

# Buckling load reduction of axially compressed cylindrical shells under the action of two interacting localized defects

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## Abstract

The effect of two localized axisymmetric initial imperfections on the critical load of elastic cylindrical shells subjected to axial compression is studied through finite element modelling. This was carried out by means of the specialized shell buckling package *Stanlax*. First, a single defect having a triangular geometry is considered in order to determine the most adverse defect configuration, then two defects having this arrangement and which are symmetrically distributed along the shell length are introduced in the problem in order to assess their global interacting effect on the buckling load reduction. A statistical approach which is based on full factorial design of experiment tables and analysis of variance is used to quantify the relative influence of all the intervening factors. It is shown that two interacting defects yield further reduction of the shell critical load.

**Keywords:** buckling, finite element method, imperfections, shells, localized defects.

## 1. Introduction

Thin shells are used in many fields of civil and mechanical engineering such as structural components like silos and tanks. Whatever the manufacturing process is used for these structures, the final geometry is never perfect. Geometric imperfections disturb always the ideal desired nominal form of the assembled shell. Control and optimization of manufacturing processes of shells make it certainly possible today to decrease imperfections amplitudes, but they could never be eliminated completely. Even if, at first guess, the geometry seems to be perfect, precise measurements enable to detect always

small geometric imperfections with magnitudes having in general the same order of scale than shell thickness.

During servicing life, shell structures may be subjected to various kind of loading, such as axial compression, external/internal pressure, flexure or torsion. For cylindrical shells under axial compression, the buckling behaviour is an important design factor. Calculation of the buckling load as it could be affected by the presence of various kinds of geometric imperfections constitutes an essential task. The objective is to know how to design with relevant margins of safety imperfect shell structures.

Several studies have been reported in the literature which deal with the effect of imperfections on strength buckling of thin shell structures. (Arbocz *et al.* [1]) have studied experimentally buckling of cylindrical shells subjected to general imperfections. They have shown that huge reduction of the buckling critical load could be obtained. (Koiter [14]) has given a review study about the effect of geometric imperfections on shell buckling strength. Other extensive investigations have considered the problem of shell buckling where analysis of relative effect of both distributed or localised imperfections on reduction of the buckling load has been performed (Yamaki [19]), (Arbocz [2]), (Bushnell [3]), (Godoy [6]) and (Gros [7]). (Kim *et al.* [13]) have considered a generalised initial geometric imperfection having a modal superposed form. By using Timoshenko shell theory (Timoshenko [18]), they have studied the buckling strength of cylindrical shells and tanks subjected to axially compressive loads on soft or rigid foundations, they have found that the buckling load decreases significantly as the amplitude of initial geometric imperfection increases.

In all cases, the previous works have assessed that imperfections reduce drastically the buckling load of elastic cylindrical shells when subjected to axial compression. The obtained reduction depends on the nature of the considered shell geometric imperfection. But, in general reduction of the buckling load is more severe in case of distributed imperfections than for localized ones.

Imperfections for which reduction of the buckling load attains a maximum might be purely theoretical like for instance the generalized Koiter imperfection (Koiter [14]) and might then never be met in practice in case of real shells. Therefore, investigation has been motivated by the analysis of buckling in the presence of typical imperfections obtained from modal analysis of measured data or by considering realistic imperfection shapes such as those resulting from welding operations performed to assemble shell structures. Steel silos and tanks are constructed from plates which are rolled to obtain the correct curvature and subsequently welded together to form strakes. The strakes are brought together then to assemble by welding the complete shell structure. At circumferential welds localised geometric imperfections form. Measurements have revealed that mostly axisymmetric imperfections occur in these structures (Ding *et al.* [5]). (Hutchinson *et al.* [9]) investigated localised axisymmetric imperfections and have shown that a single axisymmetric imperfection can have large effect on the buckling strength of thin shells. (Jamal *et al.* [10]) have analyzed the influence of localized imperfections on the buckling load for long cylindrical shells under axial compression by using an analytical method based on interaction modes. Analytical formulas were derived to predict the reduction of the critical buckling load. When considering a single localized imperfection, the strength of thin

cylindrical shell structures has been shown to be highly dependent on the nature and magnitude of imperfections. (Jamal *et al.* [11]) have investigated both the effect of distributed and/or localized imperfections on the buckling load for long cylindrical shells under axial compression. The localized imperfection they have used has yielded weaker influence on the reduced buckling load by comparison with the distributed imperfection. Circumferential weld-induced imperfections were found to have a great influence on buckling of thin-walled cylindrical shell structures. Combining shell theory with actual field imperfection measurements, (Pircher *et al.* [17]) have found that three parameters governed the shape of the surveyed weld imperfections: the depth, the wavelength and the roundness. (Khamlichi *et al.* [12]) have considered a parabolic localized imperfection and have obtained by using an analytical approach large reduction of the buckling load for thin cylindrical axisymmetric shell under uniform axial compression. (Mathon [16]) has compared the relative influence of several localised imperfections on reduction of the buckling load of shells that subjected to axial compression or to flexure. He has shown that a triangular imperfection shape has the most severe effect on buckling strength. (Hübner *et al.* [8]) have investigated the case of large steel cylinders with patterned welds by considering the interaction of localised geometric imperfection with residual stresses.

The profile of welding can vary from one shell to another but a common feature of welds is they can be characterized by only two parameters the amplitude and the width termed also wavelength. Since in almost all the previous works single geometric imperfections were considered, emphasis will be done in the following on interaction effects that could result from two geometric imperfections. The localized imperfection geometry is assumed to have a triangular entering shape form. It is considered isolated or under a situation where two imperfections having the triangular form are interacting. In this last case, the distance separating the two defects is an additional parameter which could have some influence on the shell buckling strength. One should add, to the three factors above mentioned, the classical shell aspect parameters: radius over thickness and length over radius.

The pursued objective of buckling strength analysis is to find the most adverse case for which a maximum reduction of the buckling load is attained. This load could be used to estimate within the context of reliability theories a safe design load. In the subsequent, thin cylindrical axisymmetric shells made from homogeneous and isotropic elastic material are considered. They are assumed to deform under purely axisymmetric strain state when they are subjected to axially uniform compressive loads. Investigation of the relative effect of the intervening five factors on reduction of the shell buckling load is performed by using the following methodology. At first, the shell aspect ratios for which maximum effect on the buckling strength is obtained are determined. Then with this shell configuration fixed, a parametric study is conducted by varying the three free remaining factors according to a full factorial design of experiment table. Analysis of variance is finally performed to determine the relative influence of factors.

## **2. Modelling of thin cylindrical shells with localized defects**

In order to analyse the effect of imperfections on shell buckling strength for the particular case of thin circular cylindrical shells subjected to quasi-static uniform compressive loads, shell equations corresponding to Sanders model and incorporating the effect of initial

imperfections are used (Markus [15]). A variant of this model has been used by (Gros [7]). Relevant finite element modelling of these equations was carried out by means of *Coque* element developed under Stanlax software package (Combesure [4]). Stanlax software is used in the following in order to model the imperfect cylindrical axisymmetric shell having a given number of localized geometric imperfections. Stanlax software is based on an analytical expansion in terms of circumferential variable contributions and a finite element discretisation of axial dependant quantities. The initial imperfections are included in model formulation under the assumption of small perturbations to shell geometry. Stanlax offers either a linear Euler buckling analysis or a full non linear iterative computation of the buckling load. For shells under axial compression, linear Euler mode is sufficient for buckling analysis.

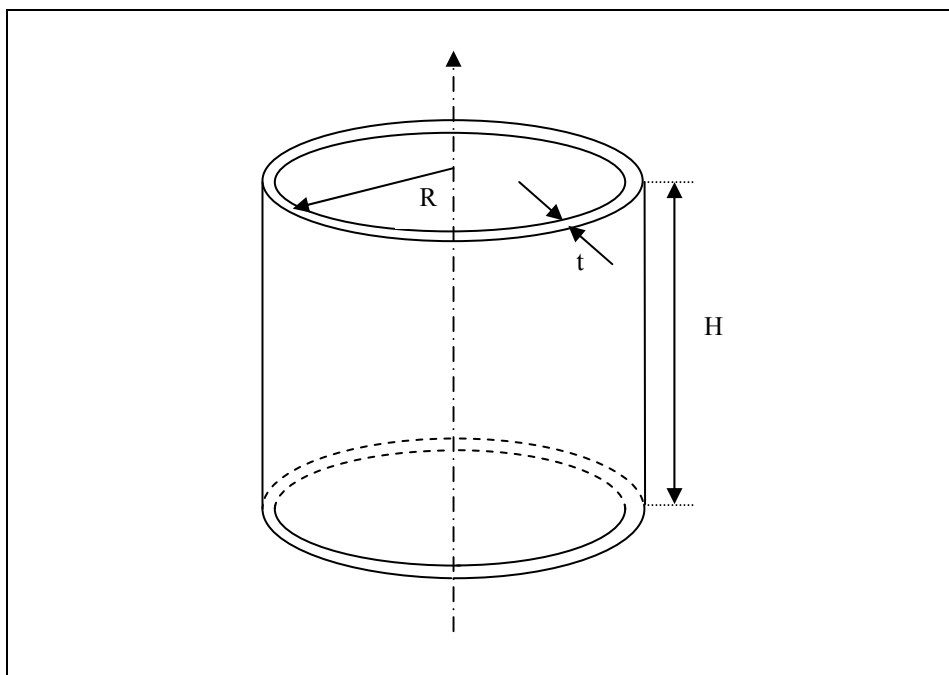


Figure 1: Shell geometry

The shell material is assumed to be linear elastic having Young's modulus  $E$  and Poisson's coefficient  $\nu$ . The geometric imperfections are supposed to be localized in the median zone of the shell length in positions that are sufficiently far from the shell ends in order to avoid significant interaction with the boundary conditions. The selected boundary conditions are those corresponding to clamped shell ends.

As shown in figure 1, parameters  $t$ ,  $H$  and  $R$  designate respectively shell thickness, shell length and shell mean radius.

Let  $A$  and  $d$  be respectively the geometric imperfection amplitude and the distance separating the two imperfections. Let  $\lambda_c = 1.72\sqrt{Rt}$ , the following non dimensionalized parameters associated to the intervening five factors are introduced:

- $R/t$  radius to thickness ratio;
- $H/R$  length to radius ratio;
- $A/t$  defect amplitude parameter;
- $H/d$  height to defect interval scale ratio;
- $\alpha = \lambda/\lambda_c$  defect wave length to critical wave length.

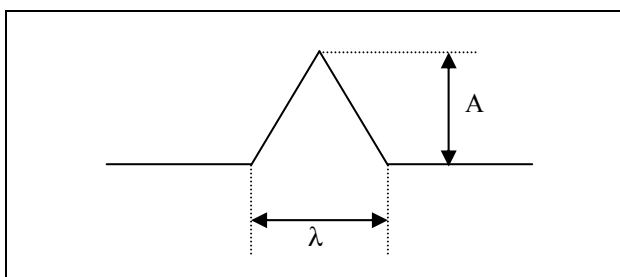


Figure 2: Shape of the localized triangular imperfection

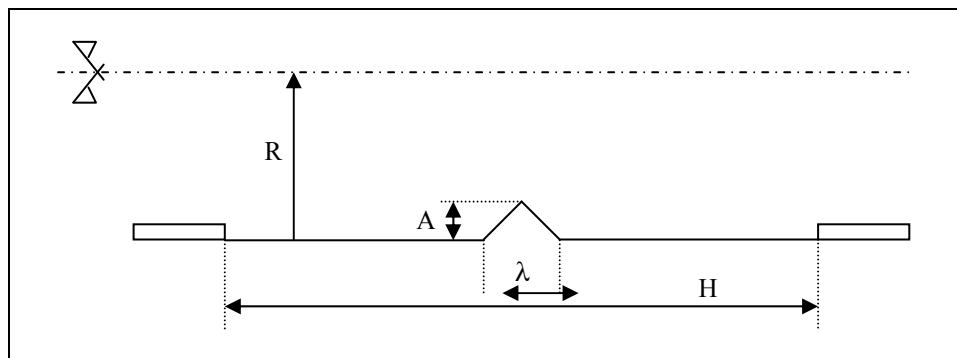


Figure 3: Configurations of triangular localized geometric imperfections  
 Single entering triangular imperfection (*dimensions  $A$  and  $\lambda$  are exaggerated*)

During the whole study, the shell radius is maintained constant at the value  $R=135$  mm while the other parameters are varied. The considered localised geometric imperfection has the triangular configuration shown in figure 2. It is directed inwards the shell radius. Combinations of the geometric imperfections for both a single imperfection of two imperfections have the configurations shown in figures 3 and 4.

Stanlax software package enables for each combination of parameters to compute the shell buckling load when it is subjected to uniform axial compression. Use is systematically made of shell element *Coque* and convergence assessment is performed in order to determine the optimal mesh size to be employed.

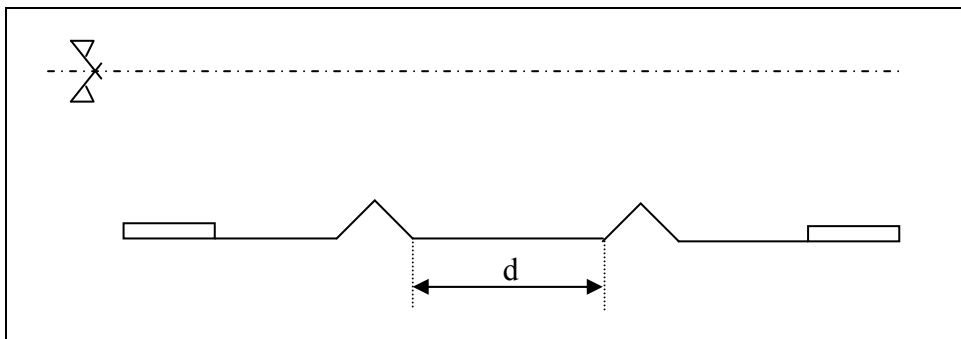


Figure 4: Configurations of triangular localized geometric imperfections  
Two entering triangular imperfections (*dimensions are exaggerated*)

### 3. Parametric study of the shell buckling load as affected by localized geometric imperfections

The effect of a single localized geometric imperfection is investigated at first in order to determine the most adverse shell aspect parameters with regards to the buckling load reduction. This enables to fix parameters  $H/R$  and  $R/t$ , and simplifies investigation about the relative influence of the geometric imperfection parameters  $\alpha$ ,  $A/t$  and  $H/d$ .

#### 2.1. Case of a single imperfection

Let's consider a single triangular geometric imperfection located at the mid height of the shell for which geometric and material properties are given by:  $R = 135$  mm,  $H = 405$  mm,  $t = 0.09$  mm,  $E = 7 \times 10^{10}$  Pa and  $\nu = 0.3$ . In this case the classical buckling load is given by  $\sigma_{cl} = 28.233 \times 10^6$  Pa. When, the imperfection amplitude is fixed at  $A/t = 1$  and its

wavelength at  $\lambda = 15\text{ mm}$ , figure 5 presents the evolution of the buckling load ratio  $K = \sigma_{cr} / \sigma_{cl}$  as function of the number of elements and the number of circumferential harmonics, with  $\sigma_{cr}$  the actual critical load and  $\sigma_{cl}$  the classical buckling load defined as

$$\sigma_{cl} = \frac{E}{\sqrt{3(1-\nu^2)}} \times \frac{t}{R} .$$

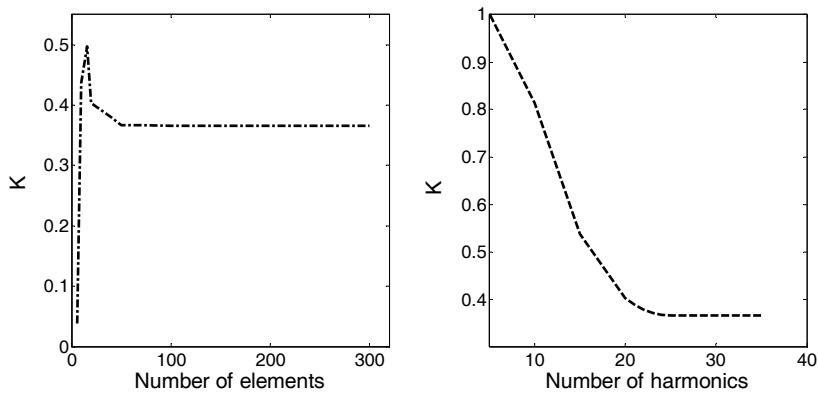


Figure 5: Convergence study of the finite element model developed under Stanlax; single entering localized geometric triangular imperfection

Figure 5 shows that a total number of 300 elements and a total number of 30 harmonics guarantee finite element model convergence. Other results not shown here have demonstrated that for all single or interacting imperfection cases a total number of 300 elements and a total number of 30 harmonics guarantee well convergence of the finite element model.

Figure 6 gives the buckling load, for the case of a single triangular entering geometric imperfection, as function of wavelength and amplitude parameters when  $H/R = 1$  and  $R/t = 750$ . Almost the same curves are obtained for the following values of shell aspect parameters:  $H/R = 1$  or 3 and  $R/t = 750$  or 1500. These results show that there is only a small effect of parameters  $H/R$  and  $R/t$  on the shell buckling load. It could be seen also from figure 6 that the wavelength  $\alpha = 2.5$  yields the most adverse case since this curve is below those associated to  $\alpha = 1$  and  $\alpha = 4$ .

Fixing now  $\alpha = 2.5$ , figure 7 gives, for  $H/R = 3$ , the buckling load as function of shell aspect ratio  $R/t$  and amplitude parameter  $A/t$ . Almost the same curves are obtained if

the shell aspect ratio  $H/R = 1$  is used. These results show that there is only a small effect associated to parameter  $H/R$  and that the most adverse case is obtained for  $H/R = 3$ . Fixing again  $\alpha = 2.5$  for the case of a single triangular entering geometric imperfection, figure 8 gives, for the shell aspect parameter  $R/t = 1500$ , the buckling load as function of shell aspect parameter  $H/R$  and amplitude parameter  $A/t$ . Here also, the results show that there is only a small effect of parameter  $R/t$  and that the most adverse case corresponds to  $R/t = 1500$ .

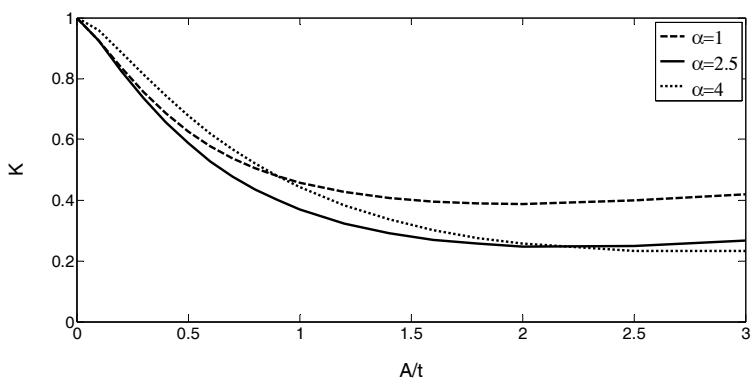


Figure 6: Effect of wavelength  $\alpha$  on the buckling load for  $R/t = 750$  and  $H/R = 1$

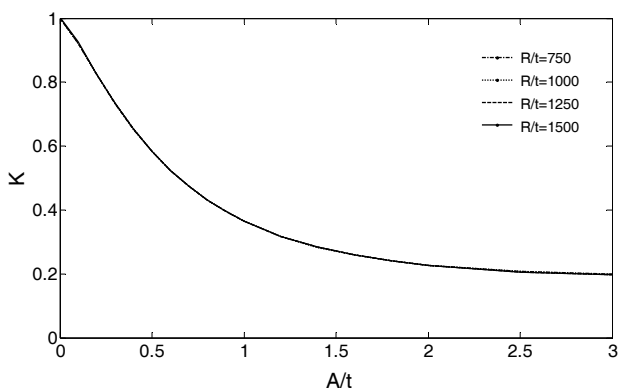


Figure 7: Effect of shell aspect ratio on the buckling load reduction for  $H/R = 3$



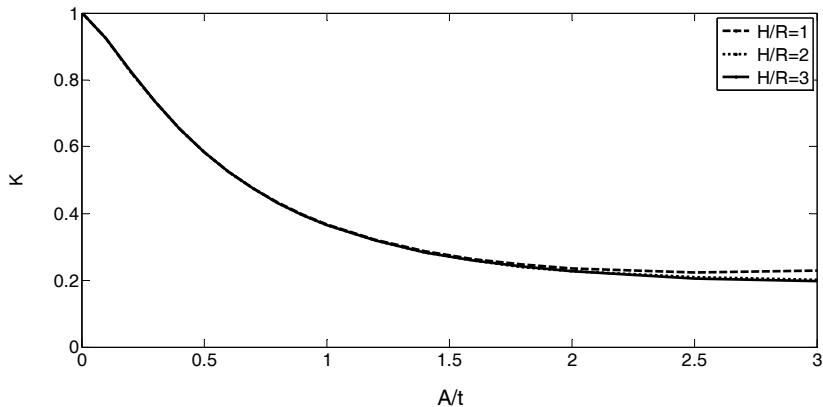


Figure 8: Effect of shell aspect ratio on the buckling load reduction for  $R/t = 1500$

### 2.1. Case of two imperfections

The previous procedure was employed also when two interacting geometric imperfections are present. The same conclusions can be assessed. So, the shell aspect parameters can be fixed at  $H/R = 3$  and  $R/t = 1500$  in order to obtain the most adverse buckling case. The influence of geometric imperfection parameters:  $\alpha$ ,  $A/t$  and  $H/d$  on the shell buckling load can now be straightforwardly investigated.

Table 2 lists parameters levels that have been considered in the analysis of geometric imperfections under the coupling situation: lower threshold, intermediate value and higher threshold. Based on this table, a parametric study regarding the influence two interacting geometric imperfections has been performed according to a design of experiment method using a full factorial table containing the three factors. A total set of 27 simulations have been conducted for each case: single or two defects.

Table 2: Ranges of variation of geometric imperfection factors

	$\alpha$	$A/t$	$H/d$
Lower threshold	1.0	0.5	81
Intermediate value	2.5	1.0	40.5
Higher threshold	3.0	2.5	20.25

For the case where  $R/t = 1500$ ,  $H/R = 3$ ,  $\alpha = 2.5$  and  $H/d = 40.5$  the obtained results in terms of  $\sigma_{cr}/\sigma_{cl}$  versus parameter  $A/t$  are presented in figure 9. It is shown that the most adverse reduction in the critical load  $\sigma_{cr}/\sigma_{cl}$  passes from 0.176 in the case of a single triangular imperfection to only 0.119 in case of two interacting imperfections.

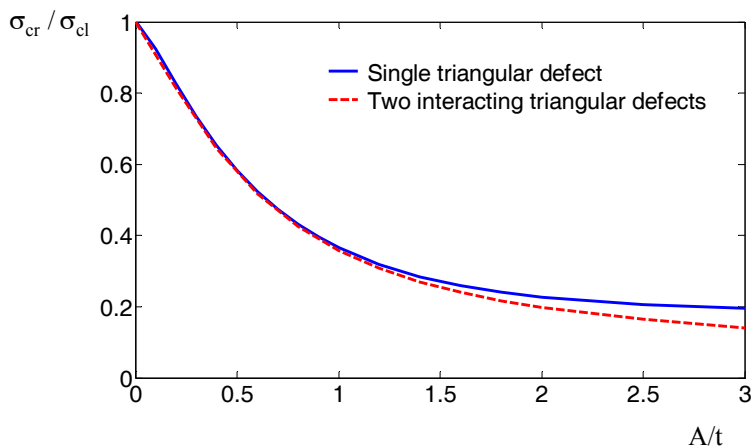


Figure 9: Reduction of the buckling load as function of defect amplitude  $A/t$

### 3.3 The special effect of distance separating geometric imperfections

It is of particular interest to investigate how the separating distance associated to parameter  $H/d$  may affect the shell buckling load value when a large range of parameter  $d$  is considered. This may be of significant help to determine the best strategy to apply when assembling shell strakes.

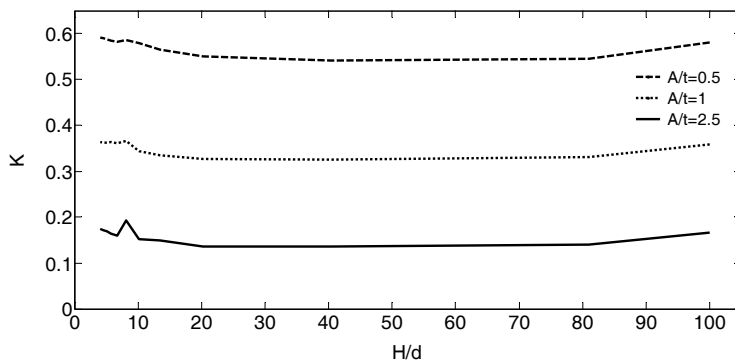


Figure 10: Effect of parameter  $H/d$  on the buckling load

Fixing parameter  $\alpha = 2.5$  and varying parameter  $A/t$ , figure 11 gives the buckling load as function of parameter  $H/d$ . It can be seen that parameter  $d$  has not an equal effect on the buckling strength. Small values of  $d$  yield higher reduction of the bulking load, while for high values of distance  $d$  the obtained buckling load is a little bit higher. A local maximum appears in case of  $A/t = 2.5$ . From a practical point of view this gives the ideal height for welding strakes during shell assembly in order to maximize the bulking strength.

### 3.4 Analysis of variance

Analysis of variance was performed on the buckling load results associated to the 27 combinations considered for two triangular localized geometric imperfections. It has given the following p-values ( $\alpha$ :0.0741;  $A/t$ : 0.0001;  $d/H$ : 0.4164). This shows that the amplitude parameter is the most significant factor. It is followed by the wavelength parameter and finally by the distance separating imperfections.

## 4. Conclusions

Numerical simulations based on the finite element method have been performed in order to quantify shell buckling load reduction in the presence of localized geometric imperfections. Elastic thin cylindrical shells subjected to axial compression and having one or two axisymmetric defects of entering triangular shape have been taken into account. A set of five factors intervening in the problem have been identified and their relative effect analysed.

The most adverse case in terms of shell aspect ratios has been first determined when a single geometric imperfection is considered to act alone. A parametric study with regards to the left factors has been performed in case of two geometric imperfections according to a full factorial design of experiment table.

It has been shown that two localized imperfections yield further reduction of the buckling load. This reduction is important and can reach 67% of the buckling load in comparison with the case of a single geometric imperfection.

Analysis of the relative effect of factors has shown that the imperfection amplitude has the greatest influence on the buckling load reduction. It is following by the imperfection wavelength and the relative distance separating the two geometric imperfections.

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