Shape and size optimization of concrete shells

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Abstract

Although concrete shells may adopt any form, it would be interesting to know to what extent changes in their shape may avoid the appearance of bending moments or reduce them. The use of optimisation techniques may be effective in providing alternative geometric forms of shells that improve their mechanical behaviour, complying with the design conditions in an optimum way. In this paper, these techniques were used to find optimum geometrical designs having an aesthetic shape similar to the form initially designed for the structure. As an example, a shell based on Candela's blueprints was optimised under a state of predominant gravitational loads. The results confirm that significant improvements in the structural behaviour of the shell may be achieved with only slight changes in its form.

Keywords: shape, size, optimisation, concrete, shells, design.

1. Introduction

Triangular distribution of stresses through a cross section is uneconomic since the maximum stress occurs on the outer fibres. This is especially true for materials such as concrete, whose resistance to tension is small compared to its resistance to compression. Therefore, the ultimate strength capacity of the cross section is bound by the former resistance.

The structural behaviour of shells, compared to that of other types of structures, is characterised by a higher mechanical efficiency. Concrete shells depend on their configuration, not on their mass, for stability. If appropriate designs are carried out, shells can support high loads and allow covering important spaces using little material and/or thickness. Moreover, shells present an attractive lightness and elegance from an aesthetic point of view, leading some authors to referring them as the "structural elegance" (Ramm, Kemmler and Schwarz [10]).

Although shells can adopt any form, double-curvature shells are, without doubt, those that present the greatest advantages, since it is possible to avoid the appearance of bending moments in them (Candela [5], Ortega and Arias [9]). Their particular behaviour is due to the arch-effect in two planes and, in contrast to the arch contained in only one plane, it allows supporting different load configurations, mainly by means of membrane internal forces, with a very low risk of bending. Moreover, these surfaces have a practically inalterable form and are in equilibrium whatever the form and distribution of the loads. This implies that shell structures, designed to act as membranes, are by themselves optimum structures. Unfortunately, as usually occurs in optimum systems, this high mechanical efficiency induces a structural behaviour, which is extremely sensitive to small changes in certain response parameters. The classic example is the reduction in the buckling load of a shell when slight geometric imperfections appear. Owing to this ambivalent characteristic, the shell has been considered the "prima donna" of structures (Ramm and Wall [12]).

Since the structural behaviour of shells is developed essentially due to their form, it would be interesting to determine if it is possible to find small modifications in its geometry that, without altering its initial aesthetic configuration too much, improve that mechanical behaviour still further, at the same time that the design conditions are complied. It could be attempted, for example, to reach a distribution of stresses in the thickness being the most uniform as possible, which would imply to have shells free of bending or with some acceptable bending values. Improving the structural behaviour of the shells by means of the shape optimisation implies a design process with a high quality, since it helps to reach structures of quasi-perfect behaviour.

The development of optimisation techniques was strongly boosted by the tremendous increase in computational and graphical capacities. These techniques represent an effective means to obtain alternative geometric forms of shells and improve their mechanical behaviour, complying with the design conditions (stress constraints, construction conditions, etc.) in an optimum way (minimum weight, maximum stiffness, minimum stress level, etc.).

In this paper, optimisation techniques were applied to find optimum geometric designs that were close to a preconceived design, i.e. the resulting geometries should have an aesthetic shape similar to the form of the initially designed structure. It is known that slight changes in the form of this type of structures can introduce important improvements in their mechanical behaviour. To achieve this task, different objective functions can be used, such as the strain energy; the weight of the structure; or the tensile stress in both faces of the shell. In this study, the parameters that govern the geometry of the structure and the thickness of the shell were used as variables. The constraints referred to the minimum thickness of the shell, the tensile stresses in concrete, and several parameters of geometric control. An actual hypar (hyperbolic paraboloid) concrete shell was used as an example.

2. Shape optimisation of concrete shells

The design process of the membrane state in a shell structure can be hindered by a series of factors such as the application of concentrated loads, the existence of free edge boundary conditions or the possible incompatibility that could appear between a given form of the shell and its thickness. Since these factors are strongly dependent on the form of the structure, it becomes essential to incorporate, in the design process, shape optimisation techniques that account for them. Therefore, the ultimate objective in shell design would be to find a form that, considering the specific properties of the material, satisfied the following design conditions (Ramm, Kemmler and Schwarz [10]):

- Stresses and displacements are enclosed inside a pre-established interval,
- membrane stress state (or a close approximation of it) is needed,
- boundary conditions are fulfilled for all load configurations,
- buckling instability phenomena are avoided,
- sensitivity of the structural response to possible variations in the geometry is minimised, and
- the shell shape complies with aesthetic criteria.

The aim of the optimal structural design is to obtain a design, a set of values for the *design variables*, which minimizes an *objective function* and complies with the *constraints* that depend on the variables.

The *design variables* of a structure can be properties of the cross-section of the elements (surface areas, thicknesses, inertia moments, etc.); structural geometry parameters; structural topology parameters (element densities in the range from 0 to 1) (Bendsoe and Sigmund [3]); and properties of the material of the structure. The type of optimisation carried out depends on the type of variables being considered. Traditionally, the design of minimum weight structures has been sought, which has led to the fact that the most common *objective function* is the weight of the structure. Nevertheless, the weight is not the determining factor in other applications, and other objective functions are used, such as cost, reliability, stiffness, etc. The *constraints* are the conditions that the design must comply with in order to be regarded as valid.

The optimum design problem can be formulated as follows:

To find the variable vector of design $\mathbf{x}$ which

$$\begin{array}{lll} \text{minimises} & f(\mathbf{x}) & \\ \text{subject to} & h_j(\mathbf{x}) = 0 & j = 1, 2, ..., m_i \\ & g_k(\mathbf{x}) \geq 0 & k = 1, 2, ..., m_d \\ & x_i^L \leq x_i \leq x_i^U & i = 1, 2, ..., n \end{array} \tag{1}$$

where $\mathbf{x}$ is the *n-dimensional* vector of the design variables; $f(\mathbf{x})$ is the objective function; $h_j(\mathbf{x})$ is the jth equality design constraint; $g_k(\mathbf{x})$ is the kth inequality design constraint; m_i is the number of equality constraints; m_d is the number of inequality constraints; n is the number of variables; and x_i^L (x_i^U) is the lower limit (upper limit) of the variable i.

This problem can be solved by several different methods (mathematical programming, genetic algorithms, etc.). The optimisation module in ANSYS [1] was used, which has a conventional first-order method using the first derivatives of the objective function and constraints with respect to the design variables. The module converts the optimisation problem with constraints into an unconstrained problem by adding penalty functions to the objective function. For each iteration, gradient calculations, which employ a steepest descent or conjugate direction method, are performed to determine a search direction. A line search strategy is adopted to minimize the unconstrained optimisation problem.

With respect to shape optimisation, several authors have proposed suitable objective functions (Ramm, Kemmler and Schwarz [10], Ramm and Mehlhorn [11], Bletzinger and Ramm [4], Martí, Tomás and Torrano [8]). As an example, in order to find a state of membrane stresses, the highest principal tensile stress can be used as the objective function. An alternative way is to substitute this condition for a constraint expressing that the appearance of tensions should be avoided in all points of the shell.

If improving the behaviour of the structure in case of instability phenomena is required, the buckling load can be used as the objective function. As the response of shell structures is very sensitive to geometric imperfections, it is recommended to include the latter into the maximisation process of the buckling load. A first approach of the buckling load can be established by analysing the initial stability through an eigenvalue analysis. However, in order to obtain a more realistic value, geometric nonlinear analysis is needed.

With the purpose of minimising displacements in the whole structure, a function called "Volumetric Displacement" (*VD*) may be defined according to the following expression (Robles and Ortega [13])

$$VD = \sum_{i=1}^{n} d_i \times S_i \times Thick_i \tag{2}$$

where d_i is the displacement vector modulus at each point i; S_i is the area of influence at this point; and $Thick_i$ is the average thickness of the structure at the mentioned area. With respect to the maximum displacement technique, the advantage of the *VD* function is to provide a wider view of displacements throughout the structure.

In order to reduce the bending in the shell, it is very appropriate to minimise the strain energy, or equivalently, to maximise the stiffness of the structure

$$S_E = \frac{1}{2}\int \sigma\varepsilon \,\mathrm{d}V \tag{3}$$

where σ are the stresses and ε the strains, both calculated in all the points of the shell. Minimising the strain energy leads to lower stresses and deformations in all points of the shell but, in an implicit way, allows to fulfil the previous objectives simultaneously. In other words, the behaviour of the structure is improved, together with (i) a higher buckling load and (ii) a "relaxed" state of stresses close to the membrane state.

Considering that all the design conditions stated above can easily be satisfied by increasing the weight of the shell, it could be convenient in some cases to limit the weight to a maximum value, or even to introduce it in the optimisation process as an equality constraint.

The inequality constraints $g_k(\mathbf{x})$, commonly used to limit the tensile stresses and the displacements, can also be necessary to limit some parameters of shape, because the value of these parameters varies in the optimisation process. A distortion can occur in the geometry of the structure in such a way that its shape differs too much from the preconceived form, and from the desired aesthetic criteria.

3. Example

The shape optimisation of a concrete shell structure is presented below. It was designed for the entrance of the Universal Oceanographic Park of Valencia, Spain (Figure 1). The roof shell is based on Felix Candela's blueprints. The geometry of the structure is the intersection of three lobes whose mid-surfaces describe the shape of a hypar (Tomás [14]). After analysing the results of the initial design, several optimisation processes under predominant gravitational loads have been carried out.

(a) (b)

Figure 1: Shell structure at the entrance of the Universal Oceanographic Park (Valencia, Spain). (a) Shell under construction. (b) The shell today

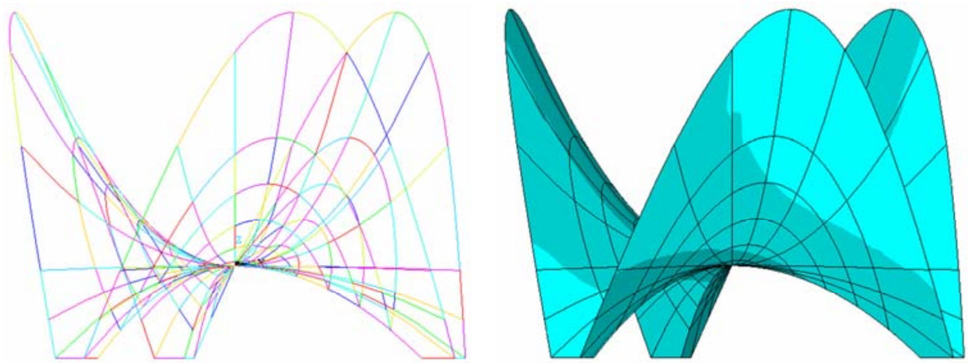
3.1. CAD model

To generate the CAD model, a global system of cylindrical coordinates with origin at the point of intersection of the three paraboloids was defined. The patches defining the mid-surface of the structure were derived from the coordinate system. The ANSYS program of finite elements was used to model the shell.

The following sequence of operations was programmed:

- Obtain a set of points (*keypoints*) contained in the mid-surface,
- link the points by curved lines (*splines*), and
- obtain ruled surfaces (*Coons patches*) from the splines to generate the geometry of the structure.

Overall, there were 1945 keypoints which, linked in sets of six, created 432 splines, which generated 216 Coons patches forming the model (Figure 2).



(a) (b)

Figure 2: CAD model of the structure. (a) Lines. (b) Areas

In addition, the shell has been strengthened by two types of ribs with similar dimensions as used by Domingo, Lázaro and Serna [6]. One of them spreads from the support to the centre of the structure (main ribs). The other one surrounds a small central hole made in the shell to avoid problems when meshing, embracing a band of 200 mm wide (hole rib).

The implementation of the CAD model in ANSYS was carried out in such a way that the form of the structure can be modified by varying the values of some design parameters. The latter referred to (i) geometric control and (ii) thickness of the elements that form the structure.

3.2. Analysis model

Due to the symmetry of the structure, the different analyses were carried out on a sextant of the shell. Prior to **meshing** the surface of the CAD model, the thickness, material, element type and geometric characteristics of the elements were defined. A small circular hole of 100 mm diameter was made at the intersection of the lobes, in order to avoid meshing problems derived from the distortion of the elements generated in the area surrounding the centre, which have very acute angles.

The following steps were implemented in ANSYS: (i) generate the mesh of finite elements; (ii) assign mechanical properties; (iii) state the boundary conditions; and (iv) apply loads on the structure.

The **material** used for the structure was concrete. The mid-surface of the shell was provided with a reinforcement, which was used to account for time-dependent effects of the concrete, since these effects can have a considerable influence when the thickness of the shell is small with respect to the other dimensions. Therefore, the contribution of the reinforcement was not considered in the analysis, excepting the effects of its density. The specific gravity of the material was 25.00 kN/m^3, a value commonly used for reinforced concrete. The mechanical properties were 30 MPa for the characteristic compressive strength of concrete (f_{ck}), 20.00 MPa for the design compressive strength (f_{cd}), 1.35 MPa for the design tensile strength ($f_{ct,d}$), 0.20 for the Poisson's ratio, and 28576 MPa for the secant modulus referred to the concrete age of 28 days.

There are essentially two different ways of formulating the **elements** used in concrete shells, those based on the degenerate solid approach and those based on a shell theory (Hofstetter and Mang [7]). We used an element belonging to the second type, called *Shell93* in ANSYS program. The element had six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the nodal *x*, *y*, and *z*-axes. The deformation shapes were quadratic in both in-plane directions. The element had plasticity, stress stiffening, large deflection, and large strain capabilities. It may have variable thickness, which was assumed to vary smoothly over the area of the element, with the thickness input at the corner nodes. The thickness at the mid-side nodes was taken as the average of the corresponding corner nodes. The normal stress for this element varied linearly through the thickness, while the transverse shear stresses were assumed constant throughout (ANSYS [2]).

Because of the symmetry of geometry and loads, the analysis of a lobe with an angle of 60º was carried out, applying symmetry boundary conditions to the nodes in the symmetry planes, and restricting the translations in the *x*, *y*, and *z* directions to the nodes in the foundation plane.

The applied loads were the weight of the structure and the distributed load of 1 kN/m^2. The action of the wind was not considered because of its slight contribution to the whole load, only 5.87% of the gravitational loads. This percentage is a maximum value obtained by adopting a simplified and safe hypothesis for introducing wind into the analysis model (Tomás [14]).

3.3. Formulation of the optimum design problem of the hypar shell

3.3.1. Objective functions

The objective functions were the following:

- Strain energy of the structure,
- weight of the hypar, and
- highest tensile stress at the nodes of the model.

3.3.2. Design variables

The following design variables whose initial values were proposed in Candela's blueprints were used for the design of the entrance to the Oceanographic Park:

- *K* Constant in the equation of the mid-surface of the hypar. In the optimisation processes, the initial value was 0.14 m^{-1}, being 0.13 m^{-1} and 0.17 m^{-1} the minimum and maximum values respectively.

- β Angle of inclination of the plane that defines the free edge of the hypar with respect to a horizontal line. Its initial value was 75º, allowing for a variation interval between 74º and 75º, since the design is very sensitive to this variable. With this interval, the structure height cannot be lower than 19 m.
- ω Angle between the two master axes of the hypar. Its initial value was 90º (equilateral hypar). The stated lower and upper limits were 84º and 91º respectively.
- e_1 Shell thickness. A minimum initial value of 60 mm for constructive conditions was chosen. In the optimisation processes, thickness was allowed to range from 60 to 80 mm.
- e_2 Hole rib thickness. The initial value was 80 mm, stating as in the previous case, a minimum value of 60 mm.
- e_3 Main ribs thickness. The initial value was 350 mm, with a variation interval between 60 and 400 mm.

3.3.3. Constraints

The maximum stresses were restricted depending on the design strength of the material of the shell:

$$\sigma_t \leq f_{ct,d} \tag{4}$$

$$\sigma_c \leq 0.85 f_{cd} \tag{5}$$

where σ_t is the tensile stress and σ_c the compressive stress. Two shape parameters of the hypar, the height of the highest point of the free edge and the radius or distance from the z-axis to the support were also restricted. This was necessary because the values of these parameters tend to decrease during the optimisation process, distorting the geometry of the structure in such a way that its shape would depart significantly from the hyperbolic paraboloid. Furthermore, its appearance would not match the design criteria. The stated minimum values were 19 m for the height of the free edge and 11.5 m for the radius.

4. Results

In the first stage, the analysis of the outputs of the model proposed by Candela for the Oceanographic Park allowed to obtain outstanding information, such as the value of stresses and displacements at the points of the structure, and its buckling load. In the second stage, several optimisation processes were executed with the purpose of improving the structural behaviour under different load combinations. The different optimisation processes of the initial model were classified into two groups, using the following criteria:

- Depending on the objective function used (strain energy, weight or tensile stress), and
- depending on the minimum thickness allowed (60 or 80 mm).

For each objective function, two optimisation processes were carried out depending on the minimum thickness allowed.

A study of the buckling of the structure was carried out for the initial model with three different thickness values and for the optimum designs, using two types of analysis: (i) initial stability or linear analysis and (ii) nonlinear analysis.

Moreover, it was considered of interest to calculate the final values of two geometric parameters: (i) the height of the free edge of the hypar and (ii) the radius or distance in ground plan from the centre of the structure to one of its supports. The comparison of these parameters in the different processes could help in visualizing and showing the changes that have taken place in the geometry of the initial model. The final values of the geometry variables in the different optimisation processes are shown in Table 1.

Process	t_1	t_2	t_3	K	ω	Radius	Height
		mm		m^{-1}	deg	m	
Initial model	60.0	80.0	350.0	0.140	90.00	13.63	24.39
SE (t_{min} = 60 mm)	61.2	60.0	362.1	0.158	85.42	11.87	19.04
SE (t_{min} = 80 mm)	80.0	95.4	400.0	0.165	84.98	11.54	19.00
W (t_{min} = 60 mm)	60.0	71.9	264.3	0.150	85.87	12.27	19.02
W (t_{min} = 80 mm)	80.0	81.0	333.1	0.141	86.45	12.79	19.05
σ_t (t_{min} = 60 mm)	159.4	80.9	384.6	0.139	86.53	12.91	19.01

SE = strain energy; W = weight; σ_t = tensile stress; t_{min} = minimum thickness

Table 1: Optimisation processes. Final values of variables of geometry

It was observed that the value of the angle ω (angle between the master axes) decreased in all the optimisation runs, implying that the hypar was no longer equilateral. On the other hand, the height of the initial model decreased in all runs tending to the stated minimum value of 19 m. Regarding the thickness of the shell, it can be highlighted that the allowed minimum value was reached when the strain energy and the weight were optimised. However, when the maximum tensile stress was optimised, the thickness of the shell was near 160 mm, indicating the high cost of a form having the membrane behaviour when geometric constraints are used.

The final values of the objective functions are shown in Table 2, together with three additional parameters whose analysis and comparison could be useful: the shell thickness e_1, the maximum compressive stress $\sigma_{c,máx}$ and the maximum vertical displacement $U_{z,máx}$.

From these results, it could be highlighted that in all the optimisation processes, the maximum compressive stresses were below 5 MPa and the maximum tensile stresses were lower than the design tensile strength of the concrete. In addition, the maximum vertical displacement of the structure was lower than 4 mm, which is in agreement with the results obtained by [3] for this type of structures, where vertical displacements are below 10% of the shell thickness.

When tensile stress was the objective function, the value of the weight approximately doubled the values obtained in the other optimisation processes. However, no substantial decrease in the tensile stress was achieved, which confirms the high cost of a form with a membrane behaviour using geometric constraints restrictions, as mentioned previously.

Process	Objective functions					
	SE	W	σ_t	t_1	$\sigma_{c,max}$	$U_{z,max}$
	N·m	kN	MPa	mm	MPa	mm
Initial model	881.77	473.83	2.19	60.0	7.89	8.73
SE (t_{min} = 60 mm)	297.94	330.17	1.33	61.2	4.96	3.90
SE (t_{min} = 80 mm)	300.66	443.28	1.29	80.0	4.22	3.25
W (t_{min} = 60 mm)	318.29	309.54	1.26	60.0	4.89	3.94
W (t_{min} = 80 mm)	367.20	431.37	1.10	80.0	4.27	3.27
σ_t (t_{min} = 60 mm)	514.36	767.87	1.05	159.4	4.75	2.26

SE = strain energy; W = weight; σ_t = tensile stress; t_{min} = minimum thickness

Table 2: Optimisation processes. Final values of objective functions, shell thickness (t_1), maximum compressive stress ($\sigma_{c,max}$) and maximum vertical displacement ($U_{z,max}$)

Finally, Table 3 shows the results of a buckling analysis by using eigenvalue and nonlinear analysis (considering geometric and material nonlinearity). These values correspond to the final designs in each process and for the initial model with a thickness value of 60 mm. The latter value was finally adopted in the construction of the hypar. The values indicate the buckling load factor expressed as a multiple of the weight of the shell.

Process	Thickness	Buckling load factor	
		Linear analysis	Nonlinear analysis
	mm		
Initial model	60.0	8.65	5.20
SE (t_{min} = 60 mm)	61.2	17.37	12.19
SE (t_{min} = 80 mm)	80.0	25.60	16.10
W (t_{min} = 60 mm)	60.0	15.82	11.01
W (t_{min} = 80 mm)	80.0	22.38	14.77
σ_t (t_{min} = 60 mm)	159.4	53.39	22.22

SE = strain energy; W = weight; σ_t = tensile stress; t_{min} = minimum thickness

Table 3: Buckling load factor (expressed as multiple of weight of the shell).

From the stability study we can highlight, on one hand, the high buckling load obtained, product of the huge stiffness of this type of structures. This was also confirmed by the maximum vertical displacement obtained of 8.73 mm in the initial model, and lower than 4 mm in the optimum designs (Table 2). On the other hand, the buckling load was approximately doubled in the optimum designs with respect to the initial models, highlighting that designing this type of concrete shell structures by using optimisation techniques provides an added value.

Lastly, the initial model and a final design are compared in Figure 3. The latter was obtained using strain energy as the objective function with a minimum thickness of 60 mm as the constraint. Both geometries are intersected to provide a better perspective and shown the slight differences between them.

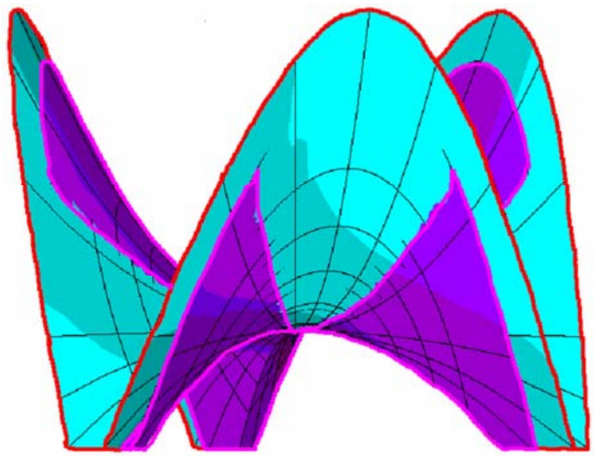


Figure 3: Intersection of initial model (light shaded) and a final design (dark shaded)

5. Conclusions

Traditionally, computers have been used, within the process of design of structures, to analyse the response of a user-defined structure and to check its safety for given applied loads. The use of optimisation techniques in the design process of structures widens the field of use of computers and allows the user to obtain optimum designs for stated design conditions.

In the present paper, we have verified that with slight changes in the shape of a concrete shell, considerable improvements are obtained in its mechanical behaviour. In particular, we can underline three interesting aspects:

- If necessary, the membrane state of compressive stresses may be achieved.
- Deformations in the optimum shells decrease considerably to values lower than half those of the initial design.
- The behaviour of the shell against instability improves significantly, since the buckling load is approximately double the initial value.

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