

## **Inelastic nonlinear analysis of beams with under-tension cables**

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### **Abstract**

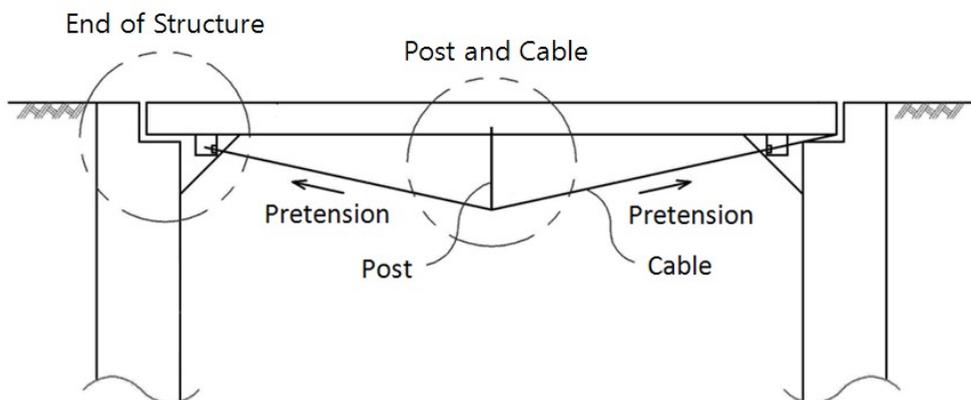
For long span structures, the size of the structural members should be determined by considering the esthetic view. As a result, the slenderness ratio of members is increased and the structures may be suffered from significant deflection as well as excessive stress level. In the under-tension systems, cables are introduced in the tensional layer of the beam. Prestress is introduced in cables to improve force distribution with some weight reduction. These assemblage of struts and cables can be applied under the beam to improve stiffness and strength under vertical load.

In this paper, inelastic nonlinear analysis of beams with under-tension cables is presented. Both Total Lagrangian (TL) and Updated Lagrangian (UL) formulations are employed for geometric nonlinearity. Material nonlinearity for cables as well as the beam is included in the analytical model. The nonlinear finite element model has been developed for the analysis of under-tension structures. The nonlinear behavior of the beam with under-tension cables is investigated addressing the effect of number of struts, length of the struts to beams, magnitude of the cable pretension, and the size of the struts on the structural behavior.

**Keywords:** Cable dome, Singular value decomposition, Form-finding, New configuration.

## 1. Introduction

As the urbanization rises and the population tend to concentrate in large cities, most large cities experience rising land prices and decreasing site tribe. In fact, it gradually gets difficult to keep the green tract of land and leisure spaces, which are the base facilities of the city. Even though the greens and rest places are secured within the city, most of them tend to show a lack of connection to their surrounding environment because each of them is created by different individual projects<sup>[1]</sup>. However, the co-existence of those areas, the greens and rest places, and urbanized cities are architectural reality, which is inevitable. This study strongly suggests the greens and resting area to be built on the upper part of the local street and the junctions, in order to efficiently understand the hectic urban environment while not disturbing local streets. From this approach, the paper will analyze the way to connect each resting area on the upper side of the street to another by building the large span structure, and will discuss about the structural behavior of this large span structure.<sup>[2]</sup>



<Fig 1> the simple version of the under-tension system

This study suggests applying under-tension system on the lower side of structure in order to control the large structure's displacement and member size. The fig.1 is the simple version of the under-tension system.

The metal strut is installed on the lower part of the structure and the cable is connecting this metal strut and the upper structure. By applying pretension force initially to the cable, this under-tension system controls the displacement of the whole structure and the member size.

If a structure is under pretension, linear analysis cannot predict the structural behavior correctly. Furthermore, the structure can undergo moderate and even large deformation.

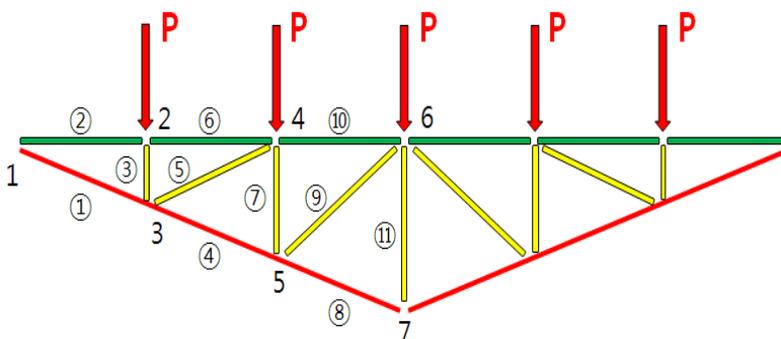
Also, the collapse of the truss structure has shows its brittle behavior. Therefore, using linear elasticity method is subject to be considered as an inappropriate method for predicting the behavior of truss structure. Hence, in order to bring out the more accurate analysis of the truss structure, considering both of geometric non-linearity and material non-linearity at the same time is fundamental.<sup>[3]</sup>

An under-tension truss analysis program has been developed based on the finite element method for nonlinear analysis **strategy**. The program traces the elastic-fully plastic load-displacement response of a structure into the post-critical range using appropriate member constitutive equations, finite element stiffness equations and an incremental-iterative solution scheme. The program is presented in the following section along with sample space truss problems to demonstrate its versatility in modeling large deformation elastic-fully plastic response of progressively failing space truss structures by using both total Lagrangian (TL) and updated Lagrangian (UL) formulation.<sup>[4],[5]</sup>

## 2. Analysis Model

It is necessary to analyze the structure with 3-D modeling, in order to get the most similar result of the actual behavior. However, this study analyzes the structure based on the simplified 2-D truss model. This helps reducing the time for structure modeling and impedes the bending moment of member so that we can more accurately affirm the effectiveness of the under-tension cable on its main structure.

As it is shown on the fig. 2, the model is 60m in span and the upper part is divided every 10m. Also, the model used in this study is composed with 12 nodes and 21 parts, and presume the same value of load which is applied from the vertical direction of the upper part of the vertical strut. All elements are made of truss and each boundary are composed with hinge



<Fig.2> Model element, node and loading point

For the case of the actual structure, the load has to be applied for all the areas of the structure on upper member to vertical direction. For the simplicity analysis, this study assumes that the load is applied throughout five points to the structure.

<Table. 1> Location of nodes and elements

Node	X-coord	Y-coord	Element	Area(m <sup>2</sup> )
1	0	0.0	1, 4, 8	$1.53 \times 10^{-3}$
2	10	0.0	2	$1.90 \times 10^{-1}$
3	10	-1.0	3, 7, 11	$6.87 \times 10^{-2}$
4	20	0.0	5	$6.87 \times 10^{-2}$
5	20	-2.0	6	$1.90 \times 10^{-1}$
6	30	0.0	9	$6.87 \times 10^{-2}$
7	30	-3.0	10	$1.90 \times 10^{-1}$

\* Symmetric for 11th element

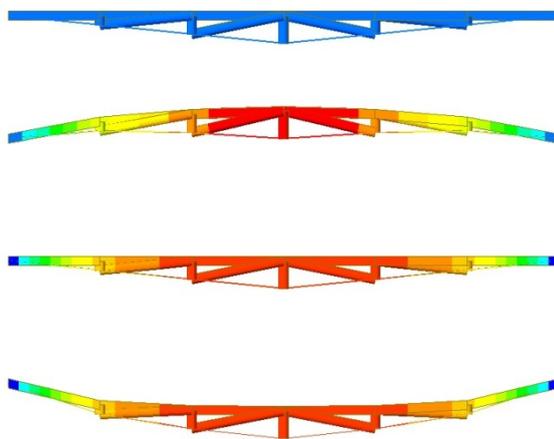
<Table. 2> Properties of Material

Material	Modulus of Elasticity	Yield Stress	Section
Steel	$2.0594 \times 10^8$ kN/m <sup>2</sup>	325 MPa	Truss (Slab)
			Truss (Strut)
Cable	$1.9025 \times 10^8$ kN/m <sup>2</sup>	1600 MPa	Cable

according to the previous research, it is known that the size of the pretension of the cable, and the number of struts are major factors of the effectiveness of the under-tension system.<sup>[6]</sup>

In order to comparing the displacement and member force, the load applied to upper part of the structure is increased in sequence from 0 kN to 100 kN. For the case of pretension applied to the cable, pretension is increasingly applied to 10 %, 30 % 50 % to the basic truss structure based on the yield force of the cable.

The fig. 3 shows the deformation of the structural behavior when the load is applied to the upper part when there pretension is also applied. When there is a pre-tension applied to the initial shape of structure, it is as if it has given the camber, because the displacement of the structure occurs from the upper part. Later, the structure changed to the early shape of it when the load is applied and when there is an increasing value of load, the structure will show the displacement to the lower part as the common structure does.<sup>[7],[8]</sup>



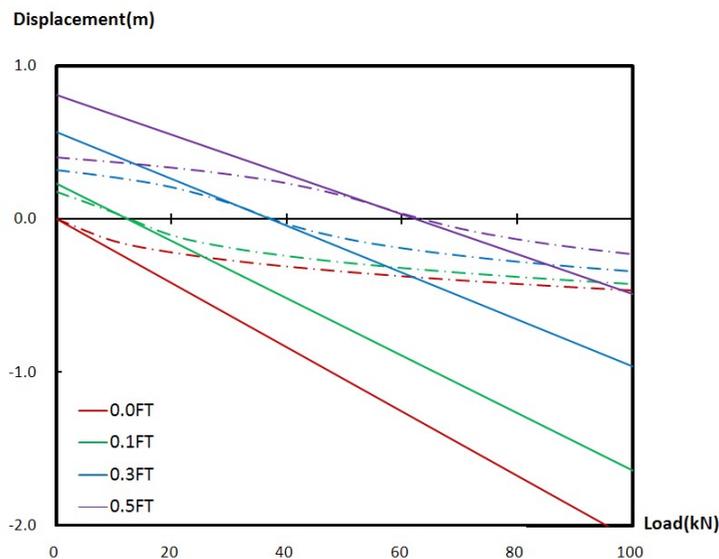
<Fig. 3> Motion of Under-tension system

### 3. Analysis result

#### 3.1 Maximum displacement

Now, compare the maximum displacement of the structure when the value of pretension is increased in sequence. The Fig.4 shows the result that when the maximum displacement is decreasing as the value of the pretension is increasing. There was no displacement occurs when the pretension is not applied, for the case of a structure with no loading. On the other hand, the displacement of 0.0262 m occurred when there is 0.1FT of pretension, 0.0787 m when 0.3 FT applied, 0.1305 m occurs when 0.5 FT applied. So this shows that if there is basically a large value of free tension, the displacement of a structure does not occur even the load is increased. On the contrary, the behavior of structure towards the upper side is shown. For this case, the value of the pretension of 0.1FT brings load of 242.2 kN, 0.3 FT brings 732.6 kN, 0.5 FT brings 1110 kN.

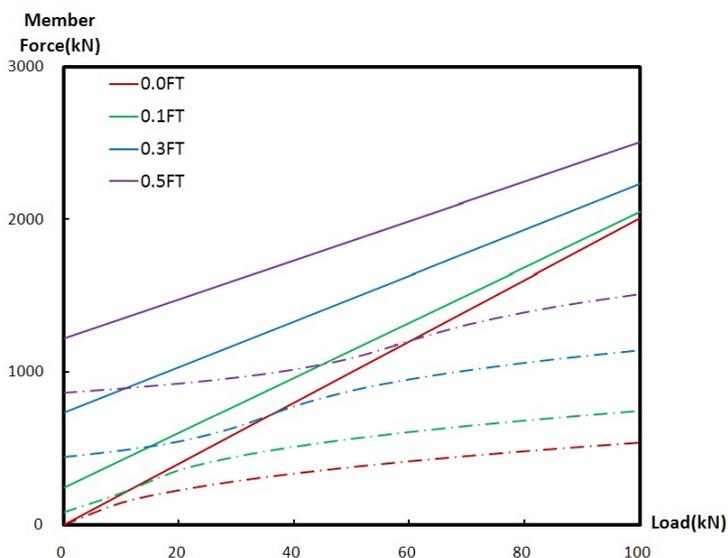
Moreover, when comparing with the result of the linear analysis, as the value of the load and pretension increase, the chance of maximum displacement, which is made by non-linear , gets bigger.



<Fig. 4> Displacement to pretension value

### 3.2 Member force

Now, with the change the value of load in sequence and the value of the pretension, compare the member force of the each member. For marking the member force, the change of stress is marked for each member, which is the major component of under-tension system. If the value of member force is positive, there is tensile force applied to that member and if the value is negative, the compressive force is applied on the contrary.

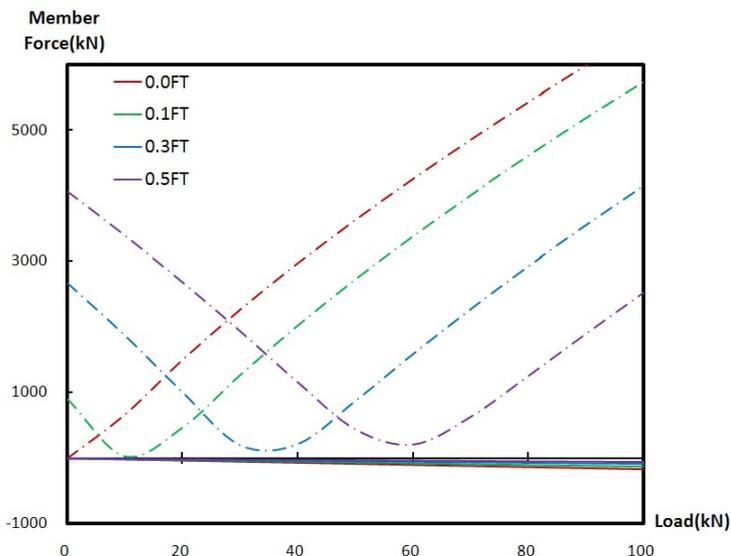


<Fig. 5> Member Force about element no.4

The fig.5 is a graph shows the element no.4's force in cable out of lower cables that get pretension. It is clear that the graph shows the nonlinear behavior in terms of member force, and as the load increases, the difference between them become more clear. With an supposition that the structure has no loading, there is no member force, when there is no pretension force is applied to the structure.

However, when the pretension is 0.1 FT with the same condition of the structure, the tension force of 244.2 kN, 739.5 kN for 0.3 FT, and 1253.6 kN for 0.5 FT occurs and as the load increases, the value of tension force increases continuously. It is considered that the tension force is created because the pretension is applied directly to the cable, and later, the cable gets the continuously tension force with the load from the upper part. Also, the differences between stress based on the changed value each pretension are actually similar to the differences between the value of early tension force applied to the real cable.

The graph which shows a necessity of using non-linear analysis in a great deal is a fig.6, which shows a member force of element no.2 of upper part. Without reference to the value of the free tension, the linear behavior, which shows that the compression force is applied as the load increases, is obtained. However, it has a very different behavior when the non-linear analysis is applied. In the other hand, linear analysis appears continuous compression force unlike nonlinear analysis appear tension force. After that, in a condition that there is no loading is applied, a different value of tension force created as the value of pretension varies : 901.6 kN for 0.1FT, 2678 kN for 0.3 FT, 4065 kN for 0.5FT. The increased load which is caused by the increased value of pretension, makes tension force to be applied to element. However, this value of tension force gradually decreases, and gets a similar amount of force with a certain load, which also a result of linear analysis has. When there is 0 value of pretension, a 0 kN of load is obtained, which means that a result of linear analysis and non-linear analysis matches with a condition where there is no loading.



<Fig. 6> Member force for element no.2

Later, the values of pretension get closer to the result of linear analysis : when there is 0.1 FT of free tension, -6.13 kN of compression force with 10 kN of load achieved, when 0.3 FT of pretension, 180 kN of compression force with 35 kN of load, and when there is 0.5 FT of pretension, it brings 201 kN of compression with 60 kN of load.

However, it is illustrated that the member which got the tension force in initial, shows compression force during the certain range, and later, as the load increases, the tension force comes back to the member. The section, which gets compression force, become

broader as the value of the pretension increases, and this increased value of pretension also causes a distinct differences between tension forces that show after compression force. For the case of each member of under-tension model used for this study, it is mostly emphasized that a non-linear analysis is crucial for the upper truss, and also, the result of non-linear analysis about upper truss shows a very dissimilar result from that of linear analysis.

## **4. Conclusion**

This study has performed a structure analysis to find a maximum displacement and member force for a structure to which the under-tension structure system is applied, by using nonlinear analysis. By gathering and comparing a whole structure's displacement and member force based on a different pretension value of the under-tension cable, the following results have been achieved.

- (1) When performing non-linear analysis by using commonly used programs, a performer wastes much time for modeling. On the other hand, when a performer uses this newly developed program, it helps to calculate the value of member force and non-linear displacement of the structure only by inputting the location of each node, the connected condition of each node, modulus of elasticity, section area.
- (2) It shows that the maximum displacement decreases as the value of the free-tension increases, for a structure which the under-tension system is applied. However, when the value of pretension increases gradually, the amount of member force applied for each node also increases. It is suggested that a performer has to consider when installing under-tension because this will bring a increased size of member, and takes more construction expenses, and extra budget.
- (3) As the value of pretension of a cable increases, unlike other members, a very different analysis is obtained from the upper truss from a analysis which is got from linear analysis. This refers that a large span structure, which has under-tension system, will need a non-linear analysis, so, more research is needed on this large span structures.
- (4) All models used for this study are considered that the component of all member they get are truss. For a real structure, it is necessary that a performer consider the bending moment, which is occurred by using of truss component and beam element.

## **Acknowledgements**

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