

Neutrino mixing and masses in a left–right model with mirror fermions

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Abstract In the framework of a left–right model containing mirror fermions with gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$, we estimate the neutrino masses, which are found to be consistent with their experimental bounds and hierarchy. We evaluate the decay rates of the Lepton Flavor Violation (LFV) processes $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$. We obtain upper limits for the flavor-changing branching ratios in agreement with their present experimental bounds. We also estimate the decay rates of heavy Majorana neutrinos in the channels $N \rightarrow W^\pm l^\mp$, $N \rightarrow Z\nu_l$ and $N \rightarrow H\nu_l$, which are roughly equal for large values of the heavy neutrino mass. Starting from the most general Majorana neutrino mass matrix, the smallness of active neutrino masses turns out from the interplay of the hierarchy of the involved scales and the double application of seesaw mechanism. An appropriate parameterization on the structure of the neutrino mass matrix imposing a symmetric mixing of electron neutrino with muon and tau neutrinos leads to tribimaximal mixing matrix for light neutrinos.

1 Introduction

The evidence for neutrino oscillations obtained in experimental results from atmospheric, solar, reactor and accelerator neutrinos leads one to conclude that the neutrinos have a mass different from zero. The current neutrino experimental data (SuperKamiokande, SNO, Kamland, K2K, GNO, CHOOZ) can be described by neutrino oscillations via three neutrino mixings [1–7]. The present data give the solar neutrino lepton mixing angle $\tan^2 \theta_{12} = 0.45 \pm 0.05$, the atmospheric angle $\sin^2 2\theta_{23} = 1.02 \pm 0.04$ and $\sin^2 2\theta_{13} =$

0 ± 0.05 [8–10]. The complex phase has not yet been measured.

The experimental information on neutrino masses and mixing points out new physics beyond the Standard Model (SM) of particle physics, with a great activity on the consequences. Among the possible mechanisms of neutrino mass generation, the most simple and attractive one is the seesaw mechanism [11–20], which explains the smallness of the observed light neutrino masses through the exchange of superheavy particles; an alternative explanation is given by extra dimensions beyond the usual three ones [21]. It has been suggested that right-handed (RH) neutrinos experience one or more of these extra dimensions, such that they only spend part of their time in our world, with apparently small masses. At the present, it is not known whether neutrinos are Dirac or Majorana fermions.

Models with heavy neutrinos of mass of order 1 TeV can give rise to significant light-heavy mixing and deviation from unitarity of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [22–24]. The nonunitarity nature of the neutrino mixing matrix due to mixing with fields heavier than $\frac{M_Z}{2}$ can manifest in tree-level processes like $\pi \rightarrow \mu\nu$, $Z \rightarrow \bar{\nu}\nu$, $W \rightarrow l\nu$ or in charged lepton decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, etc. which are flavor violating and rare and proceed at one loop level [22–26]. The TeV scale seesaw models are interesting because they can have signatures in the CERN Large Hadron Collider (LHC) in the near future [27–29].

Neutrinos also are important in astrophysics and cosmology [30] and probably they contribute to hot dark matter in the Universe and in its evolution.

Parity P violation was one of the greatest discoveries of particle physics [31–33]. Before this observation, according to Fermi's hypothesis it was believed that weak interactions have purely vectorial V or axial vectorial (V–A) parity conserving Lorentz structure [34, 35]. The theory of Lee and

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Yang in 1956 [36] proposed a fermion current with V and A structure. It is known that in the standard model (SM) the electroweak interactions have a V–A form, with only left-handed (LH) (ordinary) fermions coupling to the weak gauge boson W^\pm . But one can include also mirror fermions [37] with a V + A coupling, such that P is conserved. In this sense, the term “mirror fermion” is equivalent to “vector-like fermion”, where for a theory with gauge group G , in a representation R one has sets of LH and RH fermions.

In the literature a second meaning of that term is used. G is extended to a $G \times G$ gauge theory, and for every multiplet $(R, 1)$ a mirror partner $(1, R)$ is added, such that there is no gauge invariant mass term connecting the LH and RH multiplets [38–41]. Thus it is natural to consider the existence of mirror generations.

Masses of mirror particles arise from symmetry breaking; for mirror generation they may lie below one TeV, and feasible to be discovered in Fermilab Tevatron Collider and LHC.

A solution to the strong CP problem has been proposed within a L–R symmetric context [46, 47]. The electroweak group is extended to $SU(2)_L \otimes SU(2)_R \otimes U(1)$ including mirror fermions. These fermions are conjugated to the ordinary ones with respect to the gauge symmetry group such that a fermion representation including both of them is real and the cancelation of anomalies is automatic [48].

In this paper we consider a L–R model with mirror fermions (LRMM) with gauge group $G \equiv SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$. We discuss in Sect. 2 the formalism of mixing between standard and new exotic fermions. In Sect. 3 we present the model and discuss the symmetry breaking process with two scalar doublets.

In Sect. 4 we write the gauge invariant Yukawa couplings which after spontaneous symmetry breaking give the most general Majorana neutrino mass matrix. With a double application of the type I seesaw approximation we estimate the light neutrino masses in terms of free Yukawa couplings assuming textures for the light and mirror matrices, obtaining consistent normal hierarchical values for masses and a tri-bimaximal mixing for light neutrinos. We discuss in Sect. 4 the mixing between standard and mirror fermions. In Sect. 5 we include the radiative decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ and estimate bounds for their branching ratios. Finally, we calculate such ratios for the heavy Majorana neutrinos decays $N \rightarrow W^+l^-$, $N \rightarrow Z\nu_l$ and $N \rightarrow H\nu_l$, getting a smooth variation with the heavy neutrino mass, even when it is much larger than any of the involved masses.

2 Fermion mixing and flavor violation

To consider the mixing of fermions, we shall follow [22–24], grouping all fermions of electric charge q and helicity $a = L, R$ into $n_a + m_a$ vector column of n_a ordinary (o) and

m_a exotic (e) gauge eigenstates, i.e. $\psi_a^o = (\psi_{n_a}^o, \psi_{m_a}^o)^T$. The ordinary fermions include the SM ones, whereas the exotics include any new fermion with sequential (mirror or singlet) properties beyond the SM.

The relation between the gauge eigenstates and the corresponding light (l) and heavy (h) charged mass eigenstates $\psi_a = (\psi_l, \psi_h)^T$, $a = L, R$ is given by the transformation

$$\psi_a^o = V_a \psi_a, \quad a = L, R, \quad (1)$$

where

$$V_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix}. \quad (2)$$

In (2), A_a is a matrix relating the ordinary weak states and the light-mass eigenstates, while G_a relates the exotic and heavy states. E_a and F_a describe the mixing between the two sectors.

From the unitarity of V

$$V_a V_a^+ = 1, \quad a = L, R \quad (3)$$

it follows that the submatrix A_a is not unitary. The term $F_a^+ F_a$, which is second order in the small light–heavy fermion mixing, will induce flavor-changing transitions in the light–light sector.

The vacuum expectation values (VEV) of the neutral scalars produce the SM fermion mass terms, which together with the exotic mass and mixing matrices lead to the mass matrix M which takes the form

$$M = \begin{pmatrix} K & \hat{\mu} \\ \mu & \hat{K} \end{pmatrix}, \quad (4)$$

where K denotes the SM fermion mass matrix and \hat{K} corresponds to the fermion mass matrices associated with the exotic sector, while μ , $\hat{\mu}$ correspond to the mixing terms between ordinary and exotic fermions.

The diagonal mass matrix M_d can be obtained through a biunitary rotation acting on the L and R sectors, namely

$$M_d = V_L^+ M V_R = \begin{pmatrix} m_l & 0 \\ 0 & m_h \end{pmatrix}, \quad (5)$$

where m_l , m_h denote the light and heavy diagonal mass matrices, respectively. The form of the mass matrix will depend on the type of exotic fermion considered.

The scalar-fermion couplings within some specific Higgs sector are not diagonal in general, and one can see that the couplings are not diagonal in general; thus new phenomena associated with flavor-changing neutral currents (FCNC) will be present in such model.

3 The model

In this and next sections we follow closely [42–45]. The LRMM formulation is based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$. In order to solve different problems such as the hierarchy of quark and lepton masses or the strong CP problem, different authors have enlarged the fermion content to the form

$$\begin{aligned}
 l_{iL}^0 &= \begin{pmatrix} \nu_i^0 \\ e_i^0 \end{pmatrix}_L, & e_{iR}^0, & \nu_{iR}^0, \\
 \hat{l}_{iR}^0 &= \begin{pmatrix} \hat{\nu}_i^0 \\ \hat{e}_i^0 \end{pmatrix}_R, & \hat{e}_{iL}^0, & \hat{\nu}_{iL}^0, \\
 Q_{iL}^0 &= \begin{pmatrix} u_i^0 \\ d_i^0 \end{pmatrix}_L, & u_{iR}^0, & d_{iR}^0, \\
 \hat{Q}_{iR}^0 &= \begin{pmatrix} \hat{u}_i^0 \\ \hat{d}_i^0 \end{pmatrix}_R, & \hat{u}_{iL}^0, & \hat{d}_{iL}^0,
 \end{aligned} \tag{6}$$

where the index i runs over the three fermion families and the superscripts 0 denote gauge eigenstates. The quantum numbers of these fermions under the gauge group G defined above are given by

$$\begin{aligned}
 l_{iL}^0 &\sim (1, 2, 1, -1)_{iL}, & \nu_{iR}^0 &\sim (1, 1, 1, 0)_{iR}, \\
 e_{iR}^0 &\sim (1, 1, 1, -2)_{iR}, & \hat{\nu}_{iL}^0 &\sim (1, 1, 1, 0)_{iL}, \\
 \hat{e}_{iL}^0 &\sim (1, 1, 1, -2)_{iL}, & \hat{l}_{iR}^0 &\sim (1, 1, 2, -1)_{iR} \\
 u_{iR}^0 &\sim \left(3, 1, 1, \frac{4}{3}\right)_{iR}, & d_{iR}^0 &\sim \left(3, 1, 1, \frac{2}{3}\right)_{iR} \\
 \hat{u}_{iL}^0 &\sim \left(3, 1, 1, \frac{4}{3}\right)_{iL}, & \hat{d}_{iL}^0 &\sim \left(3, 1, 1, \frac{2}{3}\right)_{iL} \\
 Q_{iL}^0 &\sim \left(3, 2, 1, \frac{1}{3}\right)_{iL}, & \hat{Q}_{iR}^0 &\sim \left(3, 1, 2, \frac{1}{3}\right)_{iR},
 \end{aligned}$$

respectively, and the last entry corresponds to the hypercharge (Y') with the electric charge defined as $Q = T_{3L} + T_{3R} + \frac{Y'}{2}$.

A model with gauge group $SU(2)_L \times SU(2)_R \times U(1)_V \times SU(3)_H$ and the fermion content (6) was originally suggested in Z.G. Berezhiani [49] as the “universal seesaw” model which generated masses of charged fermions as well as of the neutrinos. He also worked on a $SU(5) \times SU(3)_H$ model for extension to $SO(10)$ or Pati–Salam [50, 51], predicting for instance $m_{\nu_e} = O(10)$ eV. At low (electroweak scale) energies the model simulates the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, and FCNC are suppressed naturally.

3.1 Symmetry breaking

The “Spontaneous Symmetry Breaking” (SSB) is achieved following the stages:

$$G \longrightarrow G_{SM} \longrightarrow SU(3)_C \otimes U(1)_Q, \tag{7}$$

where $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the “Standard Model” group symmetry, and $\frac{Y}{2} = T_{3R} + \frac{Y'}{2}$. The Higgs sector to induce the SSB in (7) involves two doublets of scalar fields:

$$\Phi = (1, 2, 1, 1), \quad \hat{\Phi} = (1, 1, 2, 1), \tag{8}$$

where the entries correspond to the transformation properties under the symmetries of the group G , with the “Vacuum Expectation Values” (VEV’s)

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \hat{\Phi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \hat{v} \end{pmatrix}. \tag{9}$$

The most general potential that develops this pattern of VEV’s is

$$\begin{aligned}
 V &= -(\mu \Phi^\dagger \Phi + \hat{\mu} \hat{\Phi}^\dagger \hat{\Phi}) + \frac{\lambda_1}{2} [(\Phi^\dagger \Phi)^2 + (\hat{\Phi}^\dagger \hat{\Phi})^2] \\
 &\quad + \lambda_2 (\Phi^\dagger \Phi) (\hat{\Phi}^\dagger \hat{\Phi}).
 \end{aligned} \tag{10}$$

In the last expression the terms with $\mu, \hat{\mu}$ are included so that the parity symmetry (P) is broken softly, i.e., only through the dimension-two mass terms of Higgs potential. The scalar Lagrangian for the model is written as

$$\mathcal{L}_{sc} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + (\hat{D}_\mu \hat{\Phi})^\dagger (\hat{D}^\mu \hat{\Phi}), \tag{11}$$

where D_μ and \hat{D}_μ are the covariant derivatives for the SM and the mirror parts, respectively. The gauge interactions of quarks and leptons can be obtained from the Lagrangian

$$\mathcal{L}^{int} = \bar{\psi} i \gamma^\mu D_\mu \psi + \bar{\hat{\psi}} i \gamma^\mu \hat{D}_\mu \hat{\psi}. \tag{12}$$

The VEV’s v and \hat{v} are related to the masses of the charged gauge bosons W and \hat{W} by $M_W = \frac{1}{2} g_L v$ and $M_{\hat{W}} = \frac{1}{2} g_R \hat{v}$, where g_L and g_R are the coupling constants of $SU(2)_L$ and $SU(2)_R$, and $g_L = g_R$ if we require L–R symmetry.

4 Generic Majorana neutrino mass matrix

With the fields of fermions introduced in the model, we may write the gauge invariant Yukawa couplings for the neutral sector:¹

$$h_{ij} \bar{\nu}_{iL} \nu_{jR} + \lambda_{ij} \bar{l}_{iL} \hat{\Phi} \nu_{jR} + \eta_{ij} \bar{\hat{l}}_{iR} \hat{\Phi} \hat{\nu}_{jL}$$

¹To simplify notation we drop the “0” superscript.

$$\begin{aligned}
 & + \hat{M}_{ij} \tilde{\nu}_{iL} (\hat{\nu}_{jL})^c + \sigma_{ij} \tilde{l}_{iL} (\hat{\nu}_{jL})^c \tilde{\Phi} \\
 & + \chi_{ij} \tilde{\nu}_{iR} (\nu_{jR})^c + \pi_{ij} \tilde{l}_{iR} (\nu_{jR})^c \tilde{\Phi} + h.c., \tag{13}
 \end{aligned}$$

where $i, j = 1, 2, 3$, $\tilde{\Phi} = i\sigma_2 \Phi^*$, $\hat{\Phi} = i\sigma_2 \hat{\Phi}^*$, h_{ij} , \hat{M}_{ij} , χ_{ij} have dimensions of mass, and σ_{ij} , η_{ij} , λ_{ij} and π_{ij} are dimensionless Yukawa coupling constants. When Φ and $\hat{\Phi}$ acquire VEV's we get the neutrino mass terms

$$\begin{aligned}
 & h_{ij} \tilde{\nu}_{iL} \nu_{jR} + \frac{v}{\sqrt{2}} \lambda_{ij} \tilde{\nu}_{iL} \nu_{jR} + \frac{\hat{v}}{\sqrt{2}} \eta_{ij} \tilde{\nu}_{iR} \hat{\nu}_{jL} \\
 & + \hat{M}_{ij} \tilde{\nu}_{iL} (\hat{\nu}_{jL})^c + \frac{v}{\sqrt{2}} \sigma_{ij} \tilde{\nu}_{iL} (\hat{\nu}_{jL})^c \\
 & + \chi_{ij} \tilde{\nu}_{iR} (\nu_{jR})^c + \frac{\hat{v}}{\sqrt{2}} \pi_{ij} \tilde{\nu}_{iR} (\nu_{jR})^c + h.c., \tag{14}
 \end{aligned}$$

which are written in the generic Majorana matrix form

$$(\overline{\Psi}_{\nu L}, \overline{\Psi}^c_{\nu L}) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} (\Psi_{\nu}^c)_R \\ (\Psi_{\nu})_R \end{pmatrix} \tag{15}$$

where

$$(\Psi_{\nu})_{L,R} = \begin{pmatrix} \nu_i \\ \hat{\nu}_i \end{pmatrix}_{L,R}, \quad (\Psi_{\nu}^c)_{L,R} = \begin{pmatrix} \nu_i^c \\ \hat{\nu}_i^c \end{pmatrix}_{L,R}, \tag{16}$$

$$M_L = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \sigma \\ \frac{v}{\sqrt{2}} \sigma^T & \hat{M} \end{pmatrix}, \quad M_R = \begin{pmatrix} \chi & \frac{\hat{v}}{\sqrt{2}} \pi \\ \frac{\hat{v}}{\sqrt{2}} \pi^T & 0 \end{pmatrix}, \tag{17}$$

$$M_D = \begin{pmatrix} \frac{v}{\sqrt{2}} \lambda & 0 \\ h & \frac{\hat{v}}{\sqrt{2}} \eta \end{pmatrix}, \tag{18}$$

with h , \hat{M} , χ , σ , η , λ and π unknown matrices of 3×3 dimension. By assuming the natural hierarchy $|(M_L)_{ij}| \ll |(M_D)_{ij}| \ll |(M_R)_{ij}|$ for the mass terms, the mass matrix in (15) can approximately be diagonalized, yielding

$$(\overline{\Psi}'_{\nu L}, \overline{\Psi}'^c_{\nu L}) \begin{pmatrix} M_{\nu} & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} (\Psi'^c_{\nu})_R \\ (\Psi'_{\nu})_R \end{pmatrix}, \tag{19}$$

where, neglecting $O(M_D M_R^{-1})$ terms, we may write in good approximation [52] $\Psi'_{\nu L,R} \approx \Psi_{\nu L,R}$, and $\Psi'^c_{\nu L,R} \approx \Psi^c_{\nu L,R}$. The Majorana mass matrix for the left-handed neutrinos may be written in this seesaw approximation as

$$M_{\nu} \approx M_L - M_D M_R^{-1} M_D^T. \tag{20}$$

We assume a scenario where the dominant contribution for the active known neutrinos comes from the M_L matrix having the same structure of a Type I seesaw. Then in this scenario the eigenvalues for the light neutrinos may be obtained by applying again the seesaw approximation, that is,

$$M^{\text{light}} = - \left(\frac{v}{\sqrt{2}} \sigma \right) \hat{M}^{-1} \left(\frac{v}{\sqrt{2}} \sigma \right)^T. \tag{21}$$

Taking advantage of the fact that all σ_{ij} and \hat{M}_{ij} entries in (21) are free parameters, we propose the following parameterizations for \hat{M} and M^{light} neutrino mass matrices:

$$M^{\text{light}} = \frac{Y^2 v^2}{2\hat{m}} \begin{pmatrix} 1+b & b & b \\ b & 1+b+c & b-c \\ b & b-c & 1+b+c \end{pmatrix}, \tag{22}$$

$$\hat{M} = \hat{m} \text{Diag}(Y_1, Y_2, Y_3),$$

where Y, Y_1, Y_2, Y_3, b, c are dimensionless coupling constants and \hat{m} represents the mirror scale. This parameterization for the light neutrinos mass matrix imposes a symmetric mixing of electron neutrino with muon and tau neutrinos in the first row and column of $(M^{\text{light}})_{ij}$, and the 2×2 submatrix $i, j = 2, 3$ generate maximal mixing for muon and tau neutrinos. This structure for M^{light} makes possible the diagonalization of light neutrinos by the so called ‘‘tri-bimaximal mixing matrix’’ [57], i.e.

$$\begin{aligned}
 U_{\text{TB}}^T M^{\text{light}} V_{\text{TB}} &= -U_{\text{TB}}^T \left(\frac{v}{\sqrt{2}\sigma} \right) \hat{M}^{-1} \left(\frac{v}{\sqrt{2}\sigma} \right)^T U_{\text{TB}} \\
 &= \text{Diag}(m_1, m_2, m_3), \tag{23}
 \end{aligned}$$

with

$$U_{\text{TB}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{24}$$

and the light neutrino mass eigenvalues

$$(m_1, m_2, m_3) = \frac{Y^2 v^2}{2\hat{m}} (1, 1 + 3b, 1 + 2c). \tag{25}$$

The suppression by the mirror scale \hat{m} in (25) provides a natural explanation for the smallness of neutrino masses. The allowed range of values for the square neutrino mass differences reported in PDG [56]:

$$m_2^2 - m_1^2 \approx 7.6 \times 10^{-5} \text{ eV}^2, \tag{26}$$

$$m_3^2 - m_2^2 \approx 2.43 \times 10^{-3} \text{ eV}^2,$$

with the input for normal hierarchy of the neutrino masses

$$(m_1, m_2, m_3) = (0.0865, 0.0870, .1) \text{ eV}, \tag{27}$$

fix the parameter values as $b = 0.00168$ and $c = 0.07757$. These neutrino masses are consistent with the bounds $m_{\nu} < 2 \text{ eV}$ [56], and set the mass differences

$$m_3^2 - m_1^2 \approx 2.5 \times 10^{-3} \text{ eV}^2. \tag{28}$$

So, from (25), (27)

$$\frac{Y^2 v^2}{2\hat{m}} \approx 8.65 \times 10^{-2} \text{ eV}. \tag{29}$$

Therefore, assuming $\hat{m} = m_{\hat{\nu}} = 100$ GeV and $v = 246$ GeV we obtain

$$Y \approx 5.34 \times 10^{-7}. \tag{30}$$

The matrix M_L in (17) may be diagonalized by using a unitary transformation,

$$U^\dagger M_L U = \text{Diag}(m_1, m_2, m_3, \hat{m}_1, \hat{m}_2, \hat{m}_3), \tag{31}$$

where the mixing matrix U compatible with our framework is written in good approximation as

$$U_{6 \times 6} \approx \begin{pmatrix} U_{\text{TB}} & \frac{v}{\sqrt{2}} \sigma \hat{M}^{-1} \\ -(\frac{v}{\sqrt{2}} \sigma \hat{M}^{-1})^T & I_{3 \times 3} \end{pmatrix}. \tag{32}$$

The particular numerical solution congruent with the above scenario for the neutrino masses and mixing is

$$\frac{v}{\sqrt{2}} \sigma \approx 93041.9 \text{ eV} \begin{pmatrix} -1.2001 & 0.6355 & 1.2952 \\ 0.6355 & -1.2702 & 1.3006 \\ 1.2952 & 1.3006 & 0.5389 \end{pmatrix}, \tag{33}$$

$$\hat{M} = 100 \text{ GeV Diag}(3.4918, 3.2643, 3.6043), \tag{34}$$

and

$$\frac{v}{\sqrt{2}} \sigma \hat{M}^{-1} \approx 9.3 \times 10^{-7} \begin{pmatrix} -0.3437 & 0.1946 & 0.3593 \\ 0.1819 & -0.3891 & 0.3608 \\ 0.3709 & 0.3984 & 0.1495 \end{pmatrix} \tag{35}$$

for light ν -mirror mixing. Since the light-mirror mixing is very small, the mixing matrix for light neutrinos behaves in good approximation as the U_{TB} , (24). It is worth to mention here that in the limit of very small light-mirror charged lepton mixing, $(F_L^\dagger F_L)_{ij}, (E_L^\dagger E_L)_{ij} \ll 1$, we may approach U_{TB} as the usual U_{PMNS} lepton mixing matrix for three generations. Then, we obtain $(U_{\text{PMNS}})_{e2} \simeq \frac{1}{\sqrt{3}}, (U_{\text{PMNS}})_{e3} \simeq 0$, and $(U_{\text{PMNS}})_{\mu 3} \simeq \frac{1}{\sqrt{2}}$, which give for the solar and the atmospheric neutrino mixing angles $\theta_{12} \simeq 35.2^\circ$ and $\theta_{23} \simeq 45^\circ$, with $\theta_{13} \simeq 0$ in good agreement with current data, although recent evidence [58, 59] shows that θ_{13} may have a value different from zero.

In earlier papers on the study of neutrinos and left-right symmetry [60–63] appear similar representations of the fermions and mass matrices as our in (18), but these authors obtain masses for the standard and mirror neutrinos some orders of magnitude different from ours. On the other hand, the mass generation in the LRMM here considered is achieved with the scalar fields Φ and $\hat{\Phi}$, (3), (4), transforming as doublets under $SU(2)_L$ and $SU(2)_R$, respectively, with a mirror scale much lower than 10^{12} – 10^{13} GeV.

5 Radiative decays

In this section we analyze the lepton flavor violation processes $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ arising in the model by the existence of gauge invariant mixing terms between ordinary leptons and with the mirror counterparts. The lower order contribution to these decays mediated by the neutral scalar fields comes from the Feynman diagrams where the photon is radiated from an internal line. The corresponding amplitude is proportional to the operator $u(p_2)\sigma^{\mu\nu}q_\nu\epsilon_\mu u(p_1)$, where $q = p_1 - p_2$ and ϵ_μ is the photon polarization [53–55].

In the limit $m_e \ll m_\mu \ll m_\tau$ the rate decay is given by

$$\Gamma(l_i \rightarrow l_j + \gamma) = \frac{\alpha}{512\pi^4} (G_F m_{l_i}^2)^2 \frac{m_{l_i}^5}{M_H^4} \left| \left(\ln \frac{M_H^2}{m_{l_i}^2} - \frac{4}{3} \right) \epsilon_{ij} - \sum_k x_{\nu k} V_{L,jk} V_{R,ki}^+ \right|^2, \tag{36}$$

where $x_{\nu k} \equiv \frac{m_{\nu k}^2}{M_W^2}, \epsilon_{ij} = |A_L^+ A_R|_{ij}$ represents the flavor-changing couplings, and the second term is the very small contribution from the light neutrino propagating inside the loop.

In the limit $\alpha \ll 1$ and $M_H \ll M_{\hat{H}}$ the branching ratios are, respectively,

$$B_1(\mu \rightarrow e + \gamma) = \frac{3\alpha m_\mu^4}{8M_H^4} \left| \left(\ln \frac{M_H^2}{m_\mu^2} - \frac{4}{3} \right) \epsilon_{e\mu} - \sum_k x_{\nu k} V_{L,ek} V_{R,k\mu}^+ \right|^2, \tag{37}$$

$$B_2(\tau \rightarrow \mu + \gamma) = \frac{3\alpha m_\tau^4}{8M_H^4} \left| \left(\ln \frac{M_H^2}{m_\tau^2} - \frac{4}{3} \right) \epsilon_{\mu\tau} - \sum_k x_{\nu k} V_{L,\mu k} V_{R,k\tau}^+ \right|^2 \tag{38}$$

and

$$B_3(\tau \rightarrow e + \gamma) = \frac{3\alpha m_\tau^4}{8M_H^4} \left| \left(\ln \frac{M_H^2}{m_\tau^2} - \frac{4}{3} \right) \epsilon_{e\tau} - \sum_k x_{\nu k} V_{L,ek} V_{R,k\tau}^+ \right|^2. \tag{39}$$

By using the constraints $\epsilon_{ij} < 1, i \neq j$ for the parameters in (37), (39), required by unitarity of V , see (2), (3), one gets for the above branching ratios

$$B_1 < 2.2 \times 10^{-13}, \quad B_2 < 5 \times 10^{-9} \quad \text{and} \quad B_3 < 5 \times 10^{-9} \tag{40}$$

which is congruent with the experimental bounds [56] $B(\mu \rightarrow e + \gamma) < 1.2 \times 10^{-11}$, $B(\tau \rightarrow \mu + \gamma) < 4.4 \times 10^{-8}$ and $B(\tau \rightarrow e + \gamma) < 3.3 \times 10^{-8}$ PDG [56].

6 Heavy neutrino signals

Possible new neutrinos can be detected in various ways in colliders. If these neutrinos are heavy they will be unstable and may be detected directly in their decay products.

Next generation of large colliders will probe Nature up to TeV scales with high precision, probably discovering new heavy particles. Thus, it will be a window to any new physics near the electroweak scale which couples to the SM. Such colliders can be used to produce new heavy neutrinos at an observable level to improve present limits on their masses and mixings [64–67]. These fermions with new interactions, like in the left–right models [68], can be produced by gauge couplings suppressed by small mixing angles. For the analysis of the heavy neutrinos signals it is necessary to know their decay modes, which are different in the Dirac and Majorana cases.

Heavy Majorana neutrino singlets can be produced in the process [69, 70]

$$q\bar{q}' \rightarrow W^* \rightarrow l^\pm H \tag{41}$$

with $l = e, \mu, \tau$, which cross sections depend on M_N and the small mixing V_{lN} . Heavy Majorana neutrino decays in the channels $N \rightarrow W^\pm l^\mp$, $N \rightarrow Z\nu_l$ and $N \rightarrow H\nu_l$. The partial widths for the N decays are

$$\begin{aligned} \Gamma(N \rightarrow W^+ l^-) &= \Gamma(N \rightarrow W^- l^+) \\ &= \frac{e^2}{64\pi s_{\theta_w}^2} |U_{lN}|^2 \frac{m_N^3}{M_W^2} \left(1 - \frac{M_W^2}{m_N^2}\right) \left(1 + \frac{M_W^2}{m_N^2} - 2\frac{M_W^4}{m_N^4}\right), \end{aligned} \tag{42}$$

$$\begin{aligned} \Gamma(N \rightarrow Z\nu_l) &= \frac{e^2}{64\pi s_{\theta_w}^2 c_{\theta_w}^2} |U_{lN}|^2 \frac{m_N^3}{M_Z^2} \left(1 - \frac{M_Z^2}{m_N^2}\right) \\ &\times \left(1 + \frac{M_Z^2}{m_N^2} - 2\frac{M_Z^4}{m_N^4}\right), \end{aligned} \tag{43}$$

$$\Gamma(N \rightarrow H\nu_l) = \frac{e^2}{64\pi s_{\theta_w}^2} |U_{lN}|^2 \frac{m_N^3}{M_W^2} \left(1 - \frac{M_H^2}{m_N^2}\right)^2, \tag{44}$$

where U_{lN} is the light–mirror neutrino mixing $\frac{v}{\sqrt{2}}\sigma\hat{M}^{-1}$, (35). From (32), (35) the contributions come from terms of the order $|V_{lN}| \lesssim 10^{-7}$. From these expressions we can conclude that the total branching for each of the four channels is

independent of the heavy neutrino mixing, determined only by m_N and the gauge and Higgs boson masses.

Heavy neutrino signals are limited by the small mixing of the heavy neutrino required by precision constraints [73] and masses of order 100 GeV are accessible at LHC. For this mass range, SM backgrounds are larger and, since production cross sections are relatively small, heavy neutrino singlets are rather difficult to observe. The branching ratios for different values of m_N read as in Table 1 ($M_H = 130$ GeV); and in all these cases $\sum B_i \approx 1$. Here

$$\begin{aligned} B_{W^\pm} &= B_r(N \rightarrow W^\pm l^\mp), & B_Z &= B_r(N \rightarrow Z\nu_l), \\ B_H &= B_r(N \rightarrow H\nu_l). \end{aligned} \tag{45}$$

Table 1 shows that these decays are not so sensitive to the heavy neutrino mass, such that for heavy neutrino signals it is not necessary to have center of mass energies much larger than a hundred GeV.

Among the possible final states given by (42)–(44), only charged current decays give final states which may in principle be detected. For $m_N < M_W$ these two body decays are not possible and N decays into three fermions, mediated by off-shell bosons.

Other simple production processes like

$$q\bar{q}' \rightarrow Z^* \rightarrow \nu N, \tag{46}$$

$$gg \rightarrow H^* \rightarrow \nu N \tag{47}$$

give l^\pm and l^+l^- final states which are unobservable due to the huge backgrounds. For the pair production

$$q\bar{q} \rightarrow Z^* \rightarrow NN \tag{48}$$

the cross section is suppressed by $|V_{lN}|^4$, phase space and the Z propagator, and is thus negligible.

Three signals are produced in the two charged current decay channels of the heavy neutrino

$$l^+ N \rightarrow l^+ l^- W^+ \rightarrow l^+ l^- l^+ \bar{\nu}, \tag{49}$$

$$l^+ N \rightarrow l^+ l^+ W^- \rightarrow l^+ l^+ l^- \nu \tag{50}$$

and small additional contributions from τ leptonic decays.

Heavy neutrino signals in the final state $l^\pm l^\pm$ are given in the lepton number violating neutrino decay and subsequent hadronic W decay, or leptonic decay when the lep-

Table 1 Branching ratios for different values of m_N

m_N (GeV)	B_{W^\pm}	B_Z	B_H
100	0.34	0.1	0.2
390	0.3	0.306	0.09
780	0.3	0.297	0.107
$\gg M_W, M_Z, M_H$	0.293	0.3	0.111

ton is missed. LHC present energies are enough to discover heavy Majorana neutrino with very small V_{eN} [71, 72].

7 Conclusions

Here the LRMM with gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y'}$ is applied in order to find closer values for neutrino masses fitted to experimental data. We have worked with Majorana neutrinos, which mass matrix was written in terms of blocks that stand for standard and mirror mass terms. The large number of parameters involved induces to make some simplifications on the structure of the matrix. A double seesaw approach method is used and diagonalization is performed, and with the help of neutrino data we accommodate neutrino masses with normal hierarchy of the order of $(m_1, m_2, m_3) \approx (0.0865, 0.0870, 0.1)$ eV. So, we have found a consistent smallness hierarchy for the neutrino masses. With the LRMM we have also analyzed the radiative decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow e + \gamma$ and $\tau \rightarrow \mu + \gamma$ for a Higgs mass of 130 GeV, obtaining bounds for the branching ratios congruent with the experimental ones. Decay rates for heavy neutrinos N were calculated for different channels, and we found that their BR are nearly equal for $M_N \gg M_W, M_Z, M_H$ and also that they do not change too much for other values of M_N . To find heavy Majorana neutrinos one has only a few parameter dependence (for neutrino singlets, the heavy neutrino mass and its mixing angle) and also the mass scale could be accessible at the LHC.

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