Evolution of 2D Truss Structures using Topology Optimization Technique with Meshless Method

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Abstract

This paper describes a new topology optimization (TO) technique based on meshless method to evolve two-dimensional truss structures. The meshless method has been considered as a very attractive computational technique since it does not need any mesh generation process during the analysis. It has been gradually and widely used in many engineering disciplines and a particular weak point such as the re-generation of mesh information inherit in other numerical analysis techniques have been naturally solved. However, there have been a few applications of meshless method into structural design optimization so that we here try to apply the meshless method into structural topology optimization problem. We adopt the radial point interpolation method (RPIM) which uses radial basis function (RBF) since it is stable and robust for arbitrary nodal distributions. Then, the hard kill method based on fully stressed design scheme is consistently combined with the adopted meshless method. In order to demonstrate the accuracy of the proposed topology optimization technique, several benchmark tests are tackled to investigate the accuracy and capability of the present TO technique. From numerical results, it is found to be that the proposed TO technique is very simple and easy to produce the optimum topologies of plane structures.

Keywords: topology optimization, meshless method, hard-kill method, truss structures, radial point interpolation method

1. Introduction

Structural TO has been extensively used to produce the best possible topologies for engineering structures. Figure 1 illustrates the initial and optimal density distributions for a given problem using a plane stress model. In choosing the material model, one of the important features that should be considered is that it should allow the density of material to cover the whole range of values from zero (void) to one (solid). In addition the material description should fit the periodicity assumption and should be defined by only a few parameters (as these are the design variables of the optimization algorithm).

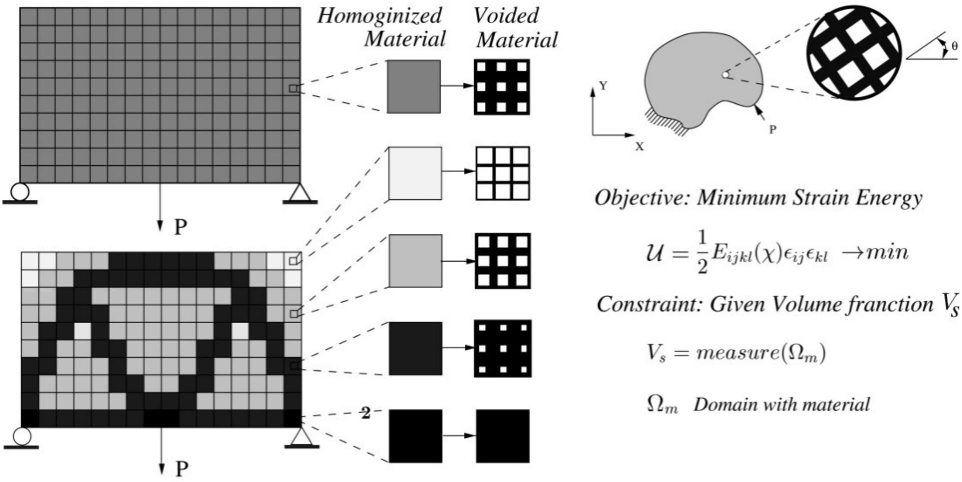

Figure 1: Basic concept of TO with homogenization using a square cell with a centrally placed rectangular hole as material model. Left-top: before homogenization (uniform, homogenized constant material properties for all elements), left-bottom: after optimization (different material densities for all elements; densities vary can vary continuously from white (no material) to black (solid)) [1].

The optimality criteria (OC) methods have been initially used [2] but mathematical programming (MP) methods are becoming one of the popular means of updating the hole size of the material [3]. Apart from OC and MP, several alternatives such as the hard-kill [4], the soft-kill [5] and the evolutionary method [6] are also introduced in TO process. The basic idea of kill method is that in a structure, material which has low strain energy (or stress) levels is used inefficiently thus, this material can be decreased. This decreasing process can be carried out by either varying the elastic modulus as a function of the hole sizes or by deleting the space occupied by the elements with the low strain energies (or stresses) from the structure. By repeating this step and removing small amounts of material density at each stage, the topology for structure gradually evolves. In particular, the elements with strain energies (or stresses) below a certain threshold strain energy (or stress) value are assigned a low elastic modulus. However, one of the main drawbacks of the kill method cannot preserve the initial volume throughout the entire optimization iteration. A set of benchmark tests is introduced to show the performance of the proposed TO technique and the numerical results are finally suggested as future reference solutions for TO with meshless method.

2. Review on the meshless method

So far, the finite element (FE) method has been widely used in various engineering displines. However, the re-generation of FE mesh is required to improve an accuracy of solution for problems with stress concentration and having crack propagation. Hence, meshless method is developed as an alternative analysis technique. In the meshless method, the problem domain and its boundary can be modeled by only using sets of field nodes scattered in the problem domain and on its boundary. Meshless method has also many different names such as SPH, diffuse element method, reproducing kernel particle method (RKPM), element-free Galerkin method (EFG), Hp clouds, radial point interpolation method (RPIM) and meshless local Petrov-Galerkin method (MLPG). Recently, meshless method has been also used to the large deformation analysis, the fracture and crack prorogation and shape design sensitivity analysis and optimization. In the FE method, the shape functions are created based on pre-defined elements, and the shape functions are the same for the entire elements. In meshless method, however, the shape functions are formed for a particular point of interest based on selected local nodes. Therefore, the shape functions can change when the point of interest changes and the accuracy of interpolation function depends on the nodes in the support domain. Note that the shape function is nonzero in the support domain and zero outside in the support domain. The most widely used support domains are circle and rectangle as shown in Figure 2.

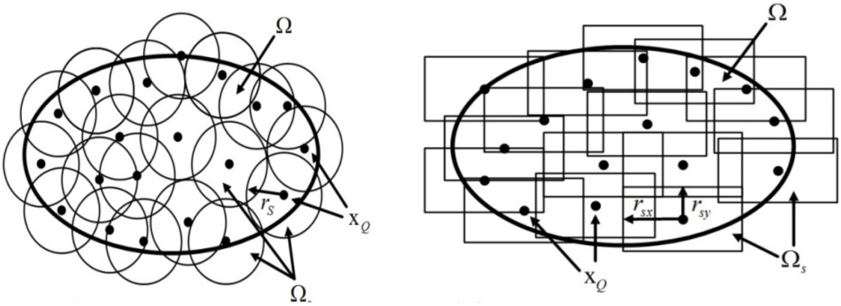

Figure 2: Support domain of point of interest at X_Q : (left) circular support domain, r_s- the dimension of the support domain ; (right) r_{sx} and r_{sy} - dimension of the support domain in x and y directions

We here adopt the radial point interpolation method (RPIM) which uses the radial basis function (RBF). The RPIM is widely used since it is stable and robust for arbitrary nodal distributions. In particular, the RPIM shape functions include the Kronecker delta function properties. So, the essential boundary conditions can be easily handled. The detailed descriptions on the meshless method used in this study can be consulted to Reference 7.

3. TO process with kill method

The kill methods have their origin in fully stressed design techniques. Several methods have been developed by various researchers such as the soft-kill method by Walther et al. [5], the hard-kill method by Hinton and Sienz [4], or the evolutionary design method by Xie and Steven [6]. The basic idea behind the different algorithms for the soft-kill/hard-kill method is that in a structure, material which has low stress levels is used inefficiently. Thus, this material can be removed. This removal process can be carried out by either varying the elastic modulus as a function of the stresses or by deleting the space occupied by the elements with the low stresses from the structure. By repeating this step and removing small amounts of material at each stage, the topology for the structure gradually evolves. This algorithm works with no instability when the amount of material removed at each stage is small. It is important to maintain a smooth transition from one topology generation to the subsequent one. This fact is reflected by the detailed algorithm.

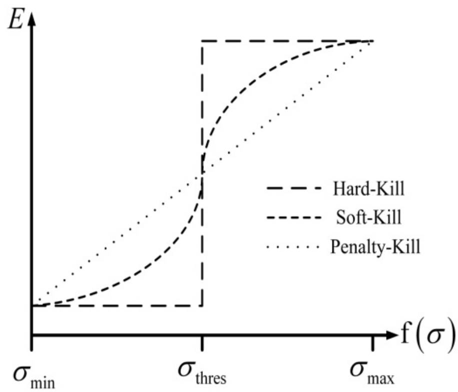

Figure 3: Different functions of elastic modulus versus a function for kill methods

The two methods using a variation of the elastic modulus do not alter the finite element mesh topology during the optimization process. Walther et al. [5] use a linear function of the elastic modulus versus a function of the stress (see Figure 3). The function of the stress can be either maximum principal stress or equivalent stress. Since the elements are removed softly this method is termed soft-kill method. If a step function is used instead of a linear function, the hard-kill method is obtained. In this method, the elements with stresses below a certain threshold stress value σ_{thres} are assigned a low elastic modulus. This level is not fixed and increases during the course of the optimization. Other functions, such as a penalty function, may be used as long as the main resulting characteristics are that lowly stressed elements are `removed' by assigning them a lower elastic modulus than to the higher stressed elements. A more detailed description of the hard-kill method is discussed in Reference 4.

The overall TO process wih hard-kill method is implemented as follows:

(a) Create the optimization model
(b) Create the analysis model
(c) Calculate the displacements u
(d) Calculate the Vonmises stress σ_{mises}
(e) Update design variables using kill method
  (e.1) Order the background cells according to their Von Mises stress values in ascending order
  (e.2) Update the values of elastic modulus using σ_{thress}
(f) If termination criteria is satisfied, then stop. Otherwise, repeat (c)-(e)

In this study, we use the following threshold stress value σ_{thres} to update the elastic modulus of background cell.

$$\sigma_{thres}^{i} = \sigma_{(p)} = \sigma_{(i\times\aleph\times N_{cel})} \tag{1}$$

where, i denotes the i^{th} iteration number, $\aleph(0 < \aleph < 1)$ is the removing rate which could be the different value for each iteration, σ_{thres}^{i} is the threshold stress value for i^{th} iteration and N_{cel} is the number of the background cells used in the analysis.

5. Numerical examples

5.1. Cantilever beam

The cantilever beam subjected to a point load at the top of the right hand edge is optimized. The dimensions of the rectangular design domain are L=3.2m H=2.0m and thickness normal to plane of structure h=1.0m. The geometry of the beam and the field nodes used in this test are illustrated in Figure 4.

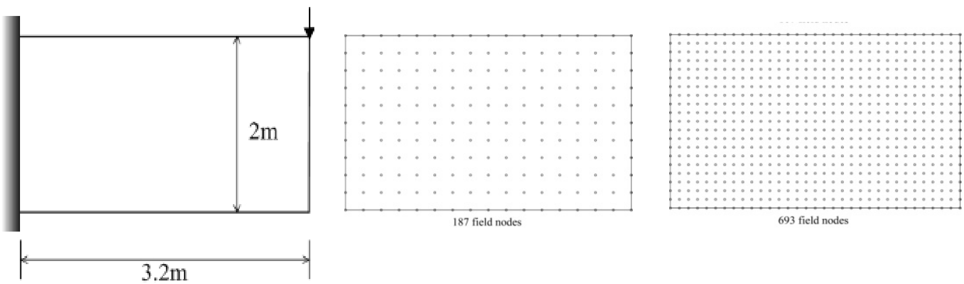

Figure 4: Cantilever beam: (left) geometry, (center) 187 field nodes, (right) 693 field nodes.

The following material properties are assumed: elastic modulus E=1.0 $\times$ 10 N/m^2 and Poisson's ratio υ =0.3. The magnitude of the load is P=1000N. The analysis is carried out using 187 and 693 field nodes with 160 and 640, background cells respectively. The problem is solved under plane stress conditions. The constant value of the removing rate $\aleph = 0.0125$ is used during whole optimization process.

In this test, we first investigate the effect of the number of field nodes on optimum topology. Two different number of field nodes such as 187 and 693 are used in the optimization with the same number of background cell such as 160. The TO process is terminated after 72 iterations. The optimum topologies obtained at 20, 40, 56, 72 iterations are provided in Figure 5.

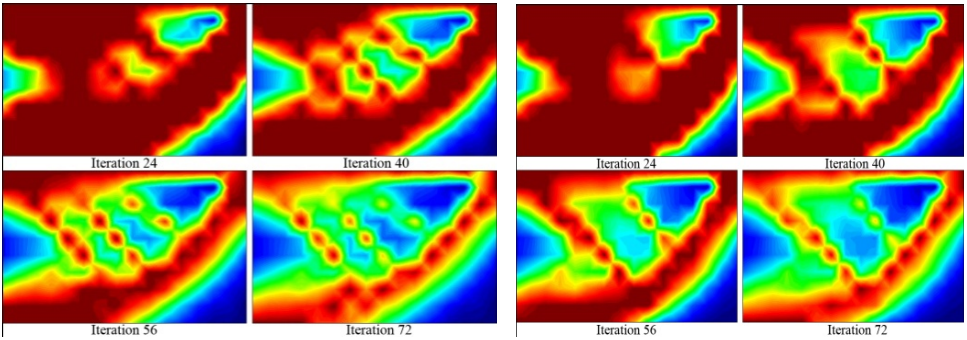

Figure 5: Cantilever beam TO results: (left) 187 field nodes, (right) 693 field nodes.

From numerical results, it is found to be that the number of field nodes can effect on the optimum topology. Specifically larger number of field node can ease some noises in the optimum topologies and produce more clear image with the fixed number of background cell. We try to optimize the same beam with 693 field nodes and 640 background cells and the results are illustrated in Figure 6.

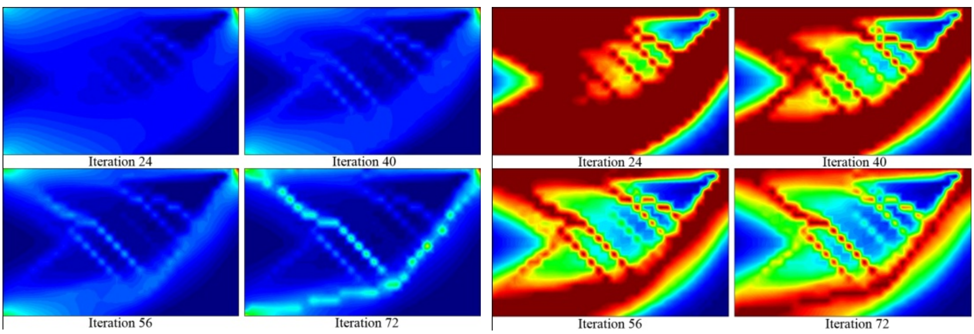

Figure 6: Cantilever beam TO results: (left) stress distributions, (right) optimum topology.

From numerical result, it is found to be that the larger number of background cells can trigger some noise in optimum topology, if we use the same number of field node in TO process.

5.2. Simply supported beam

The effect of the removing rate of hard-kill method on optimum topologies is investigated. The dimensions of the simple beam are L=10.0m and H=5.0m and the thickness of the beam is 1.0 m. The material properties used in this example are elastic modulus E=1.0 × 10 N/m^2 and Poisson's ratio υ =0.3. In this test, 1071 filed nodes with 1000 background cell is used to discretize the entire beam for the meshless analysis and TO. In this example, a range of removing rate from N=0.01 to N=0.04 is investigated. The results of the TO are very sensitive to the removing rate. The smaller values of the removing rate can trigger noises in the optimum topology. The results obtained from the different values of the removing rate are presented in Figure 7 which shows that the final topology can be affected by the removing rate values. However, it should be noted that the noise become serious with the value of removing rate less than 0.002.

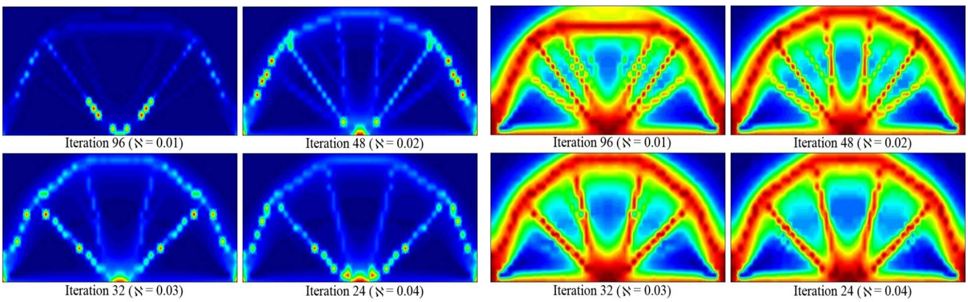

Figure 7: Cantilever beam TO results: (left) stress distribution, (right) optimum topology.

6. Conclusions

The TO is carryed out using meshless method to evlove two-dimensional truss structures. It is found to be that the present technique is found to be simple and effective to evolve the optimum topologies of plane structures such as 2-dimensional truss structures without creating any mesh during optimization process. A set of benchmarks are provided to show the capability of the proposed optimization technique for two dimensional problems.

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