles, the initial weights were in the range from 6 to 44 g and the final weights varied from 310 to 552 g. The mean value of the final density was in the 15–25 kg m⁻³ interval. Sea bream were fed, 6 days per week and two times per day, with two commercial diets (43/21 and 44/17 %protein/%lipid) using the feeding rate recommended by the suppliers. In the first phase of growth, feed was distributed by hand, and afterwards using pneumatic canyons installed in the boats.

Throughout the period from 1998 to 2000, around 300 fish from each cage were weighed every 30–60 days, and the average weight was calculated and used to test previous models (Petridis & Rogdakis 1996; Lupatsch & Kissil 1998; Lupatsch *et al.* 2003) and estimate the parameters corresponding to the growth models proposed.

The parameters for all the models were computed by means of non-linear regression analysis (least-square method) considering different periods of time between sampling in each lot, using the final weight

of fish, W_f , as the dependent variable, and the initial weight of fish, W_i , the period of time in days, n , and the mean of the water temperature of each period, T , or the sum of the daily effective temperature ($\sum T_{ef}$) for sea bream, calculated as $\sum T_{ef} = n(T - 12)$, where 12 is the minimum temperature for feeding and growth of sea bream [according to García-García (1994)], as independent variables.

In order to obtain a measure of the precision, the mean of the absolute value of the short-term prediction errors was calculated for each model. Thus, considering short periods between samplings (30–60 days), the predicted final weight for each period was computed from the real weight of the initial sampling. Next, the average of the absolute values of the difference between the predicted weight and the real weight was calculated.

The mean of absolute values of the long-term prediction errors (long-term precision) was calculated in analogous form, but considering the absolute value of the subtraction between weight estimation given by the models at the end of the growing period (229–524 days) and the real final weight. In the long-term case, a step-by-step estimation of the weight was made, computing the final weight at the end of a period and considering this weight as the initial weight for estimating the weight at the end of the next period, starting from the initial real weight at day 0. These long-term precision values were also expressed in percentages with respect to the real final weight.

To assure that the final results have not been affected significantly by random systematic errors or other external factors due to the nature of the data or the real conditions of production, weight data from another six new lots of sea bream grown from 2001 to 2004 were used to test the developed models. In this case, the cage sizes were 25 m in diameter, and every month samples of 300 sea bream were weighed. Initial weight was 10–85 g and final weight was 371–452 g.

The parameters, K_p , of the models that we will describe in the next section, were obtained using the Marquard method in the non-linear regression options of the statistical software STATGRAPHICS PLUS, version 5.1. The difference between prediction errors was analysed by ANOVA, using the Student Newman Keuls test.

Results

Table 1 shows the initial and final weights of sea bream, the duration of the growing period, the sum of effective temperatures, the SGR and the TGC values

Table 1 Real growth data from 20 lots of sea bream, farmed in marine cages under commercial production conditions

| Lot | Initial weight (g) | Final weight (g) | Days | \sum °C day ⁻¹ | SGR | TGC |
|------|--------------------|------------------|------|-----------------------------|------|---------|
| 1 | 28.2 | 335.9 | 291 | 2341.6 | 0.85 | 0.00167 |
| 2 | 28.2 | 358.4 | 269 | 2256.8 | 0.85 | 0.00180 |
| 3 | 9.0 | 379.1 | 476 | 3130.26 | 0.79 | 0.00165 |
| 4 | 11.8 | 552.6 | 524 | 4248.48 | 0.73 | 0.00140 |
| 5 | 18.1 | 333.1 | 376 | 2687.77 | 0.77 | 0.00160 |
| 6 | 10.5 | 458.5 | 467 | 3898.25 | 0.81 | 0.00142 |
| 7 | 10.5 | 310.8 | 401 | 3049.5 | 0.84 | 0.00150 |
| 8 | 11.8 | 341.8 | 391 | 2906.87 | 0.86 | 0.00162 |
| 9 | 5.7 | 381.4 | 413 | 3054.17 | 1.02 | 0.00179 |
| 10 | 10.6 | 343.8 | 383 | 2562.08 | 0.91 | 0.00188 |
| 11 | 5.6 | 421.1 | 396 | 2893.2 | 1.09 | 0.00198 |
| 12 | 25.1 | 356.2 | 356 | 2490.8 | 0.75 | 0.00167 |
| 13 | 18.2 | 349.3 | 308 | 2244.8 | 0.96 | 0.00197 |
| 14 | 27.2 | 397.8 | 303 | 2238.8 | 0.89 | 0.00194 |
| 15 | 14.6 | 320.7 | 322 | 2425.6 | 0.96 | 0.00181 |
| 16 | 22.9 | 363.0 | 272 | 2371.7 | 1.02 | 0.00181 |
| 17 | 37.9 | 367.4 | 267 | 2361.6 | 0.85 | 0.00161 |
| 18 | 32.1 | 321.8 | 246 | 2303.8 | 0.94 | 0.00160 |
| 19 | 30.8 | 344.5 | 234 | 2270.3 | 1.03 | 0.00171 |
| 20 | 43.9 | 315.6 | 229 | 2254.3 | 0.86 | 0.00146 |
| Mean | 20.1 | 367.6 | 337 | 2699.5 | 0.89 | 0.00169 |

Table 2 Short-term and long-term prediction errors (g) and long-term relative values (%) of weight estimation for 20 lots using previous models from bibliography

| Model | Short-term prediction error (g) | Long-term prediction error (g) | Long-term prediction error (%) |
|-------------------------------|---------------------------------|--------------------------------|--------------------------------|
| Petridis and Rogdakis (1996) | 18.8 | 91.8 | 25.0 |
| Lupatsch and Kissil (1998) | 20.3 | 65.6 | 17.8 |
| Lupatsch <i>et al.</i> (2003) | 18.8 | 60.9 | 16.6 |

for the 20 lots described previously. Considering the average value, gilthead sea bream grew from 20 to 368 g in 337 days or 2699 °C × day, but some variability between 20 lots was observed. The mean value of the TGC, 0.00169, was in the 95% confidence interval with end points 0.00140 and 0.00197.

The model precision, measured using the mean of the absolute values of the short-term and long-term prediction errors, i.e. the mean of the absolute value of the subtraction between real weight and estimated value, obtained from the models proposed by different authors in Eqs (3–5) (Petridis & Rogdakis 1996; Lupatsch & Kissil 1998; Lupatsch *et al.* 2003) on growth

Table 3 Values of R^2 (adjusted for degrees of freedom, d.f., of the model), estimated coefficients, standard error and lower and upper limits for each of the models

| | R^2 (d.f.) | Estimated values | Asymptotic standard error | 95% asymptotic lower limit | 95% asymptotic upper limit |
|--------------------|--------------|------------------|---------------------------|----------------------------|----------------------------|
| Model 1 (d.f. = 3) | 97.64 | | | | |
| K_1 | | 0.0156 | 0.0039 | 0.00797 | 0.02325 |
| K_2 | | 0.2884 | 0.0368 | 0.2157 | 0.3612 |
| K_3 | | 0.1332 | 0.0083 | 0.1168 | 0.1497 |
| Model 2 (d.f. = 3) | 97.60 | | | | |
| K_1 | | 3.528 | 0.273 | 2.986 | 4.067 |
| K_2 | | 0.783 | 0.014 | 0.756 | 0.811 |
| K_3 | | 0.00042 | 0.00003 | 0.00036 | 0.00048 |
| Model 3 (d.f. = 3) | 97.77 | | | | |
| K_1 | | 0.0001013 | 0.000058 | -0.0000136 | 0.0002163 |
| K_2 | | 0.2927 | 0.0351 | 0.2234 | 0.3621 |
| K_3 | | 2.5897 | 0.1598 | 2.2738 | 2.9055 |
| Model 4 (d.f. = 3) | 97.40 | | | | |
| K_1 | | 1.516 | 0.179 | 1.161 | 1.871 |
| K_2 | | 0.796 | 0.015 | 0.767 | 0.825 |
| K_3 | | 0.161 | 0.014 | 0.134 | 0.189 |
| Model 5 (d.f. = 1) | 96.57 | | | | |
| K_1 | | 0.00132 | 0.00004 | 0.00124 | 0.00139 |

data of the 20 lots appears in Table 2. Short-term precision could be acceptable, but long-term precision is poor considering absolute values, from 65 to 82 g, or relative values, from 18% to 25%, compared with the real final weight. These results justify the development of new growth models.

Five regression equations, based on previous models of cited authors, were developed. Values of coefficients K_i , asymptotic standard error and 95% asymptotic confidence intervals for each model are presented in Table 3. All regression models show a high statistical significance, with values of R^2 above 97%, so all of them seem to be adequate for predicting growth.

Model 1 was a non-linear exponential regression model with three parameters, similar to the model developed by Lupatsch and Kissil (1998), but considering the final weight (W_f) as a function of initial weight ($W_i^{K_2}$), the number of days of the period (n) and the mean of the daily temperatures of the period ($\exp(K_3 T)$), obtaining the value $R^2 = 97.6\%$ and standard error of estimation equal to 18.3.

(Model 1)

$$W_f = W_i + n \times K_1 \times W_i^{K_2} \times \exp(K_3 \times T)$$

Model 2 was a non-linear exponential regression model with three parameters, where final weight (W_f) was a function of initial weight ($W_i^{K_2}$) and the sum of the daily effective temperatures in the period

($\exp(K_3 \sum \text{ } ^\circ\text{C day}^{-1})$), obtaining $R^2 = 97.6\%$ and standard error of estimation equal to 18.5.

$$\text{(Model 2)} \quad W_f = K_1 \times W_i^{K_2} \times \exp\left(K_3 \times \sum T_{ef}\right)$$

Model 3 was a non-linear exponential regression model with three parameters, similar to the model developed by Petridis and Rogdakis (1996), but considering the final weight (W_f) as a function of initial weight ($W_i^{K_2}$), the number of days of the period (n), and the mean of the daily temperatures of the period (T^{K_3}), obtaining $R^2 = 97.8\%$ and standard error of estimation equal to 17.8:

$$\text{(Model 3)} \quad W_f = W_i + n \times K_1 \times W_i^{K_2} \times T^{K_3}$$

Model 4 was a non-linear exponential regression model, where final weight (W_f) is a function of initial weight ($W_i^{K_2}$) and the sum of the daily effective temperatures in the period ($(\sum \text{ } ^\circ\text{C/day})^{K_3}$) obtaining $R^2 = 97.4\%$ and standard error of estimation equal to 19.2.

$$\text{(Model 4)} \quad W_f = K_1 \times W_i^{K_2} \times \left(\sum T_{ef}\right)^{K_3}$$

Model 5 was based on TGC model from Cho and Bureau (1998), where final weight (W_f) is a function of initial weight ($W_i^{1/3}$) and the sum of the daily effective temperatures in the period ($\sum \text{ } ^\circ\text{C/day}$), obtaining $R^2 = 96.6\%$ and standard error of

estimation equal to 22.0.

$$\text{(Model 5)} \quad W_T = \left(W_1^3 + K_1 \times \left(\sum T_{ef} \right) \right)^3$$

Precision values obtained from the five models described for the 20 lots are presented in Table 4. Observing the statistical analysis of short-term precision values, it seems that the models satisfactorily predict the growth, with a precision measure of around 13.2–16.4 g. In long-term precision results, significant differences appeared, model 5 giving the worst value (34%), whereas the other models gave similar but significantly better values (10–11%).

Table 5 shows the precision values for the six new lots, which were not used to develop the models. The short-term prediction errors were similar for all the models, from 19.1 to 22.1 g. Long-term prediction errors did not present statistical differences for the five models, obtaining absolute values from 41.8 to 84.7 g computed at the end of the growing period, which means relative values from 10.7% to 21.8% compared with real final weight. Nevertheless, models 1 and 3 presented higher values.

In Fig. 1, the real and estimated growth curves for the five models have been drawn considering the six new testing lots.

Discussion

The growth of gilthead sea bream in the western Mediterranean Sea was lower (mean of the SGR = 0.97% day⁻¹ and mean of the TGC = 0.00169) than that cited by Petridis and Rogdakis (1996) in the Ionian sea (SGR = 1.45% day⁻¹ and TGC = 0.00199). Nevertheless, both values of TGC were higher than that cited by Kaushik (1998) for gilthead sea bream (TGC = 0.00087), because the former were calculated using effective temperature (>12 °C).

When previous regression models (Petridis & Rogdakis 1996; Lupatsch & Kissil 1998; Lupatsch *et al.* 2003) were applied to the data from the western Mediterranean Sea, the long-term prediction errors were too high (low precision), from 17% to 25%, which could be due to environment conditions, sea bream genotype, feeding, etc. Nevertheless, when the long-term precision was calculated using the model developed by Petridis and Rogdakis (1996) for the data from Ionian Sea, the relative error was very similar, around 18%. These results justify the need for calibration of previous models, calculating the coeffi-

Table 4 Short-term and long-term prediction errors (g) and long-term relative values (%) of weight estimation for the 20 initial lots and the proposed models

| Model | Short-term prediction error (g) | Long-term prediction error (g) | Long-term prediction error (%) |
|-------|---------------------------------|--------------------------------|--------------------------------|
| 1 | 13.6 | 39.8 ^a | 10.5 |
| 2 | 14.1 | 40.2 ^a | 11.0 |
| 3 | 13.2 | 39.8 ^a | 10.5 |
| 4 | 14.2 | 41.7 ^a | 11.3 |
| 5 | 16.4 | 124.6 ^b | 34.3 |

Different superscript letters show statistical differences at $P < 0.05$.

Table 5 Short-term and long-term prediction errors (g) and long-term relative values (%) of weight estimation for the six new lots and the proposed models

| Model | Short-term prediction error (g) | Long-term prediction error (g) | Long-term prediction error (%) |
|-------|---------------------------------|--------------------------------|--------------------------------|
| 1 | 20.9 | 84.7 | 21.8 |
| 2 | 22.1 | 50.5 | 12.9 |
| 3 | 20.4 | 75.0 | 19.2 |
| 4 | 21.4 | 41.8 | 10.7 |
| 5 | 19.1 | 54.3 | 13.8 |

Different letters show statistical differences at $P < 0.05$.

cients again (models 1 and 3), or developing other easily handled models (models 2, 4 and 5).

Two models, similar to Petridis and Rogdakis (1996) and Lupatsch and Kissil (1998) and Lupatsch *et al.* (2003), were developed, where SGR was a function of initial weight and temperature, but these models were rejected because of the poor values obtained for R^2 , from 74% to 81% respectively. Next, some similar regressions were tried but using final weight instead of SGR as the independent variable, obtaining model 1 ($R^2 = 97.6\%$) and model 3 ($R^2 = 97.8\%$), which provide better long-term precision errors (in both cases 10.5% in Table 4) than the original models (25% and 18% in Table 2). Likewise, when the sum of daily effective temperature was considered as a dependent variable instead of the mean of the temperature of the period, models 2 and 4 ($R^2 = 97.6\%$ and 97.4% respectively) presented similar long-term precision, 11% approximately. Model 5, i.e. the TGC model (Cho & Bureau 1998; Kaushik 1998), which is based also on the

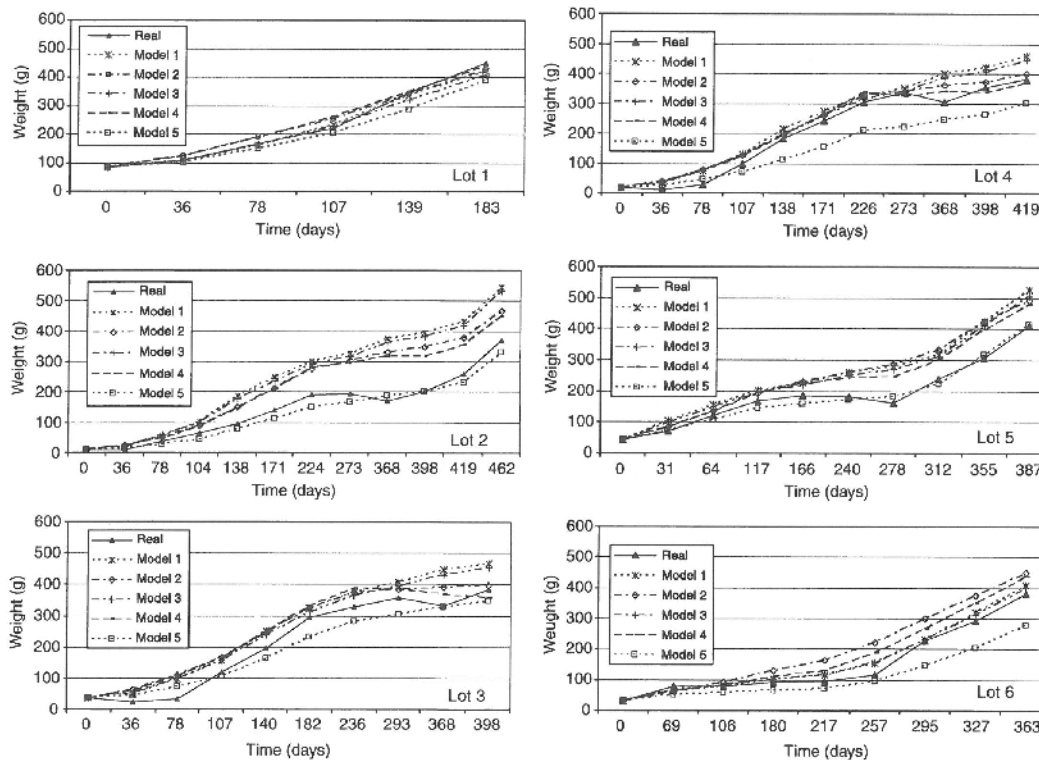


Figure 1 Comparison of estimated growth curves with real growth values for six lots.

sum of daily effective temperatures gave the worst long-term precision (34.3%).

When all models were tested considering the six news lots, models 2, 4 and 5 provide values of the long-term prediction errors lower than 14% (Table 5), although without statistically significant differences compared with models 1 and 3, which showed long-term prediction errors around 22% and 19% respectively. It should be noted that model 5 gave the worst prediction errors for 20 initial lots, but the results improved for the six new lots.

Growth curves in Fig. 1 show that models 1–4 generally tend to over-estimate the weight of sea bream throughout the growing period in different lots. Model 5 tends to underestimate growth, although in two of the lots (2 and 5) the prediction given by model 5 was almost perfect.

When a new analysis of variance was made, the different response of the six lots when considering the five regression models was classified in two groups: models based on the mean of the daily temperatures in the period and models based on the sum of the daily effective temperatures. Statistical

differences between the two groups of models were significant ($P < 0.05$), the long-term prediction errors being 48.8 and 79.9 g respectively. Models 1 and 3, based on mean of the daily temperature, gave the worst long-term predictions. In Fig. 2, the average and confidence intervals for long-term prediction errors for the five models are presented, and it can be observed that the errors for models 1 and 3 are clearly higher than the errors corresponding to models 2 and 4. These results lead us to accept the hypothesis that there are statistically significant differences between the estimation errors of the two groups of models.

A comprehensive account of the results obtained shows that models 2 and 4 would be the best because the long-term precision was the lowest in both groups of lots (Tables 4 and 5), which could be explained because these models are less sensitive when environment variations are considered. However, the important variability is obvious when comparing the results obtained for the initial 20 lots and those obtained for the other six testing lots. This variability may be caused by the different origins of fish, the

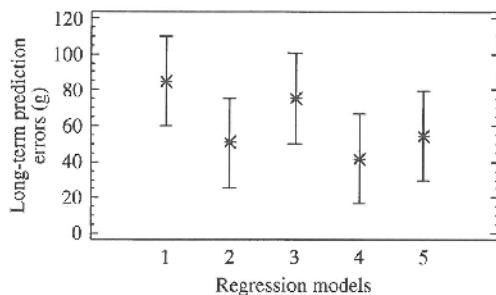


Figure 2 Confidence intervals for long-term prediction errors for six new lots.

genotype, the month of growth onset, the feeding, etc. Nevertheless, the observed variability is normal on fish farms, where, for example, the hatchery or the feed company are changed frequently, making it very difficult to control all external influences on the growth process. In spite of the fact that the determination coefficients of the five models were very significant (>97%), the contextual variability must be considered in future studies, obtaining different scenarios according to different conditions (hatchery, feed, etc.).

Previous regression models (Petridis & Rogdakis 1996; Lupatsch & Kissil 1998; Lupatsch *et al.* 2003) could be good tools for estimating the growth of gilthead sea bream, but they must be adapted to the case of each particular marine farm, obtaining an acceptable prediction of growth. However, current results indicate that the predicted weight values could be improved if models were based on the sum of the daily effective temperatures instead of the mean of the daily temperatures in the period. Therefore, models 2 and 4, developed in the present trial, improved the precision of the previous models.

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5. ARTÍCULO 2

“Use of quantile regression and discriminant analysis to describe growth patterns in farmed gilthead sea bream (*sparus aurata L.*)
Adaptación para la tesis.

Use of quantile regression and discriminant analysis to describe growth patterns in farmed gilthead sea bream (*Sparus aurata*)

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ABSTRACT

Most mathematical models of fish growth on commercial farms use the evolution of average weight during the growth period, without considering weight distribution. In this paper, quantile regression techniques were used to describe the evolution of weight distribution in 20 batches of gilthead sea bream (*Sparus aurata*) growing on a commercial fish farm in the Spanish Mediterranean. Different Thermal-unit Growth Coefficients (TGC) were obtained for several quantiles (5, 25, 50, 75 and 95%) in a group of heterogeneous batches, whereas a single TGC was observed in another group of homogeneous batches. Thus, the growth patterns observed were either homogeneous or heterogeneous. Discriminant analysis helped to identify variables that affected the classification of batches, namely, the coefficient of variation in initial weight and daily average temperature variation during the initial growth period.

Keywords:

Quantile regression
TGC-growth model
Discriminant analysis
Homogeneous growth
Heterogeneous growth

1. Introduction

Modelling fish growth with mathematical models can help to improve fish farm management, including the calculation of daily feeding rates and when to program classifications and sales. In the case of the gilthead sea bream (*Sparus aurata*), with a highly competitive and saturated market, production costs need to be reduced, and fish growth modelling may be part of the solution.

Sea bream growth in aquaculture has been described using multiple regression models (Petridis and Rogdakis, 1996; Lupatsch et al., 2003; Mayer et al., 2008) and the "Thermal-unit Growth Coefficient" (TGC), which was originally developed for rainbow trout (Iwama and Tautz, 1981; Cho and Bureau 1998), but also has been used for gilthead sea bream (Kaushik, 1998; Mayer et al., 2008). The main advantage of considering the TGC model is that the TGC value is less sensitive from the initial body weight, although recently Dumas et al. (2007) proposed three stanzas for different weight intervals in trout (<20 g, 20–500 g and >500 G), and Jobling (2003) has cautioned about the description of growth using the TGC model. However, the TGC model is simple and quite accurate in simple least square adjustments (good values for the coefficient of determination) from data obtained in previous studies (Mayer et al., 2008).

Both, multiple regression and TGC approaches, use only average weights. When a mathematical model is obtained from the average of successive weight samplings, a growth curve with a confidence interval can be drawn (Fig. 1a). In this case, there are few possibilities of using it successfully for fish farm management, because it is only possible to estimate the average weight and no information on dispersion can be obtained. Estimation of highest fish, called "heads" in aquaculture terminology, and lowest fish, "tails", is very important for making size classifications.

Mathematical growth models use information obtained from periodic samples taken at different times throughout the growth cycle (generally every month). The estimation of parameters that define the models often only considers the average fish weight for each sample, resulting in a considerable loss of information, which may affect the description of growth that the mathematical model provides. In each periodic sample, the dispersion in fish weight can be expressed by various statistical measures, such as variance or the coefficient of variation. The main problem is the heteroscedasticity of the sampled growth data, so alternatives need to be found for growth-data modelling based on simple or multiple, linear or nonlinear, regression (Cade and Noon, 2003).

Knowledge about the evolution of weight distribution over a period of time may be an essential tool to explain how fish are growing, but no published model considers it. This knowledge can provide more complete information about growth compared with simple mathematical growth models, and help to decide when to perform size classifications, separating two or more sizes that are smaller or larger than the average.

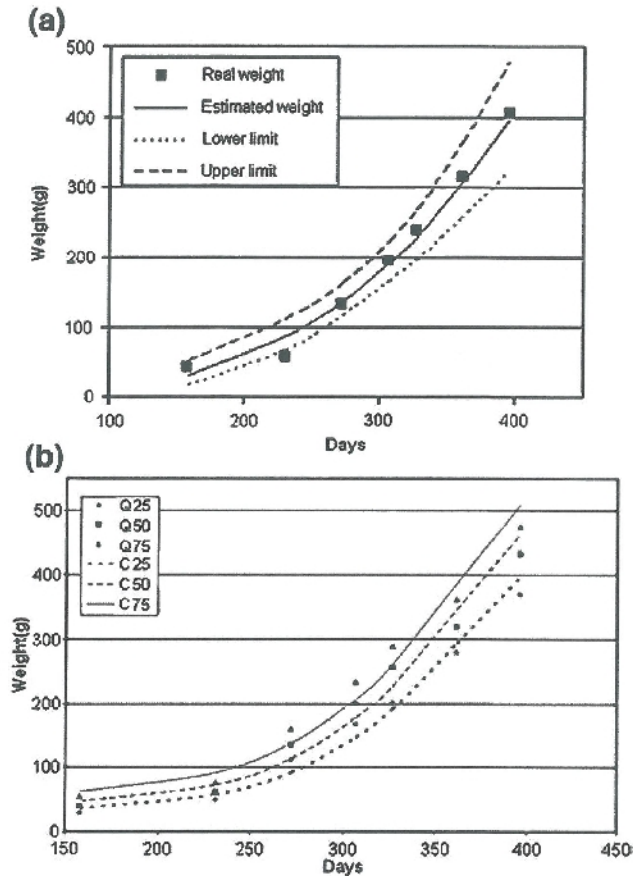


Fig. 1. Example of TGC-growth regression curve, with confidence interval for batch 11, considering (a) the mean of the weight for each sample in the batch, and (b) C25, C50 and C75 are least square TGC-growth regression curves for quantiles Q25, Q50 and Q75.

When a fish sample is considered, it is possible to approach the global distribution of weight of the cage or tank. A graphic representation of that distribution can be obtained through a histogram, a box and whisker graph, or from some representative individual quantiles (e.g., 5, 25, 50, 75, 95%) obtained from each sample. However, each sample only gives an isolated and disjointed image of the approximate distribution of the population. Even if all the images are considered, it would be difficult to obtain an overview of the evolution of weight distribution throughout the growth period.

A partial solution is to consider the individual quantiles from all samplings in the growth period to develop several regression growth curves for highest, medium and lowest fish (Fig. 1b). This is a slow and tedious method, but the idea of considering all data from all samples and the description provided by quantiles could improve the approach by using suitable statistical methods.

Statistical distributions of weight data over time are often the result of complex interactions between different factors affecting fish. Quantile regression (Koenker and Bassett, 1978) provides information about the probability distribution at each point in time and the evolution of the growth data, and is very suitable to analyse data in contexts characterized by heteroscedasticity (Yu et al., 2003). Quantile regression enables us to estimate the conditional quantiles of a response variable distribution, taking into account all the data obtained from different samples, and provides a complete view of the possible relationship between variables in the growth process. Quantile regression also allows us to consider all the sam-

pled values to estimate the evolution of the growth for the different quantiles for each batch of fish. In this manner, we obtain a comprehensive idea of the continuous-time evolution of the weight distribution.

In a previous study (Mayer et al., 2008), several simple least square regression models were considered using data from samplings for 20 batches of sea bream, in the context characterized by heteroscedasticity. The coefficient of determination was high in almost all cases, but lack of fit tests indicated that none of the models could adequately explain the variability of the data. Thus, an approach is needed that considers the evolving role of probability density of weight during growth.

The aim of this work was to obtain a methodology to describe different growth patterns of batches of gilthead sea bream growing in marine cages in the Mediterranean Sea under real production conditions, and to study the weight dispersion of fish stocks throughout the growth period.

2. Materials and methods

Weight data from 20 batches of European sea bream growing in a marine fish farm located in the western Mediterranean were considered for the current study. Characteristics of each batch and production system were published in a previous paper (Mayer et al., 2008), and some complementary data not cited there are presented in Table 1.

We can describe, in simplified form, the workflow considering three steps. In the first step we describe the growth of the batches using quantile regression. In second place, we classify the batches as homogeneous and heterogeneous using ANOVA. Finally we identify variables that influence the classification of batches as homogeneous and heterogeneous using discriminant analysis.

Linear quantile regression estimates multiple rates of change, slopes, from the minimum to the maximum growth, providing a more complete picture for each batch than the estimation of average weight through classic models. The theoretical basis of simple linear quantile regression is briefly explained. Consider a set of n ordered pairs of

Table 1

Statistical variables of weight (standard kurtosis and coefficient of variation) obtained from the first sample, and environment variables related to temperature and associated with the farming of the 20 batches of gilthead sea bream (T30=average of water temperatures in the first 30 days of the growth period, V30=average daily rates of increase in temperature in the first 30 days of the growth period).

| Batch | Standard kurtosis | Coefficient of variation (%) | Initial month | T30 (°C) | V30 (°C/day) |
|-------|-------------------|------------------------------|---------------|----------|--------------|
| 1 | -0.60 | 33.8 | June | 14.1 | 0.0006 |
| 2 | 0.77 | 31.5 | June | 14.6 | 0.0004 |
| 3 | 0.08 | 20.8 | June | 14.8 | 0.0004 |
| 4 | 0.36 | 22.7 | June | 16.5 | 0.0015 |
| 5 | 0.18 | 19.9 | August | 25.7 | 0.0012 |
| 6 | -0.42 | 16.5 | July | 23.8 | 0.0006 |
| 7 | -1.12 | 25.3 | September | 25.9 | 0.0007 |
| 8 | 1.91 | 24.8 | October | 24.4 | -0.0016 |
| 9 | 0.25 | 23.0 | December | 22.6 | -0.0022 |
| 10 | -0.86 | 32.6 | May | 12.9 | 0.0010 |
| 11 | 0.10 | 33.6 | February | 23.1 | -0.0027 |
| 12 | -0.71 | 23.2 | February | 13.9 | -0.0005 |
| 13 | 1.10 | 19.7 | February | 13.7 | -0.0005 |
| 14 | 0.38 | 29.0 | February | 13.6 | -0.0006 |
| 15 | -0.68 | 28.5 | February | 12.6 | -0.0004 |
| 16 | -0.11 | 26.1 | May | 14.5 | 0.0005 |
| 17 | 2.93 | 25.3 | May | 14.7 | 0.0006 |
| 18 | 0.91 | 25.9 | May | 15.3 | 0.0008 |
| 19 | -0.72 | 28.8 | May | 15.7 | 0.0014 |
| 20 | 9.62 | 28.2 | May | 16.0 | 0.0020 |

values (x_i, y_i) , $i = 1, 2, \dots, n$, the general quantile regression linear can be described by the optimization problem:

$$\beta(a(\tau), b(\tau)) = \arg \min \sum_{i=1}^n \rho_{\tau}(y_i - (a(\tau)x_i + b(\tau))), \quad (1)$$

where $\rho_{\tau}(\cdot)$ is a continuous real valued function of one real variable defined by

$$\rho_{\tau}(u) = \begin{cases} \tau u & \text{if } u \geq 0 \\ -(1 - \tau)u & \text{if } u < 0 \end{cases}$$

where τ is a real value, $0 < \tau < 1$. The quantile regression considers the errors of comparing the observed data y_i and the estimated values obtained from data x_i and a single linear function in which the positive errors are penalised with a weight τ (quantile) and the negative errors with a weight $1 - \tau$. When $\tau = 0.5$, a symmetrical penalty is considered and the result is a straight line that minimizes the value of the absolute errors. Therefore, solving the optimization problem, a straight line $y = a(\tau)x + b(\tau)$, for each value of the quantile τ is obtained. If the values y_i , $i = 1, 2, \dots, n$, are sampled values of a random variable y and consider different (quantile) values of τ , the lines obtained describe the evolution of the distribution of variable y conditioned by the x values.

The study of growth using quantile linear regression requires a basic growth model suitable for studies over relatively long periods of time. That model must be easily transformed into a linear model through data processing and should explain growth using a small number of parameters with intuitive meaning. In the case considered, y_i were transformed weight sampled values and x_i the sum of daily effective temperatures.

A model based on 'Thermal-unit Growth Coefficient', Iwama and Tautz (1981) and Cho and Bureau (1998), was used as the basic model to develop the quantile regression procedure (2),

$$W_f = (W_i^{\frac{1}{3}} + TGC \cdot ST)^3, \quad (2)$$

where W_f is the final weight at the end of the considered period of production, W_i is the initial weight and TGC is the 'Thermal-Unit Growth Coefficient', and ST the sum of effective temperatures, given in Eq. (3),

$$ST = (\bar{T} - 12) \times N \quad (3)$$

where N is the number of days of the period and \bar{T} is the mean temperature of the period, and 12 is the minimum temperature for feeding and growth of sea bream (according to García-García, 1994). The sampled data of the fish in the marine farms are repeated observations of weight corresponding to each time value.

Moreover, the model expressed in Eq. (2) can easily be transformed into a linear Eq. (4).

$$W_f^{\frac{1}{3}} = W_i^{\frac{1}{3}} + TGC \cdot ST \quad (4)$$

Linear models can be very helpful if the computational aspects of the quantile regression are considered. Furthermore, the slope of the linear model is the TGC . So the growth curve shown in Fig. 2a can be converted into Fig. 2b and vice versa.

A descriptive approximation of the distribution of the growth data in each batch based on quantile regression (Koenker and Bassett, 1978) was developed using the linear function from Eq. (4). The data were processed using the specific routine for quantile linear regression provided by the RDCT package (2005), obtaining several lines, corresponding to different quantiles (5, 25, 50, 75 and 95%) and where the parameters of adjustment are $W_0^{1/3}$, which corresponds to the intercept, and the values of TGC , which corresponds to the slope.

The growth of each quantile group of fish for each batch will be explained by the different values of TGC . That is, TGC 05 explains the

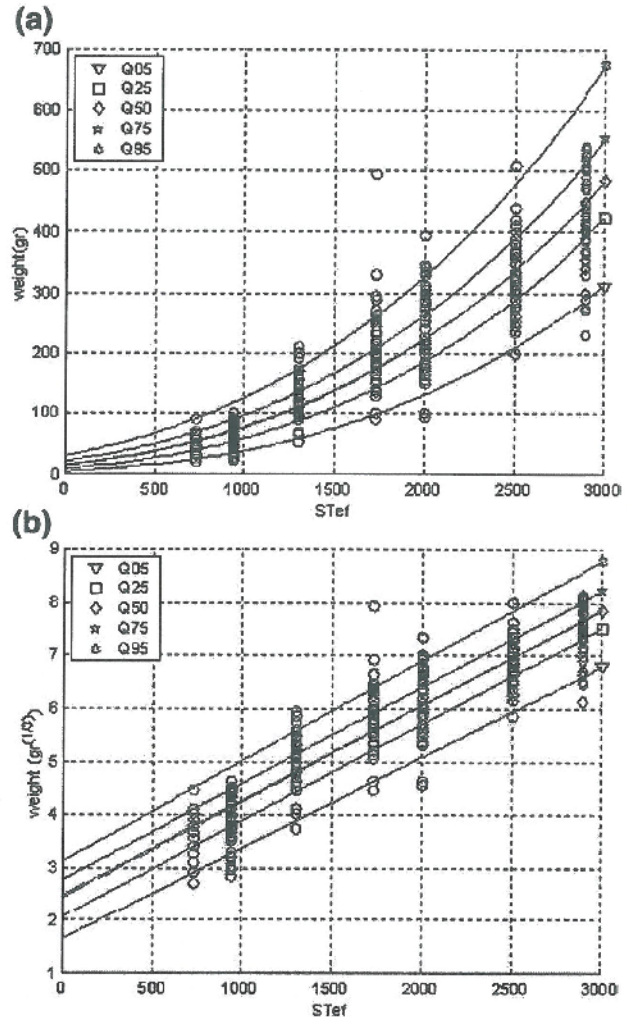


Fig. 2. Quantile regression for batch 11: (a) growth curves (5, 25, 50, 75 and 95%) considering all the samples and (b) graphic result of the linear quantile regression, considering quantiles 5, 25, 50, 75 and 95.

growth of 5% of the fish with the lowest weight; TGC 25 explains the growth of 25% of the population with lower weight; TGC 50 explains the growth of 50% of the accumulated population; TGC 75 explains the growth of 25% of the population with higher weight. Finally, TGC 95 explains the growth of the 5% of the population with the highest weight.

TGC values for each batch were compared using an analysis of variance, contrasting the slopes of the linear equations, and obtaining a classification where batches with homogeneous growth, named homogeneous batches (same slope in the lines corresponding to the 5, 25, 50, 75 and 95 quantiles), have been distinguished from batches with heterogeneous growth, named heterogeneous batches (different slopes in the lines corresponding to the considered quantiles). This ANOVA (Koenker and Bassett, 1982) was computed using the RDCT package (2005), considering a 95% confidence level for statistically significant differences between TGC values.

Discriminant analysis allows to develop a set of discriminating functions which can help to predict and to explain the classifications obtained from the ANOVA based on the values of other quantitative variables. Several statistical measures associated with the initial frequency distribution of fish weight, deduced from the first sampling and some environmental variables related to water temperature throughout the period of production (Table 1) were considered for the discriminant analysis. $T30$ (in $^{\circ}C$) is the average of water temperatures

Table 2

Estimated values of initial weight and final weight obtained by quantile regression for 20 batches (W_{in} = initial weight; W_{fi} = final weight; $\Delta 9505in = W_{in} 95^{1/3} - W_{in} 05^{1/3}$; $\Delta 9505fi = W_{fi} 95^{1/3} - W_{fi} 05^{1/3}$; Inc. 9505 fi - in = $(\Delta 9505fi - \Delta 9505in) / \Delta 9505in$; $\Delta 7525in = W_{in} 75^{1/3} - W_{in} 25^{1/3}$; $\Delta 7525fi = W_{fi} 75^{1/3} - W_{fi} 25^{1/3}$; Inc. 7525 fi - in = $(\Delta 7525fi - \Delta 7525in) / \Delta 7525in$).

| Batch | $W_{in} 5$ | $W_{in} 25$ | $W_{in} 50$ | $W_{in} 75$ | $W_{in} 95$ | $W_{fi} 5$ | $W_{fi} 25$ | $W_{fi} 50$ | $W_{fi} 75$ | $W_{fi} 95$ | $\Delta 9505$ initial | $\Delta 9505$ final | Inc. 9505 fi - in (%) | $\Delta 7525$ initial | $\Delta 7525$ final | Inc. 7525 fi - in (%) |
|-------|------------|-------------|-------------|-------------|-------------|------------|-------------|-------------|-------------|-------------|--------------------------|------------------------|--------------------------|--------------------------|------------------------|--------------------------|
| 1 | 12 | 21 | 27 | 36 | 49 | 249 | 318 | 371 | 418 | 493 | 1.37 | 1.61 | 17.7 | 0.54 | 0.65 | 18.9 |
| 2 | 14 | 21 | 30 | 40 | 53 | 254 | 332 | 378 | 418 | 483 | 1.35 | 1.51 | 12.0 | 0.63 | 0.56 | -11.5 |
| 3 | 19 | 27 | 32 | 40 | 47 | 305 | 371 | 423 | 486 | 564 | 0.91 | 1.54 | 68.2 | 0.39 | 0.68 | 74.6 |
| 4 | 20 | 30 | 40 | 47 | 54 | 505 | 615 | 713 | 808 | 948 | 1.07 | 1.86 | 74.3 | 0.50 | 0.81 | 60.7 |
| 5 | 18 | 25 | 30 | 35 | 41 | 258 | 318 | 353 | 398 | 462 | 0.83 | 1.36 | 64.9 | 0.32 | 0.52 | 66.0 |
| 6 | 11 | 15 | 19 | 25 | 41 | 382 | 466 | 527 | 586 | 679 | 1.22 | 1.53 | 25.3 | 0.47 | 0.62 | 29.9 |
| 7 | 10 | 17 | 23 | 29 | 42 | 223 | 298 | 341 | 399 | 479 | 1.29 | 1.77 | 36.7 | 0.50 | 0.68 | 36.5 |
| 8 | 10 | 16 | 21 | 29 | 42 | 272 | 341 | 380 | 418 | 475 | 1.37 | 1.32 | -3.7 | 0.57 | 0.49 | -14.0 |
| 9 | 3 | 5 | 7 | 10 | 15 | 314 | 374 | 423 | 465 | 550 | 1.21 | 1.40 | 15.9 | 0.49 | 0.54 | 10.1 |
| 10 | 5 | 9 | 13 | 16 | 22 | 219 | 268 | 297 | 331 | 390 | 1.17 | 1.28 | 9.1 | 0.44 | 0.47 | 6.9 |
| 11 | 4 | 9 | 15 | 21 | 31 | 288 | 392 | 449 | 515 | 629 | 1.52 | 1.96 | 29.2 | 0.76 | 0.70 | -7.3 |
| 12 | 12 | 19 | 25 | 29 | 36 | 258 | 320 | 370 | 425 | 501 | 0.98 | 1.58 | 61.4 | 0.40 | 0.68 | 68.0 |
| 13 | 8 | 14 | 19 | 22 | 24 | 240 | 276 | 320 | 361 | 435 | 0.88 | 1.36 | 53.4 | 0.35 | 0.61 | 74.8 |
| 14 | 10 | 17 | 24 | 30 | 39 | 270 | 350 | 400 | 440 | 520 | 1.24 | 1.58 | 27.6 | 0.54 | 0.56 | 4.2 |
| 15 | 8 | 12 | 15 | 18 | 22 | 224 | 305 | 354 | 399 | 463 | 0.76 | 1.67 | 119.6 | 0.40 | 0.62 | 57.4 |
| 16 | 11 | 17 | 22 | 26 | 36 | 240 | 315 | 356 | 408 | 465 | 1.15 | 1.54 | 34.3 | 0.44 | 0.61 | 38.4 |
| 17 | 21 | 30 | 36 | 43 | 60 | 240 | 315 | 379 | 420 | 485 | 1.18 | 1.65 | 39.6 | 0.40 | 0.68 | 68.1 |
| 18 | 19 | 27 | 32 | 40 | 52 | 228 | 297 | 344 | 400 | 473 | 1.09 | 1.68 | 54.7 | 0.39 | 0.70 | 77.9 |
| 19 | 14 | 24 | 30 | 38 | 50 | 216 | 310 | 382 | 432 | 522 | 1.25 | 2.06 | 65.0 | 0.52 | 0.79 | 52.8 |
| 20 | 22 | 34 | 43 | 53 | 69 | 189 | 289 | 333 | 370 | 434 | 1.28 | 1.83 | 42.7 | 0.49 | 0.57 | 15.1 |

in the first 30 days of the growth period, and V30 (in °C/day) is the value for average daily rates of increase in temperature in the first 30 days of the production period. The V30 was computed by numerical derivation. Discriminant analysis was carried out using the statistical package STATGRAPHICS Plus 5.1.

To appreciate the difference in increasing dispersion between homogeneous and heterogeneous batches, we compared the relative increase of the transformed weight values (1/3 power transformation) in the interquartile range (between quantiles 25 and 75) and the relative increase in the range between extreme quantiles (between quantiles 5 and 95), from the beginning to the end of the period. The comparison was made using a one-way ANOVA (STATGRAPHICS Plus 5.1) considering the homogeneous and heterogeneous classification obtained.

3. Results

Using quantile regression, different patterns of final weight (Table 2) and growth (Table 3) were detected in different batches and classified as homogeneous or heterogeneous (Table 3).

Table 3

TGC values obtained from quantile regression for 20 batches.

| Batch | TGC 05 | TGC 25 | TGC 50 | TGC 75 | TGC 95 | Sig. level | Type of batch |
|-------|------------------------|------------------------|------------------------|------------------------|------------------------|------------|---------------|
| 1 | 0.00170 | 0.00173 | 0.00178 | 0.00178 | 0.00181 | ns | Homogeneous |
| 2 | 0.00173 | 0.00184 | 0.00183 | 0.00180 | 0.00181 | ns | Homogeneous |
| 3 | 0.00129 ^a | 0.00133 ^a | 0.00138 ^b | 0.00142 ^c | 0.00149 ^d | * | Heterogeneous |
| 4 | 0.00123 ^a | 0.00127 ^b | 0.00130 ^c | 0.00134 ^d | 0.00142 ^e | * | Heterogeneous |
| 5 | 0.00139 ^a | 0.00145 ^{a,b} | 0.00147 ^b | 0.00152 ^c | 0.00159 ^d | * | Heterogeneous |
| 6 | 0.00129 ^a | 0.00136 ^b | 0.00139 ^c | 0.00140 ^c | 0.00137 ^{b,c} | * | Heterogeneous |
| 7 | 0.00128 ^a | 0.00134 ^a | 0.00136 ^a | 0.00140 ^b | 0.00143 ^b | * | Heterogeneous |
| 8 | 0.00149 | 0.00154 | 0.00154 | 0.00152 | 0.00149 | ns | Homogeneous |
| 9 | 0.00176 | 0.00181 | 0.00184 | 0.00184 | 0.00187 | ns | Homogeneous |
| 10 | 0.00169 ^{a,b} | 0.00169 ^a | 0.00168 ^a | 0.00171 ^a | 0.00176 ^b | * | Heterogeneous |
| 11 | 0.00171 | 0.00181 | 0.00179 | 0.00181 | 0.00188 | ns | Homogeneous |
| 12 | 0.00163 ^a | 0.00166 ^a | 0.00170 ^a | 0.00178 ^b | 0.00186 ^c | * | Heterogeneous |
| 13 | 0.00185 ^a | 0.00182 ^a | 0.00185 ^a | 0.00192 ^b | 0.00208 ^c | * | Heterogeneous |
| 14 | 0.00191 | 0.00199 | 0.00199 | 0.00200 | 0.00207 | ns | Homogeneous |
| 15 | 0.00167 ^a | 0.00183 ^b | 0.00189 ^c | 0.00195 ^d | 0.00203 ^e | * | Heterogeneous |
| 16 | 0.00168 ^a | 0.00178 ^{a,b} | 0.00180 ^b | 0.00187 ^c | 0.00187 ^{b,c} | * | Heterogeneous |
| 17 | 0.00146 ^a | 0.00156 ^a | 0.00166 ^b | 0.00168 ^b | 0.00166 ^b | * | Heterogeneous |
| 18 | 0.00149 ^a | 0.00158 ^a | 0.00166 ^b | 0.00171 ^{b,c} | 0.00176 ^c | * | Heterogeneous |
| 19 | 0.00157 ^{a,b} | 0.00171 ^a | 0.00182 ^{b,c} | 0.00184 ^c | 0.00192 ^c | * | Heterogeneous |
| 20 | 0.00130 | 0.00149 | 0.00151 | 0.00151 | 0.00153 | ns | Homogeneous |

*Indicates significant statistical differences ($p < 0.05$) for the values of TGC in each lot; ns indicates nonsignificant effect.

Table 2 shows the estimated initial and final weights from the quantile regression for the 20 batches. Differences between $W_i 5$ and $W_i 95$ are relevant, because in some batches $W_i 95$ is four times higher than $W_i 5$. Similarly, $W_i 75$ is twice as large as $W_i 25$, which underlines that large variability in the initial weight of fish obtained from hatcheries. With respect to final weight, $W_{fi} 95$ is twice as high as $W_i 5$, and 45% of batches had 25% of fish smaller (Q25) than commercial weight, around 350 g. Likewise, the differences between final and initial increments, both 95-5 and 75-25, were positive in most batches, and were analysed in accordance with homogeneous and heterogeneous classification (Table 5).

The statistical differences between TGC quantile values in each batch, and the classification into heterogeneous and homogeneous batches, according to ANOVA, are shown in Table 3. In the results from ANOVA significant differences in one batch (p -value < 0.05) imply that some TGC values are different, so that batch was classified as heterogeneous (i.e., heterogeneous growth). In other cases, the batch was homogeneous (i.e., homogeneous growth). For example, in batch 3, the quantile value of TGC is higher for larger fish. Batches 1, 2, 8, 9, 11, 14 and 20 are classified as homogeneous and the other batches are heterogeneous.

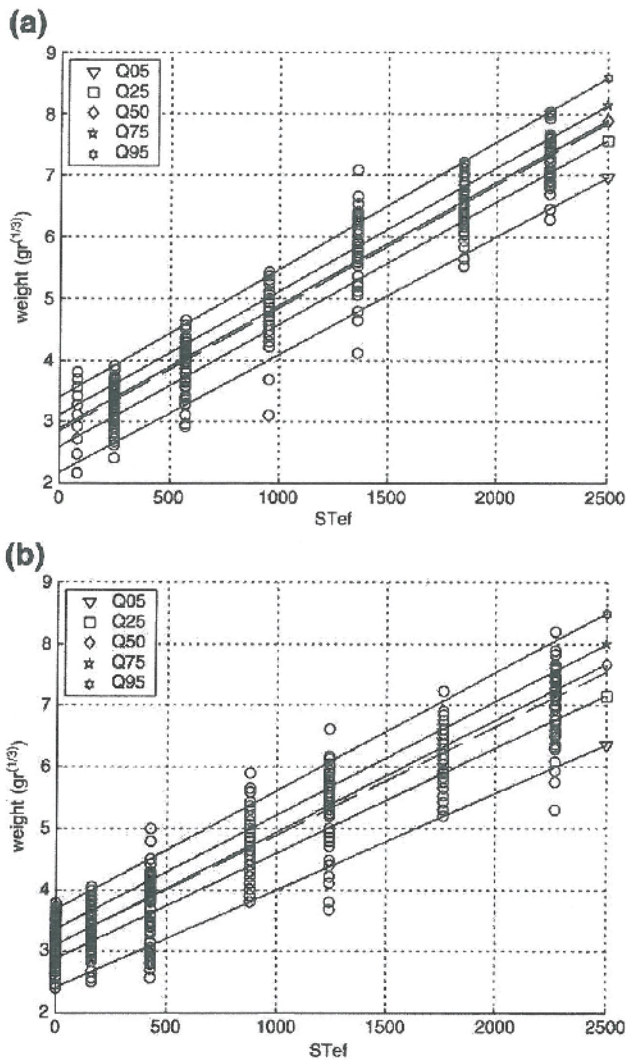


Fig. 3. Sample values and quantile straight lines (dashed line corresponds to the linear regression straight) for (a) homogeneous batch 14 and (b) heterogeneous batch 19.

Homogeneous batches do not present significant statistical differences in the slope (TGC) for the considered quantiles, which suggest homogeneous growth (i.e. the density function of the weight evolves without significant changes in its form). On the other hand, in heterogeneous batches the evolution of the density function of the weight over time is different. In other words, for a homogeneous batch, the form of the density function that describes the distribution of the weight at each time is similar and in a heterogeneous batch the density function of the weight changes with time.

In all heterogeneous batches, with the only exception of 10, appeared significant differences between "tails" (Q5) and "heads" (Q95), and also between Q25 and Q75, which prove that large fish grew more quickly than smaller fish. Differences between "tails" and Q50, or between Q50 and "heads", appeared in most batches, and particularly in batches 3, 4, 5, 15 and 18, in which TGC values were statistically different for Q5, Q50 and Q95.

In homogeneous batches, constant values and few changes in the form of distribution were observed (Fig. 3a). Heterogeneous batches are characterized by important changes in the distribution over time and specifically by a significant increase in the variability characterized by a general and significant increase in the distance between quantiles over time (Fig. 3b).

The results obtained from the discriminant analysis are shown in Table 4. The discriminating function is statistically significant ($p < 0.05$) when the batches are classified as homogeneous or heterogeneous. The coefficient of variation of initial weight had a significant effect on the discriminant analysis, explaining the homogeneous-heterogeneous classification in 75% of batches. When standard kurtosis, $V30$ and $T30$ were considered, 100% of the batches was correctly classified.

On the other hand, there are significant differences ($p < 0.05$) in the increase of dispersion between homogeneous and heterogeneous batches. This was proved by comparing the relative increase in the range between central quantiles, 25 and 75, and also in the case of extreme quantiles, 5 and 95, of the transformed weight values from the beginning to the end of the period as it is shown in Table 5 and Fig. 4. Homogeneous batches presented a lower value of final-initial increment, both 95-05 and 75-25, compared with heterogeneous batches.

4. Discussion

The different quantiles from the first sample show a large weight dispersion, since in most batches sea bream from quantile 75 weighed around 1.5–2 times more than fish from quantile 25, and Q95 around 3–4 times than Q5. That could increase the differences in fish growth (Table 2). In those cases, size classification would be necessary, because in most batches (1, 2, 5, 7, 8, 10, 12, 13, 15, 16, 17, 18, 19 and 20), 25% of fish (Q25) did not reach the minimum commercial weight of 350 g. In all batches, there is an important difference between quantile 5, the "tail" of the distribution, and quantile 95, the "head" of the distribution.

Quantile regression allowed to classify batches as either homogeneous or heterogeneous by using the slope equality tests (Table 3) to detect significant differences between TGC values at different quantiles.

In batches classified as homogeneous, the null hypothesis that TGC is equal for all concerned quantiles cannot be rejected (at confidence level of 95%) which allows us to describe the evolution of the distribution of an homogeneous batch using the initial distribution and a single value of TGC , which can be any of the TGC values obtained for any of the quantiles considered, or the mean TGC for the different quantiles, because all of them are similar (Fig. 3a). From the point of view of growth, it does not mean that all homogeneous batches show the same time evolution of the weight distribution, but that the growth in each of the homogeneous batches can be described from the distribution of the initial weight and its particular single value of TGC , which could be different for different homogeneous batches.

In heterogeneous batches, the null hypothesis that values of TGC are equal for all concerned quantiles can be rejected (at a confidence level of 95%). So, TGC values are different for different quantiles, and hence to establish a complete description of the growth of a batch, the distribution of the initial weight and the different values of the TGC corresponding to the different quantiles are required. In addition, heterogeneity is associated with an increase in the dispersion or variability associated with the increasing of the distance between quantiles throughout the growth period. The relative increase in the

Table 4
Results of the discriminant analysis for classification of batches as homogeneous or heterogeneous.

| Independent variables | p -Value | % Batches with confirmed classification |
|-----------------------|------------|---|
| • Coeff. of variation | 0.0014 | 100 |
| • Standard kurtosis | | |
| • $V30$ | | |
| • $T30$ | | |
| • Coeff. of variation | 0.0267 | 75.0 |

Table 5

Values of variables explaining differences between homogeneous and heterogeneous batches ($\text{Inc. } 9505 \text{ fi} - \text{in} = [(W_f 95^{1/3} - W_f 5^{1/3}) - (W_i 95^{1/3} - W_i 5^{1/3})] / (W_f 95^{1/3} - W_f 5^{1/3})$;
 $\text{Inc. } 7525 \text{ fi} - \text{in} = [(W_f 75^{1/3} - W_f 25^{1/3}) - (W_i 75^{1/3} - W_i 25^{1/3})] / (W_f 75^{1/3} - W_f 25^{1/3})$).

| Type of batch | Average of initial variation coefficient (%) | Average of V30 ($^{\circ}\text{C}/\text{d}$) | Inc. 7525 fi - in (%) | Inc. 9505 fi - in (%) | Average of final variation coefficient (%) |
|---------------------|--|--|-----------------------|-----------------------|--|
| Homogeneous | 29.1 ± 3.8 | -0.00058 ± 0.0015 | 2.21 ± 7.14 | 20.20 ± 9.02 | 18.0 ± 1.67 |
| Heterogeneous | 24.2 ± 2.6 | $+0.00059 \pm 0.0004$ | 54.77 ± 5.24 | 54.35 ± 6.62 | 17.1 ± 0.86 |
| Signification level | * | * | ** | ** | n.s. |

interquartile range 75-25 is on average 2.2% for homogeneous batches and 54.8% in the case of heterogeneous batches, and the relative increase in the extreme range 95-5 is on average 20.2% for homogeneous batches and 54.3% in the case of heterogeneous batches (Table 5). In both cases the differences between homogeneous and heterogeneous batches are significant, $p < 0.05$, (Fig. 4).

The majority of the batches, 65%, have been classified as heterogeneous. This suggests that fish from the highest quantiles grow faster, and, consequently, the differences between sea bream weight would be greater as fish grow, which makes size classification necessary.

The results obtained from the discriminant analysis applied to the batches classified by ANOVA suggest that homogeneity is closely related to the dispersion of the initial weight, expressed as coefficient of variation, and the initial rates of variation in temperature, expressed as the variation of temperature during the first days, V30, T30.

Once we identify the variables that influence the classification of batches as homogeneous or heterogeneous, it would be very important to determine their reference values to group batches as homogeneous and heterogeneous. The coefficient of variation of the first weight sample (Table 5) was significantly higher in batches classified as homogeneous, 29.1%, than in batches classified as heterogeneous, 24.2%, which is very surprising. That could be due to two non-exclusive hypotheses, which require a more detailed study. Batches with a higher coefficient of variation could come from a single population, whereas batches with lower values would be a mix of different populations in the hatchery, with a similar mean weight but different growth rates, and throughout the growth period one group of fish could grow faster than another and give rise to an increase in weight

dispersion, as occurs in the current trial, where heterogeneous batches had different TGC values for quantiles, and homogeneous batches presented similar TGC values for quantiles.

In this sense, Petridis and Rogdakis (1996) reported that some batches of gilthead sea bream growing in marine cages reduced the coefficient of variation (CV) of weight at the end of the period compared with the initial CV, which is in agreement with current data, where final CV were lower than initial, but without differences between homogeneous or heterogeneous batches (Table 5). Nevertheless, Irwin et al. (2002) reported that CV of turbot juveniles increased quickly in the first growth phase, as a consequence of high competition for feed and hierarchy formation, and slower afterwards; so, high fish density increased CV. Likewise, Jobling et al. (1995) found that restriction on feed or hierarchy behaviour originates differences in feed intake and consequently an increase in CV of weight.

Another hypothesis based on Irwin et al. (2002) could be that a smaller initial relative dispersion (in weight) could increase competition for food in the initial populations. In heterogeneous batches, although initial CV was lower, the hierarchy is not established as a consequence of joining two different populations with similar mean weight. In the case of homogeneous batches from the same population, the initial CV was higher, but hierarchy was established and competition for food was lower.

It seems that CV is not a good index to compare the increase of dispersion in growing period, because the higher final weight generally leads to a reduction in the value of CV. Heterogeneous batches presented a higher final-initial increment of both 75-25 and 95-05 weight than homogeneous, but final CV was similar.

On the other hand, we found statistically significant differences in the average daily change in temperature in the 30 first days, V30, between homogeneous and heterogeneous. Homogeneous batches are associated with a negative value of V30 in winter, $-0.00058 \text{ }^{\circ}\text{C}/\text{d}$, and positive values of V30 in summer time, $0.00059 \text{ }^{\circ}\text{C}/\text{d}$, are associated to heterogeneous batches (Table 5). Thus, an increase in temperature during the initial growth, positive value of V30, appears to produce heterogeneous growth, because fish would grow actively showing their different growth potentials, whereas a decrease in temperature during the same period reduces the growth for all quantiles, and differences would not appear. Batches cannot be classified using a single variable. The influence of the coefficient of variation of the initial population is very important, but the kurtosis and environmental variables in the first 30 days must also be considered.

5. Conclusions

Quantile regression provides a powerful technique for approaching the evolution of weight distribution and can help classify fish batches by growth homogeneity. The variables that influence homogeneity can be determined using discriminant analysis, resulting in highly significant variables associated with the initial weight distribution, such as the coefficient of variation of initial weight, and environmental variables such as the variation of water temperature during the first days of the growth period. Based on those results, knowledge of the initial distribution (i.e., coefficient of variation) and temperature in the early days of the cycle can help predict the subsequent evolution of the weight distribution of fish throughout the cycle. Growth heterogeneity is also associated with an increase in weight dispersion,

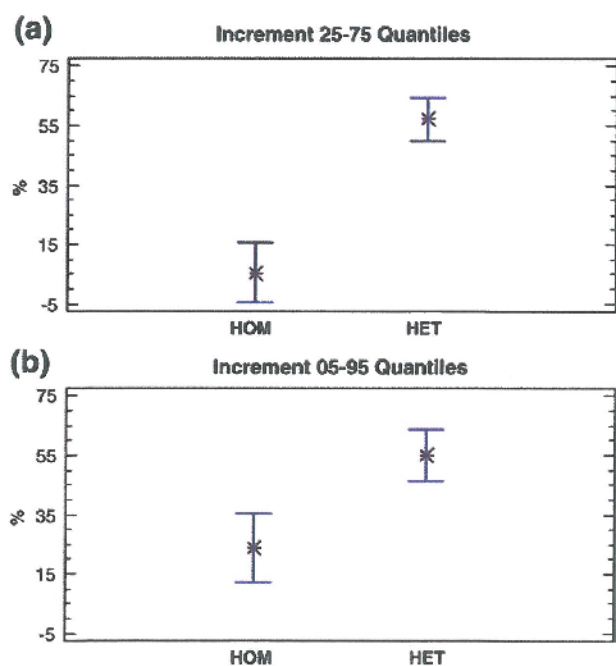


Fig. 4. Means and 95% LSD intervals for relative (%) increasing of range between quantiles 25 and 75 (a), and between quantiles 5 and 95 (b), from beginning to the end of the period, for homogeneous and heterogeneous lots.

represented as an increasing distance between quartiles, comparing initial and final weights. Knowledge about the evolution of the variability and distribution of weight and factors that influence growth patterns can provide a more accurate estimation of biomass at any time, which provides a very useful tool to improve the management of the marine farm.

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6. ARTÍCULO 3

“A two-stage growth model for gilthead sea bream (*sparus aurata L.*) based on the thermal growth coefficient”. Adaptación para la tesis.

A two-stage growth model for gilthead sea bream (*Sparus aurata*) based on the thermal growth coefficient

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ABSTRACT

Several authors have proposed models to describe fish growth taking the influence of temperature into account, and one of the most interesting is the "thermal unit growth coefficient" (TGC). Recent research has demonstrated that TGC varies throughout the growth cycle of fish, making it necessary to establish different stanzas. In this work, the original TGC model using 1/3 as an exponent was compared with a new model considering 2/3. Likewise, two stages for the growth of gilthead sea bream under commercial conditions in marine farms were detected by means of TGC seasonal models using the continuous temperature curve. A critical value for weight around 117g was obtained, which could mark the transition between two growth dynamics. To describe the weight evolution during a complete production cycle, the two growth stages were described by two separate seasonal TGC models (1/3-TGC model and 2/3-TGC model), and with an integrated model named the Mixed-TGC model, which presents interesting properties of continuity and differentiability and could be an important tool for fish farm management.

Keywords:

Mathematical model
Seasonal growth
Temperature curve
Marine cages production

1. Introduction

The importance of growth models in aquaculture has been demonstrated by the publication of a large number of papers in recent years (Baer et al., 2011; Dumas and France, 2008; Dumas et al., 2007, 2010; Libralato and Solidoro, 2008; Mayer et al., 2008, 2009; Moses et al., 2008; Seginer and Halachmi, 2008), most of which are based on the metabolic growth model developed early last century (Bertalanffy, 1938, 1957; Parker and Larking, 1959; Pütter, 1920; Ursin, 1967) to describe fish growth. Most of the classic models were based on the assumption that growth depends on live weight affected by the exponent 2/3 (surface rule), and later models (Cho, 1992; Cho and Bureau, 1998) have also used this value. Nevertheless, some authors have questioned the general use of this exponent because Ursin (1967) estimated lower values than 2/3 in some fish, and Moses et al. (2008) cited values around 3/4 in some vertebrates.

An alternative is to use the "thermal unit growth coefficient (TGC)" model reported by Iwama and Tautz (1981) in hatcheries, and developed by Cho (1992), Cho and Bureau (1998) and Dumas et al. (2007) in growing trout, and Mayer et al. (2008, 2009) in gilthead sea bream. This model is a particular version of the von Bertalanffy equation that incorporates a cumulative water temperature, which allows an estimation of fish growth in several temperature conditions, constituting an interesting tool for aquaculture management. In the case of gilthead

sea bream, growth patterns were considered as a function of the cumulative effective temperature $\Sigma (T_i - 12)$, because growth is zero, or negative, for water temperature below 12°C (Mayer et al., 2008, 2009). Other models have also considered the average temperature (Lupatsch and Kissil, 1998; Lupatsch et al., 2003; Petridis and Rogdakis, 1996) but their practical application was difficult.

Ursin (1963), Akamine (1993), Moreau (1987) and Fontoura and Agostinho (1996) studied the inclusion of a sinusoidal temperature curve in the Bertalanffy growth model, and recently Leon et al. (2006) used a temperature function applied to the growth model from Hernandez et al. (2003). Alternatively, Dumas and France (2008) proposed a model to illustrate the seasonal TGC growth of ectotherms using a one year temperature periodic function. Seginer and Halachmi (2008) also applied the sinusoidal temperature curve to the exponential growth model from Lupatsch and Kissil (1998) to study management aspects in intensive gilthead sea bream aquaculture.

Another advantage of the TGC model was the simplicity of application in aquaculture, as it was possible to estimate the weight throughout the production cycle using a single value of TGC (obtained in the same production conditions). However, Dumas et al. (2007) suggested the need to use different TGC values for different trout stages during the growth period (<20g, 20–500g, >500g). This would indicate that new studies revising the TGC model in other species are necessary.

In a previous paper Mayer et al. (2008) studied various growth models for the gilthead sea bream considering the variability of water temperature. The evolution of a set of average weights calculated from different samples obtained in 20 batches were analysed. One of the key findings of the paper was that the best models (including the

TGC model) were those that considered the accumulated effective temperature as an independent variable, instead of the time. In a second work, Mayer et al. (2009) explored full samples considering all the individual weights of sea bream from the batches studied in Mayer et al. (2008) using a discriminant analysis and quantile regression techniques, with reference to the classic TGC model. It was suggested that it was possible to differentiate two groups of gilthead sea bream with homogeneous and heterogeneous growth characterised by a different evolution of the weight dispersion. The factors that influenced the dynamics and the diversity of growth were the seasonal change of the water temperature and the weight distribution of the fishes provided by the hatchery.

The aim of this paper was to develop a new approach in the growth of gilthead sea bream under commercial production conditions with great fluctuations in the water temperature, including the sinusoidal temperature curve in the TGC model, and considering the different stages throughout the growth period, in order to improve the estimation of growth on aquaculture farms. Our initial goal was to detect the existence of significant changes in the dynamics of the evolution of the average weight of fish over a complete cycle of production considering a two-step TGC model that established the existence of a "critical or transition" live weight, which indicated a change point in the dynamics of growth of the gilthead sea bream.

2. Material and methods

2.1. Mathematical models

Considering a general model of growth given by the initial value problem,

$$\begin{cases} \frac{dW}{dt} = g(W, t), \\ W(t_0) = W_0, \end{cases} \quad (1)$$

where W is the weight and t is the time, a model that takes into account seasonal fluctuations in temperatures can be achieved by replacing in Eq. (1) the time variable t , by a function $ST(t_0, t)$ (ST was used for simplicity) which represents the accumulated temperature in the time interval $[t_0, t]$ (Akamine, 1993). Indeed, assuming that at the initial time t_0 , the accumulated temperature is zero, $ST(t_0, t_0) = 0$ we have

$$\begin{cases} \frac{dW}{dST} = g(W, ST), \\ W(0) = W_0, \end{cases} \quad (2)$$

Models (1) and (2) describe different temporal dynamics. Model (2) takes the sum of temperature as an independent variable to describe the evolution of time and the growth is described from the instantaneous rate of weight gain per unit of accumulated temperature.

Taking into account the chain rule

$$\frac{dW}{dt} = \frac{dW}{dST} \cdot \frac{dST}{dt} \quad (3)$$

and that

$$\frac{dST}{dt} = T(t) \quad \text{i.e.} \quad ST(t_0, t) = \int_{t_0}^t T(x) dx \quad (4)$$

where $T(t)$ is the continuous function that provides the temperature at any moment t , from model (2) we obtain immediately a seasonal time-dependent model

$$\begin{cases} \frac{dW}{dt} = g(W, ST)T(t), \\ W(t_0) = W_0, \end{cases} \quad (5)$$

In the case of an indeterminate allometric growth the basic model (Gamito, 1998; Parker and Larkin, 1959) is quite common,

$$\frac{dW}{dt} = kW^{1-b} \quad (6)$$

where k is a constant related with the metabolic loss of an individual unit weight and the achievement of assimilated food for growth and b is a constant ($0 < b < 1$). The model given by Eq. (6) assumes that the allometric growth rate decreases with time due to the decrease that occurs in the metabolic rate with increasing fish size and that W increases without limit (Gamito, 1998).

From Eqs. (3), (4) and (6), we obtain the associated seasonal model

$$\frac{dW}{dST} = k \cdot W^{1-b}, \quad (7)$$

i.e.

$$\frac{dW}{dt} = k \cdot T(t) \cdot W^{1-b} \quad (8)$$

In what follows, we assume that the time, t , is given in days (d), the units of the constant rate k (>0) are $g^b \cdot (^\circ C \cdot d)^{-1}$, $T(t)$ is the function that provides the water temperature at each time ($^\circ C$), and the allometric exponent b ($0 < b < 1$) is dimensionless.

If we suppose initially that $t = t_0$, $ST(t_0, t_0) = 0$ and $W = W_0$, the solution of Eq. (7) is

$$W^b(t) = W_0^b + k \cdot b \cdot ST(t_0, t). \quad (9)$$

An immediate discrete version of Eq. (9) can be obtained by considering for each day, i , $i = 1, 2, \dots, n$, the mean of the daily temperature, T_i . So we have the model

$$W_n^b = W_0^b + k \cdot b \cdot \sum_{i=1}^n T_i, \quad n = 1, 2, \dots \quad (10)$$

If $b = 1/3$ in Eq. (10), and we denote $k = TGC/b$, we obtain the classic TGC-model (Cho, 1992)

$$W_n^{\frac{1}{3}} = W_0^{\frac{1}{3}} + TGC \cdot \sum_{i=1}^n T_i \quad (11)$$

which was developed from empirical results without any previous mathematical or dynamical consideration (Dumas et al., 2007).

Either Eq. (7) or Eq. (8) allows the modelling of the indeterminate seasonal growth. The function $T(t)$ can take different expressions depending on the environmental conditions (Akamine, 1993).

The integral solution of Eq. (8) is given by the expression

$$W^b(t) = W_0^b + k \cdot b \cdot \int_{t_0}^t T(t) \cdot dt \quad (12)$$

i.e.

$$W(t) = \left(W_0^b + k \cdot b \cdot \int_{t_0}^t T(t) dt \right)^{\frac{1}{b}} \quad (13)$$

As mentioned above, the temperature function $T(t)$ depends on the context. In the case of marine farms in fixed locations, fish live in an environment where the water temperature evolves according to regular annual cycles. A simple one-year periodic expression, which allows us to include the seasonal influence of temperature on

growth in the model, is based on the sinusoidal function (Eq. (14)) used in different studies

$$T(t) = T_m + T_D \cdot \sin\left(\frac{2\pi}{365} \cdot (t - \alpha)\right) \quad (14)$$

where $t \geq 0$, and T_m is the average annual temperature, T_D is the amplitude and α is a tuning parameter. From Eq. (14), we obtain a compact expression for the cumulative temperature function in the time interval $[t_0, t]$,

$$\begin{aligned} ST(t_0, t) &= \int_{t_0}^t T(t) dt \\ &= T_m(t - t_0) - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi(t - \alpha)}{365}\right) - \cos\left(\frac{2\pi(t_0 - \alpha)}{365}\right) \right) \end{aligned} \quad (15)$$

In the case of gilthead sea bream, it is more appropriate to use the effective temperature, $T(t) - 12$, instead of $T(t)$ (Mayer et al., 2008), which only involves replacing T_m by $T_m - 12$ in Eq. (15).

$$\begin{aligned} ST(t_0, t) &= \int_{t_0}^t (T(t) - 12) dt \\ &= (T_m - 12)(t - t_0) - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi(t - \alpha)}{365}\right) - \cos\left(\frac{2\pi(t_0 - \alpha)}{365}\right) \right) \end{aligned} \quad (16)$$

By substituting Eq. (16) in Eq. (13), and by solving the integral, we obtain an expression for the weight in the instant t (Dumas and France, 2008)

$$\begin{aligned} W(t) &= W_0^b + k \cdot b \cdot (T_m - 12) \cdot (t - t_0) \\ &\quad - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi(t - \alpha)}{365}\right) - \cos\left(\frac{2\pi(t_0 - \alpha)}{365}\right) \right) \right)^{\frac{1}{b}} \end{aligned} \quad (17)$$

i.e.

$$\begin{aligned} W(t) &= W_0^b + TGC_b \cdot (T_m - 12) \cdot (t - t_0) \\ &\quad - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi(t - \alpha)}{365}\right) - \cos\left(\frac{2\pi(t_0 - \alpha)}{365}\right) \right) \right)^{\frac{1}{b}} \end{aligned} \quad (18)$$

where $TGC_b = k \cdot b$.

Dumas and France (2008) obtained good results for describing the growth of different species of ectotherms, using models analogous to that given in Eq. (18), assuming different values of b for different species and contexts, but fixing different values for different time periods under study.

From Eq. (18), three models were developed in order to simulate the seasonal indeterminate growth of gilthead sea bream. Two of them were obtained by fitting the data to Eq. (11), assuming the values $b = 1/3$ and $b = 2/3$, based on actual values of accumulated temperature. The third model is built by the aggregation of the two models mentioned above, establishing two stages of growth.

2.2. Data description

Models have been developed considering data on weight and accumulated temperature from 20 batches of farmed gilthead sea bream in real conditions of growth (Mayer et al., 2008).

To validate the models the weight data from 6 batches of gilthead sea bream (Table 1) were used. The production conditions of these 6 batches were similar to those described in Mayer et al. (2008) and

Table 1

Description data from six new batches used for the validation of models.

| Batch | Samples along the period | Initial weight, W_0 (g) | Final weight, W_f (g) | Days | T ($^{\circ}\text{C}$) | ST ($^{\circ}\text{C}$) | TGC ($\text{g}^{1/3} \cdot \text{C}^{-1}$) | SGR ($\% \text{d}^{-1}$) |
|-------|--------------------------|---------------------------|-------------------------|------|----------------------------|-----------------------------|--|------------------------------|
| 1 | 5 | 85.1 | 452.1 | 183 | 23.7 | 2141.1 | 0.001530 | 0.91 |
| 2 | 10 | 10.0 | 371.4 | 463 | 19.8 | 3611.4 | 0.001394 | 0.78 |
| 3 | 7 | 35.0 | 384.4 | 399 | 19.0 | 2793 | 0.001432 | 0.60 |
| 4 | 9 | 19.3 | 382.8 | 420 | 19.2 | 3024 | 0.001514 | 0.71 |
| 5 | 8 | 43.8 | 411.6 | 388 | 19.7 | 2987.6 | 0.001310 | 0.57 |
| 6 | 8 | 30.0 | 381.4 | 364 | 20.6 | 3130.4 | 0.001324 | 0.69 |

T : average temperature of the period, ST : cumulative effective temperature (effective temperature is the temperature in degrees Celsius minus 12), TGC : thermal growth coefficient ($\frac{W_f^{1/3} - W_0^{1/3}}{ST}$), SGR : specific growth rate ($\frac{\ln(W_f) - \ln(W_0)}{\text{Days}}$).

corresponded to an initial production period between April and October (Table 1).

2.3. Statistical analysis and design of the models

A preliminary exploratory analysis of the data from the 20 batches was performed, considering the discrete model

$$W_f = \left(W_0^b + TGC_b \cdot ST \right)^{\frac{1}{b}} \quad (19)$$

where parameters b and TGC_b were estimated from available actual data of accumulated effective temperatures, by the Levenberg–Marquard iterative method available in Statgraphics[®] plus version 5.1. The exploratory analysis studying the model (19) with both $b = 1/3$ and $b = 2/3$ was continued using a least squares fit after linearisation, obtaining the values for TGC , named $TGC_{1/3}$ and $TGC_{2/3}$, respectively.

In integrating two models it was necessary to establish the transition point of change in the dynamics of growth. We consider the expression (7) with $b = 1/3$ and $b = 2/3$, and solve the equation

$$k_{1/3} \cdot W^{2/3} = k_{2/3} \cdot W^{1/3} \quad (20)$$

Note that in Eq. (7) we must distinguish two values of k which are different for the two values of b , so $k_b = TGC_b/b$ for $b = 1/3$, $b = 2/3$, respectively. The non-zero solution for W in Eq. (20), $W_c = 1/8 (TGC_{2/3}/TGC_{1/3})^3$ is a theoretical critical value of the weight for which the instantaneous rate of change in terms of weight depending on accumulated temperature is the same for both models (see Fig. 1). We assumed the hypothesis that the critical weight obtained indicates

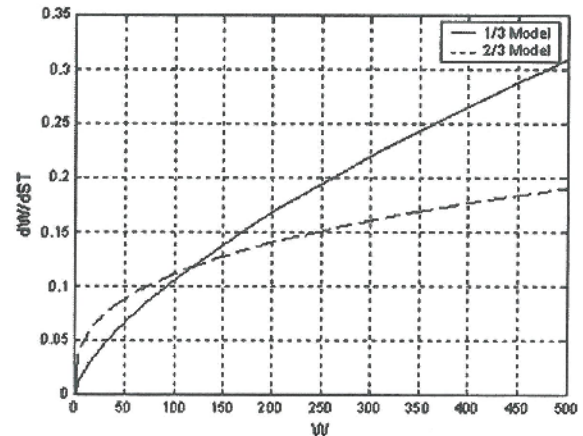


Fig. 1. Curves representing an instantaneous growth rate dW/dST for two models (1/3-TGC and 2/3-TGC).

Table 2

Thermal growth coefficients obtained using the two models ($b=1/3$ and $b=2/3$) considering growth data from 20 batches.

| Model | TGC_b | 95% TGC_b confidence interval | k_b | R^2 |
|-----------|------------------------|---------------------------------|------------|-------|
| $(b=1/3)$ | $TGC_{1/3}=0.00164561$ | 0.00156–0.00174 | 0.0049368 | 97.3% |
| $(b=2/3)$ | $TGC_{2/3}=0.0160949$ | 0.0153–0.0169 | 0.02414235 | 98.1% |

a smooth transition from the dynamics described by the model given by Eq. (19) with $b=1/3$ to the dynamics described by the model with $b=2/3$.

To estimate the final weight of gilthead sea bream, two simulation models were developed from Eq. (18) with $b=1/3$ ($TGC_b=TGC_{1/3}$) and $b=2/3$ ($TGC_b=TGC_{2/3}$), respectively, and from the temperature function, $T(t)$, given in Eq. (14). These models were designated the seasonal 1/3-TGC model and seasonal 2/3-TGC model, respectively. The parameters T_m , T_d and α , of the temperature function $T(t)$, (Eq. (14)) were adjusted for the environmental conditions where the studied batches were located. This was done using a large sample of daily temperatures of sea water for a period of 3 years (March 1998–March 2001) and the Levenberg–Marquardt algorithm available in MATLAB® v. 5.3 was used.

From the seasonal models 1/3-TGC and 2/3-TGC, taking into account the transition value of the weight obtained previously ($W_c=117$ g), a new simulation model which is a combination of the previous versions, named seasonal Mixed-TGC model, was designed. To analyse and validate the seasonal models, 1/3-TGC, 2/3-TGC and Mixed-TGC, various techniques were applied, using the statistical package Statgraphics® plus 5.1.

The three models were tested using the 6 batches described in Table 1, which were not used in model development. We have considered jointly, the actual average weight data, from different samples taken in the 6 batches obtaining a single large sample. Samples in each batch were taken at different times of the production cycle. The estimated weights for the three models, from the initial weight and for each batch, for the same times in which samples were taken, were computed. Finally the actual and estimated values were compared. On one hand, we contrasted the equality of the means of the absolute errors (absolute value of the difference between real values and the estimated values of weight) for the three models by means of an ANOVA, using the t -test. On the other hand, the differences between actual weights and estimated weights were also studied considering contrasts for paired values (using the t -test). It was thus verified whether each model estimated suitably, overestimated or underestimated the final actual weight.

Finally, by contrasting the hypotheses about the equality of standard deviations of the absolute errors, it was determined which model estimates more accurately the actual weight.

3. Results

Considering the data from the 20 batches and Eq. (19), the parameters b and TGC_b were estimated from real data of actual accumulated temperatures. A value for $b=0.6478$ very close to $2/3$ was obtained, with the 95%-asymptotic confidence interval for b , (0.5576, 0.7180), and a value for $TGC_b=0.014437$, with the 95%-asymptotic confidence interval for TGC_b , (0.007744, 0.021129) and $R^2=97.8\%$. Asymptotic confidence intervals showed that the parameters were significant and the coefficient of determination indicated a strong model fit to the data. These results led us to propose the viability of the TGC model with $b=2/3$.

The results for the value TGC_b obtained by least squares, after linearisation, for models with $b=1/3$ and $b=2/3$, respectively, are shown in Table 2. Obviously, TGC values are different in the two models, $TGC_{1/3}=0.00164$ and $TGC_{2/3}=0.01609$, but remain highly significant.

Fig. 1 shows graphs corresponding to the instantaneous rates of growth, dW/dST , depending on the weight, W , given in Eq. (6), for the cases $b=1/3$ (1/3-model) and $b=2/3$ (2/3-model), considering the values $k=k_{1/3}$ and $k=k_{2/3}$, shown in Table 2, respectively. Both curves allow us to compare the dynamics of the evolution of weight for both models. Instantaneous growth rates based on the cumulative effective temperature (dW/dST , $g^{\circ}C^{-1}$) are equal for the non-zero intercept point corresponding to the value of weight $W=117$ g (transition value of weight). From $W=0$ to $W=117$ g, the instantaneous growth rate of weight with respect to the cumulative effective temperature is higher and grows faster for the 2/3-model. After $W=117$ g, the instantaneous growth rate is higher for the 1/3-model. These results clearly suggest a pattern of gilthead sea bream growth in two stages.

The fitted values for the parameters of the temperature function $T(t)$, described in Eq. (14), are $T_m=18.8525$, $T_d=-6.6997$ and $\alpha=-312.4609$. Fig. 2 shows the temperature function $T(t)$ and the actual temperature data over a period of time established by the available actual data (available time interval started at day 69, March 10). Note that by periodicity, the first day of January would be day $1+365 \cdot j$, where j is any integer value.

So, two seasonal models were established based on Eq. (18), in order to describe the growth of gilthead sea bream; the seasonal 1/3-TGC model ($b=1/3$, $TGC_{1/3}=0.001646$) and the seasonal 2/3-TGC model ($b=2/3$, $TGC_{2/3}=0.016095$). From the former models, 1/3-TGC

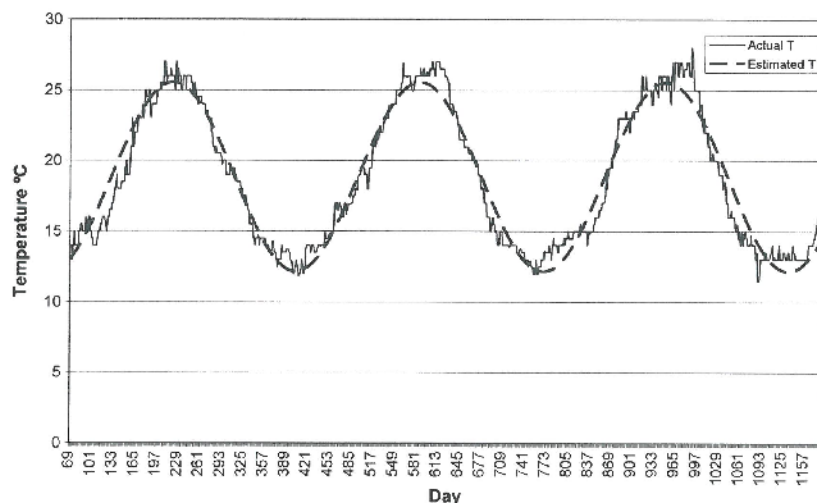


Fig. 2. The temperature curve obtained and available actual data from the Mediterranean Sea in the Spanish southwest coast.

and 2/3-TGC, we constructed the seasonal Mixed-TGC model, which is defined in Eqs. (21) and (22).

$$W_f(t) = \left(W_0^{\frac{1}{3}} + TGC_{1/3} \cdot ST(t_0, t) \right)^3, \text{ if } W_f(t) < 117 \quad (21)$$

$$W_f(t) = \left(W_0^{\frac{2}{3}} + TGC_{2/3} \cdot ST(t_0, t) \right)^{\frac{3}{2}}, \text{ if } W_0(t) \geq 117 \quad (22)$$

To estimate final weights greater than 117g from initial weight less than 117g, first we calculated the value t_1 to reach 117g using

the 1/3-TGC model and the expression (23), and then we estimated the final weight using the 2/3-TGC model and expression (24).

$$W_f(t_1) = \left(W_0^{\frac{1}{3}} + TGC_{1/3} \cdot ST(t_0, t_1) \right)^3 = 117 \quad (23)$$

$$W_f(t) = \left(117^{\frac{1}{2}} + TGC_{2/3} \cdot ST(t_1, t) \right)^{\frac{3}{2}} \quad (24)$$

Therefore, until a final weight less than 117, the Mixed-TGC model coincides with the 1/3-TGC model. In the case of an initial weight greater than or equal to 117g, the Mixed-TGC model coincides with

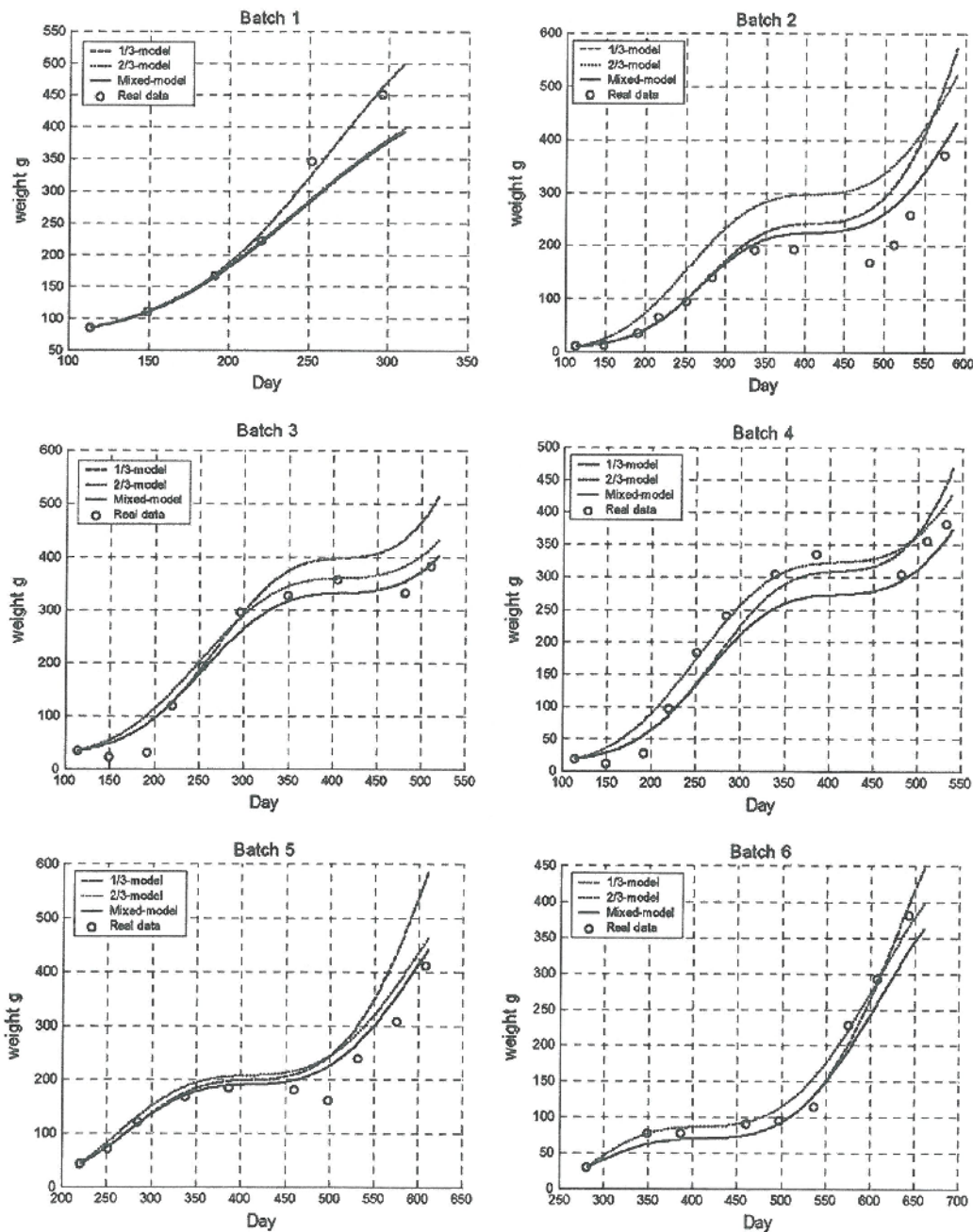


Fig. 3. Growth curves generated with three models (1/3-TGC and 2/3-TGC and Mixed) and real data from six new batches. (The abscissa axis shows the value of the time variable t day within a year. So, $t=1$ corresponds to the first day of the year, January 1 and a time value $t > 365$ indicates a transition from one year to the next).

the 2/3-TGC model. The Mixed-TGC model leads to a continuous curve in representing the final weight of the gilthead sea bream. Moreover, the curve is also differentiable at all time because the Mixed-TGC model is constructed so that when the weight is exactly 117 g, the derivatives of the functions that define models 1/3-TGC and 2/3-TGC coincide. Thus, the transition from the 1/3-TGC model to the 2/3-TGC model occurs smoothly, without sharp points.

Fig. 3 shows the actual weight points together with the estimated weight curves obtained from the three models, 1/3-TGC, 2/3-TGC and Mixed-TGC, for the six new batches reserved for validating the theoretical models.

Table 3 shows the results for the averages of the absolute errors of estimation for the complete cycle (long-term using data from all monthly samples), for the periods before the critical weight ($W_f < 117$) and after the critical weight ($W_0 > 117$) and for final weight at the end of the cycle. The estimated absolute error (absolute value of the difference between the real and estimated value), is a measure of the adjustment of the values estimated by models to the real data. The results show a lower value for the average of the absolute errors for the 1/3-TGC model than for 2/3-TGC when $W < 117$ g, and for the 2/3-TGC model than for 1/3-TGC when $W \geq 117$ g, but if the complete production cycle is considered and the Mixed TGC model is compared with the 1/3-TGC and the 2/3-TGC models, differences were not statistically significant. When the final weight was estimated from the initial weight with three models, differences were not significant.

Finally, Table 4 shows the outcomes of the hypothesis tests considering the resulting variable by subtracting the actual weight minus the estimated weight, $D = W_{real} - W_{est}$. When considering the sign of the difference between the real and the estimated weights, we can determine if a model overestimates or underestimates the real weight. Analysis distinguishes the case in which the real final weight is less than 117 g (first stage) from the case in which the real initial weight is greater than or equal to 117 g (second stage). The Mixed-TGC model does not appear in the analysis because for final weights less than 117 g the Mixed-TGC model coincides with the 1/3-TGC model, and if the initial weight is greater than or equal to 117, then the Mixed-TGC model coincides with the 2/3-TGC model.

For a significance level $\alpha = 0.05$, on one hand the results indicate that there are no statistically significant differences between the real weight and the weight estimated by the 1/3-TGC model for the first stage, and that the 1/3-TGC model tends to overestimate the final weight in the second stage of growth. On the other hand, the 2/3-TGC model overestimated the final weight in the first stage of growth while there were no statistically significant differences between the real weight and estimated weight by the 2/3-TGC model in the second stage of growth.

4. Discussion

Final weight of gilthead sea bream in real conditions of production, seems to be better explained using the TGC model with $b = 2/3$

Table 3

ANOVA results for the averages of absolute errors of the estimation (g), and the three models, for the complete cycle (long-term using data from all monthly samples), for the periods before the critical weight ($W_f < 117$) and after the critical weight ($W_0 > 117$) and for the final weight at the end of the cycle, considering data from six new batches.

| | 1/3-TGC model | 2/3-TGC model | Mixed-TGC model |
|---------------------|-------------------|-------------------|-----------------|
| Long-term | 28.9 | 31.9 | 24.4 |
| $W_f < 117^*$ | 9.5 ^a | 28.8 ^b | - |
| $W_0 \geq 117^{**}$ | 48.7 ^a | 29.0 ^b | - |
| Final weight | 64.1 | 46.6 | 34.6 |

The results must be interpreted by row. Different letters mean statistically significant differences.

* P -value = 0.0328.

** P -value = 0.0349.

Table 4

Hypothesis tests for paired variables distinguishing two stages of growth: first $W_f < 117$ g and second $W_0 \geq 117$ g.

| Model | $W_f < 117$ g | $W_0 \geq 117$ g |
|---------|---|--|
| 1/3-TGC | $H_0: D = 0$ $H_1: D \neq 0$ P -value = 0.890 Not Reject H_0 | $H_0: D = 0$ $H_1: D < 0$ P -value = 0.0005 Reject H_0 |
| 2/3-TGC | $H_0: D = 0$ $H_1: D < 0$ P -value = 0.0021 Reject H_0 | $H_0: D = 0$ $H_1: D \neq 0$ P -value = 0.60 Not reject H_0 |

than the TGC model with $b = 1/3$, because the estimated value of the exponent was $b = 0.648$, very close to 2/3. Lupatsch and Kissil (1998) developed a growth model for gilthead sea bream and obtained a coefficient for weight similar to 2/3 ($b = 0.613$), although in a new model (Lupatsch et al., 2003) the coefficient was lower ($b = 0.514$).

When the two models, 1/3-TGC and 2/3-TGC were assayed, a change in the pattern of growth for gilthead sea bream under commercial production conditions was noted, as the presence of a transition weight value from around 117 g was detected, which indicates a turning point for the dynamics of growth in the weight of fish. If we start with an initial weight of 10 g, this value can be matched with a value of the sum of effective temperatures $ST = 1670^\circ\text{C}$. We cannot explain the hypothetical physiological process of change that occurs at 117 g. The results indicate the need to address a more detailed study of allometric growth of gilthead sea bream under production conditions. Nevertheless, the reasons for the change in the pattern of growth should be related with aspects such as compensatory growth, genetic potential, allometric growth, nutrients or physiology of reproduction. Dumas et al. (2007) showed that to describe the growth of rainbow trout over a full cycle of production, there are three stanzas with different values for b . Growth changes associated with these stages are explained by morphological changes due to muscle growth dynamics, nutrient utilisation and reproduction investment. It seems clear that parameter b should not be considered a priori as a constant for a TGC model intended to explain the growth of gilthead sea bream in a full production cycle. Specifically, in the case of gilthead sea bream, when considering a complete production cycle, the TGC-1/3 model tends to overestimate the final weight (Mayer et al., 2008).

The 1/3-TGC model gives better statistically significant results for the estimated weight of fish in early stages, to lower final weights of 117 g, while the 2/3-TGC model gives better results in estimating the final weight of fish with initial weights higher than 117 g. The result is consistent with the fact that the 1/3-TGC model is based on the model proposed by Iwama and Tautz (1981) for fingerling growth in hatcheries. If we assume that the temperature varies continuously over time, therefore the model assumes that the growth rate is allometrically related to the weight, W , and the allometric constant of proportionality are directly related to temperature that varies during the rearing period.

When we compare the real weight with the estimated weights along the complete growth cycle, we cannot establish statistically significant differences, because of the large dispersion of the absolute errors corresponding to the 1/3-TGC model and the 2/3-TGC model (Fig. 4). The three models seem to provide acceptable results in estimating the long term weight, as evidenced by the analysis of the absolute errors of estimation (Table 3). If we consider only the weights at the end of the cycle, the absolute error analysis does not allow statistically significant differences between the three models, but the final error clearly seems to be lower with the Mixed or the 2/3-TGC models than the 1/3-TGC-model. In view of the graphs in Fig. 3, it seems clear that both the 2/3-TGC model and further the 1/

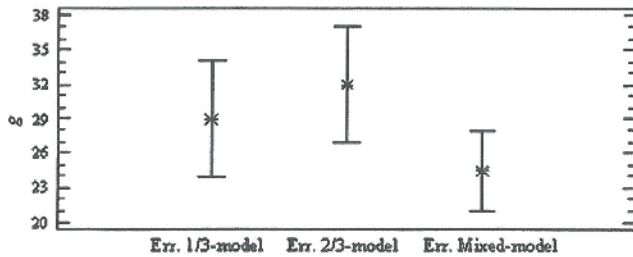


Fig. 4. Mean and standard errors for the absolute error of the long term estimation using three models and real values of new six batches.

3-TGC model, tend to overestimate the weights at the final of the cycle of production. Notably, the absolute errors for models 1/3-TGC and 2/3-TGC show a wide dispersion, which prevents us from establishing significant discrepancies in the absolute errors considering the complete cycle in the three models. In Figs. 4 and 5, the value for the standard error reflects the variation within each sample, and we can observe that the mean of the absolute errors for the Mixed-TGC model is the lowest.

It appears that the errors for the Mixed-TGC model have statistically significant lower dispersion when estimating the average absolute errors. Indeed, to test if the differences between the standard deviations of the errors are statistically significant when considering the whole cycle, hypothesis tests were performed comparing the standard deviations of the long-term absolute errors of estimation for the 1/3-TGC, 2/3-TGC and Mixed-TGC models. First, we tested the null hypothesis stating that the standard deviation from the absolute error for the 1/3-TGC model is equal to that corresponding to the Mixed-TGC model ($H_0: \sigma_{1/3} = \sigma_{Mix}$), against the alternative hypothesis stating that the standard deviation of the absolute error for the 1/3-TGC model is greater ($H_1: \sigma_{1/3} > \sigma_{Mix}$), obtaining the P -value=0.0073 which leads to the rejection of H_0 . When the null hypothesis stating that the standard deviations of the absolute errors for the 2/3-TGC model and the Mixed-TGC model are equal ($H_0: \sigma_{2/3} = \sigma_{Mix}$) was tested against the alternative hypothesis stating that the standard deviation is greater for the model 2/3-TGC ($H_1: \sigma_{2/3} > \sigma_{Mix}$), it yielded a P -value=0.0009, which also led to the rejection of H_0 . Statistically significant differences between the standard deviations of the absolute errors cannot be set for the 1/3-TGC and 2/3-TGC models. The results of these contrasts showed a lower uncertainty in the estimates from the Mixed-TGC model and confirmed what Fig. 4 seemed to show. Moreover, the 1/3-TGC model and the 2/3-TGC model tended to overestimate the weight in the second stage of growth and in the first stage of growth, respectively.

From the above considerations, the Mixed-TGC model clearly seems to be the most appropriate for describing the growth over the complete production cycle.

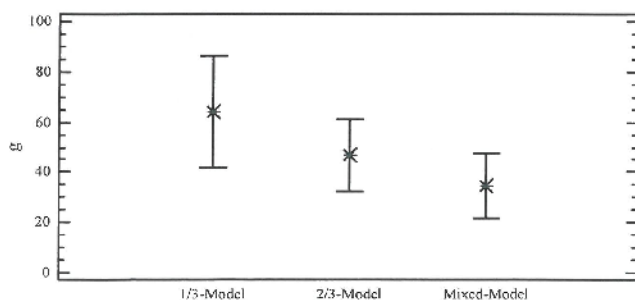


Fig. 5. Mean and standard errors for the absolute error of the final weight estimation using three models and real values of new six batches.

5. Conclusions

The family of TGC seasonal models obtained by considering different values for the b parameter in the metabolic equation provides a framework for studying and explaining indeterminate growth patterns. In the case of gilthead sea bream, the use of the 1/3-TGC model is useful in estimating the weight in the initial period of growth (in this case the 1/3-TGC model matches the Mixed-TGC model). In the case where the initial weight exceeds 117 g, it is advisable to use the 2/3-TGC model to estimate the weight (which in this case also coincides with the Mixed-TGC model). The study of the TGC models has revealed a change in the growth pattern that occurs when the fish reaches a weight around 117 g.

A continuous growth curve including the temperature function and integrating the two models was developed to establish a practical tool for fish farmers.

The results indicate that the Mixed-TGC model is the most appropriate for long-term and final weight estimations along the complete cycle of growth.

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7. DISCUSIÓN GENERAL

La adecuación de los modelos de regresión simple a la modelización del crecimiento de la dorada en condiciones de producción depende de ciertos aspectos fundamentales que condicionan el diseño de estos modelos, principalmente, las variables utilizadas y la variabilidad de los datos.

Los modelos clásicos de crecimiento, como el de Von Bertalanfy, consideran como variables principales el tiempo y la longitud (o peso) en la edad adulta, y se aplican al estudio de dinámica de poblaciones de peces en el medio natural, en el que coexisten todas las clases de edad bien diferenciadas, debido a una marcada estacionalidad reproductiva. En los sistemas de acuicultura sólo se trabaja con tamaños hasta el peso comercial (clases 0+ y 1+), que se solapan en el tiempo debido al control de la reproducción y la introducción de nuevos lotes en cualquier época del año, por lo que adquiere mucha importancia el efecto de la temperatura, al ser los peces animales poiquiloterms. En el caso del engorde de la dorada, que crece en condiciones naturales en el mar, con oscilaciones térmicas importantes (de 12 °C hasta 28 °C), ha motivado que los modelos de crecimiento en condiciones productivas hayan considerado, además del peso, la inclusión de la temperatura (Petridis & Rodgakis, 1996; Lupastch & Kissil, 1998; Lupastch y col., 2003). Cho & Bureau (1998) y Kaushik (1998) utilizaron el sumatorio de temperaturas diarias en lugar de la temperatura media, lo que supuso un avance para el uso de los modelos de crecimiento como herramienta de predicción.

Entre otras variables que también podrían ser usadas en los modelos de crecimiento, destaca la alimentación, pues resulta obvio que una mayor o menor ingestión de nutrientes (proteína y energía) originará diferentes crecimientos, como ha sido puesto de manifiesto por numerosos trabajos en los que se ha estudiado la tasa de alimentación (Jover y col., 2007; Jauralde y col., 2012) o el contenido nutritivo del alimento (Moñino y col., 2005). No obstante, la dificultad de determinar con exactitud la ingestión de alimento en jaulas marinas, y la complejidad de su control por los cambios en el tipo de pienso y en su composición nutritiva, limita su empleo en el desarrollo de modelos prácticos para la gestión de la producción.

Otra posible variable sería el origen de los alevines, pues la calidad de los reproductores (mejoramiento genético) puede dar lugar a diferentes velocidades de

crecimiento, pero resulta difícil de evaluar, debido a la escasa implantación de programas de mejora genética, y complicado de llevar a la práctica, debido a la mezcla de lotes de diferentes reproductores que se lleva a cabo en las granjas de reproducción.

El principal aspecto a tener en cuenta es la elección de las variables que se pueden controlar durante el proceso productivo. Dichas variables deben de ser lo más “fiabes” posibles y es necesario que no enmascaren a otras, y, puesto que existe la posibilidad de controlar todas las variables que afectarían al crecimiento, se deben escoger aquellas que menos incertidumbre aporten al modelo. Por tanto, se elegirán aquellas que formen parte del seguimiento y el control diario, y que puedan incluirse adecuadamente en las rutinas diarias de la empresa. También será importante detectar aquellas variables, cuyo análisis exploratorio previo demuestre que al ser añadidas a un modelo sencillo aumentan el coeficiente de determinación hasta el valor máximo.

Teniendo en cuenta todos los aspectos descritos anteriormente, se considera que la variable fundamental a considerar para describir el crecimiento es la temperatura instantánea o diaria, la cual depende a su vez del tiempo, o mejor el sumatorio de temperaturas ($\sum T_{ef}$), con la corrección de la temperatura efectiva, que considera el límite inferior térmico de crecimiento de 12 °C, establecida por García (1999).

La elección de variables fáciles de controlar y que tengan un gran peso en la explicación de los datos de crecimiento deben servir para no aportar una complejidad innecesaria a la hora de modelizar dicho crecimiento. Si además se consigue describir el crecimiento mediante un modelo sencillo, se tiene la solución a una parte fundamental del proceso productivo de la dorada en jaulas marinas. En definitiva, el objetivo es obtener un modelo sencillo que considere un número reducido de variables de entrada, y que se pueda expresar mediante relaciones funcionales sencillas, que pueda explicar la variabilidad de los datos y permita hacer “predicciones” a corto y largo plazo. La sencillez de manejo y la posibilidad de ser comparable con otros modelos basados en otros índices de crecimiento, (Petridis & Rodgakis, 1996; Lupastch & Kissil, 1998; Lupastch y col., 2003) constituye una herramienta muy interesante para la gestión de la producción.

Para la prospección inicial de diferentes modelos de crecimiento realizada en el primer trabajo (Art.1) de la presente Tesis Doctoral, se seleccionaron el peso de las

doradas (en todos ellos), la temperatura media (Modelos 1 y 3) y la suma de temperaturas efectivas (Modelos 2, 4 y 5). Los modelos basados en la temperatura media dieron los mayores errores de predicción a largo plazo al aplicarlos a los nuevos lotes de dorada (Tabla 5, Art. 1), mientras que los errores más bajos se obtuvieron con el Modelo 2 ($W_f = 3.528 W_i^{0.783} e^{0.00042 \sum T_{ef}}$) y el Modelo 4 ($W_f = 1.516 W_i^{0.796} \sum T_{ef}^{0.161}$), basados en el sumatorio de temperatura, aunque para los sucesivos trabajos se eligió el Modelo 5 ($W_f = [W_i^{1/3} + 0.00132 \sum T_{ef}]^3$), basado en Cho & Bureau (1998) y conocido como *Coficiente Térmico de Crecimiento* (CTC), pues tuvo un error similar y sólo depende de un parámetro (en vez de los tres para los Modelos 2 y 4), lo que facilitará su aplicación práctica.

En cuanto a la calidad de los datos con los que desarrollar los modelos, es necesario considerar dos problemas fundamentales, que deben ser valorados por los investigadores en el proceso de validación de los modelos, y por los técnicos de la instalación, durante su aplicación a la gestión productiva.

En primer lugar, los lotes que son introducidos en las jaulas pueden encontrarse dentro de los parámetros de una población normal, pero la evolución del peso de los peces del lote, a lo largo del proceso productivo, puede tender a la heterocedasticidad (la varianza del error debe ser constante) y en muchas ocasiones, fijado un instante del ciclo, a la pérdida de la característica inicial de normalidad en la distribución del peso, hecho condicionado no sólo por el manejo en la alimentación, sino influido también por las relaciones de competencia, agresión, o canibalismo, que puedan producirse dentro de un mismo lote. Hay que tener en cuenta que las poblaciones (lotes) que se introducen en las granjas, y que provienen de las granjas de reproducción pueden tener pesos medios similares, pero es posible que sean mezclas de “cabezas” o “colas” de varias poblaciones, con velocidades de crecimiento distintas, que provocan un incremento de la dispersión. También hay que considerar que un lote de 100.000 peces puede estar formado por unos mismos parentales, pero un lote de 1.000.000 o incluso de 300.000 de peces, nunca podrá provenir de unos mismos parentales, por lo que pueden aparecer crecimientos diferenciales por causa de su calidad genética. Por lo tanto, se debe tener en cuenta que los lotes pueden presentar gran variabilidad, y los modelos de regresión, aplicados sobre muestras obtenidas a lo largo del ciclo productivo, no superen los test de bondad de ajuste, debido a la gran heterocedasticidad y la ausencia de normalidad en las muestras.

En segundo lugar, existe una pérdida de información fundamental, ya que para trabajar con los modelos simples, sólo se tienen en cuenta los valores promedios en cada una de las muestras obtenidas a lo largo del ciclo de producción, no considerándose la variabilidad de los datos en cada muestreo, que podría aportar una interesantísima información sobre la dispersión de pesos de la población. Esto provoca que, atendiendo a la comparación entre lotes, los modelos sencillos sobre los que usualmente se trabaja no sean capaces de describir la variabilidad entre lotes, pues es difícil controlar la influencia de todas las variables externas. Es necesario establecer modelos robustos frente a la variabilidad extrema que pueda producirse puntualmente, y siempre considerando que dichos modelos serán adecuados para describir el proceso productivo, siempre y cuando se controle la información que se pierde, y se asuma lo que puede suponer dicha pérdida de información.

No obstante, la variabilidad de los datos no permite justificar un modelo de regresión simple, cuando ha de presuponerse homocedasticidad, y los datos, en general, no la cumplen. Por otra parte, valores grandes de los coeficientes de determinación en los modelos de regresión simple no tienen justificación, si no se supera el test de bondad del ajuste. Por lo tanto, se debe asumir que los datos que se han manejado sirven únicamente para predecir el crecimiento de un valor promedio del peso de los peces, lo que limita su aplicación a la gestión acuícola.

Para solucionar el problema de la variabilidad, se puede utilizar una “herramienta matemática” que permita obtener resultados robustos en poblaciones con gran variabilidad y valores extremos, caracterizadas por la heterocedasticidad y ausencia de normalidad. La regresión cuantil (Koenker y Basset, 1978) puede proporcionar información sobre la distribución del peso en cada instante del proceso de crecimiento, permitiendo estudiar la evolución de la distribución de los pesos, ya que considera todos los datos obtenidos en un muestreo en lugar de un valor medio para cada muestra. Dicha evolución de la distribución del peso es obtenida a partir del conocimiento de la evolución del crecimiento de los diferentes cuantiles para cada lote, que fue abordada en el segundo trabajo de la presente tesis doctoral (Art. 2).

La regresión cuantil presenta mayor robustez frente a valores atípicos o con gran dispersión, y permite estimar la evolución de cualquier cuantil y, por lo tanto, realizar

un seguimiento de dicha parte de la población. El modelo cuantil proporciona información sobre los diferentes índices de crecimiento para cada cuantil de un mismo lote, lo que permite hacer un seguimiento del crecimiento de los lotes y detectar la existencia de distintos patrones en dicho crecimiento, por lo que se aplicó a los datos de crecimiento de la dorada en jaulas marinas, obteniéndose diferentes valores del *Coefficiente Térmico de Crecimiento* (CTC) para los cuantiles 5, 25, 50, 75 y 95 en cada uno de los lotes (Tabla 3, Art. 2), que permitieron diferenciar entre lotes de crecimiento homogéneo y heterogéneo.

La evolución del crecimiento en cada cuantil en un lote vendrá expresado por los diferentes valores del CTC. Así, el valor del CTC 05 explicará el crecimiento del 5% de la población con el menor peso, y por otra parte, el CTC 95 explicará el crecimiento del 5% de la población que más crece. Asimismo, el CTC 25 describe el crecimiento del 25% de la población que menos crece, y el CTC 75 explicaría el crecimiento del 25% de la población con más peso. El valor del CTC 50 explica la evolución de la mediana del peso de la población con el tiempo. La evolución de los distintos cuantiles con el tiempo permite detectar si una parte de la población está creciendo mejor o peor y, también, realizar comparaciones con otros rangos poblacionales.

En aquellos lotes de doradas donde los valores del CTC pueden considerarse iguales, mediante la aplicación de una técnica ANOVA (Tabla 3, Art. 2) se tiene que asumir que dicha población tiene una evolución del crecimiento homogénea, es decir, no aumenta la dispersión del peso con el tiempo (o con el incremento del sumatorio de temperatura), por lo que se mantiene la misma pendiente, manteniéndose paralelas las diferentes rectas de evolución de los distintos cuantiles de CTC (Figura 3, Art. 2). Por el contrario, en aquellos lotes donde los valores del CTC para distintos cuantiles son estadísticamente diferentes (pendiente divergente), corresponden a una población con un crecimiento heterogéneo, en la que aumenta la dispersión de pesos. Por tanto, los valores del CTC para los cuantiles seleccionados ponen sobre aviso que la población (lote) de doradas está evolucionando hacia una mayor o menor dispersión, lo que supone una herramienta muy interesante para la gestión del lote, concretamente para decidir sobre la conveniencia de realizar una clasificación por tamaños, y también para valorar la calidad del mismo y reclamar a la granja de reproducción.

Por otra parte, el estudio de los rangos intercuantílicos de los pesos, al inicio y al final del periodo de crecimiento, permite concretar y cuantificar la evolución de la dispersión (Tabla 2, Art. 2). Los dos rangos intercuantílicos que se seleccionaron fueron el establecido entre el cuantil 05 y 95 (representativos del 90% de la población), y el que considera los cuantiles 25 y 75 (representativos del 50% de la población en sus valores centrales). La diferencia entre los rangos intercuantílicos para pesos iniciales (obtenidos con los datos del primer muestreo) y para pesos finales (obtenidos con los datos del muestreo final) informa de la evolución de la dispersión. Posteriormente, el estudio mediante ANOVA de los rangos intercuantílicos entre lotes homogéneos y heterogéneos (Figura 4, Art. 3) revela que en los lotes heterogéneos el aumento del incremento intercuantílico se sitúa en torno al 50% (independientemente del rango seleccionado), y en los lotes homogéneos apenas se produjo una variación entre un 2% (Inc. 7525) y un 20% (Inc. 9505).

Una vez realizada la clasificación en base a la evolución de los lotes, que determina qué lote es homogéneo (Lotes 1,2, 8, 9, 11, 14 y 20) y qué lote tiene un crecimiento más heterogéneo (Lotes 3, 4, 5, 6, 7, 10, 12, 13, 15, 16, 17, 18 y 19), faltaría identificar qué factores o variables son los que condicionan dicha clasificación.

La primera variable considerada, que podría condicionar la evolución del crecimiento en los lotes, fue el coeficiente de variación de los pesos de la muestra inicial (Tabla 1, Art. 2), que cuantifica la dispersión inicial del lote. Los resultados fueron concluyentes, ya que un análisis discriminante (Tabla 4, Art. 2) confirmó la clasificación del 75% de los lotes como homogéneos y heterogéneos, siendo dicho coeficiente de variación de los pesos iniciales una variable con alto poder discriminante, con valores medios de 29% para los lotes homogéneos y 24% para los heterogéneos (Tabla 5, Art. 2).

Si se consideran la inclusión de la curtosis inicial (medida de apuntamiento), la variación de la temperatura en los 30 primeros días (V30) y la temperatura media en el primer mes T30 (media), junto con el coeficiente de variación inicial, se corrobora la clasificación del 100% de los lotes en homogéneos y heterogéneos. El coeficiente de variación y la curtosis informan de las características del lote a introducir, mientras que la T30 y V30 indicarían el momento y las condiciones en que se introduce el lote en el medio.

Los valores de las características de los lotes que envía la granja de reproducción hacen pensar en la forma en que se han constituido esos lotes. Aparentemente, los lotes con coeficiente de variación inicial muy pequeño serían bien aceptados por la dirección técnica de la instalación, pero los resultados obtenidos muestran que no van a evolucionar bien a lo largo del periodo de producción, pues se producirá un aumento de la dispersión de pesos. La causa de este sorprendente comportamiento habría que buscarla en una minuciosa clasificación por tamaños realizada en la reproductora, juntando lotes de cabezas y colas de distintos lotes de alevines, pero con un mismo peso medio. Parece pues, que la tendencia de solicitar menores porcentajes de dispersión inicial por parte de las empresas de engorde debería revisarse, pues esos menores porcentajes de variabilidad inicial pueden deberse a una mayor manipulación inicial del lote en las plantas de pre-engorde.

Por otro lado, las variables ambientales muestran el efecto del momento de introducción del lote en la evolución de la dispersión del crecimiento, concretamente la variación de la temperatura del agua durante los primeros 30 días, que resultó ser significativamente diferente para los lotes homogéneos (-0.00058 °C/día) y para los lotes heterogéneos (0.00059 °C/día), de forma que, aparentemente un incremento de la temperatura durante el primer más favorecería la dispersión.

Después de observar la dinámica del crecimiento de la dorada a lo largo de todo el ciclo productivo, siguiendo el modelo del CTC de Cho & Bureau (1989), y tras profundizar en el modelo matemático que lleva desde el modelo general de crecimiento $dW/dt = g(W,t)$ a dicho modelo del CTC, se llega a una expresión genérica en la que el peso final (W_n) viene en función del peso inicial (W_o) y la suma de temperaturas ($\sum T^a$): $W_n^b = W_o^b + k b \sum T^a$. Si el exponente b se hace igual a $1/3$, se obtiene la expresión del modelo del CTC de Cho & Bureau (1989), pero en este punto hay que hacer la pregunta ¿son posibles otros valores para el exponente b ? La respuesta a esta pregunta fue abordada en el tercer trabajo de la presente tesis doctoral (Art. 3).

Al volver a profundizar en los modelos clásicos de crecimiento, se puede comprobar que muchos modelos, entre ellos el de Von Bertalanfy, resultan de la resolución de la ecuación diferencial $dW/dt = hW^d - kW^m$, en la que el primer sumando representa el anabolismo y el segundo el catabolismo, ecuación que fue resuelta por Beverton & Hold (1956), dando al exponente “d” el valor de $2/3$. Por otra parte, al

estudiar el modelo factorial de Lupastch y col. (2003), en el que, además del modelo de crecimiento, establece las necesidades de proteína y energía, obtiene el peso metabólico de la dorada, que resulta ser de 0.7 para la proteína y 0.8 para la energía, valores más próximos a $2/3$ que a $1/3$. Por tanto, se planteó la posibilidad de emplear el valor $2/3$ para el exponente b y compararlo con el valor $1/3$ del modelo del CTC, y así se obtuvieron dos modelos de crecimiento para la dorada diferentes (Figura 1, Art. 3).

Al observar que ambas curvas se cruzaban para un peso 117 g y que el modelo $1/3$ tendía a sobreestimar los pesos mayores de las doradas, mientras que el modelo $2/3$ sobreestimaba los pesos inferiores, se construyó un “modelo mixto” que reducía el error de estimación (Tabla 3, Art. 3). Este nuevo modelo mixto estima el crecimiento mediante el exponente $1/3$ hasta 117 g y mediante el exponente $2/3$ después, estableciendo la existencia de un punto “crítico” o de “transición” que indicaba un cambio en la dinámica del crecimiento de la dorada, y que estaba de acuerdo con el trabajo de Dumas & Bureau (2007), que estableció tres tramos de crecimiento en la trucha, utilizando el modelo del CTC.

Para la implementación del modelo mixto, fue muy importante la modelización de la temperatura del agua, mediante una función periódica de tipo senoidal (Figura 2, Art. 3) la cual facilita la estimación de los crecimientos de los diferentes lotes y que ya había sido usada por Dumas & France (2008).

Recientemente, Bonilla (2014) ha estudiado la relaciones alométricas de doradas en condiciones de producción en jaulas, y ha obtenido una diferencia significativa en el “índice de condición” o valor “ k ” (peso/longitud³) para doradas con un peso inferior a 117 g ($k=1.57$) y un peso superior ($k=1.81$), lo que corrobora el cambio en el patrón de crecimiento, cuyas causas habrá que buscar en futuros estudios.

La presente tesis doctoral ha desarrollado un “*modelo mixto de crecimiento cuantil*” para la dorada en base al sumatorio de temperaturas efectivas, que abre muchas expectativas para futuras aplicaciones prácticas para la gestión productiva en granjas marinas, como previsión de clasificaciones, desdobles, y ventas, pero todavía es necesario profundizar en la optimización del modelo, por ejemplo, estableciendo un método que permita ajustar, a partir de un modelo genérico de crecimiento para cada granja, una curva para cada uno de los lotes en tiempo real, que se vaya corrigiendo en

función de los sucesivos muestreos, y que permita una estimación particularizada de cada unidad de producción, lo que sin duda tendrá un importante efecto positivo en la gestión de la alimentación, en la clasificación y desdobles, y en la previsión de ventas.

8. CONCLUSIONES

- 1.- Los modelos de crecimiento que tienen en cuenta el sumatorio de temperaturas funcionan mejor que los que utilizan valores medios de temperatura.
- 2.- La linealización del modelo de Cho nos permite discriminar entre lotes de crecimiento homogéneo y heterogéneo.
- 3.- Las variables que influyen en la clasificación de los lotes son el coeficiente de variación inicial y la curtosis como variables dependientes de la hatchery, y la variación de la temperatura media en los 30 primeros días y la temperatura media en el primer mes como variables ambientales.
- 4.- El modelo de Coeficiente Térmico de Crecimiento (*CTC*) sobreestima el peso medio a medida que se desarrolla el ciclo productivo.
- 5.- Es necesario la inclusión de un modelo mixto para describir la dinámica del crecimiento a lo largo de todo el ciclo productivo.
- 6.- Existe un cambio en la dinámica del crecimiento caracterizado por un sumatorio de temperatura efectiva concreta y un valor del peso medio alrededor de 117g.

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