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Reliability-based Optimization of Spatial Structures using Approximation Model

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Abstract

This study presents an optimization approach for the reliability-based design of spatial structures subjected to severe earthquakes, while uncertainty involved in structural analysis parameters is taken into consideration. Simulated annealing method is utilized to find the structure with minimum weight from available sections, while the specified structural performances in terms of probability are satisfied. The dynamic performance of a spatial structure with uncertainty is evaluated in a probabilistic manner by applying Monte Carlo simulation (MCS). To reduce the high computational costs of MCS, Kriging method is adopted to generate the approximate response surface.

Keywords: Reliability-based design; Spatial structures; Uncertainty; Kriging method; Optimization.

1. Introduction

A structure that is built is different from that is designed, because uncertainty, in materials for example, is unavoidably involved in practical structures. Therefore, safety of the structure, subjected to extreme external loads, might not be accurately evaluated by deterministic structural analysis, as in conventional design procedure. For the purpose of designing a robust structure with reliable capability to survive severe earthquakes, irrespective of the possible uncertainty in the structure, we present in this study a reliability-based optimization design procedure. In particular, we are interested in spatial structures that can cover a large space.

In conventional (deterministic) design procedure, uncertainty involved in the structure is taken into consideration by introducing an empirical coefficient, called *safety factor*, to the structural analysis parameters to reduce their nominal values to 'dependable' values. The basic idea behind the procedure is to design a conservative structure, intended to predict the worst cases of outputs, such as maximum displacements or drift angles, by the combination of the worst cases of inputs—the dependable values of the structural stiffness/strength parameters less than those of the practical structures, and the external loads greater than that may possibly occur. However, this idea can be true only for static cases, and would mislead designers in dynamic cases. For example, the seismic responses of a structure significantly depend on characteristics of the response spectra of the input motion rather than stiffness of the structure; moreover, less strength in some structural members may lead to less responses of the structure owing to more plastic energy dissipation. Thus, conventional design procedure may not lead to conservative design as expected, and could end up with overestimating capacity of the structures, which indicates that more sophisticated approach to considering uncertainty is necessary.

To evaluate dynamic characteristics of a structure subjected to severe earthquake, time history analysis (THA) is the most reliable approach, though much higher computational cost turns out to be necessary, compared to the equivalent static analysis. Moreover, in the framework of performance-based engineering, dynamic performance of a structure subjected to possible uncertainty involved in structural analysis needs to be evaluated in terms of probability. For such purpose, Monte Carlo simulation (MCS) is the most straightforward way, however, it is unlikely to be directly applicable to complex systems, because this needs to carry out the expensive THA for many possible values of the structural parameters, which results in unacceptably expensive computational costs. Other than the direct approach, an alternative approach is to carry out MCS based on the approximate responses, predicted by a limited number of structural analyses.

Metamodels are such models that interpolate the results (dynamic responses) obtained in preliminary experiments (structural analyses) with smooth nonlinear functions, so as to predict the results at which experiments have not been carried out. There have been a number of metamodels developed so far to predict the approximate responses: for example, response surface approximation, radial basis function, artificial neural networks, Kriging method and multivariate adaptive regression splines. Among these methods, Kriging method has gained much attention in engineering literatures because of its high accuracy and low computational cost [1]. Our previous study also shows that Kriging method is of high accuracy in prediction of dynamic responses of spatial structures subjected to severe earthquakes [2].

As the dynamic performance of a structure is available in terms of probability by MCS, we are then in the position to find the optimal structure satisfying specified requirements on structural performance. In this study, we consider the problem of finding the structure with minimum weight, assembled by the available sections. For this kind of typical combinatorial optimization problem, the simulated annealing (SA) method has been proved to be an excellent heuristic method for finding the global optimum, and thus, is adopted in this study.

Following this introduction, Section 2 gives a brief introduction to Kriging method and presents the basic formulations, and briefly summarizes the simulated annealing method for the combinatorial optimization problem in the study; Section 3 uses a two-dimensional arch-type long-span structure as a numerical example to demonstrate the availability of the proposed approach for reliability-based design of spatial structures; and Section 4 concludes the study.

2. Approximate responses and optimal structure

This section gives a brief description of Kriging method for prediction of dynamic responses, and simulated annealing method for finding the optimal structure with the minimum weight while satisfying the dynamic structural performance in probability.

2.1. Kriging method

Kriging method is a spatial prediction approach based on minimization of the mean error of the weighted sum of responses at the sampling points, at which experiments are conducted. The name of 'Kriging' refers to a South African geologist D.G. Krige, who developed the method to carry out statistical evaluation of mining data. It gained further and much wider application in other engineering problems from the end of 1980s, when statisticians were involved in its development. In this section, we summarize the basic equations of Kriging method as described in [3] for the completeness of the study.

Suppose that we consider the uncertainty in n^d structural parameters, and carry out the preliminary analyses at the n^s sampling points $\mathbf{s}_i \in \mathbb{R}^{n^d}$ $(i = 1,\dots,n^s)$. The prediction points, at which responses are to be predicted, are denoted by $\mathbf{x} \in \mathbb{R}^{n^d}$.

Let $\mathbf{r}(\mathbf{x}) \in \mathbb{R}^{n^s}$ and $\mathbf{R} \in \mathbb{R}^{n^s \times n^s}$ denote the correlation vector and matrix, describing correlations of the sampling points and prediction point: the ith entry of $\mathbf{r}(\mathbf{x})$ is the correlation between the prediction point $\mathbf{x}$ and the sampling point $\mathbf{s}_i$, and the (i, j)-entry of $\mathbf{x}$ is the correlation between the two sampling points $\mathbf{s}_i$ and $\mathbf{s}_j$.

The normalized value $\hat{y}_{\mathrm{nor}}$ of the predicted (approximate) response $\hat{y}$, at he prediction point $\mathbf{x}$, is determined as follows by minimizing the mean square error (MSE) and using the best linear unbiased predictor

$$\hat{y}_{\mathrm{nor}}(\mathbf{x}) = \hat{\beta} + \mathbf{r}(\mathbf{x})^{\mathrm{T}} \mathbf{R}^{-1} (\tilde{\mathbf{y}} - \beta \mathbf{i}), \tag{1}$$

$$\hat{\beta} = \frac{\mathbf{i}^{\mathrm{T}} \mathbf{R}^{-1} \tilde{\mathbf{y}}}{\mathbf{i}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{i}}. \tag{2}$$

where every entry in $\mathbf{i} \in \mathbb{R}^{n^s}$ is equal to one. Thus, the predicted response $\hat{y}$ is

$$\hat{y} = \sigma_y \hat{y}_{\mathrm{nor}} + \bar{y}, \tag{3}$$

where σ_y and $\bar{y}$ are the standard deviation and mean of the true responses (experimental results) at the sampling points, respectively.

The correlation is usually defined as a function of correlation parameters and distances between the relevant points: the (i, j)-entry $R(\boldsymbol{\theta}, \mathbf{s}_i, \mathbf{s}_j)$ of $\mathbf{R}$ and the ith entry $r(\boldsymbol{\theta}, \mathbf{x}, \mathbf{s}_i)$ of $\mathbf{r}$ is written as

$$R(\boldsymbol{\theta}, \mathbf{s}_i, \mathbf{s}_j) = \prod_{k=1}^{n^d} R(\theta_k, d_{ij}^k), \quad r(\boldsymbol{\theta}, \mathbf{x}, \mathbf{s}_i) = \prod_{k=1}^{n^d} r(\theta_k, d_i^k), \tag{4}$$

where θ_k, d_{ij}^k and d_i^k are respectively the kth entries of correlation parameter vector $\boldsymbol{\theta}$, distances $\mathbf{d}_{ij}$ between sampling points and distances $\mathbf{d}_i$ between sampling points and prediction point.

The correlation parameters $\boldsymbol{\theta}$ are unknown and could be determined by considering the maximum likelihood estimation, which is equivalent to solving

$$\begin{aligned} &\text{Minimize } \hat{\sigma}_z^2 \, |\mathbf{R}|^{1/n^s} \\ &\text{Subject to } \boldsymbol{\theta}^l \le \boldsymbol{\theta} \le \boldsymbol{\theta}^u , \end{aligned} \tag{5}$$

where $|\mathbf{R}|$ is the determinant of $\mathbf{R}$, and $\boldsymbol{\theta}^l$ and $\boldsymbol{\theta}^u$ are the lower an upper bounds of $\boldsymbol{\theta}$, respectively; and

$$\hat{\sigma}_z^2 = \frac{(\tilde{\mathbf{y}} - \beta\mathbf{i})^{\mathrm{T}} \mathbf{R}^{-1} (\tilde{\mathbf{y}} - \beta\mathbf{i})}{n^s} . \tag{6}$$

As the existing sampling points might not predict the responses with high enough accuracy, more sampling points are then needed to improve the prediction accuracy. There are several approaches for adding new sampling points summarized in [3], and we adopt the approach that adds the point having the maximum mean square error of prediction.

2.2. Optimal structure

Using the approximate responses by Kriging method, the dynamic performances of a structure, in terms of probability, can then be easily computed by carrying out MCS, with the assumptions on probability distribution of the structural analysis parameters with uncertainty. The dynamic performances can be the probability of exceeding a specific nodal displacement, as considered in the numerical example in Section 3 for example.

Satisfying certain dynamic performances, as constraints, our next step is to find the optimal structure with the minimum weight. To be more practical, the members of a structure are selected from a given list of available sections. Thus, this is a typical combinatorial optimization problem, and we adopt simulated annealing method as summarized in the next subsection to solve it.

2.3. Simulated annealing method

As its name implies, simulated annealing (SA) method exploits an analogy between he metal annealing process and the search process of the best solution in an optimization problem [4]. It is one of the most popular approaches that can find approximate optimal solution within a practically acceptable computational cost. Gradients of the objective function are not necessary in SA, and its major advantage over other heuristic approaches is the ability to find the global optimum.

There are in total five processes involved in SA, which are (a) initial solution, (b) local search, (c) transition of solutions, (d) cooling schedule, and (e) termination condition. The typical flowchart for these processes in SA is shown in Figure 1.

Among these processes, solution transition is the key for jumping out from a local optimum, since it ensures that non-improving solution is also possible to be accepted. To be specific, solution transition will occur if a randomly generated number $\bar{P} \in (0,1)$ is less than the probability of transition calculated using increase of the objective value Δf_i and temperature t_i at the current iteration *i*:

$$P = \min\{1, e^{\Delta f_i / t_i}\}. \tag{7}$$

It is obvious from Eq. (7) that, transition to an improving solution is always accepted, and transition to a non-improving solution will be more and more difficult, as the temperature is continuously reduced according to cooling procedure. Thus, solution transition is much more active when temperature is high, and SA will gradually converge to the 'optimal' solution while the system is cooled down.

3. Numerical example

In this section, we consider a two-dimensional model of a spatial structure to demonstrate how the proposed method is used to find the optimal structure with reliable capacity.

3.1. Model description

The arch model shown in Figure 2 represents one bay of the typical spatial structure widely used for gymnasiums [5]. The model is composed of a cylindrical roof and two support columns. Span of the model is 80.0 m, and height of its support columns is 3.5 m. The lower nodes of the roof are located on a circle with radius of 80.0 m, and the half-open angle of 20 degrees. Both of the height of the roof truss and width of the column trusses are 1/40 of the span. The distance between two bays of the structure in the longitudinal direction is supposed to be 8.0 m.

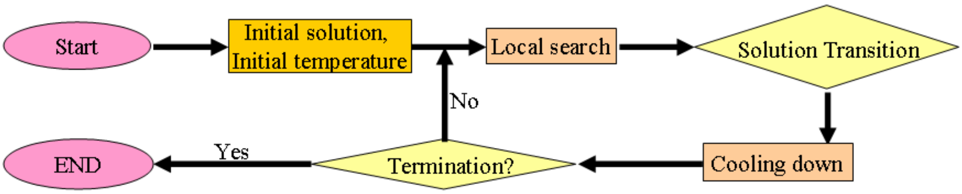


Figure 1: The flowchart of classical simulated annealing method.

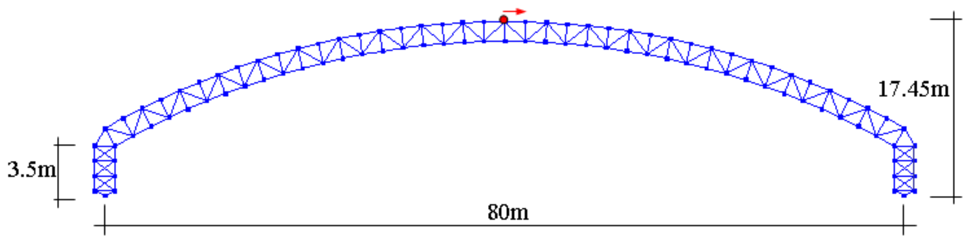


Figure 2: Two-dimensional arch model for typical gymnasiums.

The members are classified into eight groups: the two groups of struts and chords in the support columns, three groups of struts and three of chords in the roof sorted from middle to side. Cross-sectional areas of the 43 available sections for these groups of members are listed in Table 1. Our objective is to find the optimal structure, assembled by the available sections, that has the minimum weight while satisfying constraint on structural performance.

The structural performance for this numerical example is evaluated by the probability of exceedance of a specific horizontal displacement, 0.3 m or 1.7% of the height, of the central upper node of the roof, indicated as a red node in Figure 2.

The members of the arch are steel pipes and modeled as truss elements in structural analysis. The weight of the roof and the external walls are assigned as 0.98 kN/m^2 and 1.47 kN/m^2, respectively. The masses are lumped at the external nodes of the columns and the upper nodes of the roof. Young's modulus is 2.05×10^{11} N/m^2. The steel materials are idealized by a bilinear constitutive model, where the hardening coefficient is 1/100.

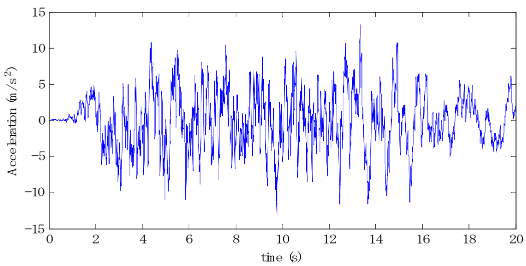


Figure 3: The artificial seismic motion applied to the arch model.

Table 1. Cross-sectional areas of available members.

Diameter (mm)	Area cm^2	Diameter (mm)	Area cm^2	Diameter (mm)	Area cm^2
89.1	6.272	165.2	19.270	355.6	70.210
	7.591		20.260		86.290
	8.638		22.720		103.300
	9.412		25.160		120.100
	11.200	190.7	30.870	406.4	80.420
101.6	9.892	216.3	29.940		98.900
	10.790		38.360		118.500
	12.850		53.610		157.100
114.3	9.808	267.4	49.270	457.2	90.640
	12.180		54.080		111.500
	15.520		75.410		133.600
	20.410	318.5	58.910		177.300
139.8	14.990		67.550		
	17.070		77.090		
	19.130		99.730		
	21.170				

The artificial seismic motion as shown in Figure 3 is generated by the standard superposition method of sinusoidal waves, corresponding to the life-safe performance level during the very rare earthquakes specified in Notification 1461 and 1457 of the Ministry of Land Infrastructure and Transport, Japan. The phase difference spectrum of El-Centro 1940(EW) has been used. This seismic motion is applied at the supporting nodes of the arch model in horizontal direction.

Time history analysis (THA) of the structure is carried out using the open source solver OpenSees [6]. The effect of geometrical nonlinearity is also taken into consideration. Rayleigh damping is adopted for THA, with the same damping ratio h=0.02 for both of the 1st and 3rd modes of the model, which are antisymmetric. The time step for integration by the Newmark-β ($\beta = 0.25$) is set as 0.01 second.

3.2. Uncertainty and optimal structure

For simplicity, we consider uncertainty in two structural parameters: yield stresses of the struts as well as the chords, because they have significant effect on the elastoplastic responses of the structure, though all other possible uncertainty in structural parameters should be included to design a structure with reliability. The nominal values of the yield stresses are 3.25×10^8 N/m^2 and 2.35×10^8 N/m^2, respectively for struts and chords. The upper and lower bounds for uncertainty of the ratios of the yield stresses to their nominal values are respectively set as 1.1 and 0.9. Furthermore, to carry out MCS, the yield stresses are supposed to have the uniform probability distribution.

To apply Kriging method for response prediction, we start from four initial sampling points, which are the combination of the upper and lower bounds of the two parameters. New sampling points, with which the maximum MSE is to be reduced, are added to existing sampling points in order to refine the response surface for improvement of prediction accuracy. MCS is then applied to compute the probability of exceedance of specified displacement of the central node. The probability should be less than a specific target value so as to ensure a safe structure, which is set as 10% as a constraint condition for searching the optimal distribution of sections in the example.

Moreover, we use the Gaussian spatial correlation function, which is preferable for a differentiable response function [7]; and lower and upper bounds of the correlation parameters are assigned as 0.1 and 1.0, respectively. The correlation parameters are found by solving the optimization problem formulated in Eq. (5), using the optimization tool provided in MATLAB [8].

The initial temperature for SA is assigned as 3.0, and coefficient for the *linear* cooling procedure is 0.97. The process of finding new solutions will be terminated when the temperature is less than 0.01.

The structure with the initially assigned sections is shown in Figure 4. Performance of SA for the two cases, with and without consideration of gravity, is demonstrated in Figure 5. Red circles and blue dots in the figures respectively represent the solutions, which are satisfactory and non-satisfactory of the specified constraint on displacement of the central upper node of the roof in the manner of probability. As can be observed from the figure, the initial solution violates the constraint on dynamic performance, although it has the smallest weight compared to its consequent solutions. For the case without gravity, SA converges at the final solution with slightly greater weight (higher volume) than the optimal solution during the searching process; the distribution of cross-sectional areas for these two solutions is illustrated in Figure 6. SA has simpler convergence performance for the case with gravity, and the optimal (final) solution is illustrated in Figure 7.

4. Conclusions

In this study, we have presented a reliability-based design methodology for spatial structures, subjected to possible uncertainty involved in the parameters of structural analysis. The optimization design of a two-dimensional arch model has demonstrated the availability of the proposed method for robust design of spatial structures.

Although only two structural parameters with uncertainty have been considered in the numerical example, the methodology can be easily extended to include any other parameters. Furthermore, any type of probability distribution, other than the uniform distribution used in the example, for uncertainty of structural parameters could be incorporated into the design procedure. Moreover, more structural performance measures and more objective functions could be considered, such that finding the optimal structure would be a multi-objective optimization problem.

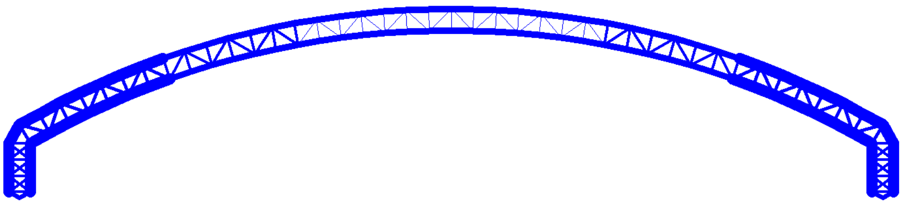
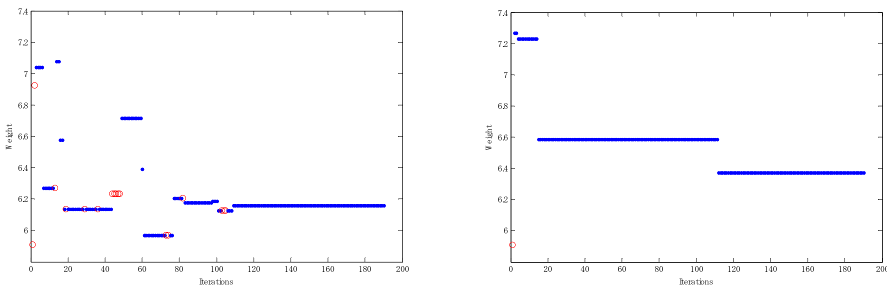
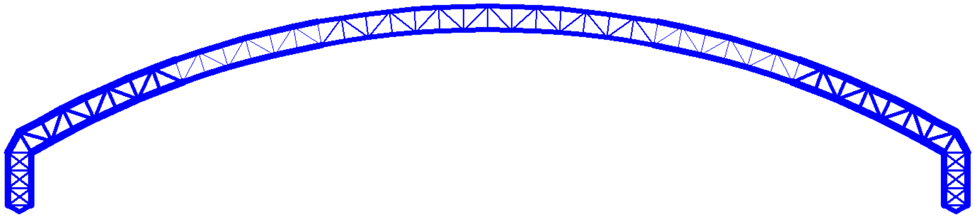


Figure 4: Initial assignment of sections (line width in proportion to cross-sectional area, total volume is 5.91m^3).

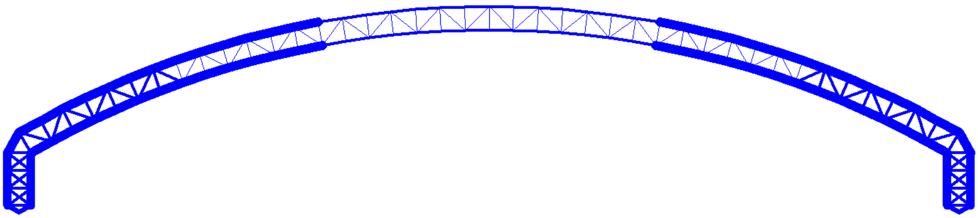


(a) Without gravity (b) With gravity

Figure 5: Performance of SA for the cases with and without gravity. (Red circle: violation of constraint; Blue dot: satisfaction of constraint)



(a) Optimal solution (total volume is 5.97m^3)



(b) Final solution (total volume is 6.16m^3)

Figure 6: Solutions for the case without consideration of gravity.

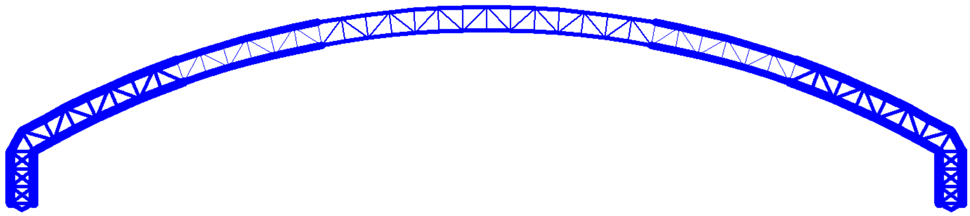


Figure 7: Optimal and final solution (total volume is 6.37m^3) for the case with consideration of gravity.

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