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Factorization of p-Dominated Polynomials through L^p-Spaces

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Introduction

Since Pietsch's seminal paper [32], the study of the nonlinear operators ideals has been strongly developed. Several polynomial ideals have been defined and studied, and it can be said that it is now a relevant topic in functional analysis and important contributions have been made by many mathematicians such as R. Alencar, R. M. Aron, G. Botelho, V. Dimant, M. Matos, Y. Meléndez, D. Pellegrino, J. Santos, J. Seoane-Sepúlveda, and A. Tonge, among others. The hope of these nonlinear ideals is that they keep the main properties of the corresponding linear operator ideals, and so the linear theory can be lifted and extended to more general settings. To establish the relationship between an operator ideal and its natural polynomial extensions, the notions of a coherent sequence of polynomial ideals and of compatibility between polynomial and operator ideals were introduced in [18]. A variant of these notions that involves pairs formed by polynomial and multilinear ideals was considered in [27]. All these concepts are related to the concept of Property (B) and to holomorphy types (see [6]).

The search for such good nonlinear extensions was carried out with considering several different approaches (as the linearization and the factorization introduced by Pietsch in [32] or the more recent one introduced in [3], based on locally $\mathcal{I}$-bounded sets). In particular, the ideal of absolutely summing linear operators has been extended in a wide range of possibilities that have been faced by comparing which properties are satisfied in each class (see [16; 31]). A relevant polynomial ideal is the one formed by all p-dominated polynomials, which play a central role in the theory and have been intensively studied [7; 8; 9; 10; 12; 13; 24]. They were introduced as a generalization of absolutely p-summing linear operators, and their interest lies in the fact that they fulfill a Pietsch-type domination theorem. In fact, more than ten different generalizations of the original Pietsch domination theorem to nonlinear operator ideals have been obtained in the last 10–15 years (see [11; 19; 28; 30] and the references therein).

The theory of multilinear summing operators has found applications to quantum information theory. For example, very recently, estimates for the constants of the multilinear Bohnenblust–Hille inequality (case of real scalars) were used to

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solve problems in quantum information theory (see [15; 25]). The hypercontractivity of the Bohnenblust–Hille inequality for complex homogeneous polynomials was proved in the remarkable paper [20].

The classical Pietsch domination theorem in tandem with Pietsch factorization theorem is in the basis of the linear operator theory. This is why a factorization theorem for p-dominated polynomials has been pursuit concomitantly (see [23; 24; 10]). However, the search of a canonical prototype of a p-dominated polynomial through which any p-dominated polynomial could factor in the spirit of the linear theorem for absolutely summing operators has turned out difficult and tricky. The factorization of p-dominated polynomials requires new techniques, which have been recently developed and can be found in [10] and [14]. These results allow us to prove that the factorization of p-dominated polynomials needs to consider spaces based on $L^p(\mu)$ for a Pietsch measure μ but endowed with a different norm in a way that they do not coincide with the $L^p(\mu)$-norm.

In this paper we isolate the class of p-dominated polynomials that satisfy a Pietsch-type factorization theorem, paying regard to both required ingredients: on one hand, we determine the canonical prototype of a polynomial in the class, and, on the other hand, the factorization is given through a subspace of an L^p-space endowed with the L^p-norm. These polynomials are defined by means of a summing inequality and form an ideal of polynomials, which we call an ideal of factorable p-dominated polynomials. Significantly, factorable p-dominated polynomials are then an appropriated generalization of absolutely summing operators rather than p-dominated polynomials.

1. Definitions and Notation

We use standard notation. Let $m, n \in \mathbb{N}$, and let X, Y, Z, G be Banach spaces over $\mathbb{K} := \mathbb{R}$ or $\mathbb{C}$. Let $1 \leq p < \infty$. As usual, $\ell_p^n(X)$ is the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\|(x_i)_{i=1}^n\|_p = \left(\sum_{i=1}^n \|x_i\|^p\right)^{1/p},$$

and $\ell_{p,w}^n(X)$ is the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\|(x_i)_{i=1}^n\|_{p,w} = \sup_{\|x^*\|_{X^*} \leq 1} \left(\sum_{i=1}^n |\langle x_i, x^*\rangle|^p\right)^{1/p},$$

where X^* is the topological dual of X. The closed unit ball of X^* is denoted by B_{X^*}.

Let us introduce now some basic definitions regarding polynomials. A function $P : X \to Y$ is an *m-homogeneous polynomial* if there exists a unique symmetric m-linear operator $\check{P} : X \times \overset{(m)}{\cdots} \times X \to Y$ such that $P(x) = \check{P}(x, \overset{(m)}{\ldots}, x)$ for every $x \in X$. As usual, $\mathcal{P}(^m X; Y)$ denotes the space of all continuous m-homogeneous polynomials from X to Y endowed with the sup norm. We refer to [21; 22; 26] for the main definitions and properties of these polynomials.

Recently, the notion of ideal of m-homogeneous polynomials has been intensively studied (see, e.g., [2; 3; 5]).

A polynomial $P \in \mathcal{P}(^mX; Y)$ is *p-dominated* if there is a constant $C > 0$ such that for every $(x_j)_{j=1}^n \subset X$, we have

$$\|(P(x_j))_j\|_{p/m} \le C[\|(x_j)_j\|_{p,w}]^m. \tag{1}$$

We write $\mathcal{P}_{d,p}(^mX; Y)$ for the space of all p-dominated m-homogeneous polynomials. It is well known that $(\mathcal{P}_{d,p}, \|\cdot\|_{d,p})$ is a Banach ideal of polynomials if $p \ge m$. The norm for this space is given by the infimum of all constants C appearing in (1). Some fundamental results on p-dominated homogeneous polynomials between Banach spaces can be found in [5; 23; 24].

A polynomial $P \in \mathcal{P}(^mX; Y)$ is *weakly compact* if it maps bounded sets to relatively weakly compact sets.

2. Factorable p-Dominated Polynomials

Recently, some factorization theorems have been obtained for p-dominated polynomials and related classes of operators (see [1; 14] and the references therein). However, although these results are quite general and hold for broad classes, they are not giving factorizations through L^p-spaces in the fashion of Pietsch's factorization theorem that one could expect. In [4] it is shown that dominated polynomials are not always weakly compact. This fact obligated to consider factorizations through linear subspaces of L^p-spaces conveniently renormed. Our aim is to determine the class of homogeneous polynomials that factor through subspaces of L^p-spaces keeping the L^p-norm and characterize them by means of a summability property.

We will show in this section that the following restricted class of p-dominated m-homogeneous polynomials is exactly the one whose polynomials satisfy the desired factorization theorem.

Given $x \in X$, by $\langle x, \cdot\rangle^m$ we mean the m-homogeneous polynomial $x^* \mapsto \langle x, x^*\rangle^m$. It is well known that for dual Banach spaces $X = Y^*$, the supremum given in the definition of $\|\cdot\|_{p,w}$ can be taken over all elements in B_Y. Therefore, when considering the space $\mathcal{P}(^mX^*; \mathbb{K})$ as a dual space, it can be proved that

$$\left\|\left(\sum_{i=1}^k \lambda_j^i \langle x_j^i, \cdot\rangle^m\right)_j\right\|_{p,w} = \sup_{\|x^*\|_{X^*}\le 1}\left(\sum_{j=1}^n\left|\sum_{i=1}^k \lambda_j^i \langle x_j^i, x^*\rangle^m\right|^p\right)^{1/p}$$

for any $x_j^i \in X$ and scalars λ_j^i, $1 \le j \le n$, $1 \le i \le k$, $n, k \in \mathbb{N}$.

Definition 2.1. Let $p \ge 1$. A polynomial $P \in \mathcal{P}(^mX; Y)$ is *factorable p-dominated* if there is a constant $C > 0$ such that for every set of vectors $x_j^i \in X$ and scalars λ_j^i, $1 \le j \le n$, $1 \le i \le k$, $n, k \in \mathbb{N}$, we have

$$\left\|\left(\sum_{i=1}^k \lambda_j^i P(x_j^i)\right)_j\right\|_p \le C\left\|\left(\sum_{i=1}^k \lambda_j^i \langle x_j^i, \cdot\rangle^m\right)_j\right\|_{p,w}. \tag{2}$$

We write $\mathcal{P}_{Fd,p}(^mX; Y)$ for the space of all factorable p-dominated m-homogeneous polynomials. The factorable p-dominated norm $\|P\|_{Fd,p}$ is given by the infimum of all $C > 0$ appearing in (2).

REMARK 2.2. Note that we could have taken for each j different many summands, which turns out to be equivalent to consider k summands for every j.

Clearly, the class of factorable p-dominated polynomials is included in the one formed by all pm-dominated polynomials. Indeed, if $P \in \mathcal{P}(^mX; Y)$ is factorable p-dominated, given $x_1, \dots, x_n \in X$, taking $k = 1$ and $\lambda_1^1 = \cdots = \lambda_n^1 = 1$, it follows that

$$\begin{aligned}\|(P(x_j))_j\|_{pm/m} &= \|(P(x_j))_j\|_p \\ &\le \|P\|_{Fd,p} \sup_{\|x^*\|_{X^*} \le 1} \left(\sum_{j=1}^{n} |\langle x_j, x^* \rangle^m|^p \right)^{1/p} \\ &= \|P\|_{Fd,p} \|(x_j)_j\|_{mp,w}^m.\end{aligned}$$

We will see that this inclusion is strict. A similar notion of weighted summability is considered in [29] when characterizing arbitrary nonlinear mappings that satisfy a Pietsch domination-type theorem around a given point.

The next proposition shows that factorable p-dominated polynomials satisfy the ideal property for polynomials. We omit the easy proof.

PROPOSITION 2.3. *Let $p \ge 1$, and let G, X, Y, Z be Banach spaces. If $P \in \mathcal{P}_{Fd,p}(^mX; Y)$, $v : Y \to Z$, and $u : G \to X$ are continuous linear operators, then $v \circ P \circ u \in \mathcal{P}_{Fd,p}(^mG; Z)$ and $\|v \circ P \circ u\|_{Fd,p} \le \|v\| \cdot \|P\|_{Fd,p} \cdot \|u\|^m$.*

Factorable p-dominated polynomials form a polynomial ideal (we refer to [5] for the definition) that generalizes the ideal of absolutely p-summing linear operators. The next theorem shows that these polynomials satisfy the corresponding domination theorem. To avoid a repetition of standard arguments to prove it, we will use the abstract and general theorem given in [11] that unifies known domination results in several different linear and nonlinear operator ideals. A simplified version of this general result appears in [28], and a more abstract version has been considered in [30].

THEOREM 2.4. *Let $p \ge 1$. An m-homogeneous polynomial $P \in \mathcal{P}(^mX; Y)$ is factorable p-dominated if and only if there are a regular Borel probability measure μ on B_{X^*}, endowed with the weak-star topology, and a constant $C > 0$ such that*

$$\left\| \sum_{i=1}^{k} \lambda^i P(x^i) \right\| \le C \left(\int_{B_{X^*}} \left| \sum_{i=1}^{k} \lambda^i \langle x^i, x^* \rangle^m \right|^p d\mu \right)^{1/p}$$

for all $x^1, \dots, x^k \in X$ and scalars $\lambda^1, \dots, \lambda^k$. In this case, the factorable p-dominated norm of P coincides with the infimum of the constants C above.

Proof. A factorable p-dominated m-homogeneous polynomial P is RS-abstract p-summing (see [11; 28] for the definition) for $R : B_{X^*} \times (\mathbb{K} \times X)^{(\mathbb{N})} \times$

$\mathbb{R} \to [0, \infty)$ (where $(\mathbb{K} \times X)^{(\mathbb{N})}$ denotes the finite sequences in $\mathbb{K} \times X$) and $S : \mathcal{P}(^m X; Y) \times (\mathbb{K} \times X)^{(\mathbb{N})} \times \mathbb{R} \to [0, \infty)$ given by

$$R(x^*, (\lambda^1, x^1, \ldots, \lambda^k, x^k), b) := \left| \sum_{i=1}^{k} \lambda^i \langle x^i, x^* \rangle^m \right|$$

and

$$S(Q, (\lambda^1, x^1, \ldots, \lambda^k, x^k), b) := \left\| \sum_{i=1}^{k} \lambda^i Q(x^i) \right\|,$$

$(\lambda^1, x^1, \ldots, \lambda^k, x^k) \in (\mathbb{K} \times X)^{(\mathbb{N})}$, $x^* \in B_{X^*}$, $b \in \mathbb{R}$, $Q \in \mathcal{P}(^m X; Y)$. Theorem 2.2 in [11] or Theorem 3.1 in [28] gives the result. □

Let us show in what follows the factorization theorem for the class of the factorable p-dominated polynomials. Let $i_X : X \to C(B_{X^*})$ be the evaluation map given by $i_X(x) := \langle x, \cdot \rangle$, $x \in X$. Given a Borel measure μ on B_{X^*}, let $j_p : C(B_{X^*}) \to L_p(\mu)$ be the canonical map that identifies each continuous function with itself when considered as an element of $L_p(\mu)$. Using this linear map, Meléndez and Tonge [24, Theorem 13] prove a factorization theorem for dominated polynomials P of the form $P = Q \circ j_p \circ i_X$, where the map Q that closes the factorization is a homogeneous polynomial. Let us define the canonical factorable p-dominated m-homogeneous polynomial through which any other polynomial of the class must factor. Define $j_p^m : i_X(X) \to L_p(\mu)$ as

$$j_p^m(x) := (j_p \circ i_X(x))^m = \langle x, \cdot \rangle^m, \quad x \in X.$$

Some efforts have been made to get a factorization through the canonical homogeneous polynomial $j_{p/m}^m$ in [10; 14]. For instance, in [14, Theorem 4.7] it is proved that assuming that (B_{X^*}, w^*) is separable, a polynomial $P \in \mathcal{P}(^m X; Y)$ is p-dominated if and only if P factors as $P = u \circ j_{p/m}^m \circ i_X$, where the linear operator u is continuous for a suitable norm on the domain.

PROPOSITION 2.5. *The m-homogeneous polynomial j_p^m is factorable p-dominated, and its factorable p-dominated norm is less than or equal to* 1.

Proof. Consider $x_j^i \in X$ and scalars λ_j^i, $1 \le j \le n$, $1 \le i \le k$, $n \in \mathbb{N}$. Then

$$\begin{aligned} \left\| \left(\sum_{i=1}^{k} \lambda_j^i j_p^m(x_j^i) \right)_j \right\|_p &= \left(\sum_{j=1}^{n} \left\| \sum_{i=1}^{k} \lambda_j^i \langle x_j^i, \cdot \rangle^m \right\|_{L_p(\mu)}^p \right)^{1/p} \\ &= \left(\int_{B_{X^*}} \sum_{j=1}^{n} \left| \sum_{i=1}^{k} \lambda_j^i \langle x_j^i, x^* \rangle^m \right|^p d\mu \right)^{1/p} \\ &\le \sup_{\|x^*\| \le 1} \left(\sum_{j=1}^{n} \left| \sum_{i=1}^{k} \lambda_j^i \langle x_j^i, x^* \rangle^m \right|^p \right)^{1/p} \\ &= \left\| \left(\sum_{i=1}^{k} \lambda_j^i \langle x_j^i, \cdot \rangle^m \right)_j \right\|_{p,w}. \end{aligned}$$

□

To obtain the factorization of factorable p-dominated polynomials through j_p^m, we use a direct argument that in a sense is simpler than the one that is used in [14, Proposition 3.4]. The reason is that the linear operator v that closes the diagram is automatically well defined without further requirements on the Pietsch measure involved, due to the stronger domination that is assumed for P.

THEOREM 2.6. *Let $m \in \mathbb{N}$ and $p \geq 1$. A polynomial $P \in \mathcal{P}(^m X; Y)$ is factorable p-dominated if and only if there exist a regular Borel probability measure μ on B_{X^*} with the weak* topology, a closed subspace $\overline{S}$ of $L^p(\mu)$, and a continuous linear operator $v : \overline{S} \to Y$ such that the following diagram commutes*:

$$\begin{array}{ccc} X & \xrightarrow{\ P\ } & Y \\ \big\downarrow{\scriptstyle i_X} & & \big\uparrow{\scriptstyle v} \\ i_X(X) & \xrightarrow{\ j_p^m\ } & \overline{S} \\ \big\downarrow & & \big\downarrow \\ C(B_{X^*}) & & L_p(\mu) \end{array} \tag{3}$$

Proof. Assume that $P \in \mathcal{P}_{Fd,p}(^m X; Y)$ and consider the polynomial $j_p^m \circ i_X : X \to L_p(\mu)$, where μ is a Pietsch measure for P given by Theorem 2.4. By Proposition 2.5, this polynomial is factorable p-dominated. Consider the subspace S of $L_p(\mu)$ given by the linear span of $j_p^m \circ i_X(X)$ and the linear operator $v : S \to Y$ given by $v(z) := \sum_{i=1}^n \lambda_i P(x_i)$ for $z = \sum_{i=1}^n \lambda_i \langle x_i, \cdot\rangle^m \in S$. Let us show that this map is well defined. Consider another representation for z, $z = \sum_{i=1}^k \tau_i \langle y_i, \cdot\rangle^m$, that is, $\sum_{i=1}^n \lambda_i \langle x_i, \cdot\rangle^m$ and $\sum_{i=1}^k \tau_i \langle y_i, \cdot\rangle^m$ are equal μ-a.e. Consider the element $w = \sum_{i=1}^n \lambda_i \langle x_i, \cdot\rangle^m - \sum_{i=1}^k \tau_i \langle y_i, \cdot\rangle^m$. Then w equals 0 μ-a.e. We need to show that $v(w)$ equals 0 too. Since by Theorem 2.4 we have that

$$\left\| \sum_{i=1}^n \lambda_i P(x_i) - \left(\sum_{i=1}^k \tau_i P(y_i) \right) \right\| \\ \leq C \left(\int \left| \sum_{i=1}^n \lambda_i \langle x_i, \cdot\rangle^m - \left(\sum_{i=1}^k \tau_i \langle y_i, \cdot\rangle^m \right) \right|^p d\mu \right)^{1/p} = 0,$$

we have that $v(w) = 0$ and so v is well defined. Moreover, also by Theorem 2.4 we obtain that

$$\|v(z)\| = \left\| \sum_{i=1}^n \lambda_i P(x_i) \right\| \leq C \left\| \sum_{i=1}^n \lambda_i \langle x_i, \cdot\rangle^m \right\|_{L_p(\mu)} = C \|z\|_{L_p(\mu)}.$$

Therefore, v is continuous, and $\|v\| \leq C$. To get the desired factorization, it suffices now to extend v to the completion $\overline{S}$ of S in $L^p(\mu)$.

The converse is a consequence of Proposition 2.5 and the ideal property given in Proposition 2.3. □

REMARK 2.7. Let us consider $1 < p < \infty$. Botelho [4] (see also [17]) proved the existence of a p-dominated polynomial that is not weakly compact. This example was used in [10] to prove that p-dominated m-homogeneous polynomials could not be expected to factor through a subspace of an L^p-space, and it justified the introduction of a new norm in order to obtain a factorization theorem for all p-dominated polynomials. The fact that we keep the $L^p(\mu)$-norm on the subspace $\overline{S}$ guarantees that the m-homogeneous polynomial $j_p^m : i_X(X) \to \overline{S}$ is weakly compact. Actually, since $\overline{S}$ is reflexive, the bounded set $j_p^m(B_{i_X(X)})$ is relatively weakly compact. The factorization given in Theorem 2.6 allows us to conclude that every factorable p-dominated polynomial is weakly compact. Botelho's example [4] proves now that the class of factorable p-dominated polynomials does not coincide with the class of pm-dominated polynomials.

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