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Steel cylinders on local supports with rigid stiffeners

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Abstract

The main part of a traditional silo is the cylindrical shell placed upon a limited number of support columns in order to facilitate emptying operations. These local supports lead to stress concentrations in the cylinder and can cause local instabilities of the shell. In order to prevent this failure phenomenon, the cylindrical shell can be reinforced by means of two longitudinal stiffeners of limited length above each support in combination with a ring stiffener above and below the vertical stiffeners.

In this paper, the longitudinal stiffeners are assumed to be flexurally and axially rigid in order to prevent the stiffeners from buckling and to allow the study of the structural behaviour of the cylinder. The results of an initial numerical study with the finite element package ABAQUS are presented.

Keywords: cylindrical shells, buckling, Eurocode, local supports, imperfections.

1. Introduction

Thin-walled shells form challenging structures which are the subject of numerous studies as their behaviour is still not fully understood. These structures are prone to instability and geometric imperfections, which are unavoidable, can have an extremely detrimental influence on the buckling load.

An example of such a thin-walled shell structure is an elevated steel silo. The main part of the silo is the cylindrical body which is supported by a limited number of columns. These discrete supports lead to a local force introduction in the cylindrical shell which in turn causes axial compressive stress concentrations in the cylindrical wall above the supports. As the shell is thin-walled, these concentrations can lead to instability in the silo shell. In the past, several proposals have been made to deal with the stress concentration. In the simplest approach, the wall thickness of the entire cylinder is increased so that instability is prevented (Guggenberger *et al.* [3]). However, elevated stresses only appear in the lower part of the silo, shortly above the supports. This means that a more economical solution should be able to be found by only increasing the wall thickness of the lower strakes of the silo. This idea, bottom course wall thickening, was investigated by Rathé *et al.* [4].

Even more material can be saved if, instead of increasing the wall thickness, the cylinder is reinforced by attaching stiffeners of limited length to the cylindrical wall above the supports. In this proposal, the material is added only in the regions where the stress concentrations occur (Figure 1). In this stiffener configuration, two flat rectangular plate longitudinal stiffeners of limited length are placed above each support. Above and below these stringer stiffeners, a ring stiffener, also made from a flat plate, is placed. This concept was investigated by Vanlaere *et al.* [7]. The study showed however that, for these stiffened cylinders on local supports, instability can also occur in the flat longitudinal stiffeners because they buckle as plates with only one edge supported.

Here, it is supposed that these longitudinal stiffener plates are made stiff enough to resist instability, so they are treated as flexurally and axially rigid. This allows the study to focus on the structural behaviour of the cylinder. In this paper, the results of initial numerical simulations are given. In the next stages of the study, numerical simulations will lead to design rules for this structure and the behaviour of the stiffeners will be investigated separately to determine the required cross-section.

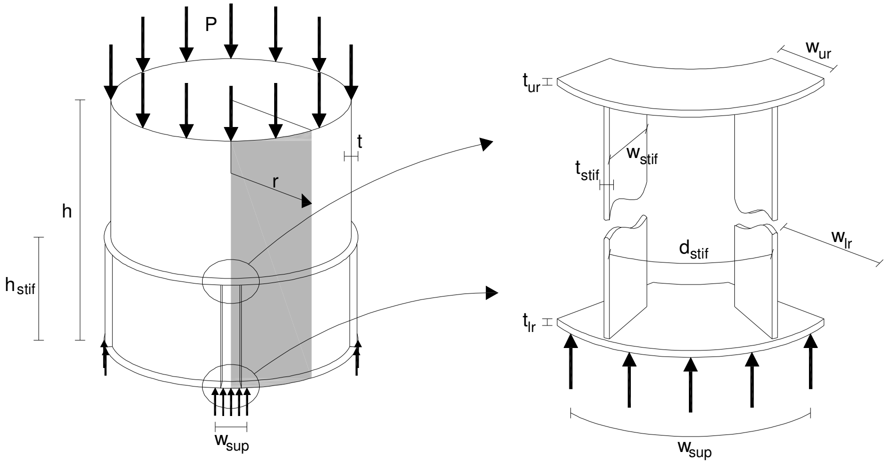

Figure 1: The stiffened cylinder on local supports

2. Investigated geometry

The results of an initial numerical study are presented here. A single cylinder geometry has been investigated with the dimensions of a scale model that will shortly be tested in the laboratory at Ghent University.

The stiffened cylinder on local supports is shown in Figure 1. The example cylinder studied here rested on n=4 supports and had a radius-to-thickness ratio r/t=500 with r=350mm and t=0.7mm. The thickness of the stiffeners was taken as equal to the cylinder wall thickness:

$t_{stif}=t_{ur}=t_{lr}=t$. The ratio of the length of the longitudinal stiffeners to the cylinder radius was $h_{stif}/r=0.75$. The stiffener length to cylinder length ratio was $h_{stif}/h=0.286$. The width of the longitudinal stiffeners to the cylinder radius was $w_{stif}/r=0.06$. The width of the ring stiffeners was $w_{ur}=0.5w_{lr}=w_{stif}$. The ratio of width of the supports to the cylinder radius was $w_{sup}/r=0.15$. The distance between the two stringers above each support was made equal to the support width: $d_{stif}=w_{sup}$. The cylinder was subjected to a uniform line load around the upper edge (Figure 1). The calculations are reported in terms of the total load applied to the complete top edge.

3. Numerical simulations

3.1. Numerical model

The structural behaviour of the stiffened cylinder on local supports was studied by means of numerical simulations with the finite element package ABAQUS . The stiffened cylinder and the ring stiffeners were modelled using shell elements (S8R5). The flexurally and axially rigid longitudinal stiffeners were modelled using rigid elements (R3D4). In order to reduce the computational time needed, only a segment of 45° was modelled (grey shaded in Figure 1). Symmetry boundary conditions were applied to the longitudinal edges of the model. On the top edge and the supported part of the bottom edge the radial displacements were restrained. The boundary conditions on the supported part of the bottom edge of the cylinder also reflected the effect of an axially rigid support. The properties of the steel were taken as Young's modulus $E=200$ GPa, Poisson's ratio $\nu=0.3$ and yield stress $\sigma_y=235$ MPa. A perfectly elastic-plastic model was adopted in the materially nonlinear analyses.

3.2. Results of the analyses

In EN 1993-1-6 [2], different types of numerical simulations are described. The most sophisticated simulation is the geometrically and materially nonlinear analysis of the imperfect shell (GMNIA). However, this standard requires that other less sophisticated analyses are undertaken to assess the relative importance of buckling and plasticity in the failure phenomena.

The first analysis undertaken was a linear bifurcation analysis (LBA). The failure load obtained was 203.9 kN (eight times the load applied in the 1/8 model) at the time of failure. When geometrical nonlinearity was taken into account (GNA), the failure load dropped to 152.9 kN, which is a reduction of 25%. This reduction is significantly larger than that commonly quoted for the case of uniformly compressed cylinders, where 15% is a common value (Yamaki [8], Rotter [5]).

The materially nonlinear analysis (MNA) produced a failure load of 234.0 kN. This plastic collapse load can be compared to an analytical estimate found by the condition of membrane yield along any kinematically admissible path. This follows the logic used by Doerich and Rotter [1] in determining the collapse load of bracket-supported cylinders. With such a complex local geometry, there are several possible plastic mechanisms that can occur. For this chosen geometry the critical mechanism is yield in shear of the shell on the outside of each stiffener and yield of the shell in compression at the top of the upper ring.

$$F_{pl} = n \cdot \left(d_{stif} \cdot t \cdot \frac{2}{\sqrt{3}} \cdot \sigma_y + 2 \cdot h_{stif} \cdot t \cdot \frac{1}{\sqrt{3}} \cdot \sigma_y \right) = 239.3kN \qquad (1)$$

When both geometrical and material nonlinearity were included in the simulation of the perfect structure (GMNA), the failure load was 137.8 kN. Since this is only 10% smaller than the failure load predicted by the GNA analysis, it is clear that plasticity only plays a minor role for this geometry and the failure is dominated by nonlinearity and buckling.

The effect of two imperfection shapes on the buckling strength was investigated. The amplitude δ for both of these shapes was taken as equal to the wall thickness (δ/t=1.0). The first assumed imperfection had the shape of the first eigenmode (Figure 3b) as recommended by EN 1993-1-6 [2]. The failure load found with this shape (GMNIA) was equal to 113.9 kN. The presence of this imperfection therefore led to a further reduction of the failure load by 17%. The second chosen imperfection had the shape of a weld depression Type A (Rotter *et al.* [5-6]), which represents a practical imperfection seen in full scale structures. Calculations were performed with this weld depression at different locations to identify the most critical place. With this imperfection at the critical location, the failure load of the GMNIA analysis was calculated at only 89.0 kN, which is 35% lower than that for the perfect structure (GMNA). Clearly a weld depression imperfection of this amplitude is much more detrimental than the eigenmode in this instance.

The results of the different analyses are given in Figure 2 and the corresponding failure patterns are shown in Figures 3 and 4. For all the analyses except the weld depression imperfection GMNIA, the buckles were located in the cylindrical shell near the top of the rigid stringer stiffener. In the GMNIA analysis including the weld depression imperfection, the buckle appeared at the location of the weld depression at the critical location 11λ above the stiffener termination, where $\lambda = 2.44\sqrt{rt}$ is the linear bending half-wavelength of the cylinder.

Type of Analysis	Failure load [kN]
LBA	203.9
GNA	152.9
MNA	234.0
GMNA	137.8
GMNIA	
first eigenmode (δ/t=1)	113.9
weld depression (δ/t=1)	89.0

Figure 2: Overview of the results of the numerical simulations

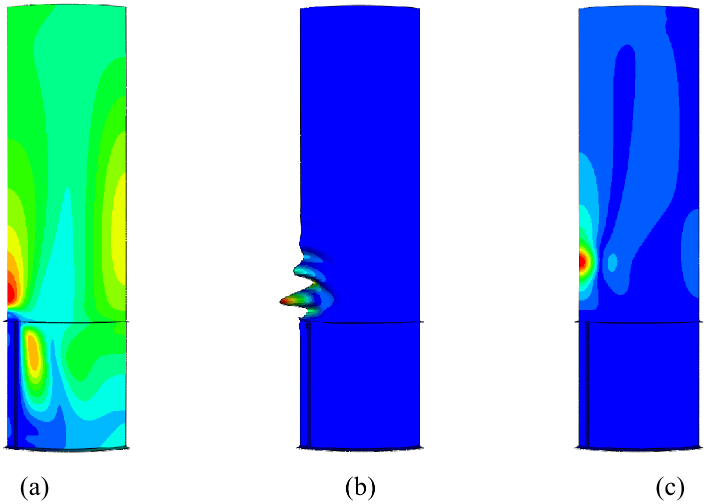

Figure 3: The deformation patterns of the different numerical simulations for (a) MNA, (b) LBA, (c) GNA.

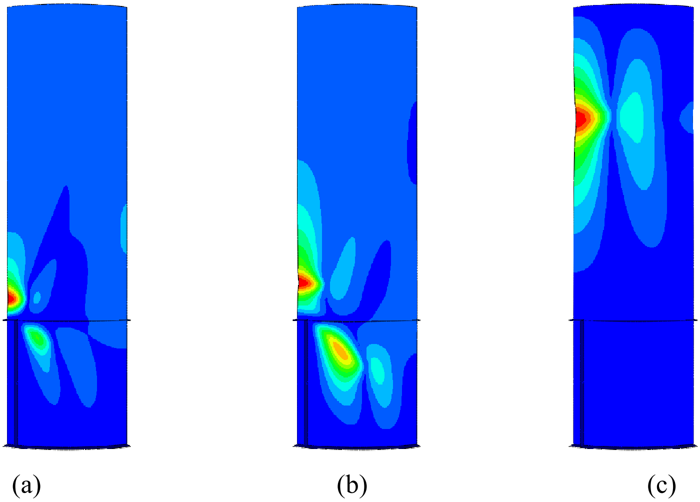

Figure 4: The deformation patterns of the different numerical simulations for (a) GMNA, (b) GMNIA with the first eigenmode as imperfection shape and (c) GMNIA with a weld depression as imperfection shape.

3.3. Effect of imperfections

Two imperfection shapes were investigated in this initial study. As stated above, the first imperfection was in the form of the first linear eigenmode of the perfect structure. In

Figure 5, the effect of the amplitude of this imperfection is given. A small imperfection amplitude reduced the failure load by about 17%. A further increase of the amplitude seemed to have no significant effect on the failure load. The second imperfection shape that was investigated was a weld depression. For this shape, both the location of the centre of the depression and its amplitude are independent parameters that can be varied. Figure 6 shows the variation of the buckling load with the location of the centre of the weld depression.

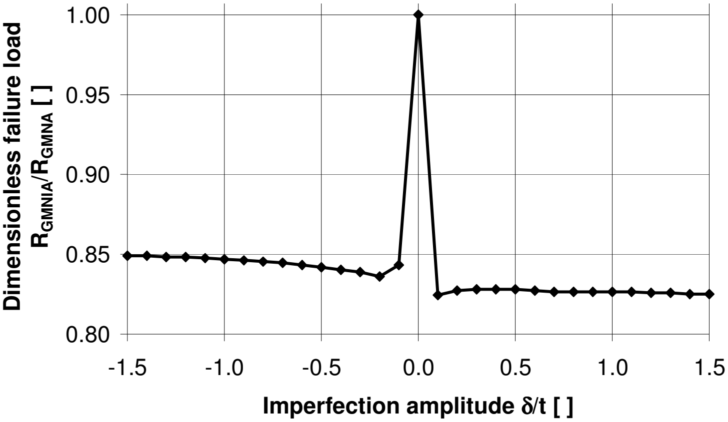

Figure 5: The effect of the amplitude of the first eigenmode on the failure load of the GMNIA analysis.

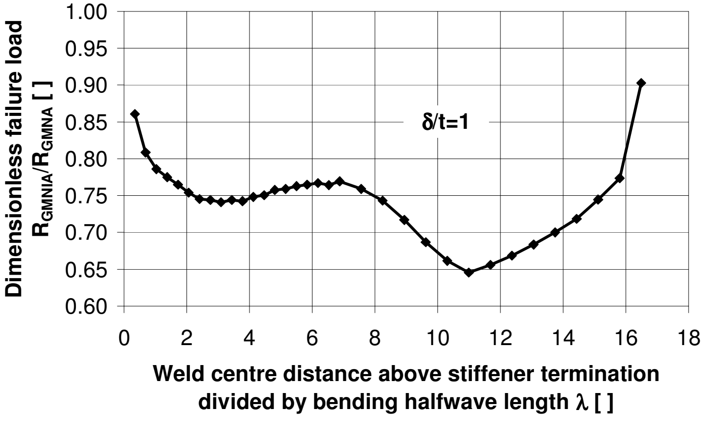

Figure 6: The effect of the location of the weld depression on the failure load of the GMNIA analysis.

At this imperfection amplitude (δ/t=1.0), the most detrimental location for the weld depression is not near the top of the rigid stiffener but at a point 11λ above the stiffener termination. The compressive stresses take their highest values just above the top of the stiffener, but the highly stressed zone there is too small for a critical buckle to form. As the stresses disperse upwards into the shell, the potential size for a buckle grows, and the buckling load declines.

Thus, at a greater distance from the stiffener termination the compressive stresses may be lower but these stresses spread over a larger region, making it easier for the buckle to form. The failure load of the GMNIA analysis given in Figure 2 is for a weld depression at 11λ above the stiffener termination.

For this most detrimental position, the effect of changing the imperfection amplitude was studied. The results are summarised in Figure 7. At a small amplitude, the weld depression has no effect, but at amplitudes larger than 30% of the wall thickness a significant reduction of the failure load occurs. This can be explained by examining the deformation patterns in Figures 4a and 4c. Where no imperfection is included in the numerical simulation, the failure pattern is characterised by a buckle just above the stiffener.

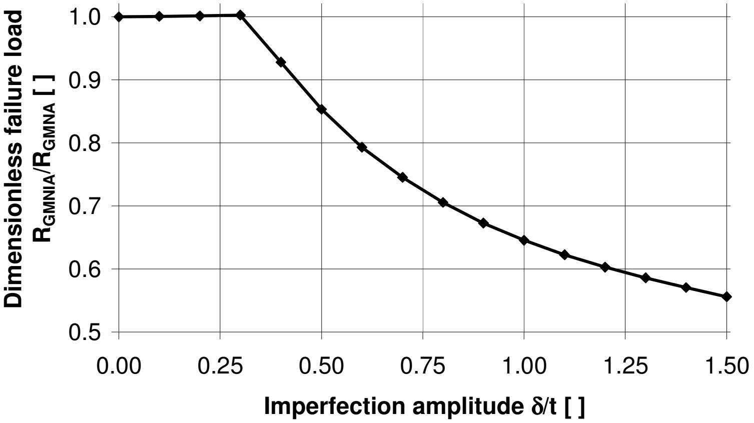

Figure 7: The effect of the amplitude of a weld imperfection located 11 λ above the termination of the support on the failure load of the GMNIA analysis.

A weld depression with a small amplitude located at 11λ above the stiffener termination is insufficiently damaging to become more critical than a buckle at the location appropriate to the perfect structure. But for all realistic imperfections (amplitudes larger than 30% of the wall thickness), the weld depression causes a reduction in the buckling strength so that the critical buckle occurs at the weld depression. The failure load decreases further with increasing imperfection amplitude.

4. Conclusions

An initial study has been presented of the buckling behaviour of stiffened cylinders on local supports with rigid stiffeners. The vertical stiffeners were assumed to be rigid in order to prevent failure by instability of the stiffeners. This allowed an investigation of the structural behaviour of the cylinder alone. In this initial study, different analysis types were performed to quantify the effects of geometrical nonlinearity, plasticity and geometrical imperfections. For the studied geometry, the results show that geometrical nonlinearity has a larger effect than is typically found for uniformly supported cylinders. The effect of plasticity for this geometry was found to be rather limited, but geometric imperfections were found to have a strong impact on the failure load. In particular, the weld depression imperfection reduced the failure load by more than 35%, even when the amplitude was only equal to one wall thickness. Future studies will focus on the development of a design rule for these stringer stiffened cylinders on local supports and will investigate the behaviour of the stiffeners.

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