Multi-objective optimization of membrane structures using Pareto-based Genetic Algorithm

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Abstract

A multi-objective optimization method is presented for membrane structures to answer the question "which shape is optimal" directly. Multi-objective Genetic Algorithm is applied to solve the Pareto set, which includes all solutions with different weights and can be provided designers to select with regard to their specific objectives and liking. And the applications on conical membrane structures are carried out. The results demonstrate that the proposed method is effective and accurate.

Keywords: membrane structure; multi-objective optimization; Pareto solutions; multi-objective Genetic Algorithm

1. Introduction

Membrane structure is a kind of tension structure, the performance of which is highly dependent on its shape and prestress. At present, the methods of form finding and load analysis for membrane structures have been well developed. However, it is still a difficult work to select optimal shapes efficiently from millions of available ones. To get better performance of structure, a number of shapes with different combinations of parameters, have to be tried and modified for many times relying much on the experience of designers. Such an empirical procedure is feasible for small size structures however inadequate to medium or large size structures. It not only leads to the unnecessarily cost in time and finance but also failures of structures. For instance, at least three membrane structures with medium to large size collapse in Brazil during the year of 2005 because of lack of conscious analysis(Pauleti *et al* [1]). Optimization of membrane structures is an efficient method to deal with this problem, which is expected to answer the question "which shape is optimal" directly.

In the recent past, the optimization problem of membrane structures has been studied by some researchers. Sindel and Nour_Baranger [2] optimized membrane structures taking maximum stiffness as objective and choosing structure parameters such as pretension, height-to-span ratio as optimization variables. Maximum stiffness is described by the sum of node displacements. The Conjugate Gradient Method is adopted to solve this problem. Uetani and Fujii [3] proposed an objective to get a specified shape with respect to stress ratio in two directions. Derivative is calculated to solve this problem. In Qian *et al*. [4], the membrane structure is optimized by minimizing strain energy for maximum stiffness. In conclusion, the above literature survey indicates that most of researches focused on single objective optimization of structures. But the real-world design of a membrane structure is actually governed by multiple requirements, which are often conflicting and should be treated simultaneously to obtain a desired compromise. San *et al*. [5] considered the multiple optimization objectives of membrane structure and used the weighting method to change the several objectives into a single one; the big trouble of this method is how to determine the weighting coefficients, which affects the results of optimization severely. Sometimes it happens that the decision maker is not able to assign priorities to the objectives and optimizations have to be carried out for many times with different weighting factors.

Therefore, in this paper a multi-objective optimization method based on Pareto Multi-objective Genetic Algorithm is proposed for design of membrane structures. Help is supposed to be provided to the designers by presenting set of Pareto optimal solutions. They can pick up that solution out of this set with regard to the specific objectives and liking.

2. Multi-objective Genetic Algorithm

As discussed above, optimization of membrane structures are generally multi-objective, which can be formulated as:

$$\begin{aligned} &\min[f_1(X), f_2(X), \cdots f_n(X)]^T \\ &s.t. g_j(X) \le 0 (j = 1,2,\cdots J) \\ &h_k(X) = 0 (k = 1,2,\cdots K) \end{aligned} \tag{1}$$

Where, X is optimization variable vector; f_i is *ith* objective function; $g_j(X)(j = 1,2,\cdots J)$, $h_k(X)(k = 1,2,\cdots K)$ are constraint functions.

Much different form single-objective problem, it is difficult to optimize all objective functions simultaneously when they are in trade-off relationship. To be more clear, the Pareto optimal solution is introduced here.

For multi-objective optimization problems there is not a single solution, but a set of non-dominated solutions which is called Pareto set, which is sketched in Figure 1. A solution belongs to Pareto set if and only if there does not exist another solution that is no worse in all objectives and is strictly better in at least one objective (Deb [6]). In the present study, the global Pareto set is focused on, in which no assumption about the relative importance of different objective criteria is made a priori. To obtain the whole Pareto set directly, Genetic Algorithms (GAs) are considered one of the most powerful methods.

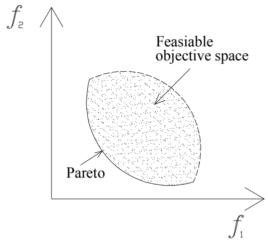


Figure 1: A two objective search space and the corresponding Pareto set

GAs are stochastic search techniques based on the mechanism of natural selection and natural genetics, and do not need a differentiable function to solve problems. Different from traditional optimization algorithms, GAs focus on a population of solutions instead of a single one, which is the main reason that GAs are efficient to solve the whole Pareto set.

The process of GA is initialized with a randomly generated group of trial solutions, i.e., the initial population. For each trial solution in a population, the relevant objective functions are evaluated and a fitness value is obtained to reflect its relative merit standing in the current population. Based on these fitness values, GAs perform a series of operations of selection, crossover, and mutation to create a new offspring generation. The GA process continues until prescribed stopping criteria are satisfied. In previous researches, Single-objective Genetic Algorithms (SGAs) have been introduce a lot (San [5], Gen [7]). Accordingly, this paper focuses on the characteristics of Multi-objective Genetic Algorithms (MGAs) different from SGAs, which include two aspects: the evaluation of fitness and the requirement of diversity.

1) Evaluation of fitness in MGA

In this study the evaluation of fitness is based on non-domination ranking which depend on the locality of solutions in objective function space (Carlos Fonseca and Peter Flemming, 1993). In this method, a design is denoted rank-1 if it is Pareto-optimal in the generation. For other designs, if the number of its domination solutions is *p*, its rank is defined as: rank=1+p (as Figure 2). It is obvious that the designs with smaller rank numbers have higher fitness. The fitness function is obtained by interpolation by the rank number from 0 to 1.

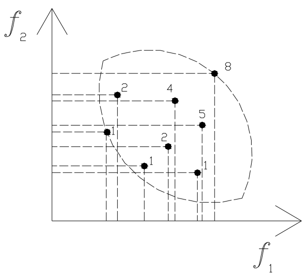


Figure 2: Diagrammatic sketch of ranking for a two-objective optimization

2) Maintenance of diversity in MGA

MGAs have higher requirement in maintaining diversity of individuals than SGAs because of the aim to guarantee uniform distribution of Pareto solutions. An efficient approach is the use of niche sharing technique (Goldberg, 1989), where assessment of each individual solution is also depended on its immediate neighbors. Fitness sharing consists in the reduction of the fitness of an individual proportionally to the number of nearby individuals. The shared fitness f_i' of the *i-th* individual is given by

$$f_i' = \frac{f_i}{m_i} \tag{1}$$

where f_i is the original fitness of the *i-th* individual and m_i is the niche count:

$$m_i = \sum_{j=1}^{N} sh(d_{ij}) \tag{2}$$

m_i is the niche count that takes into account the whole population in relation to the *i-th* individual, degrading the fitness of this individual according to the nearness of the others. Variables N and d_{ij} are, respectively, the population size and the distance between individuals i and j. The function that quantifies this proximity is the following:

$$sh(d_{ij}) = \begin{cases} 1 - d_{ij} / \sigma_{share}, if \quad d_{ij} < \sigma_{share} \\ 0, otherwise \end{cases} \tag{3}$$

Where σ_{share} is niche radius, which is generally defined by empirical formulas or trial methods.

3. Examples

3.1 Structure models

In this section, the proposed multi-objective method is applied to optimize conical membrane structures, which have simple shapes and have been used a lot . However, it is still not clear which shapes are optimal. For example, the “Centro Cultural CBI” in Brasil ((Pauleti *et al* [1]), which is a kind of conical type collapse because of unreasonable design.

Here consider a single conical membrane structure supported by a frame. The boundary shape and the layout of cables are as illustrated in Figure 3, in which dimensions of plan is denoted as $a \times a$ and height of conical is denoted as H. In addition, conical structures are generally combined to form more complex shapes, such as double conical structures(shown in Figure 4). The structure has a rectangle plan with the dimensions of $2a \times a$ and is symmetrical with respect to x and y axis. The mast rings are located eccentrically. The eccentric distance is denoted as e, where $e = a/2 - b$. If $e > 0$, the mast ring is close to the out boundary; if $e < 0$, the mast ring is close to the center point o.

The optimal solutions of single conical and double conical membrane structure both will be obtained. The material properties are shown in Table1

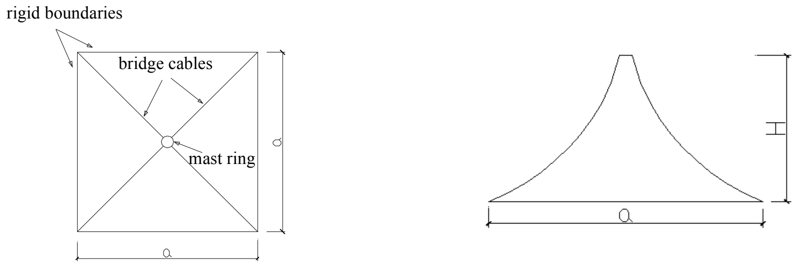


Figure 3: Sketch of single conical membrane structures

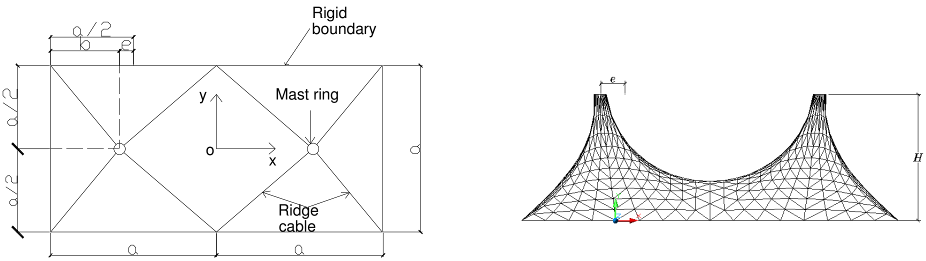


Fig.4 Sketch of double conical membrane structures

Table1 Material properties

Membrane				Cable
Extensional Rigidity (Ext,Eyt)	Shear rigidity (Gxyt)	Poisson's Ratio (μ)	Thickness (t)	Tensile rigidity (EA)
1200.0 kN/m	120.0 kN/m	0.2	1mm	6.28×10^4kN

3.2. Optimization model

3.2.1. ***Optimization Variables***

In engineering applications, optimization variables are generally the spatial control node positions of the design model and prestress which define shapes of membrane structures. For clear expression, they are defined as dimensionless parameters. The spatial control node positions of conical membrane structures are represented as height-to-span ratio:

$$\lambda = \frac{H}{L} \tag{4}$$

Where, H and $L = \sqrt{2}a$ are height and span of structures, respectively.
and the location of central masts, which is for double conical:

$$\delta = \frac{e}{0.5 \times a} \tag{5}$$

Distribution of prestress is described by two functions, which respectively express the prestress distribution of membrane surface and the ratio of cable pretension to membrane stress. As most membrane structures are designed as uniform stress structures, i.e. the prestress of membrane and pretension of cables are constant, the distribution of prestress can be described only by the prestress density ratio of cable to membrane. The prestress density of cable is expressed as

$$C_p = T/l \tag{6}$$

Where, T and l are respectively pretension and length of cables.
The prestress density of membrane is denoted as

$$M_p = ST = \sigma * t \tag{7}$$

Where, σ and t are respectively prestress and thickness of membrane; M_p represents the pretension per unite width.

Consequently, the prestress density ratio of membrane structures is obtained as following

$$\gamma = \frac{C_p}{M_p} \tag{8}$$

3.2.2. ***Optimization Objectives***

Optimization objectives play key roles in an optimization problem, which represent the evaluation criteria of a structure and affect the optimization results directly. In this example, shapes of membrane structures are discussed to be evaluated in three aspects, deformation, stress as well as reaction, and three corresponding objective functions are proposed to describe these performances.

1）Objective1: Maximization of stiffness

Different from traditional structures, deformation is main response of membrane structures against external loads. The strain induced in membrane is usually larger than that in traditional structure elements by several orders of magnitude. Consequently, the designing of membrane structures is usually oriented by stiffness rather than strength. A lack of stiffness will result in a lot of problems such as wrinkling, fluttering and water accumulation, which are apt to decrease the safety of structures dramatically. Therefore, large stiffness is always an important objective of shape design of membrane structures.

Here, the objective function for stiffness is represented by strain energy, which is defined as the energy stored in structure via elastic deformation. Maximization of stiffness is realized by minimizing strain energy, which is written as

$$\min f_1 = U_{Mem} + U_{Cab} \quad (9)$$

Where, U_{Mem} and U_{cab} are respectively the strain energy induced in membrane and cables by applied loads.

2) Objective2: Maximum uniformity of stress under loads

Stress is also important in describing structural performance, including its magnitude and distribution under external loads. However, magnitude of stress is not supposed to be considered in the process of shape optimization because the maximum stress is mainly controlled by the level of prestress instead of the shape for membrane structures. On the other hand, the distribution of stress mainly depends on the shape of structures. And it is generally desired to be as uniform as possible in order to ensure materials strength utilized sufficiently and avoid tearing caused by stress concentration (localized high stresses). Therefore, maximum uniformity of stress under loads is put forward as an objective in this paper. And stress fluctuation coefficient is proposed to evaluate the uniformity degree of stress. The maximum uniformity of stress means minimum fluctuation coefficient:

$$\min f_2 = D(\sigma) / E(\sigma) \quad (10)$$

Where $$E(\sigma) = \frac{1}{m}\sum_{i=1}^{m}\sigma_i \qquad D(\sigma) = \sqrt{\frac{\sum_{i=1}^{m}(\sigma_i - E(\sigma))^2}{m-1}}$$

in which σ_i is maximum principle stress of the *i*th membrane element, *m* is the total number of membrane elements.

3) Objective3**:** Minimization of reactions under load

Conical membrane structures are usually anchored in foundations or other structures. It is obvious that large reactions will increase the difficulty and cost of construction. Different from traditional structures, the reaction of each support is not only related to the spans of each structure unit but also heavily affected by pretension of cables. That is to say, reaction can be modified by changing the pretension of cables even in the case where the spans are specified. Thus it is practical to select minimization of reaction as an objective. Here the maximum value of all reaction is given as objective function:

$$\min f_3 = R_{\max} = (\sqrt{R_{ix}^2 + R_{iy}^2 + R_{iz}^2})_{\max} \qquad (11)$$

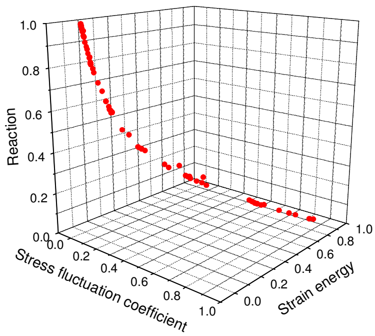
Where, R_{ix}, R_{iy}, R_{iz} are reactions at the *ith* support in *x,y,z* directon, respectively.

3.3. Optimization Results

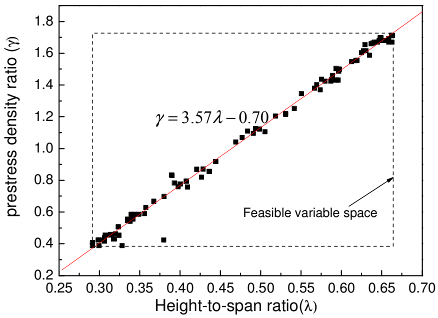
First, a single conical structure with respect to $a = 15m$ and $ST = 2.0kN/m$ is optimized and the Pareto set is obtained, which presents a clear tradeoff among the three objective functions with all kinds of weights (Figure 5a). The optimization variables of Pareto solutions are also plotted in Figure5b and the optimal relationship between λ and γ is derived

$$\gamma = 3.57\lambda - 0.70 \quad (0.30 \le \lambda \le 0.68) \qquad (12)$$

The above formula can guide designers to obtain Pareto solutions. And the final structure which meets the specified demand of design can be easily and effectively selected from Pareto set.



a) Pareto set



b) the corresponding design parameters of Pareto set

Figure5 Multi-objective optimization results conical membrane structure ($a = 15m$, $ST = 2.0kN/m$)

Subsequently, the single conical membrane structures with the various span of L=21m and L=25.4m, and various prestress of membrane of $ST = 2.5kN/m$ are respectively optimized in the proposed method. The similar results are obtained. It is demonstrated that the optimization solution is independent on the span and stress level.

In order to demonstrate the advantages of the proposed multi-objective optimization, the solutions of the single-objective optimization that respectively minimize strain energy, stress fluctuation coefficient and reaction, are indicated for comparison, denoted as SO1, SO2 and SO3, which of course are included in Pareto set. The objective functions of SO1, SO2 and SO3 and other Pareto solutions are shown in Figure6. Apparently, when the structure is optimized only considering one index, other performance indexes are probably sacrificed. For instance, the SO1 obtain the minimum strain energy, however results in too large reaction. Whereas, the shapes with good tradeoff, such as the shapes with respect to $0.35 \le \lambda \le 0.45$, all objective functions of which are at a good level, are totally omitted, which are usually more acceptable for practical design. In contrast, multi-objective optimization enables designers to select the final design by actively compromising all objectives in a preferred manner.

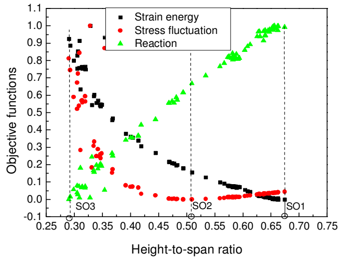


Figure 6 Objective functions of Pareto solutions

In the same way, the optimal solutions of double conical membrane structures are obtained.

The optimal δ : $0.0 \le \delta \le 0.33$

The optimal λ: $(0.24 + 0.7\delta) < \lambda < (0.47 + 1.4\delta)$

The optimal γ: $\gamma = 3.57\lambda - 0.70$

The optimization result is in good agreement with mechanical characteristics of double conical membrane structures. It is observed that a double conical membrane structure actually consists of two cones and a saddle which is located between two cones. With the fixed height and prestress, the saddle has larger curvature than conical structures. The saddle covers larger space with respect to $\delta > 0$ than $\delta < 0$. Therefore the structures with $\delta > 0$ is optimal. In addition, because the conical parts are vulnerable sections, the optimal relationship of the double conical membrane structure is oriented by the behavior of conical structure. Accordingly, the optimal relationship between height-to-span ratio and prestress density ratio is similar with the single conical structures (Eq. (12)). Similarly to example of single conical membrane structures, this optimization also shows a good tradeoff among three objectives.

4. Conclusions

A multi-objective optimization method is presented for membrane structures to answer the question "which shape is optimal" directly. Multi-objective Genetic Algorithm is applied to solve the Pareto set, which includes all solutions with different weights and can be provided designers to select with regard to their specific objectives and liking. And the applications on conical membrane structures are carried out. The results demonstrate that the proposed method is effective and accurate.

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