Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Comparing Structural Systems For Free-Forms

Wim M.W.KAMERLING

*ass. professor
Technical University of Delft, Faculty of Architecture,
Delft, The Netherlands
M.W.Kamerling@tudelft.nl

Abstract
For free-forms as well as shells the choice of the grid of the structural system will be decisive for the architectural and structural design. This paper analysis several grids concerning the load transfer and construction. To generalize this analysis, the surface of the free-form is designed by stretching a half sphere, so the ground plan is an ellipse.

Keywords: free-form, ellipsoid, load transfer, grid, structural system, optimization, design, construction.

1 Introduction
The choice of the grid will effect the design of the structure and envelope. Generally the grid is chosen at the first stage of the process of design. Then the available information is minimal, but the need for information and the effect of the decisions is maximal. Due to the complex geometry it is hard to find a strategy to design the structure of a blob optimal. The structure can be designed conform the grid made by the architect to visualize the free-form. Sometimes the structure is designed as a reticulated network, see for example the structure above the great court yard of the British Museum in London (Veltkamp [4]). For blobs the variety of forms is numerous so these buildings are hard to classify. Nevertheless we can distinguish a special category of blobs, which are designed by transforming a regular form as a cylinder, cube, cone or a sphere by stretching, pushing or truncating. For these free-forms, designed by transforming a regular form, it can be profitable to apply a structural system developed for the regular form. In the past some structural systems were developed for domes. Engel describes for example the Schwedler, the lattice, the hexagonal lamella and the geodesic dome (Engel [1]). For blobs, designed by transforming a sphere, these systems can be suitable too. In this paper several structural systems are analyzed and evaluated for a free-form, constructed by deforming a sphere, with respect to the load transfer and construction. To generalize the analysis, the surface is constructed by stretching a half sphere, so the ground plan is an ellipse. For ellipsoids, many problems concerning construction, design and load transfer are similar to the problems rising for the

described category of blobs. The following three grids are analyzed: Firstly a grid composed of meridians and parallels, as applied for the rigid airships known as Zeppelins. Secondly a grid composed of vertical and horizontal parallels, the parallel grid. Finally a grid composed of meridians and parallels, conform the grid on a globe.

The coordinates of the ellipsoid are calculated with the well known following expression:

$$(x/a)^n + (y/b)^n + (z/c)^n = 1 \quad (1)$$

The investigated ellipsoid is oblong with the parameters n = 2, a = 10, b = 20 and c = 10. The structure is subjected to an unit load of p = 1 kN/m^2. The structural systems are composed of bars jointed at the vertices and subjected to concentrated loads acting on the nodes. The concentrated loads are acquired by multiplying the unit load times the area of the surface supported by the corresponding node. The bending moments, forces are calculated with the program Matrix-Frame. Further the bending moments, forces and deformations are analyzed for 2D-frames identical loaded as the structural elements of the three spatial grids. Comparing the results of the grids and the 2D-frames will show the effect of the rings and hoops for the spatial grids.

2 Zeppelin-grid

This grid, composed of circular parallel arches and meridians was in the past applied for the rigid airships known as Zeppelins. At the bow and the rear the meridians are jointed at the supports with the coordinates (0, ± 20, 0).The center to center distances of the parallels are constant. Figure 1 shows the radius and the area of the surface for the four vertical parallels. The parallel at the center (N1-N7) has a maximal span and maximal loading. The last parallel, next to the support at the bow and rear (N22-N28) has a minimal span and minimal loading.

Parallel	Radius	Area
N1-N7	10	13,1163
N8-N14	9,6825	12,8182
N15-N21	8,6603	11,9096
N22-N28	6,1438	11,8565

Figure 1: Radius of the vertical parallels and the area of the surface supported by the nodes.

Figure 2 shows the reaction forces acting on the supports of the grid. The magnitude of the vertical reaction force F_z acting on node N29 (0, ±20, 0) is considerately, so the meridians starting at this node transfer a major part of the load and support the parallells quite well.

Node	F_x	F_y	F_z
center, N7	-23.7	-0.2	-76.1
N14	-19.0	-1.0	-70.7
N21	-12.6	-1.1	-62.2
N28	-14.1	-9.7	-33.3
bow, N29	-13.1	-30.8	-67.8

Figure 2: Forces [kN] acting on the supports from center to the bow.

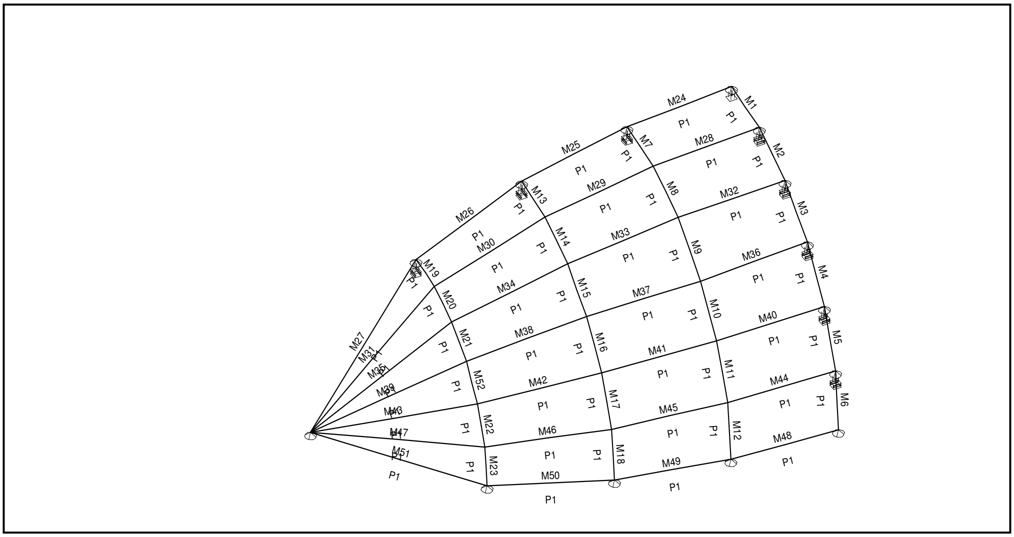

Figure 3: A quarter part of the grid of the Zeppelin.

The radius of the meridians is decreasing from the center to the bow. At the top the meridians are compressed, at the bottom the meridians are tensioned just conform the normal forces acting on the hoops of a spherical dome (Shodek [3]), see figure 4.

Meridians	bay at the end	bay at the center
first, at the top	-17.2	-32.2
second	-25.4	-27.3
Third	-16.6	-27.2
fourth	-1.5	-4.0
Fifth	21.5	14.0
sixth, just above ground floor	16.0	27.9

Figure 4: Normal Forces [kN] acting on the 6 meridians from top to bottom for the last bay (column at the left) and the forces in the bay at the center (column at the right).

Due to the varying radius of the meridians, the meridians will support the arch next to the bow, N22-N28, quite well, but the arch at the center, N1-N7, with the maximum span is supported poorly by the meridians. Consequently the bending moments acting on the arch N1-N7 are considerately, see table 5.

parallel N1-N7	parallel N8-N14	parallel N15-N21	parallel N22-N28	2D frame
5.9	2.6	-1.0	-9.5	0
5.4	1.8	2.1	6.6	-5.6
-10.5	-5.7	0.4	8.1	-24.0
-24.4	-13.8	-4.6	15.8	-49.8
-33.8	-22.8	-7.4	14.2	-62.9
-35.0	-25.6	-11.2	16.6	-53.0
5.6	-2.7	-0.3	-12.0	0

Figure 5: Bending moments [kNm] acting on the parallels and the bending moments acting on the 2D frame.

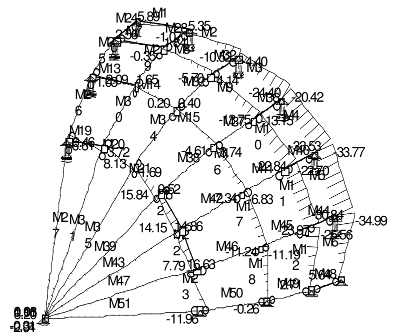

Figure 6: Bending moments on the elements of the grid.

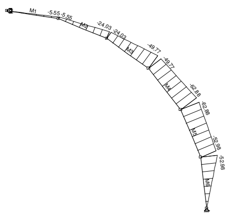

Figure 7: Bending moments on the 2-D frame.

The bending moments are also calculated for an independent 2D frame, not connected with meridians, subjected to an identical loading as the parallel at the center. Comparing the bending moments acting on the grid and the 2D frame shows that the bending moments in the grid are remarkable smaller than the bending moments acting on the 2D frame. The ratio is equal to 35.0/62.9= 0.56.

3 Parallel-grid

This grid is composed of vertical parallel frames and parallel hoops, see figure 8. Due to the decreasing span and the decreasing center to center distance the forces acting on the supports are decreasing from the center to the end, see figure 9.

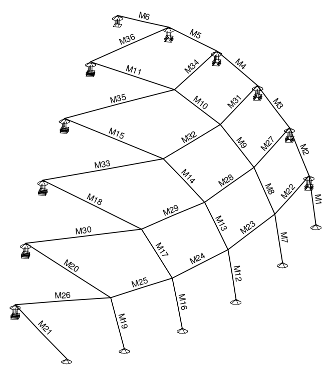

Figure 8: A quarter of the grid composed of horizontal and vertical parallels

Support	Fx	Fy	Fz
Center N1	-23.1	0.8	-75.8
N8	-24.8	7.7	-73.3
N14	26.9	5.5	-72.6
N19	-30.8	9.0	-71.8
N23	-19.8	9.7	-18.4
end support N26	-3.9	-4.0	-0.1

Figure 9: Table 5 Forces [kN] acting on the supports

Figure 10 shows the bending moments acting on the vertical parallel at the center of the grid with the maximal span. Due to the supporting hoops the bending moments in the grid are smaller than the bending moments in the 2D frame which is subjected to the loads identical to the parallel of the grid at the center. The maximal moment acting on the parallel of the grid is smaller than the maximal moment acting on the independent arch the ratio is equal to: 39.8/56.3 = 0.7, so the hoops support the vertical parallels well.

Node	arch at the center	independent arch
N7	0	0
N6	1.2	2.2
N5	-14.0	-14.8
N4	-30.0	-40.9
N3	-39.8	-56.3
N2	-36.3	-49.6
N1	0	0

Figure 10: Bending moments in the arch at the center and the independent 2D frame. The nodes are numbered from the top to the support.

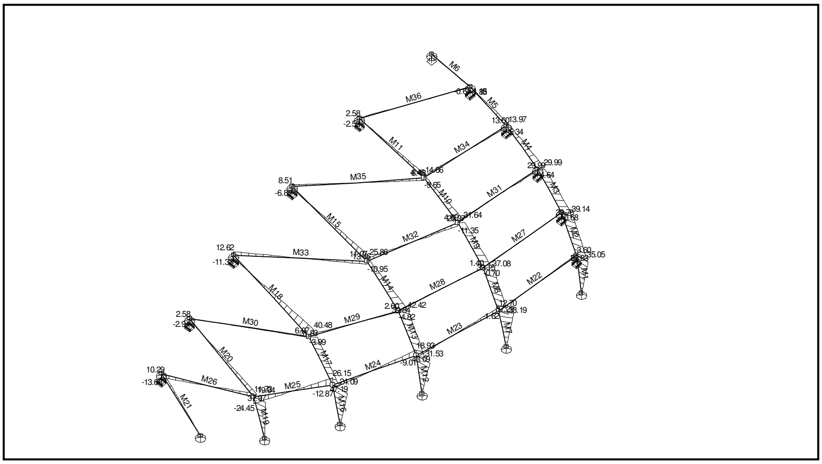

Figure 11: Bending Moments I the grid composed of parallel arches and hoops.

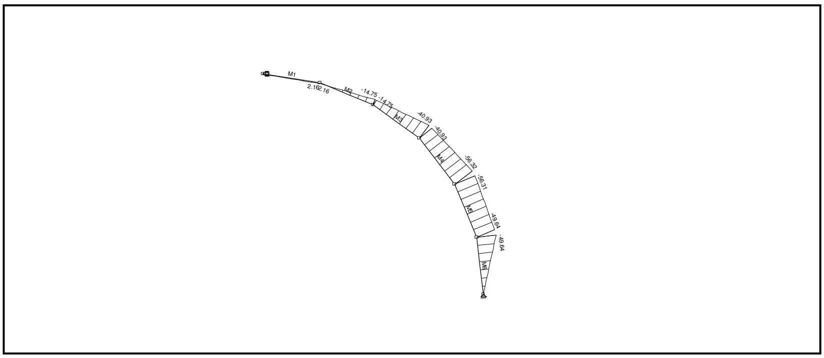

Figure 12: Bending moments in the independent arch.

Figure 13 shows the normal forces acting on the hoops. At the top the parallels are compressed and at the bottom the parallels are tensioned just as the parallels of a shell of rotation [2]. The normal forces are decreasing from the center to the end just because the center to center distance of the arches and the radius of the hoops is decreasing from the center to the end.

	bay 3		bay 2		bay 1
member	normal force	member	normal force	member	normal force
N24- N27	0.1				
N20 –N24	16.7	N21-N25	5.3		
N15 – N20	22.7	N16-N21	5.5	N17-N22	-0.5
N9 - N15	22.2	N10-N16	24.6	N12-N17	-11.9
N2 - N9	26.1	N3-N10	33.7	N4-N12	-7.0

Figure 13: Normal forces[kN] acting on the hoops, bay 3 is near the center, bay 2 is second to the end and bay 1 is at the end.

4 Globe-grid

The grid is composed of vertical meridians jointed at the top and horizontal parallels just as the gridlines on a globe. This grid was applied for Schwedler domes, which were invented during the 19^{e} century. Mostly these domes were braced with diagonals to resist asymmetrical and lateral loads. Grids composed of meridians and parallels are still constructed, see for example the dome on the top of the Reichstag in Berlin.

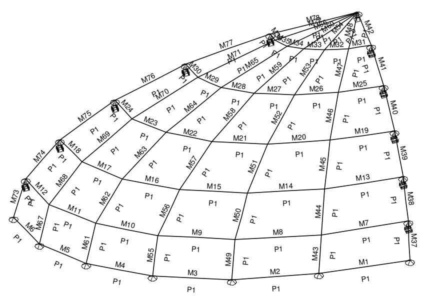

Figure 14: A quarter part of the ellipsoid with meridians and parallels

The analysis of the reaction forces acting on the support shows the load transfer by the meridians. The loads on the nodes are proportionally with the area of the surface supported by the node, so the loads acting on the meridians are proportionally with the center to center distance and the span. The vertical reaction acting on the support of the meridian with the maximal span is much smaller than the reaction of the meridian with the minimal span so the better part of the loads are transferred from the meridian with the maximal span to the neighboring meridians. The meridians are connected by the parallels, due to the support of the parallels a part of the loading is transferred from the meridian with maximal span to the neighboring meridians.

Node	Fx	Fy	vertical Force Fz
center, N1	-7.8	0.2	-50,4
N2	-5.5	-4.5	-52.6
N3	-6.5	-7.4	-53.6
N4	-5.3	-9.1	-49.1
N5	-2.5	-11.6	-45.0
N6	-0.3	-9.5	-40.1
end support, N7	0.6	-10.8	-35.4

Figure 15: Forces [kN] acting on the supports.

	end 6	5	4	3	2	center 1
top ring	-18.9	-21,1	-25.2	-28.9	-31.0	-32.5
	-19.0	-21.7	-26.0	-29.5	-32.8	-34.3
	-5.2	-4.9	-5.6	-6.8	-7.7	-7.8
	15.6	18.3	22.2	24.2	26.8	27.9
bottom ring	27.2	29.6	33.6	40.4	44.6	45.5

Figure 16: Normal forces acting on the parallels in the bays from the end to the center.

	meridian XZ-face	Meridian YZ-face	2D –frame
top 1	-1.6	0.5	0
2	1.2	-0.9	0.6
3	3.0	-1.1	-0.7
4	2.4	-1.0	-5.7
5	-1.5	0.6	-14.0
down 6	-6.7	4.4	-13.9
7	0.3	-0.8	0

Figure 17: Bending moments acting on the meridian in XZ-face with the minimum span and the meridian in the YZ-face with maximum span compared with the bending moments in the independent 2D-Frame.

Figure 16 shows the normal forces acting on the parallels. The parallels are compressed at the top and tensioned at the bottom. The parallels are at the top shorter than at the bottom, so the stiffness of the parallels is increasing from the bottom to the top. The radius of the parallels is decreasing from the center to the end, consequently the normal force in the parallels are decreasing from the center to the end too. The bending moments acting on the meridians are smaller than the bending moments in the parallel and zeppelin grid. The maximal moment acting on the bars of the grid is half of the maximal moment acting on the flat 2D-frame, the ratio is equal to: 6.7/14 = 0.48. The loads are mainly transferred by the normal forces acting on the meridians and parallels. The structure approaches a vector active structural system.

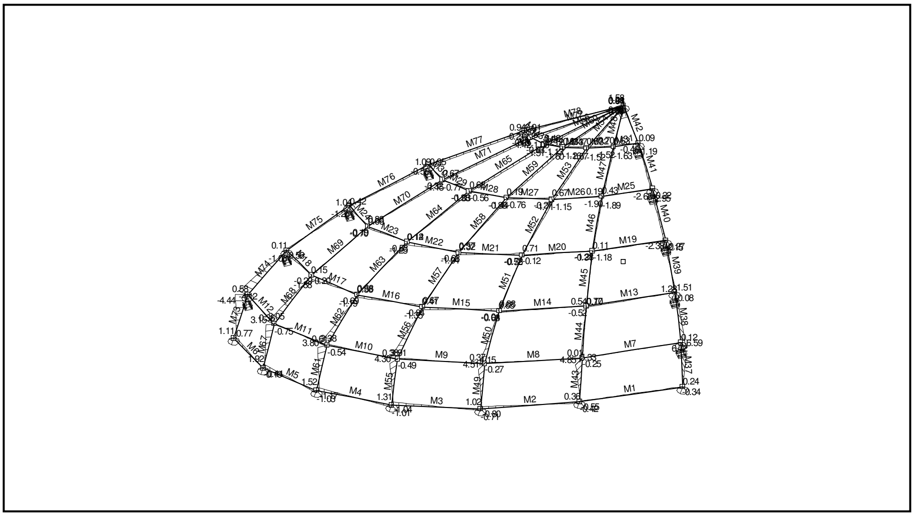

Figure 18: Bending moments acting on the elements of the grid

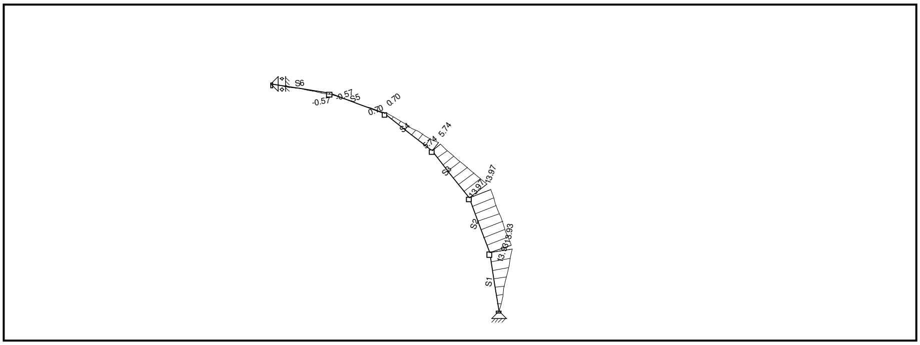

Figure 19: Bending moments acting on the 2D-frame

5 Construction

For the choice of the grid the load transfer is important but construction, production, aesthetics and cost are important too. Many domes were constructed with a spatial grid composed of small members jointed at the nodes. Specially during the middle of the XX-century the geodesics were quite popular for spherical domes. Concerning the aesthetics the geodetics perform quite well. Many architects like the idea to compose a geometric complex structure with one element and one joint, designed parametrically to meet the varying lengths and angles. Nevertheless the vertical loads are transferred well by vertical elements and showing the load transfer will contribute to aesthetics too. For free-forms these systems developed for geodetics can be applied too, but due to the complex geometry the variation of edges, nodes and faces will be very huge. Nowadays, thanks to the CAD-CAM technology, customized elements can produced without increasing the cost tremendously but the construction of systems composed of varying elements is still laborious.

Figure 20: Rebuilding the I-Web at the campus of Delft University.

For example, for the I-Web firstly a spatial frame was designed. Later this design was altered into a system of radial frames connected with diagonals. Still the construction was time-consuming and expensive. Concerning the construction the described three systems can be composed of prefabricated vertical 2D-frames jointed on the side with the bars of the horizontal elements. Prefabrication of the vertical frames will simplify the construction and reduce the costs. The construction effects the load transfer too: a prefabricated frame made in the factory will be stiffer than a frame composed of small parts jointed at the site with hinges.

6 Conclusions

For the investigated three grids the globe-grid performs optimal concerning the load transfer. Figure 21 shows the ratio of the maximal bending moment acting on the grid and the identical loaded 2D-frame. For spherical domes Torroja noticed that the grid composed of meridians and parallels performs well just because this grid follows the stresses consequently (Torroja [3]). Nevertheless we have to be carefully to extrapolate the conclusion and recommend the Globe-grid generally. The system of meridians and parallels will be less effective in case the ellipsoid is stretched further. Evidently for a cylinder a system of parallel arches will optimal above the globe-grid, so then the Zeppelin-grid and the parallel-grid will be very efficient. The described grids can be composed of vertical prefabricated elements jointed at the site, consequently the construction will be less laborious and the costs will drop down.

Grid	Maximal moment	ratio bending moment grid/2D-frame
Zeppelin-grid	35.0	0,56
Parallel-grid	39.8	0,7
Globe-grid	6.7	0,48

Figure 21: The maximal moment [kNm] and the ratio moment acting on the grid with respect to the moment acting on the 2D-frame for the zeppelin-, parallel- and globe-grid.

References

[1] Engel H., Structure Systems, (2^{nd} edition), Verlag Gerd Hadje Ostfildern-Ruit, Germany, 1999, page 152-155.

[2] Shodek D.L. , Structures, (4^{nd} edition), Prentice-Hall Inc. New Jersey, USA, 2001, page 432-437.

[3] Torroja E. Logic der Form, Verlag, G.D.W.Callway, Munchen, 1961, page 108.

[4] Veltkamp M., Free Form Structural Design, IOS press, Delft, The Netherlands, 2007 page 56.