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Comparison of nonlinear solutions between tangential stiffness equation and third order nonlinear equation on bifurcation load of space trusses

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Abstract

This paper studies approaches to improving the convergence properties of nonlinear stiffness equations. A space truss is large structure that must be built to be quite thin, and for this reason a structure stability review is a critical part of the structural design process. The stability of the shelled structure is sensitive to diverse conditions. This leads to a nonlinear problem with concomitant large deformation. To examine structural stability, the accuracy of nonlinear stiffness equations must be improved. In this study, the space truss is an analysis model. Tangent stiffness equation and nonlinear stiffness equation is using nonlinearity analysis program. The study compares analysis results to investigate the accuracy and the improvement in the convergence properties of nonlinear stiffness equations.

Keywords:

Space Truss, Instability, Nonlinear, Convergence, Accuracy

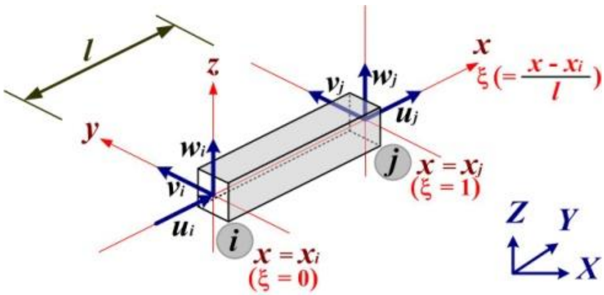
1. Introduction

In large space structures, a space truss structure, in which a stiffness structure system is dispersed, is made in the form of a shell in order to maintain stability by lowering the weight of a roof. As the structure must be thin, a structural stability review is an important step in the structural design of a large space structure. The structural stability of a shelled structure is sensitive to a diversity of conditions. This paper aims to examine the improvement in the accuracy and the convergence of the nonlinear stiffness equation that considers geometric nonlinearity. For this research goal, nonlinear analysis will be performed, using the space truss as its analysis model. For nonlinear analysis, the nonlinear stiffness equation will be made into a program and its result will be compared using NASS, the analysis program for spatial structures that is made on the basis of the tangential stiffness equation. The analysis results of the two theories will be comparatively examined, through which we will examine improvement in the accuracy and the convergence of the nonlinear stiffness equation.

2. Nonlinear FE formulation

2.1 Tangent FE formulation

Figure 1 shows the element coordinates of the tangent truss element in the local coordinate system.



$$\begin{cases} x, y, z : \text{element coordinate system} \\ u, v, w : \text{displacement} \end{cases}$$

Figure 1 : Coordination system and nodal displacements

Using the principle of virtual work, the tangent stiffness equation of element is,

$$f^{(0)} + f = Al\left(A_1{}^T\sigma_x^{(0)}\right) + Al\left(\sigma_x^{(0)}B^TB\right)d + AlE\left(A_1{}^TA_1\right)d + \text{Higher order terms} \tag{1}$$

In Eq. (1), the residual force arising from the elimination of higher-order terms on d is,

$$r = AlA_1{}^T\sigma_x^{(0)} - f^{(0)} \tag{2}$$

Using Eq. (2), the incremental equation can be expressed as,

$$f - r = AlE\left(A_1{}^TA_1\right)d + Al\left(\sigma_x^{(0)}B^TB\right)d = (k_E + k_G)d \tag{3}$$

where

$$k_E = AlE\left({A_1}^T A_1\right) : \text{Elastic stiffness matrix of element}$$

$$k_G = Al\left(\sigma_x^{(0)} B^T B\right) : \text{Geometric stiffness matrix of element}$$

Using transformation matrix T, the tangent stiffness matrix can be expressed in terms of the global coordinate system as,

$$F - R = [K_E + K_G]D \tag{4}$$

where

$$K_E = T^T k_E T : \text{ Elastic stiffness matrix}$$

$$K_G = T^T k_G T : \text{ Geometric stiffness matrix}$$

2.2 Nonlinear FE formulation

Choose a local coordinate system (xyz) and a global coordinate system (XYZ) as shown in Figure 2, and express the nodal force vector {f} and the nodal displacement vector {d} of the local coordinate system, and the nodal force vector {F} and nodal displacement vector {D} of the global coordinate system as follows:

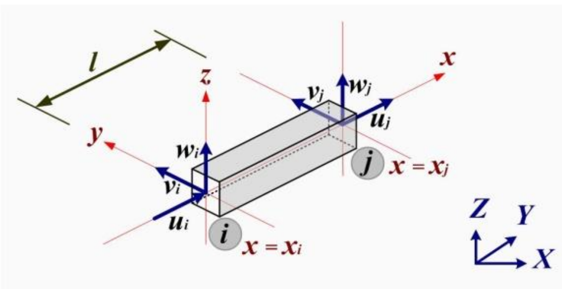


Figure 2 : Coordination system and nodal displacements

$$T = Tf \ , \ D = Td \ , \tag{5}$$

$$\{F\} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \begin{Bmatrix} f_i^x \\ f_i^y \\ f_i^z \\ f_j^x \\ f_j^y \\ f_j^z \end{Bmatrix}, \quad \{F\} = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \begin{Bmatrix} F_i^x \\ F_i^y \\ F_i^z \\ F_j^x \\ F_j^y \\ F_j^z \end{Bmatrix}, \quad \{d\} = \begin{Bmatrix} d_i \\ d_j \end{Bmatrix} = \begin{Bmatrix} u_i \\ v_i \\ w_i \\ u_j \\ v_j \\ w_j \end{Bmatrix}, \quad \{D\} = \begin{Bmatrix} D_i \\ D_j \end{Bmatrix} = \begin{Bmatrix} U_i \\ V_i \\ W_i \\ U_j \\ V_j \\ W_j \end{Bmatrix}$$

T is transformation Matrix, and in the coordinate system of Figure 2, it becomes as follows:

$$T = \begin{bmatrix} T_i & 0 \\ 0 & T_j \end{bmatrix},\ T_i = T_j = \begin{bmatrix} a_1 = l_0 & a_2 = \frac{m_0}{\sqrt{l_0^2+m_0^2}} & a_3 = \frac{l_0 n_0}{\sqrt{l_0^2+m_0^2}} \\ b_1 = m_0 & b_2 = \frac{l_0}{\sqrt{l_0^2+m_0^2}} & b_3 = \frac{m_0 n_0}{\sqrt{l_0^2+m_0^2}} \\ c_1 = n_0 & c_2 = 0 & c_3 = \sqrt{l_0^2+m_0^2} \end{bmatrix} \tag{6}$$

where,

$$l_0 = \cos(X.x),\ \ m_0 = \cos(Y.y),\ \ n_0 = \cos(Z.z)$$

Introduce non-dimensional quantity ξ, which is $\xi = \frac{x}{l}(0 \le \xi \le 1)$, and assume that the displacement in elements is changed in a linear manner.

$$u(\xi) = \alpha_1 + \alpha_2\xi,\ \ v(\xi) = \alpha_3 + \alpha_4\xi,\ \ w(\xi) = \alpha_5 + \alpha_6\xi \tag{7}$$

Using the secondary nonlinear item of strain, the strain-displacement equation becomes as follows:

$$\epsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2,\ \ \delta\epsilon = \frac{d\delta u}{dx} + \frac{du}{dx}\frac{d\delta u}{dx} + \frac{dv}{dx}\frac{d\delta v}{dx} + \frac{dw}{dx}\frac{d\delta w}{dx} \tag{8}$$

where,

$$\frac{du}{dx} = \frac{1}{l}\frac{du}{d\xi} = \frac{1}{l}(u_j - u_i),\ \ \frac{dv}{dx} = \frac{1}{l}\frac{dv}{d\xi} = \frac{1}{l}(v_j - v_i),\ \ \frac{dw}{dx} = \frac{1}{l}\frac{dw}{d\xi} = \frac{1}{l}(w_j - w_i)$$

It can be expressed in the form of a matrix, as follows:

$$\begin{aligned} \frac{du}{dx} &= \frac{1}{l}[-1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \cdot d = UT^tD = A_1,\quad U = \frac{1}{l}[-1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \\ \frac{dV}{dx} &= \frac{1}{l}[0 \quad -1 \quad 0 \quad 0 \quad 1 \quad 0] \cdot d = VT^tD = A_2,\quad V = \frac{1}{l}[0 \quad -1 \quad 0 \quad 0 \quad 1 \quad 0] \\ \frac{dw}{dx} &= \frac{1}{l}[0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 1] \cdot d = WT^tD = A_3,\quad W = \frac{1}{l}[0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 1] \end{aligned} \tag{9}$$

Mark $\delta\epsilon$, the increment of ϵ, in the matrix also.

$$\frac{d\delta u}{dx} = UT^t\delta D,\ \frac{d\delta v}{dx} = VT^t\delta D,\ \frac{d\delta w}{dx} = WT^t\delta D \tag{10}$$

Assume that all materials are within the range of elasticity ($\sigma = E\epsilon$). In order to calculate the stiffness matrix of the element, introduce the principle of virtual work.

$$\delta D^t F = EAl \cdot \delta\epsilon^t \epsilon \tag{11}$$

Then, substitute equation (8) for equation (11), and the following fundamental equation can be obtained.

$$F = EAl \cdot [TU^t A_1 + TU^t A_1^2 + TV^t A_1 A_2 + TW^t A_1 A_3 + TU^t \frac{1}{2}(A_1^2 + A_2^2 + A_3^2) \tag{12}$$
$$+ TAU^t A_1 \frac{1}{2}(A_1^2 + A_2^2 + A_3^2) + TAV^t A_2 \frac{1}{2}(A_1^2 + A_2^2 + A_3^2) + TAW^t A_3 \frac{1}{2}(A_1^2 + A_2^2 + A_3^2)]$$

In conclusion, the following nonlinear equation can be obtained.

$$\mathbf{F = K_1 D_1 + K_2 D_2 + K_3 D_3} \tag{13}$$

where,

$$K_1 = \frac{EA}{l}\begin{bmatrix} -a_1^2 & -a_1 b_1 & -a_1 c_1 \\ -a_1 b_1 & -b_1^2 & -b_1 c_1 \\ -a_1 c_1 & -b_1 c_1 & -c_1^2 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ a_1^2 & a_1 b_1 & a_1 c_1 \\ a_1 b_1 & b_1^2 & b_1 c_1 \\ a_1 c_1 & b_1 c_1 & c_1^2 \end{bmatrix}, \quad K_2 = \frac{EA}{2l^2}\begin{bmatrix} -3a_1 & -a_1 & -a_1 & \vdots & -b_1 & 0 & -c_1 \\ -b_1 & -3b_1 & -b_1 & \vdots & -a_1 & -c_1 & 0 \\ -c_1 & -c_1 & -3c_1 & \vdots & 0 & -b_1 & -a_1 \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ 3a_1 & a_1 & a_1 & \vdots & b_1 & 0 & c_1 \\ b_1 & 3b_1 & b_1 & \vdots & a_1 & c_1 & 0 \\ c_1 & c_1 & 3c_1 & \vdots & 0 & b_1 & a_1 \end{bmatrix}$$

$$K_2 = \frac{EA}{2l^2}\begin{bmatrix} -1 & 0 & 0 & \vdots & 0 & 0 & \vdots & -1 & 0 & \vdots & -1 & 0 \\ 0 & -1 & 0 & \vdots & -1 & 0 & \vdots & 0 & 0 & \vdots & 0 & -1 \\ 0 & 0 & -1 & \vdots & 0 & -1 & \vdots & 0 & -1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \vdots & 0 & 0 & \vdots & 1 & 0 & \vdots & 1 & 0 \\ 0 & 1 & 0 & \vdots & 1 & 0 & \vdots & 0 & 0 & \vdots & 0 & 1 \\ 0 & 0 & 1 & \vdots & 0 & 1 & \vdots & 0 & 1 & \vdots & 0 & 0 \end{bmatrix},$$

$$D_1 = \begin{Bmatrix} \breve{u} \\ \breve{v} \\ \breve{w} \end{Bmatrix}, \quad D_2 = \begin{Bmatrix} \breve{u}^2 \\ \breve{v}^2 \\ \breve{w}^2 \\ \breve{u}\breve{v} \\ \breve{v}\breve{w} \\ \breve{w}\breve{u} \end{Bmatrix}, \quad D_3 = \begin{bmatrix} \breve{u}^3 \\ \breve{v}^3 \\ \breve{w}^3 \\ \breve{u}^2\breve{v} \\ \breve{u}^2\breve{w} \\ \breve{v}^2\breve{u} \\ \breve{v}^2\breve{w} \\ \breve{w}^2\breve{u} \\ \breve{w}^2\breve{v} \end{bmatrix},$$

$$\breve{u} = u_j - u_i, \quad \breve{v} = v_j - v_i, \quad \breve{w} = w_j - w_i,$$

3. Numerical example

3.1 Making out an analysis program using Fortran

The program, which was developed on the basis of the tangential stiffness equation described in 2.1 above, is called NASS (Nonlinear Analysis for Space Structures). NASS is a nonlinear analysis program that was developed using the finite element method considering geometric nonlinearity. It is used for the structural analysis of hybrid-type spatial structures. It is an analysis program that can handle the nonlinear structural analysis of shell structures, space frame structures, membrane structures, space cable structures, and hybrid structures. The operation flowchart is shown in Figure 3. Here, $\alpha = 0$ indicates the convergent analysis process based on Newton-Raphson law, and $\alpha = 1$ indicates the sequential analysis process based on the incremental method.

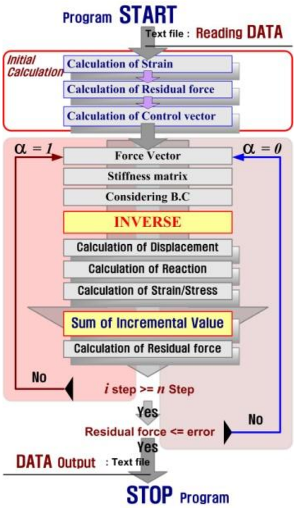


Figure 3 : NASS Flow Chart

The program, which was developed on the basis of the nonlinear stiffness equation described in 2.2 above, is called NonT (Nonlinear Truss). NonT, a nonlinear analysis program, does not omit higher-degree terms and includes third-degree terms of displacement.

3.2 Accuracy of the solution through plane truss model

Using the unit plane truss of 1-free node as the analysis model, this paper compared the tangential stiffness equation with the nonlinear stiffness equation in order to examine the accuracy of the nonlinear stiffness equation. The model looks like Figure 4. For the analysis, NonT, which was made with Fortran, and NASS described above 3.1, are used. Displacement incremental method is performed.

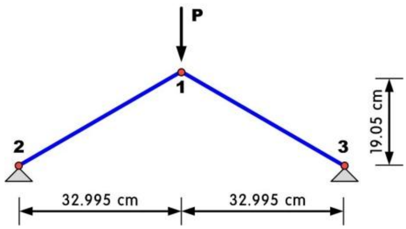


Figure 4 : Shape of the plane truss model

The unit plane truss, which is the analysis model, has 3 nodes and is connected with 2 elements. In consideration of the symmetry of the structural model, it is a 1-free node structure. It has been used by many researchers as an aid to understanding the nonlinear behaviors of a unit structure. It is known that its path after buckling shows very sensitive behaviors, depending on the stress-strain. The boundary conditions of the analysis model are that no. 2 node and no. 3 node are fixed, and no. 1 node is free. The load conditions are that the vertical node load, P, works on no.1 node. The specifications of the elements are as follows:

. Cross-section of the elements : $A = 96.77\ (\mathrm{cm}^2)$
. Elasticity : $E = 7.03\ \times\ 10^6 (\mathrm{kgf/cm}^2)$

Perform analysis through displacement-increment for the vertical displacement 50cm at intervals of 1mm on no. 1 node of 1-free node plane truss model, and show its result in Figure 5. The solid line and the broken line show the relationship between the vertical displacement and the load of no. 1 node.

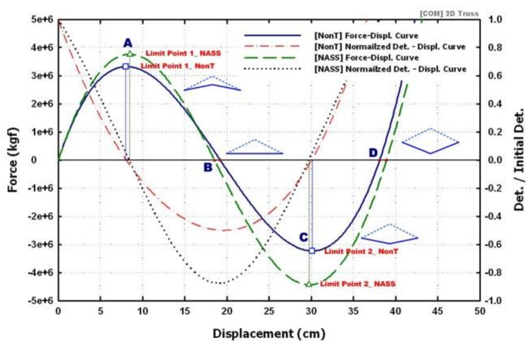


Figure 5 : Changes in the nonlinear behaviors of the plane truss model and in determinant values

In Figure 5, A and C show the strain status at the first and the second limit points, and B and D show the strain status at the point where the load becomes 0. In the shape figure of each point, the dotted line shows the first shape and the solid line shows the strained shape. Short dash line and the dotted line are the results of the determinant values of the nonlinear stiffness matrix and the tangential stiffness matrix, respectively. They show the relation between the value, which has generalized the determinant obtained from each increment with the determinant obtained from zero-load level, and displacement. The determinant becomes zero at the limit point, and the structure can cause the unstable snap-through phenomenon. In addition, until the first limit point A is reached after load-giving starts, it becomes the balanced path of stability, and from the first limit point A to the second limit point C, it becomes an unstable path. After the second limit point C, it becomes the balanced path of stability.

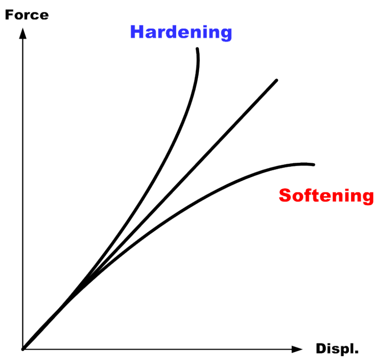


Figure 6 : Behavioral characteristics of nonlinear structures

Figure 6 shows nonlinear behavioral characteristics and the plane truss structure has "softening" characteristics.

The limit point load obtained from the analysis of this study is 3.318×10^6 (kgf) for NonT and 3.756×10^6 (kgf) for NASS. At this time, the vertical dispacement of no. 1 node is 800cm for NonT and 860cm for NASS. The balanced path of Figure 5 includes the path after the limit point that cannot be analysis by the load incremental method, in addition to the balanced path before the limit point. Therefore, it can be said that the displacement incremental method used in this paper can trace even the balanced path after buckling. Furthermore, it can be said that as a result of NonT analysis based on the "softening" phenomenon, accuracy is improved.

3.3 Analysis of convergence through the stiffness equation

In order to examine the convergence of the nonlinear stiffness equation, 1-free node truss structure and 2-free nodes truss structure are used as analysis models, and NonT and NASS are used for analysis.

3.3.1 One-free node model

The analysis model is a 1-free node unit structure that has 7 nodes and is connected with 6 elements. The model is composed of 1 free node and 6 boundary nodes, and the load condition is such that compressive force works at node-1 in a vertical direction. The form and the node number of the model are as shown in Figure 7. The sectional shape and the material properties are the same in all elements, the coefficient of elasticity(E) is 2.1×106kgf/ cm^2, and the density (ρ) is 7.85×10-3kgf/ cm^2.

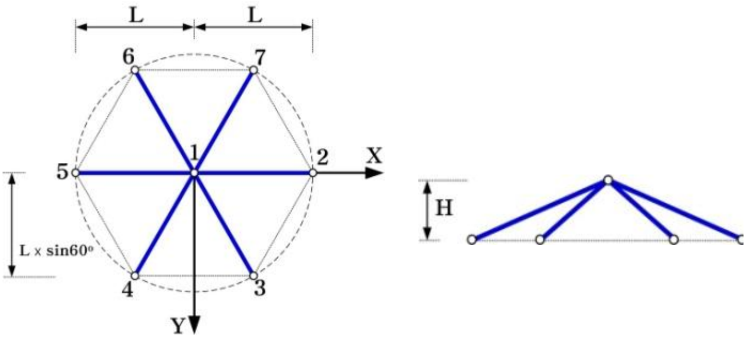


Figure 7 : Shape of a One-free node

Using NonT and NASS for analysis, perform displacement-increment for the vertical displacement of node-1 in intervals of 1mm.

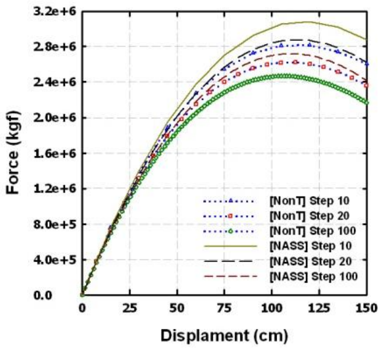
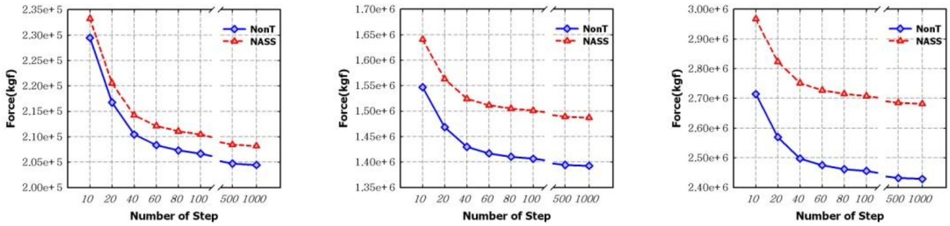


Figure 8 : Load-displacement curved line based on the number of steps of One-free node truss (H 250)



(a) H 100 (45cm) (b) H 200 (85cm) (c) H 250 (110cm)

Figure 9 : Convergence curved line based on the number of steps of One-free node truss (H 250)

Figure 8 is the load-displacement curved line based on the number of the steps of 1-free node truss. The cases in which the numbers of steps are 10, 20, and 100 are analysis and compared. The number in parentheses shows the convergence displacement. Figure 9 is the convergence curved line based on the increase in the number of steps. The analysis result of NonT shows an improvement in the convergence, and also shows that as the height of the model (H) increases, convergence is improved further.

3.3.2 Two-free node model

This model uses a triangle as the basic module, and is connected by 10 nodes and 11 elements. The heights of both models are H. The heights of node-1 and node-2 are the same. The boundary condition is that node-1 and node-2 are free nodes, while the remaining nodes are boundary nodes. The shape of the model is as shown in Figure 10.

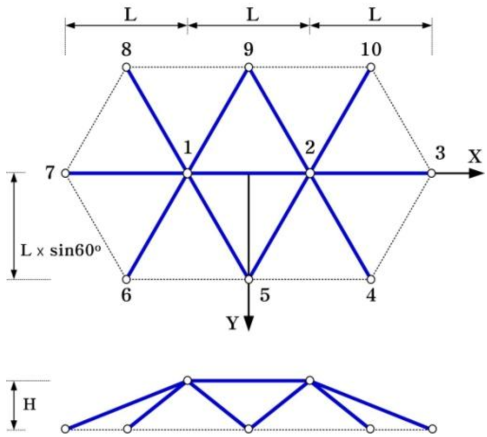


Figure 10 : Shape of a two-free node model

The sectional area, the coefficient of elasticity, and the density of the two-free nodes structure are the same as those of a one-free node structure. The load condition for the structure is such that the same level of load works at node-1 and node-2 in the vertical direction.

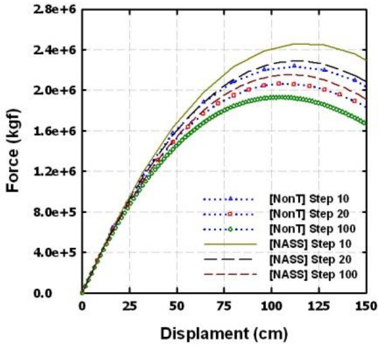
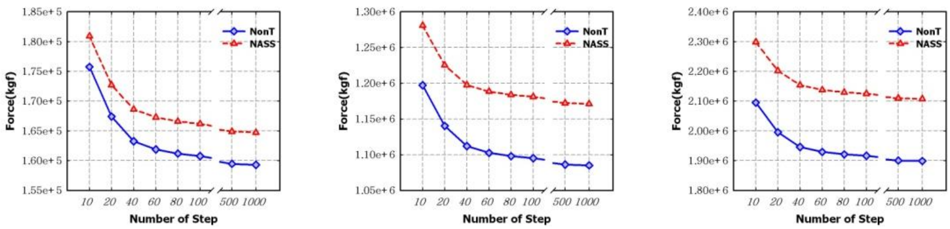


Figure 11 : Load-displacement curved line based on the number of steps of 2-free nodes truss (H 250)



(a) H 100 (45cm) (b) H 200 (80cm) (c) H 250 (100cm)

Figure 12 : Convergence curved line based on the number of steps of 2-free nodes truss (H 250)

Figure 11 is the load-displacement curved line based on the number of steps of the 2-free nodes truss. The cases in which the numbers of steps are 10, 20, and 100 are analysis and compared. The number in parentheses shows the convergence displacement. Figure 12 is the convergence curved line based on the increase in the number of steps. The analysis result shows that the convergence has improved.

4. Conclusion

This paper made out analysis programs through the tangential stiffness equation and the nonlinear stiffness equation, and comparatively examined the analysis results through the space truss analysis model, particularly considering the accuracy and the convergence of the two equations. The conclusions are as follows:

1. As a result of the analysis of the tangential stiffness equation and the nonlinear stiffness equation, softening was rapidly promoted and accuracy was improved in the nonlinear stiffness equation.
2. Through the space truss structural analysis, the nonlinear stiffness equation had better convergence than the tangential stiffness equation. When the structures were unstable, there was no significant difference in the convergence.
3. NonT (Nonlinear Truss), which was made out with Fortran, could efficiently access nonlinear structural behaviors, and could achieve very sensitive nonlinear solutions.

Acknowledgement

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