Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Structural behaviour of spatial arch bridges

Juan José JORQUERA LUCERGA*

* Dr. Ing. de Caminos, CC y PP.
Madrid (Spain).
jjorqueral@gmail.com

Abstract

From an aesthetical point of view, spatial arch bridges are a consequence of new architectural demands for bridges in urban environments. They also arise to meet functional requirements when arch structures are the most suitable for supporting horizontally curved decks.

In these cases, in so-called spatial arch bridges, their structural behaviour extends from the original vertical plane to a three-dimensional configuration.

For single arch bridges with only one deck, this spatial behaviour emerges, mainly, in the following cases:

- when arch springing and deck abutment are not at the same location.
- if arch (or deck) is curved in plan.
- if arch is out of from the vertical plane of symmetry of a straight deck.
- in the case of the arch plane is leaning away from the vertical plane or rotated around a vertical axis.
- if the arch axis is an out-of-plane curve.

Existing bridges where these factors appear have been studied, and parametric and theoretical studies (Jorquera [2]) have been carried out.

Main conclusions are presented, which can be generalized to very complex spatial arch-deck configurations, including a new method for finding antifunicular three-dimensional shapes, even in the most general case of spatial arches with clamped ends.

Keywords: leant arch, spatial arch, antifunicular, form-finding, curved girder.

1. Spatial arch bridge composed of a plane arch and a lower deck.

In these cases, arch shape is a parabola resting in a plane, not necessarily vertical. The deck girder is suspended by hangers pinned at both ends and attached to its longitudinal axis. Movements of springings of arch and abutments of deck are restrained.

1. Arch behaviour in its own plane and in its perpendicular one is uncoupled. The same happens to the deck. Arch and deck behave, simultaneously, like an arch in its plane and a curved girder (an *encastré* beam) in perpendicular plane.
2. For permanent loads, vertical components of hangers' prestressing loads are equal to deck reactions over fixed bearings.

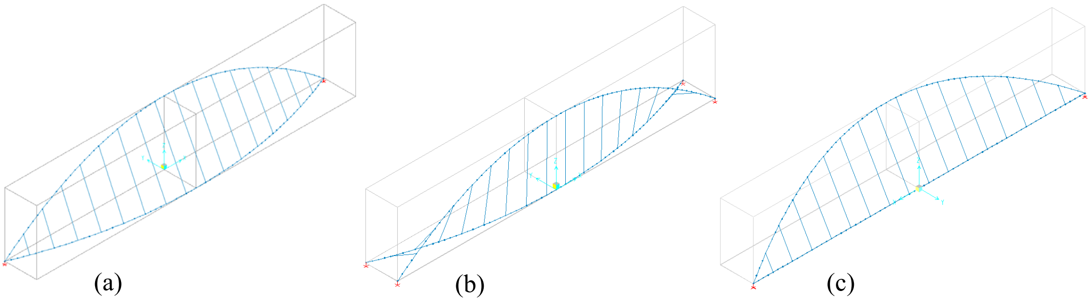


Figure 1: Geometrical parameters: (a) Curved deck, (b) Arch-girder lateral relative displacement, (c) Arch leant away from vertical plane.

2.1. Horizontally curved girder.

If the deck's horizontal curvature (Figure 1, case a) increases, hangers lean, and some internal forces appear that can't be found when the deck is straight: vertical axis bending and torsional forces appear at both arch and deck, and axial forces appear at deck.

For dead loads (Fig. 2) the components of hangers' prestressing loads (N_P) are R, vertical and depending on deck's curvature, and H, divided into N_{PX} and N_{PY} along global axes X and Y, in horizontal plane.

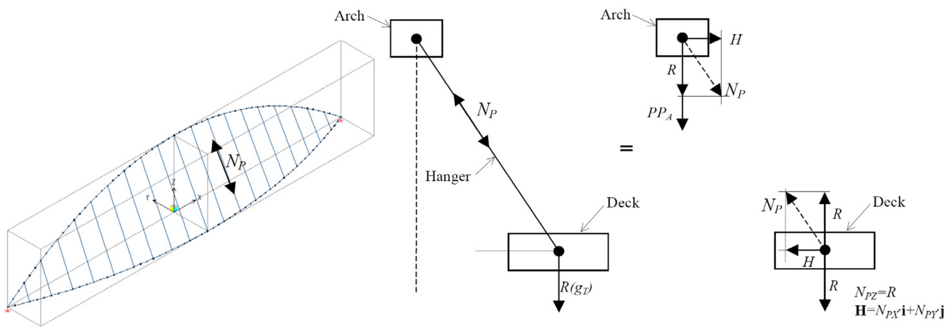


Figure 2: Horizontally curved girder: dead load.

N_{PX}, placed in the arch`s plane, modifies classical vertical plane arch behaviour, and N_{PY} produces curved girder behaviour. Torsional moments appear because of coupling caused by curvature. The deck now behaves as a circular horizontal arch loaded by horizontal components of hangers, H. Vertically, is a curved girder over fixed bearings.

Under live loads (Figure 3), these fixed bearings become elastic springs, and their vertical stiffness depends specially on the lateral stiffness of the arch, working as a curved girder. The deck`s horizontal stiffness is much higher because of its curved shape (a horizontal arch) in plan and because it is much wider than the arch due to functional requirements.

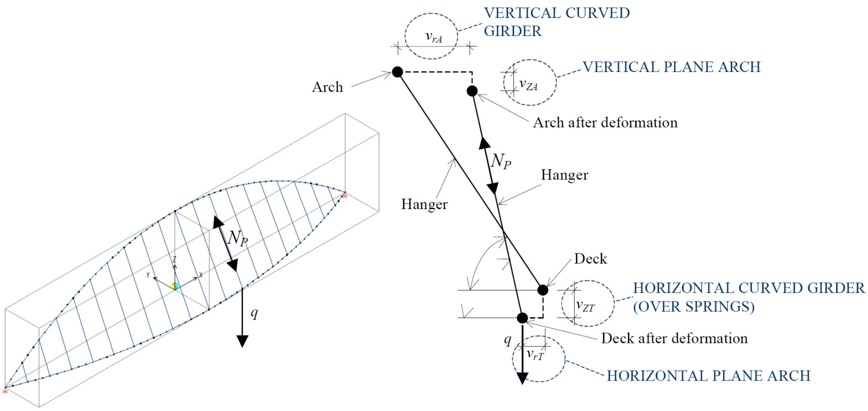


Figure 3: Vertical live loads: deformation and associated structural systems.

2.2 Arch horizontal stiffness.

So, structural systems that remain inactive in classical plane arches are now 'activated' by the deck's curvature (Manterola [3]). Configurations as shown, where all hangers are attached on the same side, make the arch sensitive to lateral actions. In these cases, horizontal stiffness must be increased and correctly distributed, which means cross sections must be horizontally oriented, stiffer (to limit deflections), and wider at springings than at the crown, to follow the strong variation of bending moment law. All these requirements are simultaneously fulfilled, for instance, in the Gateshead Footbridge (Figure 4).

Figure 4: Gateshead Footbridge (Johnson and Curran [1]).

2.3. Effect of arch-deck transversal relative position.

In this case, the deck's curvature is kept constant and laterally displaced under a vertical plane arch (Figure 1, case b).

For dead loads, the following considerations can be made:

1. The components *R* of hangers are constant, since deck does not change either.
2. Longitudinal projection of hangers are constant, and so are N_{PX}.

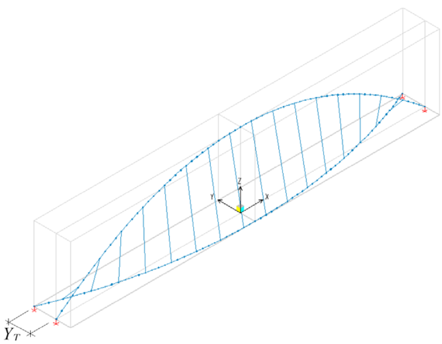
Therefore, since *R* and N_{PX} do not change, prestressing hangers' projection over the arch's plane is independent of transversal relative position between the arch and the deck. Behaviour of the arch in its own plane is constant (axial and bending moment), and behaviour of the deck as a curved girder is also constant, with coupled bending and torsional moments.

For symetrical live loads, horizontal displacements of the arch are smaller when it is at an intermediate position than when the whole arch is at the same side of deck. The arch can now be less stiff because loads at each side balance one another.

Not always can the arch be freely laterally displaced in relation to the deck. But, whenever it is possible, it is much better to search for their optimum relative position than simply increasing arch's stiffness. This is more efficient in order to limit horizontal displacements, and consequently, internal forces in all the bridge.

As a preliminary guideline to help find that optimum, let's say there's always a specific relative arch-deck position in which the horizontal displacement of a given point of the arch is zero for a uniform load acting upon the whole deck.

The range of positions that minimize internal forces in the arch correspond to positions that make horizontal displacement at least at one point of the arch to be zero. This is because loads have to balance at both sides of arch. Relative position that make spandrels or the crown not move when the deck is loaded could be a good starting point (Figure 5).



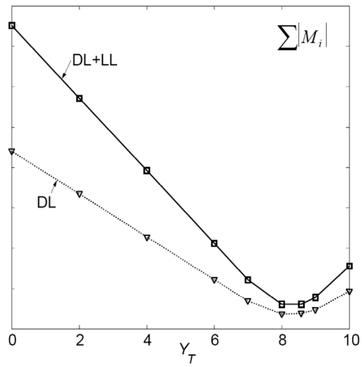


Figure 5. (Left) Arch-deck transversal relative position, defined by Y_T. (Right) Y_T vs. $\Sigma|M_i|$ (surface under vertical axis bending moment law at arch) for dead load and dead load + live load. Optimum is near positions where displacement is zero for crown or spandrels.

Just shown configurations are as sensitive as classical vertical arch planes to asymmetrical live loads.

2.4. Rotation of arch´s plane around longitudinal axis.

If arch is rotated at an angle ω (Figure 1, case *c*), its plane leans away from the vertical. For dead loads, this rotation has the following consequences:

1. Although vertical reactions R are independent of ω, a new horizontal component, H, appears. This causes vertical axis bending moment at the deck.
2. Internal forces in arch's plane increase since axial forces of hangers do.
3. Arch works as a curved girder, when the out-of-plane projection of self weight of the arch and hangers act upon arch.

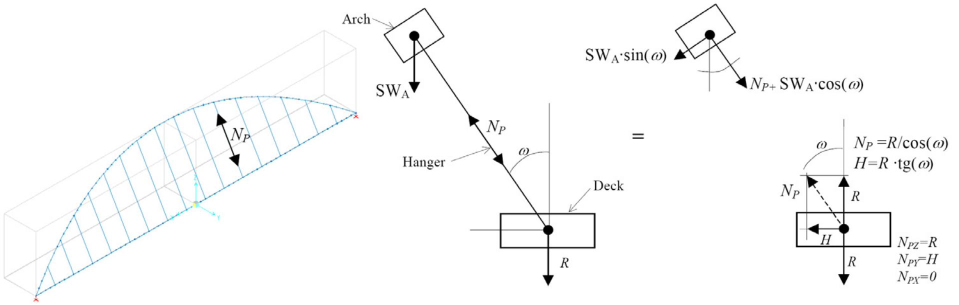


Figure 5: Leant arch under dead loads.

Under live loads, axial forces transferred to arch by hangers increase according to ω. It's necessary to enlarge cross sections of hangers and arch to keep their contribution to staying system stiffness.

Hangers cannot be attached to the axis of a lower deck without widening it (Strasky [4]), for example, in Calatrava's Ondarroa Bridge (Tzonis [6]). To avoid this, this type of bridge (for example, many other of Calatrava's bridges) usually move arch to one edge, 'activating' the deck´s torsional stiffness.

3. Spatial arch with antifunicular out-of-plane shape.

A new iterative method has been developed in order to find antifunicular spatial shapes of arches with clamped ends. Shapes returned by the algorithm are not only highly efficient from the structural point of view, but also aesthetically challenging.

3.1. Description of method.

The method is based on two main concepts:

1. *To generalize to three-dimensional shapes the classical methods for plane arches*. For instance, one well-known way to find the antifunicular shape of a two-hinged arch is to calculate its vertical coordinate at a given point, Z, as M/R_X, where M is the bending moment at that point, and R_X is the longitudinal reaction at springings. It´s very important to remark that, in this process, zero-moment points keep their position during iterations.

2. *To use fictitious jacks at springings during the form-finding process*. An arch with clamped ends cannot be antifunicular because of axial shortening when loaded. To correct this, hydraulic jacks must act upon the structure to make springings become zero-moment points, at least in theory.

   In the method presented, jacks act upon the structure *before* it is finished, in order to make desired points (for example, springings) be zero-moment during iterations. As each jack acts biexcentrically, three degrees of freedom are controlled, and not only both moments at springings can be countered by jacks, but also, for example, longitudinal bending moment at the crown.

   So, even when the arch is clamped, for a specific iteration, both moments are zero at springing, and also is longitudinal moments at crown. The original clamped arch transforms into a vertical three-hinged arch (in elevation) *and* a horizontal two-hinged arch (in plan). So, next iteration will correct arch`s shape, but will keep positions of springings and vertical coordinate Z of crown.

The antifunicular shape may be determined for dead load plus half of live loads. But the problem would not be completely solved if anchorage points at deck had uncontrolled deflections for dead load. To solve this, a thermal load (uniform cooling), that simulates hanger prestressing, is introduced at each hanger to contrarrest this undesired movements.

Teherefore, for each iteration, a system of equations must be solved to determine the values for:

- Axial forces and two bending moments at each springing.
- Value of prestressing force for each hanger.

If any kind of non-linearity is going to be considered, for example, geometrical non-linearity, this system turns non-linear.

Figure 6 shows the process for an iteration, which begins with an initial geometry for the arch, shown in (a). The longitudinal and vertical axis moment due to dead load plus half live load and the deflected shape due to dead load are shown in (b). To nullify bending moments and deflections at desired points load case PRESTR, shown in (c), must act upon the structure. It is composed of prestressing loads acting upon hangers and hydraulic jacks upon springings. As a result, as shown in (d), a three-hinged arch in elevation and a two-hinged arch in plan is obtained. Therefore, next iteration will keep position of springings and coordinate Z of crown. Now, geometry can be corrected until difference between two consecutive iterations may be neglected.

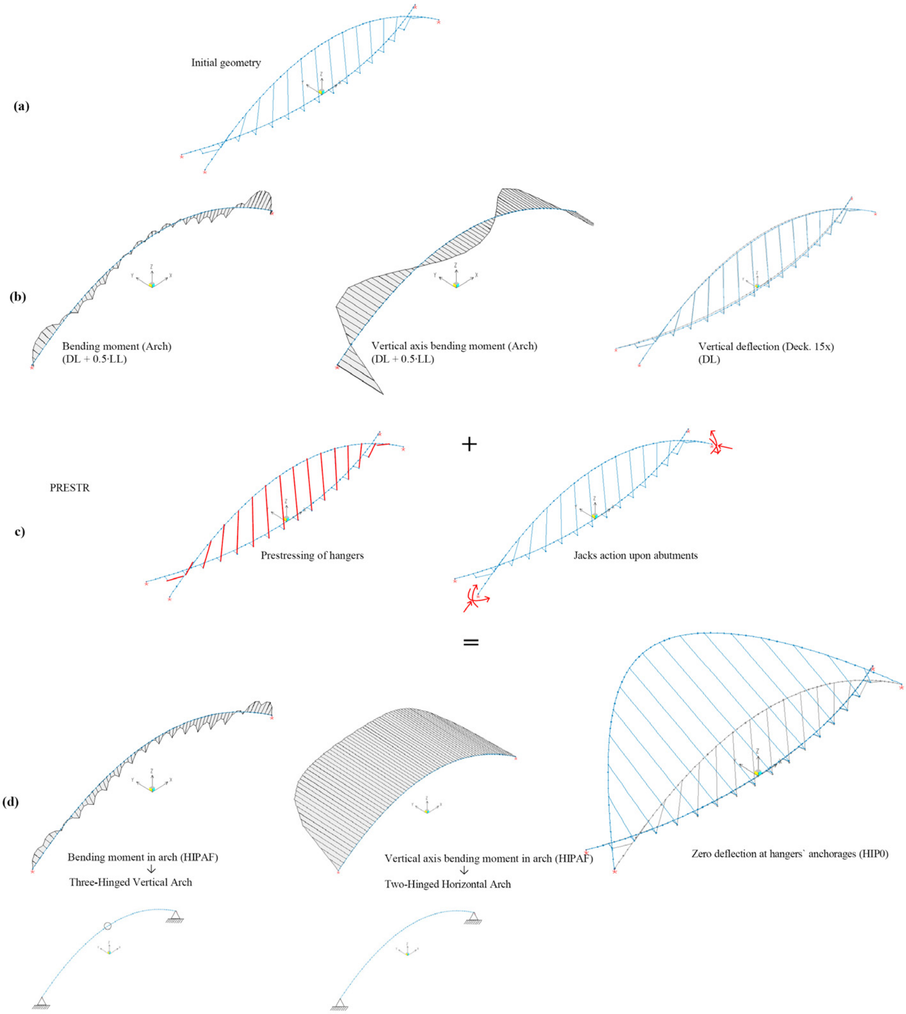


Figure 6: Antifunicular spatial iterative form-finding process for clamped arches.

3.2. Correction of geometry.

To correct geometry, two alternative methods have been developed. Since antifunicular shape is unique, results are equal.

1. *Global reactions method*, where bending moment increment at a given point is assumed to depend only on the change of their relative coordinates in relation to

springing point, upon which reactions act. It generalizes the two-hinged arch case presented previously.

2. *Local eccentricities method*, where the arch must pass through the centre of gravity of axial loads. It generalizes the 'equal resistance arch' concept by Mörsch.

Coupled and uncoupled formulations have been developed for both methods. Coupled formulations enable us to obtain the antifunicular shape for both bending principal directions simultaneously.

The uncoupled formulation only enable us to find antifunicular shapes for one direction, but any other geometrical restriction may be imposed on the other direction. For example, an antifunicular shape for in-plane bending moment can be found for a leant-arch bridge, and at the same time, the arch shape can be forced to rest in that plane.

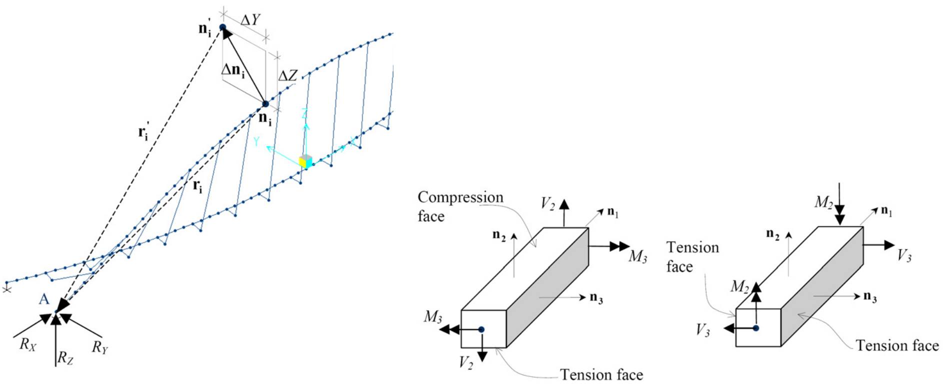


Figure 7: Global reactions method: Notation and positive internal forces.

For the global reactions method, coupled formulation can be written, at any point of the arch, as the following system of equations. The unknown solution are ΔY and ΔZ, which are the necessary increments of coordinates to balance external bending moment $\boldsymbol{M}_{ext}=\boldsymbol{M}_{2,ext}+\boldsymbol{M}_{3,ext}$. Each equation is obtained projecting along the local axis of cross section.

$$\begin{bmatrix} R_x \cdot n_{3Z} - R_z \cdot n_{3X} & -R_x \cdot n_{3Y} + R_Y \cdot n_{3X} \\ R_x \cdot n_{2Z} - R_z \cdot n_{2X} & -R_x \cdot n_{2Y} + R_z \cdot n_{2X} \end{bmatrix} \cdot \begin{bmatrix} \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} M_{3,ext} \\ -M_{2,ext} \end{bmatrix} \tag{1}$$

Where R (Figure 7) is the reaction upon springing, and $\boldsymbol{n_2}$ y $\boldsymbol{n_3}$ are unitary vectors of frame local axis whose components are $[n_{2X}\ n_{2Y}\ n_{2Z}]$ and $[n_{3X}\ n_{3Y}\ n_{3Z}]$.

At a classical plane vertical arch, $M_2=0$, $n_3=[0\ -1\ 0]$ and $R_Y=0$, and (1) will be reduced to

$$\Delta Z = \frac{M_{3,ext}}{R_x} \tag{2}$$

which is the expression for two-hinged plane arches.

Uncoupled formulation for both local axes is:

$$\Delta Y = \frac{-M_{2,ext}}{R_x \cdot n_{2Z} - R_z \cdot n_{2X}} \tag{3}$$

$$\Delta Z = \frac{M_{3,ext}}{-R_x \cdot n_{3Y} + R_Y \cdot n_{3X}} \tag{4}$$

For local eccentricities method, uncoupled formulation is:

$$\Delta Y = -\frac{M_{2,ext}}{N_{ext}} \frac{1}{n_{3Y}} \tag{5}$$

$$\Delta Z = -\frac{M_{3,ext}}{N_{ext}} \frac{1}{n_{2Z}} \tag{6}$$

where N_{ext} is the axial force.

An example of the complete process is shown in Figure 8. In this case, geometrical non-linearity has been considered.

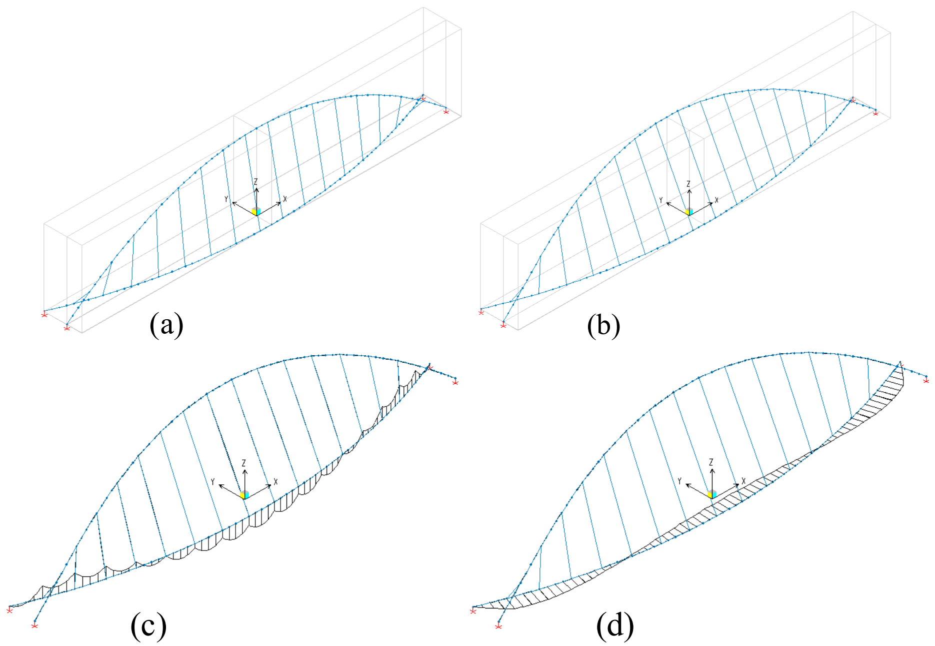


Figure 8: Spatial antifunicular shape: (a) Initial geometry, (b) Antifunicular geometry, an out-of-plane curve (c) bending moment, (d) vertical axis bending moment. (Internal forces at arch and deck drawn at same scale).

3.3. Effect of arch-deck transversal relative position.

For antifunicular spatial arches, internal forces in the arch can be drawn in a diagram similar to the one shown in Figure 5 for plane arches. The form and conclusions are very similar, and the smallest internal forces are obtained when live loads balance one another at each side of the arch.

The effect of the relative position is presented at figure 9. The vertical initial arches transform into out-of-plane antifunicular curves (upper row, cases Y_T=5 and Y_T =8.5). The center diagram shows their frontal views (from X axis). Beside it, a third arch is also represented (case Y_T=7.35), where internal forces are minimum. Plan view for the three cases is shown at the bottom of the figure. It's very interesting to notice that, for the optimum case Y_T=7.35, antifunicular shape crosses over the deck, and its plan is S-shaped.

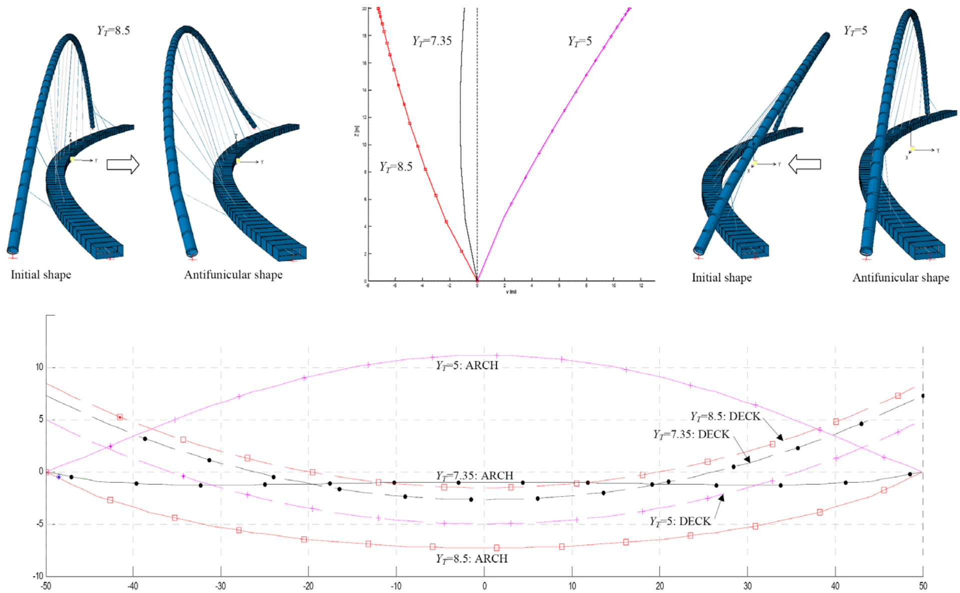


Figure 9: Effect of arch-deck transversal relative position. Optimal positions generates S-shaped antifunicular arches in plan.

3.4. Spatial arch bridge composed of an antifunicular arch and an upper deck.

As for plane arches, optimum transversal relative position may force abutment and springing to be too far. In these cases, the deck will not be able to tie arch springing and bow-string behaviour will not be possible. So, for lower deck bridges, a balanced solution must be found. This problem simply disappears in upper deck arch bridges, where optimum solutions can be found if the arch can be freely displaced.

The main differences between lower and upper decks are:

1. Tensioned both end pinned hangers now transform into compression rods.
2. These rods, under the deck, do not interfere with traffic, and can be attached to the deck axis, even when are leant.

Form-finding methods presented are fully applicable for upper deck bridges if rods are hinged at their tops. If not, best results are obtained finding antifunicular shapes as if rods were hinged at their tops, and clamping them after the process.

As before, lateral sensitiveness to horizontal actions depends on arch stiffness, but, above all, it depends on correct lateral position.

An example of these spatial arches is shown in Figure 10, where out-of-plane shape of the arch reminds us of Ripshorst Footbridge (Schlaich [5]), one of the masterworks of J. Schlaich.

Figure 10: (Left) Example of spatial arch bridge composed of an antifunicular arch and an upper deck (Right) Ripshorst Footbridge, by J. Schlaich.

Acknowledgements

This paper compiles some of the conclusion obtained at the reference 2, under the direction of Prof. Javier Manterola, to whom I would like to express my gratitude.

References

[1] Johnson, J.,Curran P. Gateshead Millennium Bridge—an eye-opener for engineering. 2003. Proceedings of ICE. *Civil Engineering*, nº 156, 16-24.

[2] Jorquera Lucerga, J.J. *Estudio del comportamiento resistente de los puentes arco espaciales.* Tesis Doctoral E.T.S.I. Caminos, Canales y Puertos, UPM. 2007. (Full version available at http://oa.upm.es/812/)

[3] Manterola Armisén, J. Puentes arco con tablero inferior. *Revista de Obras Públicas*, 2003, nº 3436, 7-30.

[4] Strasky, J. *Stress ribbon and cable supported pedestrian bridges.* Thomas Telford. Londres. 2005.

[5] Schlaich, J. Urban footbridges. *16th congress of IABSE*. Lucerne. 2000

[6] Tzonis, A, Caso Donadei, R. *Santiago Calatrava. Los puentes.* Electa.2005.