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Computational Morphogenesis of Free Form Shells

H. OHMORI* , T. KIMURA[a] and A. MAENE[b]

*Graduate School of Environmental Studies, Nagoya University
Furo-cho, Chikusa-ku, Nagoya 464-8603, JAPAN
Email : hero@dali.nuac.nagoya-u.ac.jp

[a] Technical Staff, Sasaki Structural Consultants
[b] Graduate School of Environmental Studies, Nagoya University

Abstract

A new method of computational morphogenesis for the shell structures with free curved surface is proposed, where not only the shape of shell surface but also the topology of the curved surface can be obtained at the same time. For the topology optimization, the optimization process for the shell thickness is used, where the topology of the curved surface with a multi-connecting surface is dealt with as a limit in which the shell thickness vanishes. For mathematical expression for the shape of the shells, NURBS (Non Uniform Rational B-Spline) is used, by which the number of unknowns can be effectively controlled while the high degree of freedom for the expression of the shape of shell surface as well as the shell thickness are kept.

This paper shows the fundamental formulation of the proposed method for the optimization problem for both the shape and the topology of the free-form shell, followed by numerical examples in which the effectiveness of the proposed scheme is investigated through the application to several existing shell structures.

Keywords: shell thickness, NURBS, shape optimization, topology optimization, strain energy

1. Introduction

As for the shell structures, various dynamic designs can be realized because an architectural form and a structural configuration are closely related with each other. In addition, forming the opening area by operating the topology invents the space over which the soft light pours down from a top light, which gives us architectural degree of freedom. However, the mechanical behavior of the shell structures with free curved surface is complicated. It is difficult to resolve the best curved surface shape and to optimize the area of shell thickness and the apertural area at the same time. However, complex free-form structure can be constructed even if it does not have structural rationality owing to the recent improvement of the computer, the construction technology and the material science. Therefore, a new method of computational morphogenesis for the shell structures with free curved surface is requested, by which structurally rational shell form can be realized starting from the original shell given by architects or engineers.

Recently, we proposed the scheme of computational morphogenesis in which the shape, the thickness as well as the topology of the free-form shells can be obtained at the same time [9]. The purpose of the present study is to demonstrate effectiveness as well as usefulness of the proposed method through several numerical examples and to investigate the mechanical properties of those structures which are resulted from the proposed optimization process.

2. Optimization with respect to shape, thickness and topology

In this proposed scheme, the strain energy of the structures has been adopted as the quantitative scale for the mechanical measurement. For mathematical expression for the shape of the shells, NURBS (Non Uniform Rational B-Spline) is used, by which the number of unknowns can be effectively controlled while the high degree of freedom for the expression of the shape of shell surface as well as the shell thickness are maintained. Shape, thickness and structural topology can be simultaneously optimized by introducing a method in which both the deletion and the addition of the domain according to the shell thickness are continuously carried out in the optimization process.

2.1. Formulation of optimization problem

In the present method, an optimization problem with respect to shape, thickness and topology of the curved surface for the shell structures is dealt with as a strain energy minimization problem. When the control point coordinates of decision vector $\boldsymbol{\varphi}$ and NURBS control point of thickness $\boldsymbol{q}_h$ is adopted as the design variable, the nonlinear programming problem in question can be expressed as the following form;

$$\begin{aligned} &\text{minimize} && f(\boldsymbol{w}) = \frac{1}{2}\boldsymbol{d}^{\top}(\boldsymbol{w})\boldsymbol{K}(\boldsymbol{w})\boldsymbol{d}(\boldsymbol{w}) \\ & && \boldsymbol{L}\boldsymbol{w} \leq \boldsymbol{h}^{u} \\ &\text{subject to} && \boldsymbol{h} \in \boldsymbol{\Omega} \quad \boldsymbol{\Omega} = \{\boldsymbol{h} \mid \boldsymbol{\varepsilon} \leq \boldsymbol{h}\} \\ & && \Delta V^{l} \leq \Delta V(\boldsymbol{w}) \leq \Delta V^{u} \end{aligned} \tag{1}$$

where $\boldsymbol{L} = [\boldsymbol{0} \quad \boldsymbol{B}] \in R^{3n \times 6n}$, $\boldsymbol{B}$ represent a constant matrix connecting the coordinates of the control point with the nodal coordinates of the finite elements. $\boldsymbol{d} \in R^{6n}$ and $\boldsymbol{K} \in R^{6n \times 6n}$ represent the nodal displacement vector and the total stiffness matrix, respectively. $\boldsymbol{h}^{u} \in R^{n}$ and $\varepsilon \in R^{n}$ represent the vector of the upper boundary with respect to the shell thickness and the criterion value for the topological optimization, respectively. Ω represents the class which satisfies $\boldsymbol{\Omega} = \{\boldsymbol{h} \mid \varepsilon \leq \boldsymbol{h}\}$. $\Delta V(\boldsymbol{w})$ represents the increment size of the total volume. ΔV^{u} and ΔV^{l} represent the maximum increasing and decreasing amount of the total volume, respectively. In the method of this problem, Sequential Quadratic Programming (SQP) is adopted to solve the posed problem.

3. Numerical examples

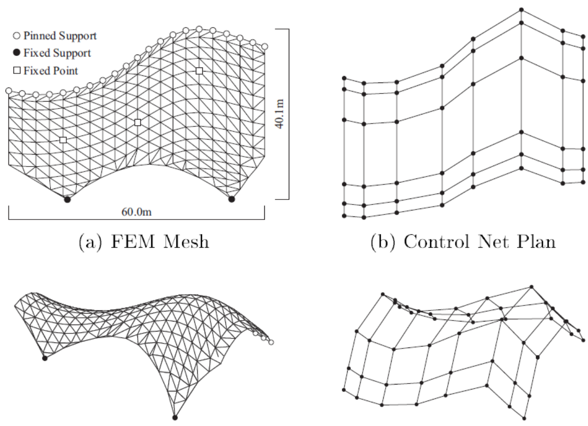
Several existing shell structures are adopted as numerical examples in order to investigate its effectiveness.

3.1. Tachira Club

3.1.1. Numerical Model

The first numerical example is taken from the proposal for of Tachira Club which is well known as an unrealized project designed by Eduardo Torroja.

The original shape is as shown in Fig. 1 (a) and (c), and the NURBS control points are arranged on the x-y plane as shown in Fig. 1 (b), which realize the original curved surface including the edge curved lines as well as the supporting conditions. Thickness of the shell is assumed to be uniformly 0.1 m and the fixed point throughout the modification process of the shape is set to the supporting points as well as the specific points as shown in Fig. 1 (a). As the external load, the own weight of 24 kN/m^3 and the live load of 1 kN/m^2 have been adopted and Young's modulus and Poisson's ratio are assumed as 21 GPa and 0.17, respectively. Modification of the curved surface is made only on the vertical direction and the vector $\boldsymbol{\varphi}$ from which z-coordinate of the position vector of the objective surface is obtained is taken as the unknowns. As the inequality constraint conditions, the upper boundary with respect to the shell thickness is given as 0.120m, the criterion minimum value for the topological optimization is given as 0.020m, the upper and lower boundary with respect to the total structural volume are given as ±0.21m^3 (0.1 percent of that of the initial shell), respectively. The numerical result is shown as follows.



(a) FEM Mesh (b) Control Net Plan

(c) Initial Shape (d)Control Net

Figure 1 Numerical Example (1) : Tachira Club

3.2.2. Numerical result

In Fig. 2, the shapes of the shell obtained through the proposed optimization process are depicted, where the isometric view is shown and $f(\boldsymbol{w})$ represents the magnitude of the strain energy. In Fig. 2, the diagram of distribution of the shell thickness are shown, where the thickness distribution is shown by using the contour lines which has been shown by dividing 30 levels between the maximum value and the minimum value of the shell thickness of all steps. Moreover, the grayed contour lines represent the thickness thinner than initial one, the blacked ones represent the thickness thicker than initial one. Heavy black lines represent the maximum value of the shell thickness and heavy gray lines show the minimum value of the shell thickness. In Fig. 3, the membrane stress and the bending stress distribution on the shell surface are depicted. Continuous lines and dotted lines show the compressive stress and the tensile stress at the center of the element, respectively, and the length of the lines show the magnitude of the principal stress. In these figures, σ^t and σ^c represent the maximum tension and compression stress over the whole surface and σ^b represents the maximum bending stress, respectively. In addition, $w_{\max}$ shows the maximum value of the vertical displacement over all nodes on the shell surface. Fig. 4 shows the transition of strain energy that is the target function during the numerical calculation, where the horizontal axis shows the number of steps and the vertical axis corresponds to the strain energy. Fig. 5 shows the transition of various mechanical quantities, where the horizontal axis shows the numbers of steps, the left vertical axis presents the principal energy, and the right vertical axis corresponds to the displacement.

In Fig. 2, it can be observed that the shape of the free edge transforms into catenary arch through optimization process, and the edge with high rise appears. In Fig. 2, it is possible to confirm that the shell thickness at the vicinity of the supporting points gradually become

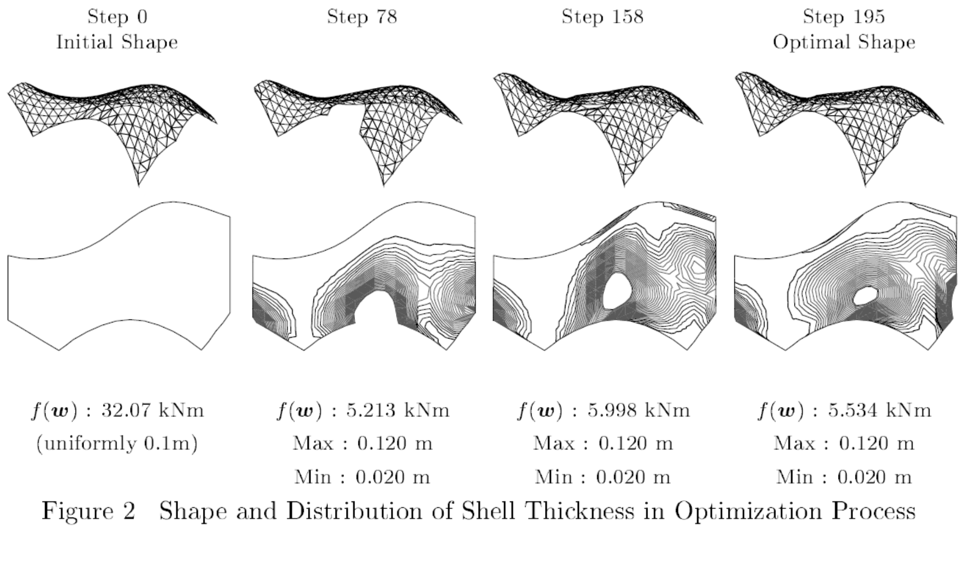
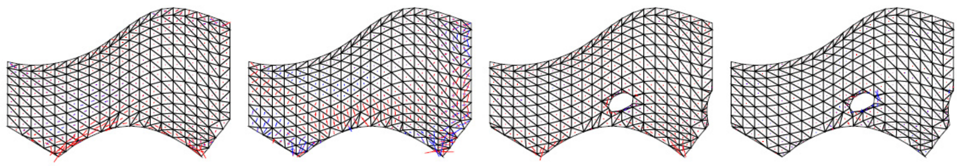


Figure 2 Shape and Distribution of Shell Thickness in Optimization Process



(a-1) Membrane Stress σ^t : 2.646N/mm^2 σ^c : 11.57N/mm^2
(a-2) Bending Stress σ^b : 13.75N/mm^2 $w_{\max}$: 166mm
(a) Initial Shape

(b-1) Membrane Stress σ^t : 1.456N/mm^2 σ^c : 7.786N/mm^2
(b-2) Bending Stress σ^b : 6.740N/mm^2 $w_{\max}$: 12.0mm
(b) Optimal Shape

Figure 3 Distribution of Principal Stress

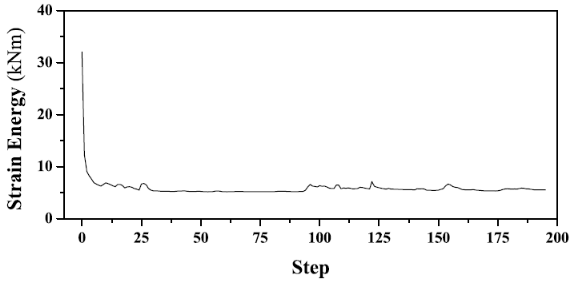


Figure 4 Iteration History of Objective Function

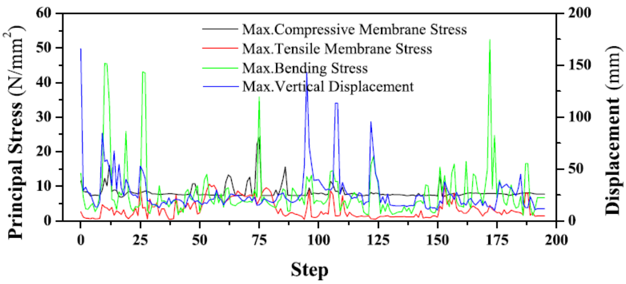


Figure 5 Iteration History of Mechanical Quantities

thick. Moreover, it can be also confirmed that the hole is generated, which shows that a new topology can be obtained through the proposed approach. Fig. 3 shows that all principal stresses have become small in the optimal shape. In addition, one can also observe that a large amount of the bending moment at the vicinity of supporting points of the initial shape of shell has become small in the optimal shape. It shows that the proposed optimization process suppresses an increase of the bending stress which is an inefficient way in transmitting the stress. In Fig. 4, it can be observed that the strain energy greatly decreases as the numerical calculation advances. In Fig. 5, one can see that a large amount of the bending stress and the tensile stress have been generated at the step where the topology has changed.

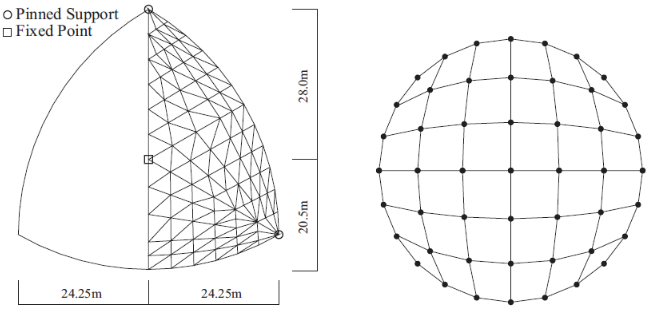
3.2. Kresge Auditorium

3.2.1. Numerical Model

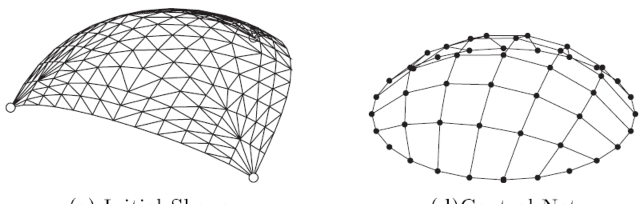
A numerical example (Fig. 6) is taken from Kresge Auditorium (designed by Eero Saarinen), which is a hall in M. I. T. opened in 1955. This hall has a geometrical shell roof, which arose by cutting the spherical shell into 8 equal parts. The radius of curvature is 34.29m, the apex point of the shell is 14.5m high and the center of a free edge is 8.23m high. The curved shell surface is framed in the edge beams. The shell thickness of the real structure takes the maximum value of 0.495m at the support points and the minimum value of 0.089m at the apex point.

In this section, the stress is calculated by a linear-static analysis of the finite element analysis. The half part of the shell is adopted as the objective structure. As the external load, the own weight of 24 kN/m^3 and the live load of 1 kN/m^2 have been adopted; the Young's modulus and the Poisson's ratio are given as 21GPa and 0.17, respectively. The initial shell thickness is given as uniformly 0.292m, which is the average of the maximum and the minimum one of the real structure. The apex point of the shell is treated as the fixed point throughout the modification process of the shape (not in the stress analysis).

As the inequality constraint conditions, the upper boundary with respect to the shell thickness is given as 0.495m, the criterion value for the topological optimization is given as 0.030m, the upper and lower boundary with respect to the total structural volume are given as ±0.28m^3 (0.1 percent of that of the initial shell), respectively. The numerical result is shown as follows.



(a) FEM Mesh (b) Control Net Plan



(c) Initial Shape (d)Control Net

Figure 6 Numerical Example (2) : Kresge Auditorium

3.2.2. Numerical result

In Fig. 7, the shape of the shell obtained through the optimization are depicted, where the isometric view is shown, where *f*(***w***) represents the magnitude of the strain energy. In Fig. 7, the diagram of distribution of the shell thickness are shown, where the thickness distribution is shown by using the contour lines which has been divided into 30 levels between the maximum value and the minimum value of the shell thickness of all steps. Moreover, the grayed contour lines represent the thickness thinner than initial one, the blacked contour lines represent the thickness thicker than initial one. In Fig. 8, the membrane stress and the bending stress are depicted. Continuous lines and dotted lines show the compressive stress and the tensile stress at the center of the element, respectively, and the length of the lines show the magnitude of the principal stress. In these figures, σ^t and σ^c represent the maximum tension and compression stress over the whole surface and σ^b represents the maximum bending stress, respectively. In addition, $w_{\max}$ shows the maximum value of the vertical displacement over all nodes on the shell surface.

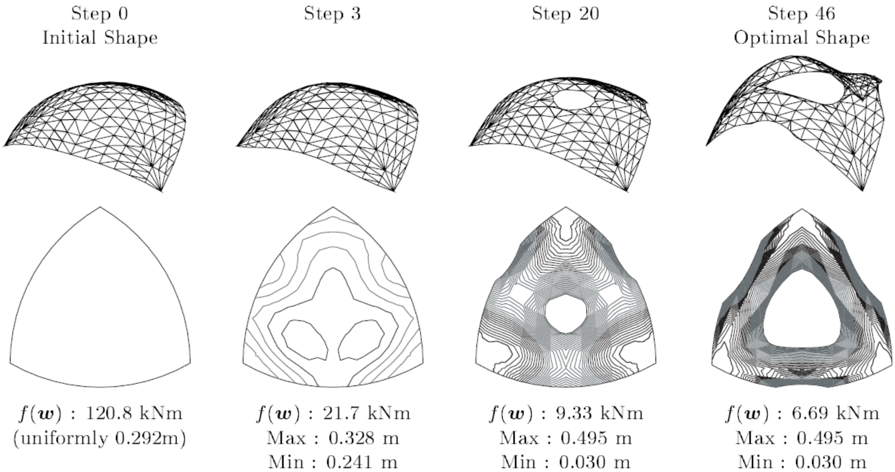
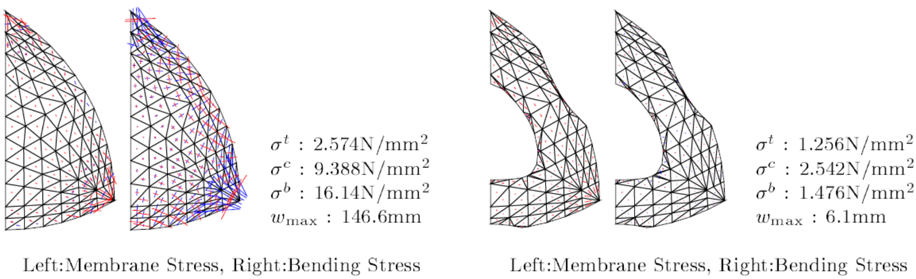


Figure 7 Shape and Distribution of Shell Thickness in Optimization Process



(a) Initial Shape (b) Optimal Shape

Figure 8 Distribution of Principal Stress

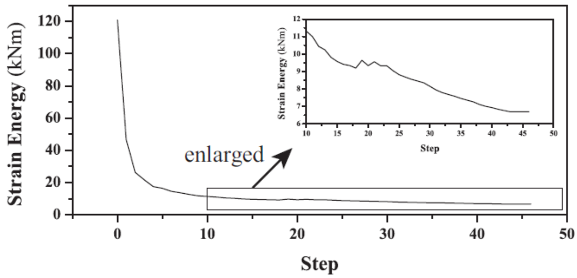


Figure 9 Iteration History of Objective Function

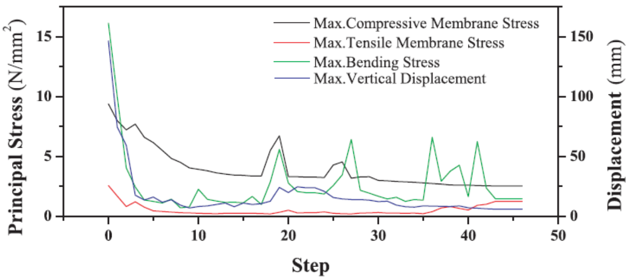


Figure 10 Iteration History of Mechanical Quantities

Fig. 9 shows the transition of strain energy that is the target function, where the horizontal axis shows the number of steps and the vertical axis show the strain energy. Fig. 10 shows the transition of various mechanical quantities, where the horizontal axis shows the numbers of steps, the left vertical axis corresponds to the principal energy, and the right vertical axis represents the displacement.

In Fig. 7, it can be observed that the shape of the free edge is changed into catenary arch through optimization, and the edge with high rise appears. In Fig. 7, it is possible to confirm that the shell thickness at the vicinity of the supporting points and the free edge gradually become thick. Moreover, it can be also confirmed that the hole at the vicinity of the apex is generated, which shows that a new topology can be obtained through the proposed process. The number of holes in the center part of the shell sensitively changes caused by the topology manipulation which is repeated based on a small difference of shell thickness though the thickness is almost constant over all of those area. Fig. 8 shows that all principal stresses have become small in the optimal shape. In addition, one can observe that a relatively large amount of the bending moment at the vicinity of supportings of the initial shape of shell has become small in the final optimal shape. It shows that the proposed optimization process surpress an increase of the bending stress which is an inefficient way in transmitting the stress. In Fig. 9, it can be observed that the strain energy greatly decreases as the numerical calculation advances. In Fig. 10, one can observe that the bending stress and the tensile stress have been greatly generated at the step where the topology is drastically changed. We can understand mechanical quantities of the shell change very sensitively that according to the change of the shell topology.

4. Conclusion

In the present paper, the effectiveness of the computational morphogenesis of shells with free curved surface through the simultaneous optimization of shape, thickness and structural topology has been confirmed. Through the numerical analysis of the models adopted from well-known shells such as Tachira Club and Kresge Auditorium, the optimization of those shells based upon strain energy minimization leads the shells which can resist against the external loads mainly through membrane resultants. Additionally, we have confirmed that topology optimization can be realized through the appropriate control of the shell thickness, which implies that shape and topology optimization of shells can be effectively carried out by taking two fundamental parameters such as shape and thickness of shells.

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