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Propagation of intense acoustic waves in sonic crystals

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Abstract

The propagation of intense acoustic waves in a periodic medium (sonic crystal) is numerically studied. The medium consists in a structured fluid, formed by a periodic array of fluid layers with alternating linear acoustic properties and quadratic nonlinearity coefficient. The spacing between layers is of the order of the wavelength; therefore Bragg effects such as band gaps appear. We show that the interplay between strong dispersion and nonlinearity influences wave propagation. The classical waveform distortion process typical of intense acoustic waves in homogeneous media can be strongly altered when nonlinearly generated harmonics lie inside or close to band gaps.

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1. Introduction

The control on wave propagation is one of the main challenges in different areas as optics, acoustics, phononics,... Since in the late 80's, Yablonovich (1987) introduced the concept of photonic crystal to light waves, periodic media have been studied for this purpose, introducing the concept of phononic or sonic crystal. Sonic crystals (Sigalas et al. (1992)) consist of an inhomogeneous periodic distribution of scatterers in a host medium that is a fluid. In mathematical terms, thinking in them as infinite media, sonic crystals are periodic media obtained by the translation of a base and the sites where this base is placed are called lattice. Propagation of waves through a periodic system are mainly characterized by dispersion, i.e., the pass of waves throughout these systems is governed by the dispersion relation that present propagation bands and bands of frequency for which the propagation is forbidden (band gaps). It is precisely this characteristic the one that has been exploited for obtaining structures based on sonic crystals for focalization, self-collimation, negative refraction, and many others effects on waves and beams. Up to now, the majority of the studies have been done in the regime of low-amplitude waves. Propagation of intense waves throughout

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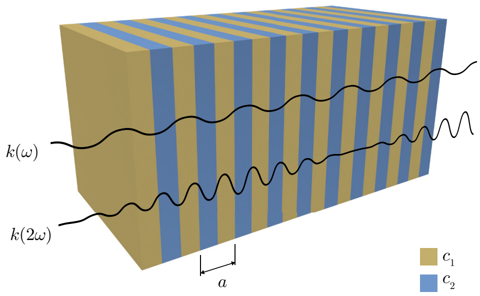

Fig. 1. Layered acoustic system with two different layers, the lattice period being $a = a_1 + a_2$. A second harmonic generation scheme is also represented.

nonlinear periodic media has been much less explored. We are interested in studying new phenomena related to sound wave propagation in nonlinear medium with periodically modulated properties.

In this work we consider waves with enough intensity to produce non-linear effects but sufficiently weak to keep the validity of the dispersion relation. Our aim is to study different effects that appear as a consequence of the interplay between the nonlinearity and the periodicity of the medium.

We consider a system formed by alternation of layers of fluid with different properties. This system can be considered as a 1D Sonic Crystal (this concept includes more exotic structures as the granular crystal or lattice; see Sanchez et al. (2013)), although in other context such a structure has been named a multilayer system, a superlattice (particularly in the context of semiconductors). This system will be the most simple for studying effects as the harmonic generation.

2. Harmonic generation in layered media

As we have mentioned in the previous Section 1, for the sake of simplicity we are going to consider a medium made of a periodic distribution of two homogeneous fluid layers of thickness a_1 and a_2 with different material properties. Only longitudinal waves under normal incidence impinge. A scheme of such medium can be seen schematically in Fig. 1.

The main advantage of such infinite periodic medium is that the propagation of waves (with small amplitude) in it is completely described by a dispersion relation given by an analytic formula (see Kosevich (2006))

$$\cos(ka) = \cos(k_1 a_1)\cos(k_2 a_2) - \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)\sin(k_1 a_1)\sin(k_2 a_2) \tag{1}$$

also known as the Rytov's formula, where k is the Bloch wave-number, $a = a_1 + a_2$ is the lattice period, and $k_i = \omega/c_i$ is the local wavenumber, with c_i the sound speed in the i-th layer. This equation gives a band structure of propagating and non-propagating (band gap) regions. Under the assumption that we are in the regime for which the amplitude of waves is high enough to produce nonlinear effects but lower enough that the dispersion relation remains valid, Eq. (1) gives us the effect of periodicity on the different harmonics as they propagate throughout the multilayer. The ratio between layer thickness can be defined as $\alpha = a_1/a$, leading to $a_2 = (1-\alpha)a$.

Now the nonlinear propagation of sound in the multi-layered media (acoustic inhomogeneous media) is considered. We are going to consider that the layers have quadratic nonlinearity and only corresponding to the layer's bulk. We are neglecting the nonlinear effects occurring at the boundary between adjacent layers.

We start from the continuity equation for mass conservation (Naugolnykh et al. (1998)),

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{2}$$

and the equation of motion that follows from conservation of momentum

$$\rho \frac{D\mathbf{v}}{Dt} + \nabla p = 0 \tag{3}$$

where ρ is the total density, $\mathbf{v}$ is the particle velocity vector over a Eulerian reference frame, p is the acoustic pressure, t is the time and D is the material derivative operator. We are dealing with inhomogeneous media (layered), i.e., the total density can be expressed as $\rho(t, \mathbf{x}) = \rho'(t, \mathbf{x}) + \rho_0(\mathbf{x})$, where $\rho_0(\mathbf{x})$ is the spatially dependent ambient density and $\rho'(t, \mathbf{x})$ is the perturbation of the density or acoustic density, that is space and time dependent. Then, using the material derivative, Eq. (3) becomes

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} + \nabla p = -\rho' \frac{\partial \mathbf{v}}{\partial t} - (\rho' + \rho_0) (\mathbf{v} \cdot \nabla) \mathbf{v} \,. \tag{4}$$

The first two terms in the left-hand-side account for linear acoustic propagation and the terms in the right-hand-side introduce nonlinearity in the Eulerian reference frame through momentum advection processes. We can expand Eq. (2) for nonhomogeneous media as

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla \rho_0 = -\rho' \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \rho' \,, \tag{5}$$

where the first two terms on the left-hand-side account for linear acoustic propagation, the third, also linear, accounts for the magnitude of the changes in the ambient layer properties. Notice that this term is space dependent but only changes at the interface between adjacent layers. For density matched layers, $\rho_i = \rho_{i-1}$, this terms vanishes. The terms on the right-hand-side are nonlinear and accounts for mass advection. In order to close the system of equations, we need to consider the state equation $p = p(\rho, s)$, where s is the entropy. We can express the local nonlinear medium response relating density and pressure variations as:

$$p = c_0^2 \rho' + \frac{B}{2A} \frac{c_0^2}{\rho_0} \rho'^2 \,, \tag{6}$$

where we have retained up to second order terms in the expansion in terms of powers of ρ'. $B/A(\mathbf{x})$ is the quadratic nonlinear parameter and $c_0(\mathbf{x})$ is the sound speed that in general can be considered spatially dependent. In this system of equations, the quadratic nonlinearity is present in the equation of motion (4) and in the continuity equation (5), in the momentum and mass advection terms respectively, and also in the equation of state, Eq. (6). We solve numerically the full constitutive relations.

As it can be seen in Rytov's formula (1) in the very low frequency regime the dispersion curve has a slope nearly constant and the layered medium behaves as an homogeneous medium. In this case the dispersion of the different harmonics is negligible and all of them propagate throughout the medium with practically the same velocity and are phase matched. For higher frequencies the slope of the curve changes and dispersion of harmonics appears, affecting the harmonic generation process. If we consider an intense monocromatic wave that travels throughout our layered medium, due to the quadratic nonlinearity of the layers, this wave generates the second harmonic. The resonant condition for a cumulative generation are $2\omega_1 = \omega_2$ and $2k_1 = k_2$ that hold in the case of non-dispersive media. But these conditions are not fulfilled in general in dispersive media and *free* and *forced* waves are mismatched,.i.e, there exists a detuning $\Delta k = |k_2 - 2k_1| = |k(2\omega) - 2k(\omega)|$. The waves with frequencies that lie in the band gap are evanescent, that is, the wave number k is complex whose imaginary part is non negligible. The wave decays exponentially in space according to the value of this imaginary part. If the frequency of the nonlinear generated second harmonic falls in the band gap, its amplitude does not decay but it reaches a constant values. It can be understood in terms that the first harmonic is transferring energy to the second one at every point in the space but, due to the evanescent character of this second harmonic, the cumulative process with distance does not take place. The "pumping"rate, depends on the ratio between the layer thickness and the shock distance in a layer. It also strongly depends on the ratio between the characteristic exponential decay length (related to the imaginary part of the wave number) and the shock distance in a layer. Finally, it depends also on the detuning of real part of the wave number. These two last factors depend on the dispersion relation.

In Fig. 2(a) and (b) we can observe the differences between the second harmonic generation when its frequency is just above the band gap or in the middle of the band gap. Frequencies are given in normalized units with respect to

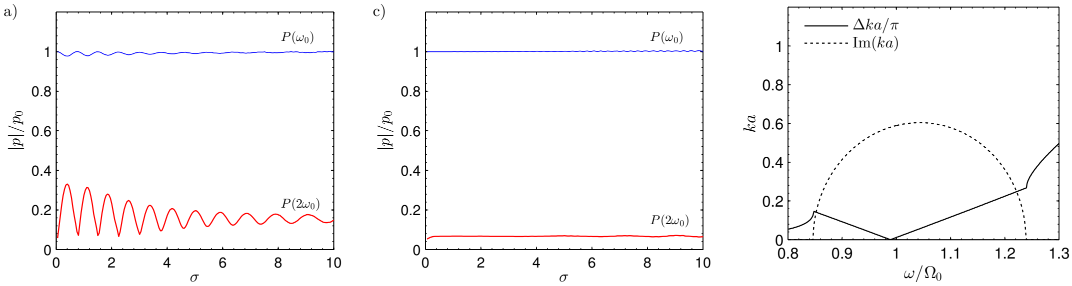

Fig. 2. For an incident wave of frequency $\omega_0 = 0.84\Omega_0$, evolution of the second harmonic field propagation when its frequency is (a) just above band gap, (b) in the middle of the bandgap. Results are for a layered medium with $\alpha = 1/2$ and $c_1/c_2 = 1/2$. (c) Detuning of the second harmonic (continuous line) and imaginary part (dotted line) in function of the normalized frequency.

$\Omega_0 = \pi\tilde{c}_0/a$ where $\tilde{c}_0$ is an effective velocity related with the low frequency regime. Figure 2(c) shows the detuning of the second harmonic with respect to the imaginary part of the wave vector as a function of the frequency for a medium with $\alpha = 1/2$ and $c_1/c_2 = 1/2$, being the ratio of the velocities of sound in each layer. At the middle of the band gap the detuning is null (phase matching condition) when the evanescence is nearly maximized, and viceversa. However, the magnitude of the effects can be very different. As the rate of the second harmonic generation is independent on the detuning, and the evanescence implies that the wave decays after few layers, there is not compensation of the effects at the center of the band gap.

A modification of the waveform is also possible via the tuning of the parameters of the medium, modifying, for example, the sawtooth profile that is observed in nondispersive media to other tailored profiles (in Jimenez et al. (2015)).

3. Conclusions

Due to the fact that periodic media are dispersive, the interplay between dispersion and nonlinearity has a strong influence on the acoustic waves that propagate through them. In particular, here we analyze the case of a multilayered periodic media, a particular case of a one-dimensional phononic crystals or superlattice. The nonlinear generated harmonics propagate at different velocities and, as a consequence, are phase-mismatched, altering the transfer of energy between the different harmonics and also modifying the waveform itself and avoiding the shock formation that is typical in homogeneous media. In summary, we report here the possibility of an effective control of the spectrum of acoustic waves in a nonlinear regime tuning the dispersion relation of the medium.

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