

Effects of member loss on the structural integrity of tensegrity systems

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Abstract

Tensegrity systems are statically and kinematically indeterminate systems. It may be mistakenly believed that this inherent redundancy provides a large measure of safety against collapse. However, a number of members are critical to system integrity, with the loss of any of them likely to produce serious strength reductions. Furthermore, when these members are lost suddenly, their forces are shed into the structure in a dynamic manner, leading to yet more severe damage. This paper presents a numerical investigation into the static and dynamic response of plane tensegrity systems due to the gradual and sudden member loss. According to the results of this study, in some cases, the dynamic effect of member loss caused the occurrence of progressive collapse. It was shown that in several cases, static analysis cannot provide a correct and realistic picture of the behavior of the damaged tensegrity system and would lead to a significant overestimate of the load carrying capacity of the structure. The conclusions drawn from such a study can in turn, lead to the suggestion of some simple guidelines for the design of such systems.

Keywords: tensegrity systems, self-stress levels, structural integrity, collapse mechanisms.

1. Introduction

Tensegrity systems are a class of truss structures consisting of a continuous set of cables and a set of struts. They are stabilized by a self stress state that is the internal stress state established during assembling.

In tensegrity systems researchs, more attention has been paid so far to the form-finding of these systems than to their static and dynamic behaviours. There are few studies undertaken on the effect of member loss on tensegrity systems. In this regard, we can refer to the “effect of cable rupture on tensegrity systems” which was performed by Ben Kahla and Moussa [3], in which, the behavior of a beam-like tensegrity system was investigated without applying external loads.

The effect of member loss on space trusses was studied by many researchers such as Murtha-Smith [8], El-sheikh [5] and Malla [6]. It was illustrated that a loss of a member in a critical truss area was more serious than a member loss in another area. Further to this, it was indicated that when a truss member is buckled, it will be snapped through to a low post-buckling load. Since this phenomenon was rapid, dynamic effects could develop, leading to a further damage in the space truss. Nevertheless, so far no study was conducted to confirm and examine the effect of member loss on nonlinear behavior of double layer tensegrity systems under external loads.

Tensegrity systems are statically and kinematically indeterminate systems. It may be mistakenly believed that this inherent redundancy provides a large measure of safety against collapse. However, a number of members are critical to system integrity, with the loss of any of them likely to produce serious strength reductions. Furthermore, when these members are lost suddenly, their forces are shed into the structure in a dynamic manner, leading to yet more severe damage.

This paper presents a numerical investigation into the static and dynamic response of plane tensegrity systems in the event of gradual and sudden member loss. The emphasis was given to account for the dynamic nature of the member loss. The response and characteristic of the structure investigated include load carrying capacity in static analysis and displacement time-history of the configurations in dynamic analyses. The effect of the self-stress level and damping ratios was also investigated in the sensitivity analyses of tensegrity system.

2. Method of analysis

The tensegrity system was analyzed using ABAQUS, a nonlinear finite element software package. The analyses considered both geometric and material nonlinearities. The cables and struts were modeled as simple two-node truss elements with unilateral rigidity of tension and compression, respectively. The tension and compression characteristics of truss members considered in the present study were as shown in Figures 1 and 2. There are several main causes of geometrical and material nonlinearity in tensegrity structures. Therefore, in the collapse analysis of these structures, material and geometrical nonlinearity should be considered [1].

Analysis of the system before member removal was static, and followed the well-known equilibrium equation [2]:

$${}^{t+\Delta t}K^{(i-1)}\Delta U^{(i)} = {}^{t+\Delta t}\lambda^{(i)}R - {}^{t+\Delta t}F^{(i-1)} \quad (1)$$

$${}^{t+\Delta t}U^{(i)} = {}^{t+\Delta t}U^{(i-1)} + \Delta U^{(i)}$$

where R is the externally applied nodal load vector, F the nodal forces that correspond to the element stresses, K the tangent stiffness matrix, λ a scalar load factor and ΔU the incremental displacement vector.

When a member was removed gradually, its contribution to K disappeared and its internal force transmitted to the remaining structure in small steps. Having stabilized under this force, the incomplete tensegrity system was then further loaded and its performance

obtained using the same Equation 1. In this case, to trace the non-linear behavior of tensegrity system, Riks method was adopted.

However, in the case of sudden member loss, the following equation of motion was used to determine tensegrity system response [6]:

$$M\ddot{U} + C'\dot{U} + K'U = R \quad (2)$$

where M is the mass matrix, \ddot{U} the system acceleration vector, \dot{U} the system velocity vector, K' and C' the stiffness matrix and damping matrix of the structure without the contribution of the removed member. It is worth noting that while K and C was affected by member losses, M remained unchanged to indicate that damaged members disappeared structurally but remained physically to be part of the structure [5].

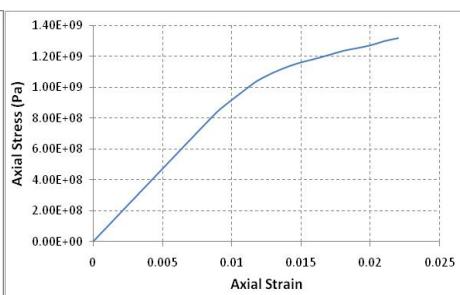
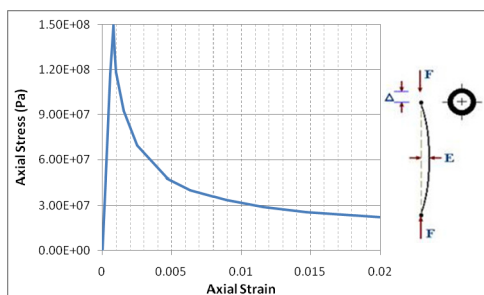


Figure 1: The axial strain-axial stress responses of the struts with the slenderness ratio of $L/r=100$ ($\epsilon=0.001L$)

Figure 2: Stress-strain relationship for cable [4]

In each analysis only one member was removed from the overall tensegrity system, and the system was analyzed to determine the effect of member removal. Member loss could be realized either gradually over part of loading history on the model, or suddenly at any load level. If the loss of member is gradual, then the redistributions will be gradual, and static analysis should be adequate. However, if the member loss is sudden and the load on the model is nonzero, then dynamic effects can come into play [5]. In this paper, for every member considered to be removed, two analysis was performed.

- Firstly, with a member loss that took place gradually. In this stage, static analysis was performed and for tracing the non-linear behavior of the model, Riks method was adopted. As the damage is gradual, it does not differ to remove the member at any load level.
- Secondly, with a sudden member loss that was triggered at design load level (as mentioned by SLS) and at 60% of ultimate load level. The last high load level was chosen because it could produce a large internal force in the member under consideration. In addition, prior to this load level, the local collapse due to buckling of struts or rupture of cables did not occur.

The analysis described in the first stage was nonlinear static analysis and accomplished using Arc-length (Riks) method. However, the analysis mentioned in the second stage involved four steps. In the first, the complete tensegrity system (i.e. without any member losses) was pre-stressed through applying initial stresses to cables. Then the system was loaded with either design load level (SLS) or 60% of the ultimate load. This was followed by the sudden removal of a truss member in the third step. Upon member removal, a sudden redistribution of internal member forces took place. Following that, nonlinear dynamic analysis was carried out keeping constant the loads and the masses. It is worth noting that after third step, in the strained configuration, an eigenvalue analysis was performed to determine incremental step and damping ratios. The damping matrix was formed using Raleigh type damping. This is achieved by introducing two factors α_m and β_s , which are constants to be determined from two given damping ratios that correspond to two unequal frequencies of vibration. The time increment was chosen such that $\Delta t \leq \frac{1}{20} T_{co}$, in which $T_{co} = 2\pi/\omega_{co}$ and $\omega_{co} = 4\omega_0$, ω_0 being the system first natural frequency [2]. In the performed analysis, nodal masses corresponding to the applied load was also included.

3. Double-layer tensegrity system

The studied system is a square grid 9m long constituted of 36 square truncated pyramids. This system was formed by node-to-node connection of modules (Figures 3, 4 and 5). The height of the grid is 1.15m, giving an aspect ratio of about 1/8. Therefore, the assembled grid has 133 nodes and 516 components (144 struts and 372 cables). The grid is supposed to be resting on the external nodes of the lower layer. The design process of this system is composed of two stages [7]:

- i. A Service Limited States (SLS) design ensures that the deflection criterion is met which is not larger than 1/200th of the span. Also local stability of the elements and overall stability of the structure is verified such that none of the cables in the structure must be slackened.

For this stage, the combination of $\mathbf{G} + \mathbf{Q} + \mathbf{S}$ is used in which (G) is the self weight of the structure, (Q) the active loads and (S) the self-stress level. As for the choice of the self-stress level, 50% of the critical load in simple compression is chose for the struts. A self weight load of 250 N/m² was considered which includes the self weight load of the elements and a potential surface of cladding. The live load was taken as a load of snow which is 1100 N/m².

- ii. An Ultimate Limited States (ULS) verification ensures the overall stability of the structure under extreme loading. The loads to be taken into account are as follows:

1.35 G + 1.5 Q + 0.8 S for the resistant self-stress and **1.35 G + 1.5 Q + 1.2 S** for the acting self-stress. In the ULS, the slackness in the cables were accepted if there is no doubt about the overall stability of the structure.

With these conditions, a cross-section of 4.14 cm² for the struts and of 0.875 cm² for the cables was obtained.

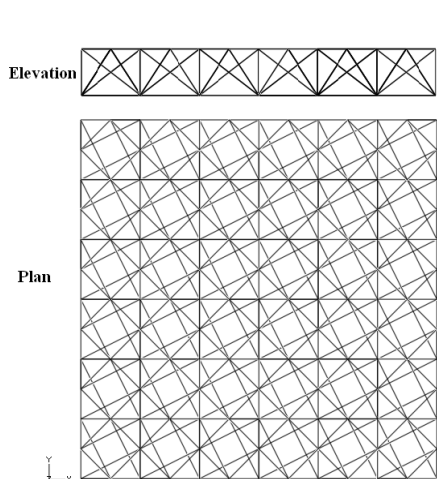


Figure 3: Tensegrity grid formed of 36 square truncated pyramid (6×6)

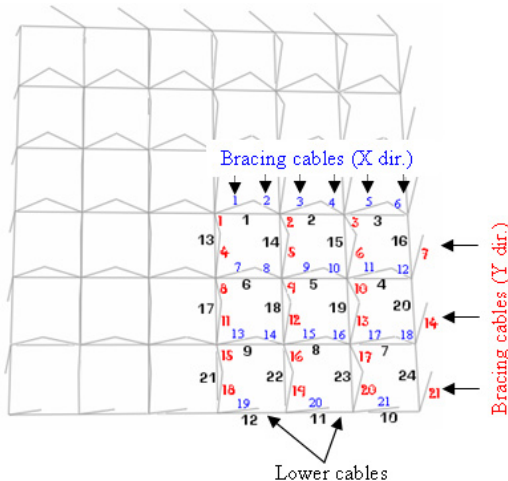


Figure 4: Lower and bracing cables numbering

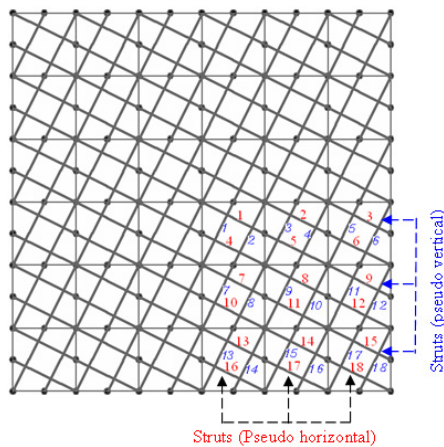


Figure 5: Strut numbering

Fig. 6 shows the value of the stresses in the elements of tensegrity system after applying self-stress. The self-stress levels are corresponding to the configurations in which struts have slenderness ratio of 100. In this figure, the struts are numbered from 1 to 144 and the cables from 145 to 516.

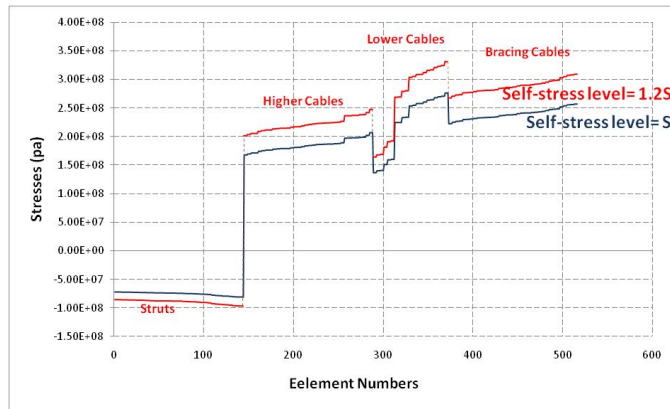


Figure 6: Value of the stresses in the elements of tensegrity system after applying self-stress.

4. Numerical results

In this section, the results of the numerical analyses undertaken on the tensegrity system are presented. Several cases involving gradual and sudden loss of members were considered. In the performed analysis, the effect of gradual and sudden loss of cables and struts of the whole systems were investigated. The effect on the strength due to gradual member loss is illustrated in Tables 1 and 2, which due to symmetry of system, include only quarter of the system. These Tables are corresponding to self-stress level=S and 1.2S, respectively. It should be emphasized that the effect of higher layer cable loss is unimportant.

4.1. Effect of gradual member loss

Table 1 shows clearly that the gradual loss of an edge bottom member produced a small reduction in tensegrity system strength (maximum 8% due to member loss 10, 11, 12, 16, 20 and 24 in Table 1). However, the gradual removing another bottom member except edge members leads to large reduction in strength (as high as 35% in cable 18).

According to table 1, the effect of gradual losing of an edge-bracing member is smaller than that of an edge lower member (maximum 5% due to removing member 19, 20, 21 in the X direction and 7, 14, 21 in the Y direction). However, it was found that the strength reductions due to the gradual loss of another bracing cable rather than edge one were up to 33% (at bracing cable 17 in the Y direction).

The influence of member loss become more serious with losing of pseudo horizontal struts 3, 5, 9, 11, 15, 16, 17 and pseudo vertical struts 1, 7, 14. Losing of pseudo horizontal strut 15 located at the corner module has the worst effect on truss behavior (30.87% strength reduction and more brittle behavior) compared to other critical struts.

Unlike configuration corresponding to self-stress level=S, in this case, as illustrated in Table 2, the effect of removing an edge member is relatively important (up to 30.5% strength reduction in member 11). The effect of losing of another member rather than edge one on the behavior of this configuration is similar to the previous one. In addition, the influence of removing some central and closer to central member is almost smaller than that of previous configuration.

Table 1: Strength reductions due to gradual member losses in tensegrity system corresponding to self-stress level=S.

Removing Lower Cable	1	2	3	4	5	6	7	8	9	10	11	12
	27.85%	32.28%	15.63%	26.54%	29.06%	31.70%	29.00%	34.26%	18.92%	7.85%	3.84%	2.52%
	13	14	15	16	17	18	19	20	21	22	23	24
	27.85%	29.18%	20.75%	2.34%	32.39%	35.00%	24.72%	3.78%	15.63%	11.53%	18.21%	4.31%
Removing Bracing Cable in the X.dir	1	2	3	4	5	6	7	8	9	10	11	12
	6.88%	3.72%	7.65%	3.71%	15.07%	0.10%	4.33%	14.04%	6.12%	3.79%	20.23%	0.05%
	13	14	15	16	17	18	19	20	21			
	3.64%	16.97%	6.69%	5.88%	6.67%	0.22%	0.00%	0.00%	4.63%			
Removing Bracing Cable in the Y.dir	1	2	3	4	5	6	7	8	9	10	11	12
	6.88%	6.18%	9.09%	3.72%	2.73%	11.17%	-0.02%	7.65%	7.30%	25.86%	3.71%	1.66%
	13	14	15	16	17	18	19	20	21			
	6.81%	0.00%	15.07%	3.75%	32.83%	0.10%	0.00%	1.21%	3.37%			
Removing Pseudo Horizontal Strut	1	2	3	4	5	6	7	8	9	10	11	12
	12.28%	9.39%	26.91%	10.25%	22.19%	1.82%	8.89%	14.86%	29.27%	15.75%	26.40%	1.98%
	13	14	15	16	17	18						
	8.82%	8.92%	29.74%	18.57%	22.99%	2.56%						
Removing Pseudo Vertical Strut	1	2	3	4	5	6	7	8	9	10	11	12
	19.57%	13.26%	8.61%	11.10%	17.42%	7.44%	26.81%	17.83%	8.74%	12.47%	13.77%	2.76%
	13	14	15	16	17	18						
	0.87%	24.13%	0.62%	13.42%	5.31%	0.55%						

Table 2: Strength reductions due to gradual member losses in tensegrity system corresponding to self-stress level=1.2S.

Removing Lower Cable	1	2	3	4	5	6	7	8	9	10	11	12
	26.74%	33.78%	15.57%	26.74%	30.13%	31.35%	24.31%	36.46%	25.20%	18.33%	30.47%	26.26%
	13	14	15	16	17	18	19	20	21	22	23	24
	26.75%	26.74%	32.30%	10.31%	33.81%	30.37%	29.47%	4.57%	15.56%	10.50%	19.90%	4.30%
Removing Bracing Cable in the X.dir	1	2	3	4	5	6	7	8	9	10	11	12
	8.09%	1.35%	5.11%	4.34%	31.03%	0.26%	4.43%	8.82%	6.51%	6.72%	26.33%	0.03%
	13	14	15	16	17	18	19	20	21			
	4.37%	19.94%	1.29%	1.83%	1.74%	0.34%	0.11%	0.04%	1.12%			
Removing Bracing Cable in the Y.dir	1	2	3	4	5	6	7	8	9	10	11	12
	8.09%	6.29%	7.67%	1.35%	2.49%	7.73%	0.28%	5.11%	3.28%	26.44%	4.34%	4.84%
	13	14	15	16	17	18	19	20	21			
	0.96%	0.00%	31.03%	1.66%	41.74%	0.26%	0.56%	3.49%	0.84%			
Removing Pseudo Horizontal Strut	1	2	3	4	5	6	7	8	9	10	11	12
	12.29%	9.89%	28.65%	10.67%	27.35%	2.15%	9.82%	10.81%	39.00%	15.31%	26.02%	2.05%
	13	14	15	16	17	18						
	5.13%	9.98%	36.29%	13.66%	24.72%	3.44%						
Removing Pseudo Vertical Strut	1	2	3	4	5	6	7	8	9	10	11	12
	17.55%	12.30%	8.43%	12.24%	12.28%	10.00%	27.44%	16.64%	11.40%	8.77%	7.90%	4.62%
	13	14	15	16	17	18						
	0.98%	29.03%	0.69%	9.63%	1.55%	0.64%						

The effect of any bracing cable loss is more important than of the previous configuration. The worst effect is belonged to the losing of bracing cable 17 in the Y direction. This includes a severe 41.74% strength reduction in the overall strength of this configuration. By comparing this result with Table 1, it is observed that with increasing self-stress level, the effect of critical member loss of struts on the strength reduction becomes also more serious.

4.2. Effect of sudden member loss

As explained before in section 2, sudden member loss has dynamic effect on the structure. In addition, since this phenomenon is rapid, dynamic effects could develop, leading to a further damage in tensegrity system. When member loss occurs in the structure which is under load, energy stored in the structure is released and this induces a state of transient vibration in the structure. Results of analyses and discussions are presented in this section.

Figures 7 to 10 show time-vertical displacement responses of top central node of the tensegrity system corresponding to self-stress level=S during the time period from 2 s (removing time) to 7 s. Figures 7 and 8 represent the influence of individual lower cable and bracing cable loss at design load level (SLS), whereas Figures 9 and 10 represent the influence of sudden loss of an individual lower cable and strut on the behavior of the tensegrity system at 60% ultimate load level.

It is observed that, as shown in Figures 7 and 8, sudden loss of any individual lower cable and bracing cable corresponding to self-stress level=S at design load level (SLS), causes an oscillation response of the structure, and owing to the damping properties of the structural system, this oscillation gradually damps out. Consequently, this configuration after removing any member at design load level can be overloaded statically up to the overall collapse of the structure.

By performing a dynamic analysis on this configuration at 60% ultimate load level, in which an individual lower cable or bracing cable is suddenly removed, it is found that the effect of member loss may cause serious damage to the integrity of the system. As shown in Figure 9, with removing a lower member (e.g. 22), the tensegrity system may show an oscillation behavior which damps out roughly about its static equilibrium position. However, the dynamic effect of losing of some cables such as lower cable 2, 5 and 17 causes the occurrence of partial progressive collapse. In these cases, upon removing member, the structure becomes unstable and snaps to another configuration by experiencing large deflection, and then begins to oscillate and gradually damp out about a stable configuration. In some cases, the dynamic effect of losing of a lower cable (e.g. 8) is so severe. In such a case, a wide propagation of the collapse was always achieved and removing a member was led to overall collapse. Figure 10 shows that the influence of sudden strut loss is not as severe as that of lower cable. In this case, it was observed that sudden loss of pseudo vertical strut 2, 10, 14 and pseudo horizontal strut 2 showed an oscillation behavior whereas sudden losing of pseudo horizontal strut 9 caused overall collapse to be occurred.

The results show that in partial progressive collapse, after damping out, the structure is stable and can continue to carry additional load; whereas in the overall collapse, upon removing member, the whole structure is collapsed.

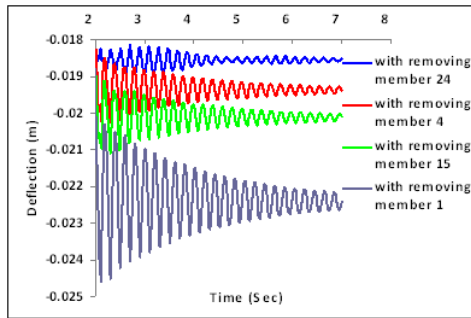


Figure 7: Time-vertical displacement response of the tensegrity system corresponding to self-stress level=S due to lower cable loss

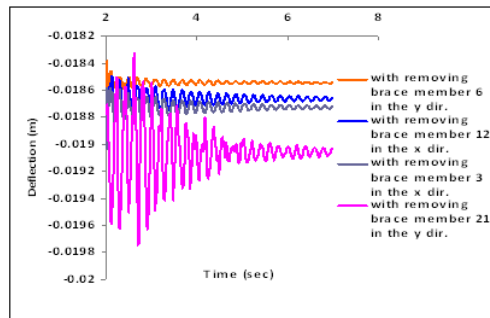


Figure 8: Time-vertical displacement response of the tensegrity system corresponding to self-stress level=S due to lower cable loss

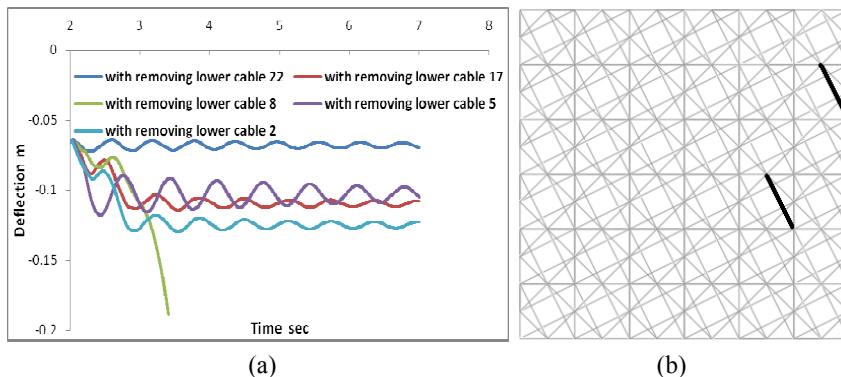


Figure 9: Effect of lower cable loss on the system corresponding to self-stress level=S; a- Time-vertical displacement of the system at 60% ultimate load level; b- Sequence of member failure due to sudden loss of lower cable 2.

Figures 11(a) shows time-vertical displacement response of the tensegrity system corresponding to self-stress level=S and 1.2S under 60% ultimate load level, comparatively. Considering Figure 11(a), it is observed that, in the case of self-stress level=1.2S, losing of some cables leads to partial or overall collapse, whereas removing them in the self-stress level=S, had not the same effect (e.g. member 11). It was indicated that once lower cable 6 is lost, the configuration corresponding to self-stress level=S, showed overall collapse, whereas sudden loss of this member in the case of self-stress level=1.2S, caused the

occurrence of partial progressive collapse. It is also noticed that the change in the deflection of this configuration due to loss of cable 4 is roughly independent of self-stress level. According to results of the performed analyses, with increasing self-stress level, sensitivity of the tensegrity system to the loss of an edge members becomes severe. This may be attributed to the self-stress level and its distribution. In addition, increasing self-stress level caused that slackening of cables to be postponed and rigidity of the systems to be enhanced. By improving rigidity of the systems, the kinetic energy released due to failure of a member increases and therefore, sensitivity of the system increases. Therefore removing a member, especially those located at edge region, cause serious problem. However, through inspecting the results, it was found that as self-stress level is increased, the effect of losing of a member located at center or parallel with central member passing through the center is not as severe as the previous configurations corresponding to the self-stress level (S).

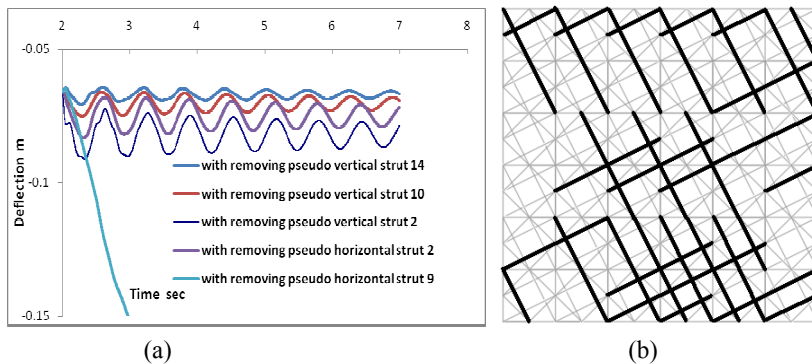


Figure 10: Effect of strut loss on the system corresponding to self-stress level=S; a- Time-vertical displacement response of the system at 60% ultimate load level; b- Sequence of member failure due to sudden loss of pseudo horizontal strut 9

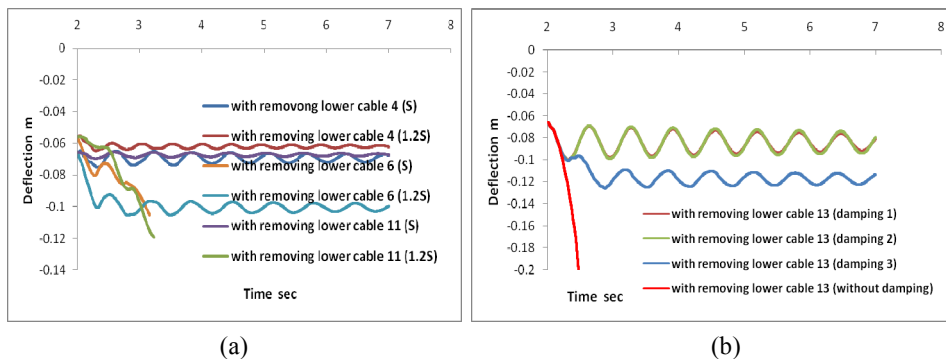


Figure 11: Comparison of the effects of lower cable loss on tensegrity system; a- Time-vertical displacement response of the system with different self-stress level; b- Time-vertical displacement response of the system with different damping ratios

Figure 11(b) indicates that for damping 1 ($\xi_1 = 1.5\%$, $\xi_5 = 2.5\%$) and damping 2 ($\xi_1 = 1.0\%$, $\xi_5 = 1.5\%$), when lower cable 13 is lost suddenly, the tensegrity system showed oscillation behavior, whereas for damping 3 ($\xi_1 = 0.5\%$, $\xi_5 = 1.0\%$) and without applying damping, showed partial progressive collapse and overall collapse, respectively. It must be emphasized that damping ratios have an important role in the sensitivity of tensegrity systems to sudden member loss. In several cases, decreasing damping ratios changes the oscillatory behavior to partial or overall collapse. This issue presented a clear warning to the vibration behavior of tensegrity systems.

5. Conclusions

The study reported herein is concerned with the investigation into the effect of member loss on the integrity of plane tensegrity systems. Emphasis was given to the dynamic nature of the member loss and effects of such dynamic member loss on the overall structural behavior. Responses of a plane tensegrity system with two different self-stress levels were investigated during gradual and sudden loss of members. The results show that the static member loss has generally localized effects, whereas dynamic consideration in the member loss may have wide spread effects in the structures. It was shown that gradual member loss can cause reduction of strength up to 35% and 41.74% in configurations with self-stress level=S and 1.2S respectively. The results presented also show that with increasing self-stress level, tensegrity systems became more sensitive to member loss. Those members especially located at the edges of the tensegrity system have more effect.

According to the results, the dynamic effect of losing of some members under 60% ultimate load level caused the occurrence of partial or overall collapse. In partial progressive collapse, after experiencing large deflection, the structure begins to oscillate and gradually damp out, therefore the structure is stable and can continue to carry additional load, whereas in the overall collapse, a wide propagation of the collapse was always achieved and a member loss was led to overall collapse. However, it was illustrated that a nonlinear static collapse analysis which predicts that a damaged tensegrity system should be safe and can sustain additional load, in several cases cannot provide a correct and realistic picture of the behavior of the damaged tensegrity system. It was observed that using a static approach would lead to a significant overestimate of the load carrying capacity of the structure. In determining effect of sudden member loss on the behavior of tensegrity systems, damping ratio is the most significant parameter. According to the results, in most cases by decreasing damping ratios from 1.5%, 2.5% to low amounts, the oscillatory behavior was changed to partial or overall collapse.

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