Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Combined Timber Plate and Branching Column Systems – Variations and Development of System Interaction

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Abstract

The theme of this paper is interaction between and within plate-based roof structures braced by branching columns and tensile members. Interaction of cross-laminated timber plate structures and branching column systems have been investigated in a previous paper [1] by utilising physical models and Genetic Algorithms. Based on this study the concept of interaction and the utilisation of compressed and tensioned members in combination with folded and/or single- and double-curved plate assemblies are treated and discussed for development of structural and architectural applications designed for rational construction.

Keywords: Timber plate structures, branching columns, system interaction, assembly geometries, Evolutionary Computation, Genetic Algorithms

1. Introduction

The previous study regarded a folded plate assembly supported by a stiff column structure of steel pipes. In computer models, depending on the loading conditions and the arrangement of the columns, column members were shown to carry both tension and compression. In such cases, in response to the occurring forces stiff branching members in the supporting structure may be replaced by tension members and the overall structural shape and layout could be rearranged in analogy with a form finding approach where the forces and the member behaviour is allowed to steer and develop the overall shape.

The geometrical/topological features of the basic unit can furthermore be utilized to a wider extent. In the previous study the relation between structural complexity and production/assembling rationality was of interest and a basic rhombus and a trapezoid unit were chosen for and tested in 3-dimensional assemblies with branching column supports. A Miura-Ori pattern with trapezoids was tried, but rejected since it provides a mechanism as an effect of its geometry not being locked. The Miura-Ori pattern may be described as a developable surface [2], denoting an unfolded or unfoldable plane. This is not a property of the facetted rhombus-based units chosen for further studies in this context.

To develop properties of structure and form of folded plate assemblies, experiments have been initiated where altered angles of the rhombuses are introduced in the periphery of the previously treated units 2x2 to obtain curved structures. With a plane plate assembly as point of departure the assembly types studied in this paper are single and double curved roof structures where different parts of the supporting system are developed to respond to the overall load configuration and act in either tension or compression.

2. Branching system

2.1. Support structures in tension and compression

Combinations of structural properties and of materials are frequently used in structural and architectural solutions. In the first modelled plate-based vault the support structure is chosen as a combination of a compressed tree structure in the centre of the section and branching tension cables on each side of this. This is basically the principle of a "flying mast" which is a well-known rigging technique of which various architectural examples can be found (see Figure 1). The principle is most commonly utilised in structures with membranes as covering surfaces where the tension cables bracing the columns are fixed to mast structures in the periphery.

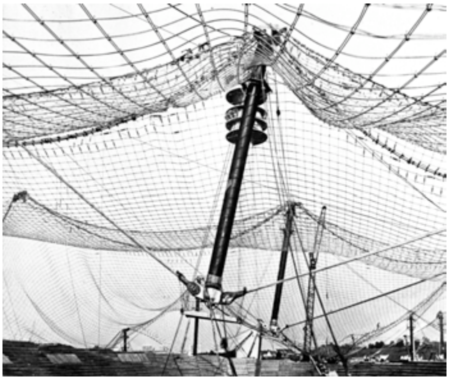

Figure 1. Cable braced columns or "flying masts" used by Frei Otto on the Olympic Stadium in Munich, 1972 (from IL archive, Univ. of Stuttgart)

The plate shells supported by the currently treated tree structures show in comparison different properties and need to be tailored in quite another way since they are neither flexible nor elastic. The sub-division of the covering surface into elements for fabrication is carried out following basically one of two main principles: geometric stiffness through folding or mapped and/or projected meshes.

2.2. Struts, rods and cables

Struts and cables may be utilised in different combinations to suit different structural prerequisites resulting in varieties of shapes. Several varieties of tension tree structures have been described, e.g. by Saitoh et al. [3], where replacement of compression members with tension members leads to reinforcement of columns and reduction of beam sections. Other gains from introducing stay cables and bracing are reduced column stresses and obtained efficient self-equilibrium systems. Rods and/or cables may be utilised for staying, bracing and tying down column structures and may be combined both in connection with each other and isolated as in tensegric structures. Tensegrity is a strictly defined structural principle but there are numerous varieties on the tensegric theme including the grouping of compressed members circumscribed by tension, segregation of tension and compression in more or less conventional truss systems, introduction of plate elements for shear capacity in the basic tensegrity unit etc. The structural essence in these tensegric typologies, as well as in pure tensegrity structures, is the stabilising of structural members.

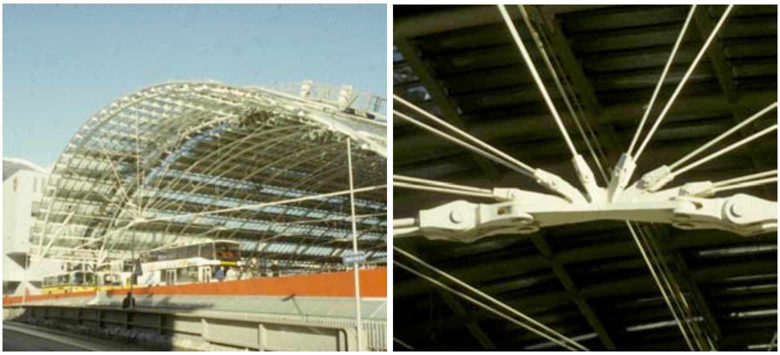

Figure 2: The steel rod structure of the bus terminal in Chur

The roof structure over the central railway and bus terminal in Chur, Switzerland (by Brosi/Orbist/Rice in 1988-92) may be referred to in this context (Figure 2). The roof in Chur is constructed with a glazed single-curved vault supported by steel arches spanning across a terminal floor over-decking a train station. The steel arches are complemented with sets of steel rods stabilising the curve. In each set of bracing rods, two main rods are acting as bottom chord between the support points on each side at the vault base; between the support points six double tension rods stabilise the curved steel tubes that act as primary structure. The rods come together in a joint on the centre line elevated above the base line, seen to the right in Figure 2.

Another example is the atrium of DZ Bank in Berlin (Gehry/Schlaich in 1998), where the shape of a free-form double-curved glass roof is stabilised by a series of steel rods oriented like a fan as seen in Figure 3.

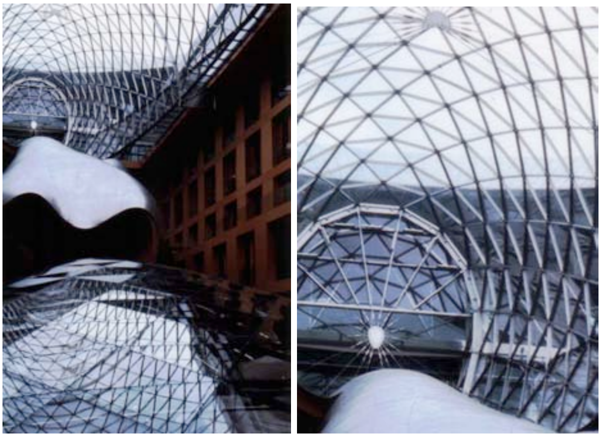

Figure 3: The stabilising structure of the atrium roof at DZ Bank

Both of these examples use only cable bracing to stiffen the arch in a manner similar to a bicycle wheel with the arch in compression and the cables in tension. Both of these examples use only cable bracing to stiffen the arch in a manner similar to a bicycle wheel with the arch in compression and the cables in tension. In the Berlin Hauptbahnhof (Gerkan, Marg und Partner/Schlaich in 2006) a rigid strut and rod structure stiffens the vault, as seen in Figure 4. The members perpendicular to the vault surface are all in compression while the force is transferred by a tension rod pressing the vault inwards at each side and upwards in the middle section.

Figure 4: Berlin Hauptbahnhof, its vault and the stabilising undulating rod structure

3. Plate assembly

3.1. Defining a pattern. 1: Mapped and projected meshes

By mapping a regular mesh across a curved surface a varied pattern with a multiplicity of the element dimensions and low repetitiveness may be obtained; the resulting element shapes in the pattern are slightly changed where the curvature is small and are more transformed the more the curved surface deviates from the plane of projection. Such facetted translation surfaces can be defined through a bar-and-node approach describing a triangulated space-frame where the bars can be translated to plate edges. Even facetted translation systems with quadrilateral facets in a bar and node systems are equivalent to triangulated bar and node systems as long as the surface of each facet transmits shear [4]. Shear capacity is in the current context provided by the cross-laminated timber plate elements showing satisfying shear stiffness and enabling a number of rational joint solutions [5]. The choice of principle for and pattern of sub-division determines how close to an even curvature the faceting gets. It also guides the repetitiveness across the structure. In the tessellation of a single-curved surface a single quadrilateral or triangular element may be repeated as a tiling [6], forming plane strips along the sides of the vault.

3.2. Defining a pattern. 2: Folded assemblies

Geometric sub-division of curved surfaces can be carried out in different ways. Different methods have different effect on the structural-architectural relation in the constructed object. The sub-dividing grid may be translated into a regular mesh following the curved plane, whereas its projection on a plane surface is changing. For unfolded plane undulating surfaces a triangulated or hexagonal mesh as defining pattern for bars and nodes is often preferred from a structural point of view. In this case elements corresponding to a quadrilateral mesh are chosen to build up a geometric stability through a structural depth of the assembly section.

In the previous study of plate-column interaction a folded surface composed of rhombic elements was used. The structural pattern is basically pre-defined and provides a strong impact on the resulting architectural expression, which in this case was regarded as a positive effect.

In the facetted units composed of four rhombic elements (2x2) the resulting interface angles between the base plane and the unit interfaces in a vertical section was 90° thus resulting in a plane repetitive assembly of units. In a cardboard model in the current study the angles of the basic element were changed by modifying two of the element sides. One of the angles was kept as a = 85° (to the left in Figure 5), whereas the angle of the opposite corner was changed into b = 89°, resulting in the shape seen to the far right in Figure 5.

By varying the angles of the rhombuses the interface angles of a 2x2 unit deviate from I = 90° to the left in Figure 6. The resulting angle between interface and base plane is II = 76.4° as seen to the right in Figure 6 and the facetted assembly turns into a curved shape (Figure 6 below). This choice of angles has the advantage that the surface sheds water.

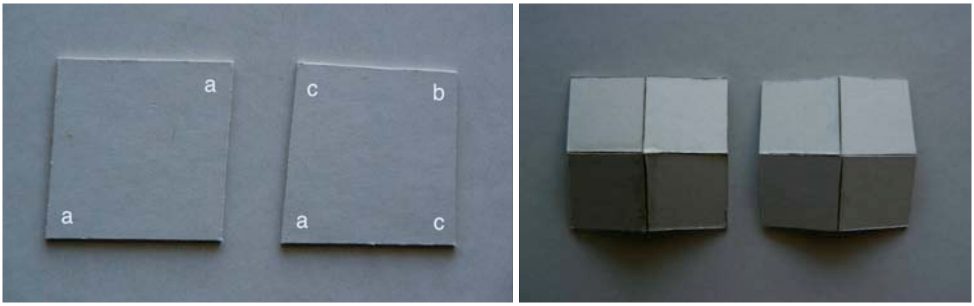

Figure 5: Model photo with element angles and resulting 2x2 units

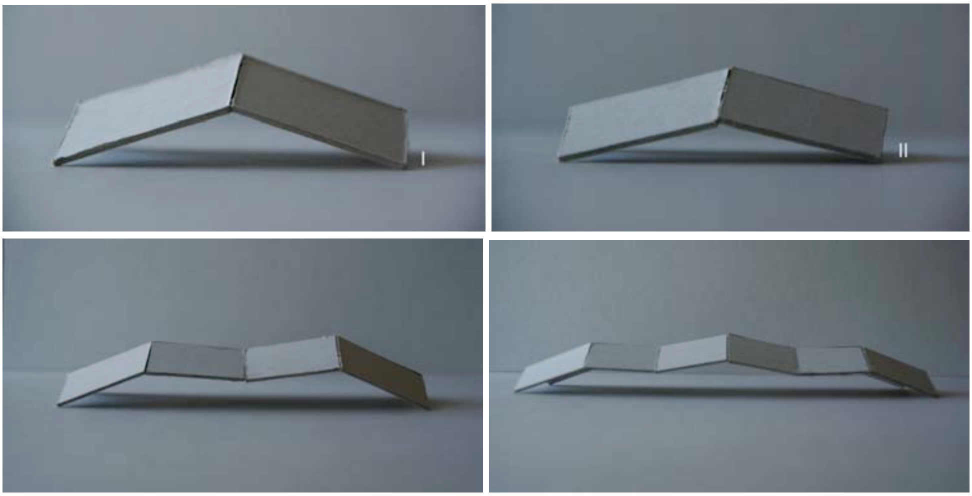

Figure 6: Model photo with interface angles (above), resulting curved assemblies (below)

The curved assemblies may as a first step be studied in single-curved versions, where the basic unit is designed with interface angles = 90° on two opposite sides and interface angles < 90° on the other opposite sides in the perpendicular direction initially generating an arch and through repetition of the arch a vault (Figure 7). The change of the interface angles follows the change described above in the corners of the rhombus plates.

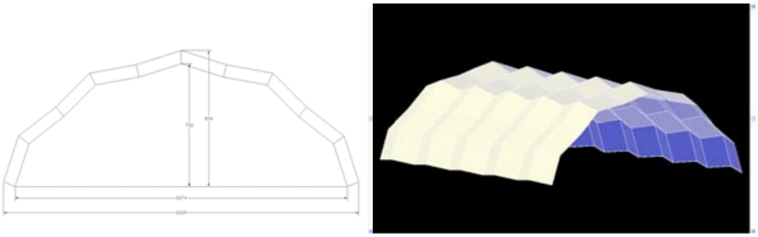

Figure 7: Computer model of the single-curved folded plate assembly without additional bracing

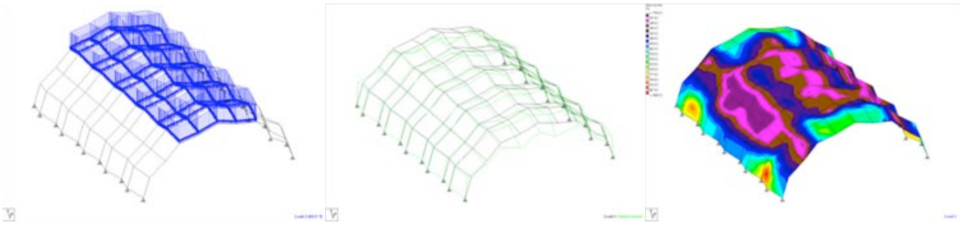

Figure 8: Analysis based on snow load (right), magnified deflection (centre) and stress distribution in the plates (right)

Like in the previous study, the currently studied elements are based on a square cross-laminated timber-plate element measuring 2.95 x 2.95 m with a thickness of 146 mm. The curved assembly was analysed using STAAD-Pro and the applied load was 1.5kN/m^2 on top representing a snow load and self-weight of the timber plates with 146 mm thickness. The analysis shows small deflections (less than 1 cm) and low von Mises plate stress, as shown in Figure 8, and high stiffness compared to a non-folded single-curved surface.

Through further modification of the unit interface angles, assemblies of double-curved character were studied in cardboard models (Figure 9), showing structural behaviour similar to the one observed in the plane plate assembly.

Figure 9: Model photos of a plane assembly (left) and a curved assembly (middle and left)

3.3. Stabilizing geometries and patterns

Meshes for sub-division may be designed differently depending on parametric approach, overall geometry and structural/architectural aim. A mesh may be viewed as a framing vessel, a space-enclosing cage, or as a "template of action", as Cecil Balmond puts it in [7] referring to a diagram allowing for different structural variations on a given theme. Both approaches provide input to the design task and may be utilized while generating the form [8] in a process of architectural and structural design interaction.

The structural performance differs depending on the pattern and surface design, and the relationship between shell and support needs adjustment to varying degrees. To a single- or double-curved non-folded surface with a smoothly curved shape, the mesh needs to be optimized to provide stiffness through the 2-dimansional pattern for load-transfer and the dependence on the support structure for stabilizing and fixing the curve increases. A 3-dimensional geometry/topology of folds provides higher stiffness and increased rigidity and thereby decreased dependence on the support structure for stabilization and fixation of the shape.

3.4. Free-form versus form-free

The difference between patterns providing stiffness and stability through geometric properties/topology and patterns providing optimal stress distribution to a curved surface through irregular changes of a specific mesh should be noted. When dealing with e.g. NURBS structural patterns may be developed and designed through densification of the mesh or grillage, through locking the basic grid shape of e.g. a hexagon, but varying its proportions and frequency when applying it and modifying it to suit the desired shape. The basic pattern is allowed to transform and – even though optional – the structural cross-sections may vary too, to save material and to avoid over structuring of the form [7]. To define a rigid pattern on a free-form shape where e.g. a hexagonal mesh has been mapped across the surface, a Voronoi diagram is applicable to obtain a triangulation of the mesh. Parametric design tools and principles are steadily developed which are rather easily applied and utilized by both engineers for structural layouts and architects for definition of form and surface [9]. The parametric approach may be efficiently used both early in the design process while defining the first basic constraints and parameters for the conceptual design process, and in the finishing structural design phase when the system detailing is set.

The different principles for sub-division and the design and optimisation of the supporting tree structure provide a variety of topologies and resulting potential typologies: arches, irregular grids, asymmetric curves, convex and concave vaults as well as hyperbolic shells.

The term free-form is common today denoting a form experimenting approach to the defining and creation of an architectural object/shape. The counterpart could in this context be a structural system or principle – like a principle of sub-division – which initially to some extent is form-free. The two opposite phenomena describe a duality in form-finding: the form seeking its structure, and the structure seeking its form.

4. Plates with tensioned and compressed branches

4.1. A strut and cable stayed single-curved vault

When regarding the relation between tension and compression forces in a vault structure there are also other possible options. In the Berlin Hauptbahnhof the choice was made to fix the curve of the roof solely with compression bars and letting the support structure change from outside to inside and back. To support the structure from only one side both tension and compression need to be considered and struts may be partly replaced by rods or cables. The tree structures presumably show similar configurations in both load-configurations as shown in the previous study where loads were applied from below in both digital and physical models. The stress distribution may be elaborated by varying the number of support points and changing the number of branching node levels and the number of braches per node.

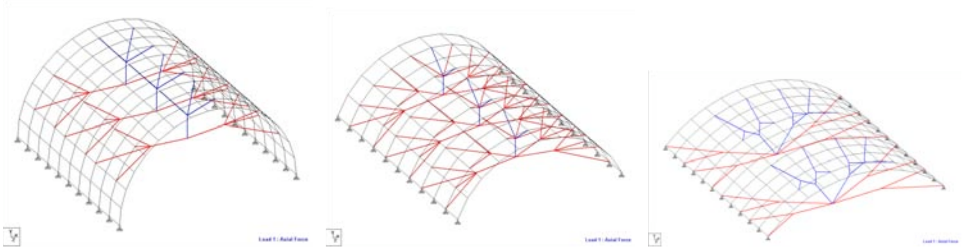

Figure 10: Three possible bracing patterns for single-curved roof with different span/height ratios. Tension (red) and compression (blue) under symmetric snow load of 1.5 kN/m^2

Figure 10 shows three constant radius configurations with different span to height ratios. From right to left they are (A) 2:1, (B) 3:1 and (C) 4.8:1. For comparison each structure was loaded with a 1.5kN/m^2 uniform load over the area with slope less than 45° plus dead load. The bottom chord in each case was pre-tensioned. The laminated wood plates are each 2650 mm in length and 146 mm thick. Out of plane moments were released at the plate nodes. Figure 11 shows a comparison of deflections magnified by 100:3. Configuration A has the largest deflections of the shell, 6.6 cm at the centre. Case B has the lowest deformation of the shell with about 1cm at the centre. The more shallow case C has a little more deflection at 16 mm. Case B is more similar to the tensile stay arrangement noted earlier in the Chur bus terminal. Case C acts more like a flying mast. In case C the low location of the tree base is a potential architectural disadvantage since it may interfere with and disturb the utilization of the created/covered space.

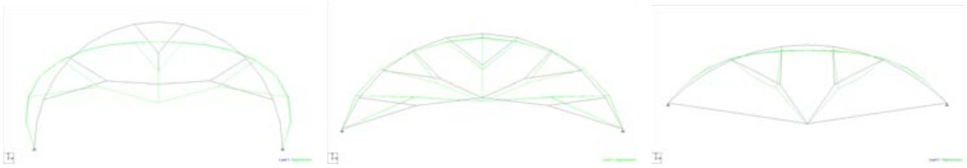

Figure 11. A comparison of the deformation of the three configurations. Scale = 3:100

The forces in the bracing members (Figure 12), as well as the stress levels in the wood plates (Figure 13), were also compared for the three configurations. The centre tree was placed in compression in all three cases. The shell stress was relatively low but higher in the taller arch toward the base. The large outward deformation of the first case also caused larger force in the bracing members. The pre-stress was actually higher in case C which can be noted in the bottom members of that arch.

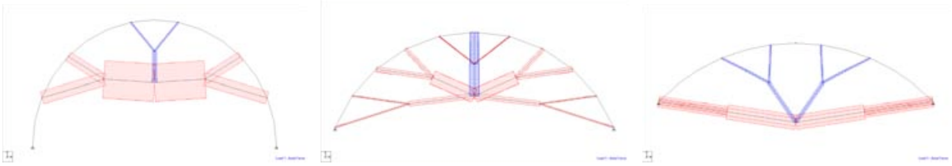

Figure 12. Force levels shown at the same scale. Red is in tension and blue in compression

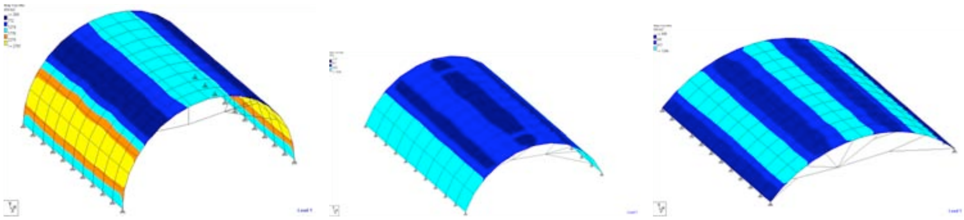

Figure 13: Maximum von Mises plate stress levels in the wood plates

4.2. A double-curved folded plate vault stayed by struts and cables

Results from physical and computer models show increased structural capacity in both plane and curved folded assemblies. The combination of plane folded assemblies and branching column supports should be developable into a structurally efficient double-curved type where the overall shape is more easily stabilised depending on the increased rigidity of the shell, compared to a non-folded surface. The combination of tension cables and compressed bars in the supporting tree structure presumably differ from the initially tried tree design for the single-curved non-folded roof. The first computer model of a non-folded surface points at needed changes of the configuration of supports structures and depending on the stiffness of the curved folded surface the support design may be further modified, unless they are redundant. The results of the actual application are, however, still hypothetical and further studies are needed.

5. Discussion

For the previously studied plane assemblies the branching supports may be composed of both tension and compression members and this is an interesting issue for further elaboration in both plane and curved plate structures. So far in the current study model studies have been performed on a single-curved non-folded shell with branching supports, and of a single-curved and a double-curved folded roof structure, not yet combined with a branching structure. The differences between tree configurations in single- and double-curved assemblies should be analysed, and the design of the plate and tree sub-systems could in all cases be further elaborated for optimised interaction. To continue and fully evaluate the study of structural development from simple single-curved vaults without folds to double-curved folded surfaces with tension and compression tree supports there are a number of combinations of interest, to study, model and compare:

I: In single-curved non-folded assemblies the effect of a stabilising structure is notable and the supports may be composed with a combination of tension and compression members, even though the first model was inefficient in stabilising the overall behaviour. Changed tree configurations and pre-tensioning levels may be tried to increase the structural efficiency and the architectural utility.

II: Double-curved non-folded surfaces have not yet been regarded in the study but should be modelled for comparison, both without supports and interacting with a tension and compression tree structure. In this case the sub-dividing grid design easily increases in complexity and a number of varieties could be tried for reference.

III: Single-curved folded surfaces perform with increased stability, in the models studied so far, as could be expected compared to non-folded surfaces. The analyses of the arch and vault shaped folded assembly show that sufficient stiffness is obtained by the folded plate structure itself, with the dimensions, element angles and resulting pitch that were used. In these cases the supporting tree structures become redundant. This is presumably the case also for a double-curved folded assembly. The element angles and interface angles have so far, however, been chosen through an analogue model procedure and further analyses may very well indicate optimizing steps of interest implying other pitches of the basic unit and thereby changed structural depths of the assembly.

Optimisation of the structure may imply modifications of the element angles and interface angles, e.g. decreasing the structural depth and thereby decreasing the stiffness of the plate shell and increasing the effect of a supporting structure. Studies may be complemented by studies using Genetic Algorithms to vary and optimise the design and properties of the branching structure, the relation between tension cables and compressed bars and the configuration and number of branches and nodes.

6. Conclusions

In the current study, issues of interest are basically the same as in the previous study but regarding the difference in focus between the two cases the studies are considered to complement each other. In the previous study the design of the plate assembly was initially defined and kept the same, whereas the branching and inclination of the supporting columns were varied and tested. In this paper modification of the shell is introduced.

The relationship between plate assemblies and support structures varies depending on the radius and structural depth of the assembly and so does the very need for supports. The results so far show that curved folded surfaces based on repetition of a single element or a small number of element types are possible and provide notably increased rigidity to the assembly. Combinations of cables and struts furthermore enable a developable variety of the branching support structures, potentially increasing the means of optimization of combined plate-shell and branching tree structures for a satisfying behaviour.

Thus, though not completed, the modelling of folded surfaces and combinations of tension and compression members in the supporting structure performed so far, and the discussion of different topologies show developable aspects of interplay and sub-system optimization with prerequisites for further studies.

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