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Response Evaluation of Seismically Isolated Lattice Domes using Amplification Factors

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Abstract

Seismic responses of raised lattice domes with substructures are known to be very complicated, and it is known that not only horizontal response but also vertical response is excited under horizontal seismic input. Such action causes damage on structures and non-structural elements such as ceiling and suspended lightning equipments. A seismic isolation system is known to be effective to reduce such responses. In this paper, simple response evaluation methods for lattice domes supported by substructures with seismic isolation bearings are proposed using response amplification factors and linearization techniques with the same concept proposed by authors. Their validities are discussed against the results of time history analyses.

Keywords: Lattice dome, Seismic isolation, Equivalent linear methodResponse Evaluation, Elasto-plastic damper, Viscous damper

1. Introduction

Lattice domes generally have large number of parallel vibration modes, and their amplitude against seismic input changes drastically along the relationship between domes and substructures, this leads the seismic responses of raised domes with substructures to be complicated. Such action causes damage to both structures and non-structural elements such as ceiling and suspended lightning equipments. Recently it is indicated introducing a seismic isolation system between the roof and substructure is effective to reduce such responses by (Kato *et al.* [1]). Also, it has been made clear that the main vibration modes of medium-span latticed domes with substructures are condensed to several modes when the out-of-plane building stiffness increased, and simpler formulas for the distributions of maximum response accelerations were proposed using amplification factors by authors (Takeuchi *et al.*[2]). In this paper, analytical models of domes supported by seismic isolation bearings with elasto-plastic dampers or viscous dampers are constructed, and response of the dome without substructure is examined firstly, and evaluated by using response amplification factors and equivalent linearization techniques. Next, simple response evaluation methods of lattice domes supported by substructures with seismic isolation system between the roof and substructure are proposed. Their validities are discussed against the results of time history analyses including the accuracies for using them as equivaent static loads.

2. Analytical Model

The analytical models are rigidly jointed medium-span latticed domes with substructures, as shown in Figure 1, and the dimensions of the dome are given in Table 1. The elasto-plastic dampers or viscous dampers are inserted beneath the tension ring of the dome. The diameter of the dome is 60m, and its half-subtended angle θ is 30°. The out-of-plane bending stiffness are 100 times of the single layer dome roof, which are equivalent to double layered domes with depth/span ratio of 1/50. The members of the dome are tubular sections as listed in Table 2. The line connects A-O-A’ in Figure 1 is called as the ridge of the

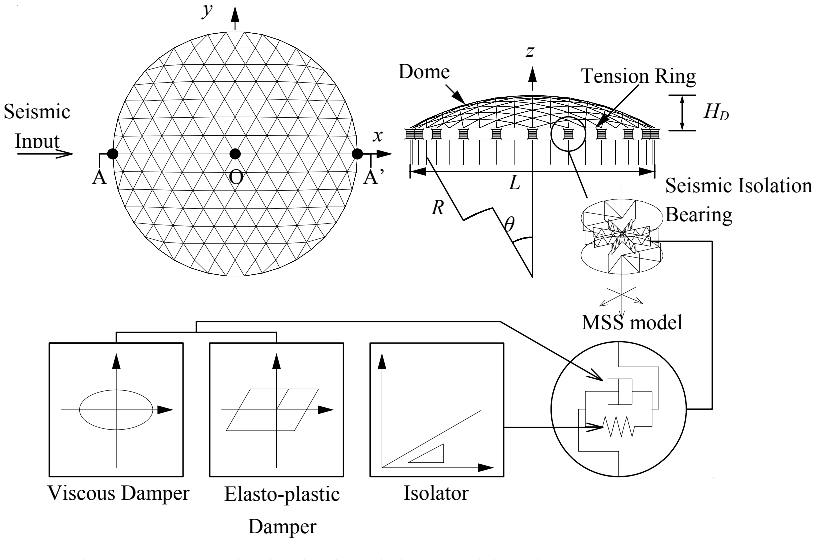


Figure 1: Seismically isolated dome with substructures

Table 1: Dimensions of the dome

Half Subtended Angle θ (deg.)	30
Span of the Dome L (m)	60
Radius of the Dome R (m)	60
Rise of the Dome H_D(m)	8.04
Ridge Member Length l (m)	5.23

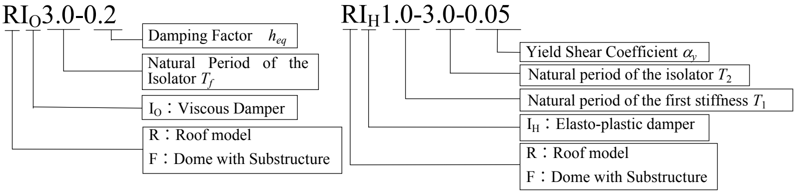


Figure 2: Analytical model index

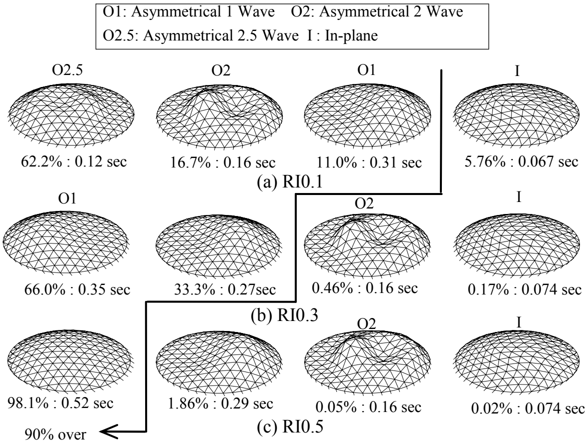


Figure 3: Principal modes along seismic isolation periods

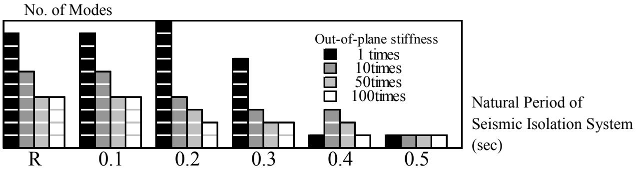


Figure 4: Number of modes satisfying total effective mass ratio as 90% of total

dome. The seismic isolation system is composed of elastic spring of isolator and viscous or elasto-plastic dampers. Viscous damper has linear viscous, and elasto-plastic damper has bi-linear hysteresis, and they are laid out as Multi Shear Spring (MSS) elements. Their stiffness and damping ratio are changed, and the names of these analytical models are defined in Figure 2.

The major natural vibration modes for R0.1, R0.3 and R0.5 are shown in Figure 3 together with effective mass ratios and natural periods. The major vibration modes for R0.1 are governed by out-of-plane deformation. On contrary, in R0.5, the major vibration modes become sway in the horizontal direction. Also, the out-of-plane bending stiffness of the dome is increased by 10, 50 and 100 times of single layer dome, which is equivalent to double layered domes of depth/span ratio of about 1/170, 1/70, 1/50, respectively. When the natural period of isolation system exceed 0.5 sec, the numbers of modes satisfying their total effective mass ratio as 90% of total are unified to sway mode regardless the out-of-plane stiffness as shown in Figure 4.

3. Response of the Dome without Substructures

Firstly, time history response analyses considering geometrical nonlinearity are carried out for various seismic isolated domes without substructure as shown in Table 3. The input earthquake motions are artificial earthquake BCJ-L2, El Centro NS (1940), Taft EW (1952), Hachinohe NS (1968) and JMA Kobe NS (1995), all adjusted to the target spectrum as shown in Figure 5.

Figure 6 shows the maximum response accelerations along the ridge of the dome. The horizontal response accelerations in RI_O with viscous dampers are reduced as the damping factor is increased. However, the vertical response accelerations are not decreased despite the increasing damping factor. In RI_H with elasto-plastic dampers, the vertical response accelerations are increased as initial stiffness of dampers become larger. This is because of the out-of-plane deformation mode being excited by the elastic natural period of the damper.

Table 3 Damper parameters for roof model analyses

Model Name	h_{eq}	C_d (kNsec/m)	T_f (sec)	K_f (kN/m)
RI_O1.5-0.02	0.02	66.6	1.5	6.970×10^3
RI_O1.5-0.2	0.2	665.6		
RI_O1.5-0.3	0.3	998.3		
RI_O1.5-0.4	0.4	1331.1		
RI_O2.0-0.02	0.02	49.9	2.0	3.921×10^3
RI_O2.0-0.2	0.2	499.2		
RI_O2.0-0.3	0.3	748.8		
RI_O2.0-0.4	0.4	998.3		
RI_O2.5-0.02	0.02	39.9	2.5	2.509×10^3
RI_O2.5-0.2	0.2	399.3		
RI_O2.5-0.3	0.3	599.0		
RI_O2.5-0.4	0.4	798.7		
RI_O3.0-0.02	0.02	33.3	3.0	1.742×10^3
RI_O3.0-0.2	0.2	332.8		
RI_O3.0-0.3	0.3	499.2		
RI_O3.0-0.4	0.4	665.6		

Model Name	α_y	δ_y (mm)	T_1 (sec)	K_1 (kN/m)	T_2 (sec)	K_2 (kN/m)
RI_H0.1-3.0-0.01	0.01	0.02	0.1	1.568×10^6	3.0	1.742×10^3
RI_H0.1-3.0-0.05	0.05	0.12				
RI_H0.1-3.0-0.1	0.1	0.25				
RI_H0.1-3.0-0.2	0.2	0.50				
RI_H0.1-3.0-0.3	0.3	0.75				
RI_H0.1-3.0-0.5	0.5	1.24				
RI_H0.3-3.0-0.01	0.01	0.22	0.3	1.742×10^5		
RI_H0.3-3.0-0.05	0.05	1.12				
RI_H0.3-3.0-0.1	0.1	2.24				
RI_H0.3-3.0-0.2	0.2	4.47				
RI_H0.3-3.0-0.3	0.3	6.71				
RI_H0.3-3.0-0.5	0.5	11.18				
RI_H1.0-3.0-0.01	0.01	2.48	1.0	1.568×10^4		
RI_H1.0-3.0-0.05	0.05	12.42				
RI_H1.0-3.0-0.1	0.1	24.84				
RI_H1.0-3.0-0.2	0.2	49.68				
RI_H1.0-3.0-0.3	0.3	74.52				
RI_H1.0-3.0-0.5	0.5	124.20				

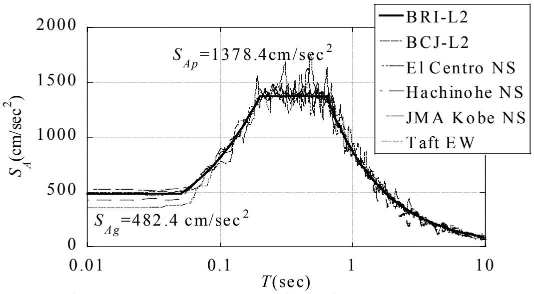
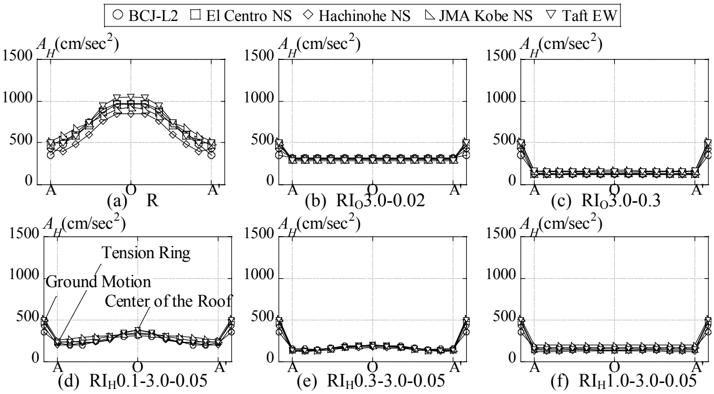


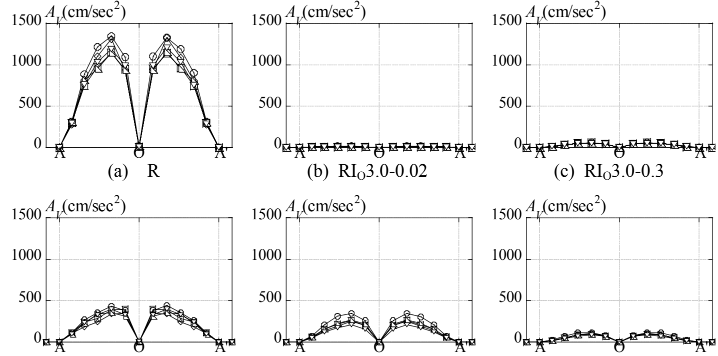
Figure 5: Target acceleration spectrum (h_0=0.02)



(a) R (b) RI_O3.0-0.02 (c) RI_O3.0-0.3

(d) RI_H0.1-3.0-0.05 (e) RI_H0.3-3.0-0.05 (f) RI_H1.0-3.0-0.05

(A) Horizontal



(a) R (b) RI_O3.0-0.02 (c) RI_O3.0-0.3

(d) RI_H0.1-3.0-0.05 (e) RI_H0.3-3.0-0.05 (f) RI_H1.0-3.0-0.05

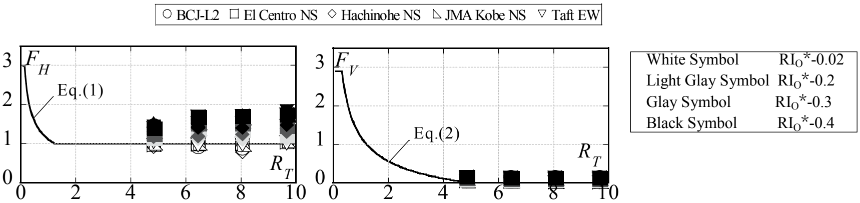
(B) Vertical

Figure 6: Acceleration distributions for roof models

These models are applied to the already proposed response evaluation method for ordinary domes as in previous studies [2]. In these studies, the amplification factors to estimate maximum roof acceleration are expressed in the following equations for relatively shallow (θ<40°) domes.

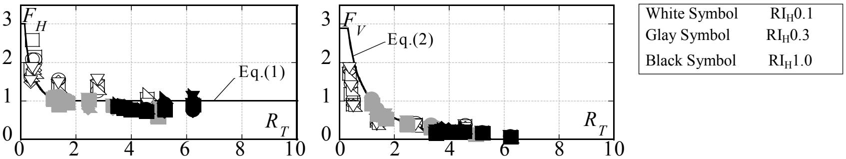
Horizontal:
$$F_H = \begin{cases} 3 & (0 \le R_T \le 5/36) \\ \sqrt{\dfrac{5}{4R_T}} & (5/36 < R_T \le 5/4) \\ 1 & (5/4 < R_T) \end{cases} \tag{1}$$

Vertical:
$$F_V = \begin{cases} 3C_V\theta & (0 \le R_T \le 5/16) \\ \left(\sqrt{\dfrac{5}{R_T}} - 1\right)C_V\theta & (5/16 < R_T \le 5) \\ 0 & (5 < R_T) \end{cases} \tag{2}$$



(a) Horizontal (b) Vertical

(A) RI$_O$ model (Viscose damper)



(a) Horizontal (b) Vertical

(B) RI$_H$ model (Elasto-plastic damper)

Figure 7: Amplification factors along R_T

Where C_V=1.85, and $R_T=T_{eq}/T_R$ is ratio of natural periods between the roof and seismic isolation system. The relationship between the response amplitude factors F_H, F_V calculated from Eq. (3) (4) and the period ratio R_T are plotted in Figure 7,

Horizontal: $$F_H = A_{H\max} / A_{eq}(T_{eq}, h_{eq}) \tag{3}$$

Vertical: $$F_V = A_{V\max} / A_{eq}(T_{eq}, h_0) \tag{4}$$

Where $A_{H\max}$, $A_{V\max}$ are the maximum horizontal and vertical response accelerations in the dome roof respectively, $A_{eq}(T_{eq}/h_{eq})$ and $A_{eq}(T_{eq}/h_0)$ are accelerations estimated by response spectrum in the SDOF model. The equivalent natural period T_{eq} is calculate from isolator's stiffess T_f for viscous dampers. For elasto-plastic damper, however, we porose to use the average of integration between T_1 and $T_{\max}$ which is expressed in Eq.(5), where T_1 is the natural period derived from the elastic stiffness, and $T_{\max}$ is those from the stiffness lead by maximum shear force divided by maximum displacement. This reflects that the equivalent natural perid varies depending on the amplitudes.

$$A_{eq} = \frac{1}{T_{\max} - T_1} \int_{T_1}^{T_{\max}} S_A(T, h_{eq}) dT \tag{5}$$

For this range of present study, Eq. (5) corresponds roughly to A_{eq} calculated by using T_{eq} that is the mean value of T_1 and $T_{\max}$.

$$T_{eq} = (T_1 + T_{\max})/2 \tag{6}$$

The acceleration A_{eq} for the vertical amplitude factors ignores the influence of additional damping in Eq. (4). The proposed methods and time-history analyses on the amplitude factors are compared in Figure 7. They agree each other and the proposed response evaluation method is considered to be applicable.

4. Response of the Dome with Substructures

Next, the response characteristic of those lattice domes supported by substructures with a seismic isolation system is investigated. The beams of the substructure are modeled as rigid, and columns are designed as dimensions in the Table 4. Then the column stiffness K_S and mass ratio R_M are changed as shown in Table 5, where T_S is the natural period of total mass as rigid being supported by substructures. Here the mass ratio R_M is defined as the ratio between the mass of the dome (M_R), and the total mass of dome (M_R) and substructure (M_S).

$$R_M = M_{eq}/M_R = (M_R+M_S)/M_R \tag{7}$$

Table 4 Member size of substructure column

Diameter D (mm)	914.4
Tickness t (mm)	16
Section Area A (cm^2)	452
Moment of Inartia I (cm^4)	4.56×10^5
Young's Modulus E (N/mm^2)	2.05×10^5

Table 5 Parameters of substructure

K_S	R_M	T_S(sec)
1.0	1.2	0.18
0.1	1.2	0.58
10	1.2	0.06
1.0	3	0.29
1.0	22	0.78

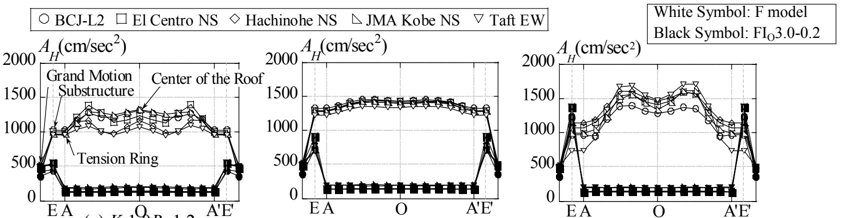
Table 6 Damper parameters for substructure model analyses

(a) Viscous damper

Model Name	h_{eq}	C_d(kNsec/m)	T_f(sec)
$FI_O3.0\text{-}0.2$	0.2	332.8	3.0

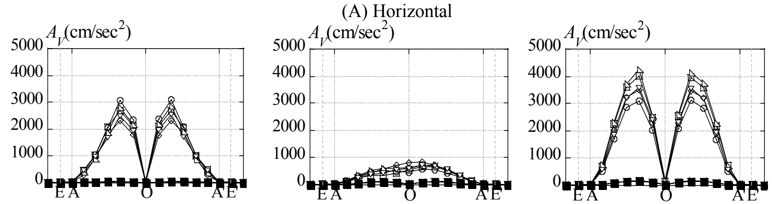
(b) Elasto-plastic damper

Model Name	α_y	δ_y(mm)	T_1(sec)	K_1(kN/m)	T_2(sec)	K_2(kN/m)
$FI_H0.1\text{-}3.0\text{-}0.05$	0.05	0.12	0.1	1.568×10^6	3.0	1.742×10^3
$FI_H0.3\text{-}3.0\text{-}0.05$		1.12	0.3	1.742×10^5		
$FI_H1.0\text{-}3.0\text{-}0.05$		12.42	1.0	1.568×10^4		



(a) K_S1.0R_M1.2 (b) K_S0.1R_M1.2 (c) K_S1.0R_M3

(A) Horizontal



(a) K_S1.0R_M1.2 (b) K_S0.1R_M1.2 (c) K_S1.0R_M3

(B) Vertical

Figure 8: Acceleration distributions for substructure models

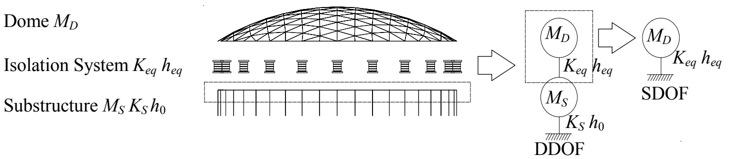


Figure 9: Conversion to simplified model

The parameters of dampers are shown in Table 6, and time history analyses were carried out for these models. Figure 8 shows the maximum response accelerations along the ridge of the dome. The response of F models without a seismic isolation system are heavily affected by substructure characteristics. On the other hand, the responses of the domes with a seismic isolation system are insensitive against substructure, not only acceleration distributions are reduced.Response evaluation of these models is also investigated in the following.

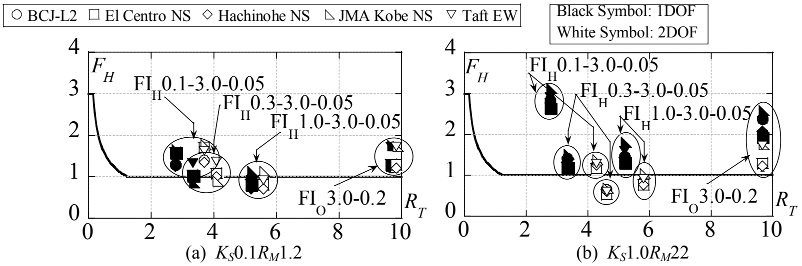
Firstly, the amplitude factors evaluated in the SDOF model in Figure 9 regarding the substructures as rigid as are shown in Figure 10 by black marks. In R_M1.2 models, this method is roughly appreciable. However, in heavy substructures as R_M22 models, the error increases because of responses of the roof being amplified by the response of the substructure.

For the model with the mass ratio exceeding R_M1.2 and the natural periods ratios between isolation system and substructure $\beta = T_{eq}/T_S < 5.0$, the responses should be calculated in DDOF models as proposed by Matsui et.al. [3], as following process.

1) Estimate the natural period T_{eq} and equivalent stiffness K_{eq} of a seismic isolation system. For the elasto-plastic damper, assume a response displacement δ_S in seismic isolation layer, and obtain the equivalent natural period $T_{\max}$ by the equivalent stiffness at the point of maximum amplitude.

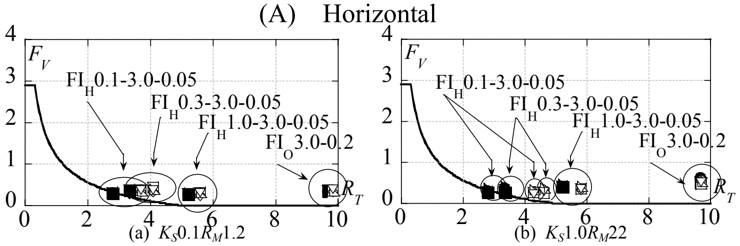
$$T_{eq} = (T_1 + T_{\max})/2 \tag{8}$$

$$K_{eq} = 4\pi^2 M_R / T_{eq}^2 \tag{9}$$



(a) K_S0.1R_M1.2 (b) K_S1.0R_M22

(A) Horizontal



(a) K_S0.1R_M1.2 (b) K_S1.0R_M22

(B) Vertical

Figure 10: Amplification factors along R_T

2) Calculate the response of the roof (the upper part mass) of DDOF as shown in Fig. 9 by combining two mode responses obtained by response spectrum with SRSS.

3) If the response displacement in the seismic isolation layer calculated by 2) does not correspond to assumed displacement, repeat the steps from 1) to 2).

4) Calculate period ratio R_T $(=T_{eq}/T_R)$ of natural period of 1st modes (T_{eq}) to natural period of asymmetric 1 wave mode for dome roof (T_R)

5) Evaluate the maximum acceleration in the dome, by multiplying the amplification factors calculated by Eq. (1)(2) on $A_{eq}(T_{eq},h_{eq})$ for horizontal, and $A_{eq}(T_{eq},h_0)$ for vertical response.

Comparison of the above method and time-history analyses in the amplitude factors are compared in Fig 10 by white marks. In $R_M1.2$ models, F_H and F_V calculated in DDOF generally correspond to those calculated in SDOF, however horizontal errors are mitigated in R_M22 models.

5. Response Evaluation for Detailed Examples

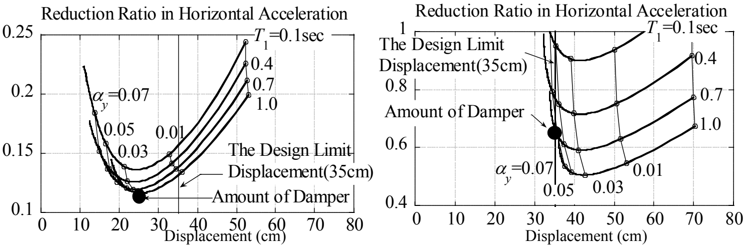
The proposed methods are applied to two detailed design examples; a model with a light, stiff substructure ($R_M1.2T_S0.2$) and another with a heavy substructure having a low degree of stiffness ($R_M22T_S1.0$), and the response reduction effect of the seismic isolation system is evaluated. The natural period of the isolator (T_f) is set as 3.0 sec and the maximum displacement in seismic isolation system is assumed to be 35cm. In $R_M1.2T_S0.2$ model, the effects of substructures are ignored because the stiffness of the substructure is sufficiently greater than the stiffness of seismic isolation system. In $R_M22T_S1.0$ model, where $\beta<5.0$, the response of the roof is calculated in consideration of amplification by the substructure. Figure 11 shows the relation between the horizontal response acceleration reduction rate and the displacement of a seismic isolation layer in the case with elasto-plastic dampers. The response acceleration distributions along the ridge line in each model are shown in Figure 12. The horizontal and vertical response decreased in $R_M1.2T_S0.2$. In $R_M22T_S1.0$ model, in spite of the horizontal response accelerations are reduced, the vertical accelerations are increased because of the close natural period between substructure and seismic isolation system. However, the vertical response of the roof has already been reduced before the introduction of the seismic isolation devices.

From evaluated maximum acceleration $A_{H\max}$ and $A_{V\max}$ with the modified amplification factors, acceleration response distribution can be calculated by the following equations as indicated in the previous paper [2].

Horizontal: $$A_H(x,y) = A_{eq}\left\{1+(F_H-1)\cos\frac{\pi\sqrt{x^2+y^2}}{L}\right\} \tag{10}$$

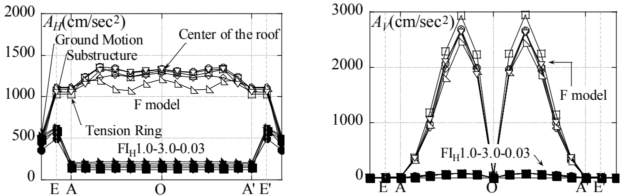
Vertical: $$A_V(x,y) = A_{eq}F_V\frac{x}{\sqrt{x^2+y^2}}\sin\frac{2\pi\sqrt{x^2+y^2}}{L} \tag{11}$$

By using Eq.(10)(11), the maximum acceleration distribution in each model is estimated, and displacements and each member forces are calculated by using the acceleration as the equivalent static load coefficient. The results at all the connectiones in the roof are shown in Figure 13,



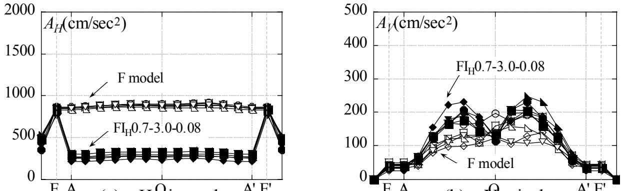
(a) $R_M1.2T_S0.2$ (b) $R_M22T_S1.0$

Figure 11: Response evaluation in FI_H model



(a) Horizontal (b) Vertical

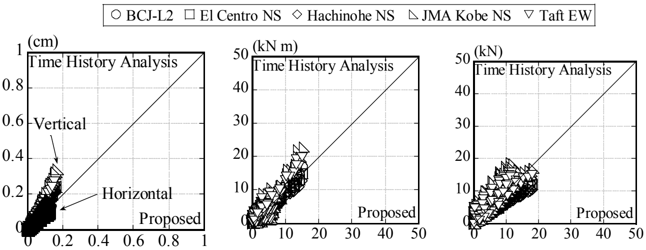
(A) $R_M1.2T_S0.2$



(a) Horizontal (b) Vertical

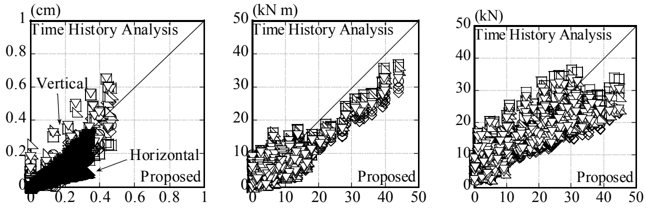
(B) $R_M22T_S1.0$

Figure 12: Acceleration distributions for detailed examples



(a) Displacement (b) Bending Moment Amplitude (c) Axial Force Amplitude

(A)$R_M1.2T_S$



(a) Displacement (b) Bending Moment Amplitude (c) Axial Force Amplitude

(B)$R_M22T_S1.0$

Figure 13: Accuracies of proposed evaluation method

compared with those of time history analysis. As observed in Figure 13, although the results in axial forces and bending moments are distributed relatively widely, the proposed method is considered to be valid for rough estimations.

6. Conclusions

Seismic response of raised domes supported by substructures with a mid-story seismic isolation system was investigated, and the following results were obtained.

1) For the roof model with seismic isolation system, the vertical response accelerations are reduced by extending the natural period, however, little reduction effect by additional damping was observed for vertical response. For the seismic isolation system with elasto-plastic dampers, the vertical response is more reduced when the elastic stiffness of damper is reduced. By considering these effects, the proposed method can evaluate the response more accurately.
2) When the substructure is light and stiff, the effect of the substructure is negligible and the amplitude factor can be estimated by SDOF model regarding the substructures as rigid.
3) When the substructure is much heavier than the roof, and natural period of the substructure approaches that of the seismic isolation system, the maximum response of the roof can be evaluated by combining the equations of amplification factors and a predictive method using DDOF models.

Acknowledgement

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References

[1] Kato S., Nakazawa S., Uchikoshi M., Ueki T., Osugi F.: Earthquake Response of Domes Implemented by Hysteresis Dampers for Earthquake Isolation, *Proccedings of IASS1998 (Sydney)*, 451-459

[2] Takeuchi T., Kumagai T., Ogawa T.: Seismic Response Evaluation of Lattice Shell Roofs with Substructures, *Journal of IASS*, Vol.48, No.3, 2007, 197-210.

[3] Matsui T., Furuike H.: Seismic Response Prediction for Base-Isolated Spatial Structures by the Response Spectrum Method, *Summaries of technical papers of Annual Meeting Architectural Institute of Japan. B-1*, 781-784, 2006 *(in Japanese)*