Nonlinear analysis of carrying capacity of reinforced concrete smooth and ribbed shells under action of concentrated load

Vladimir V. SHUGAEV

Professor, Dr. Sc. (Eng), NIIZHB, 2nd Institutskaja str.,6, 109428 Moscow, Russia
Fax: 499-174-74-50, E-mail: moo-pk@yandex.ru

Abstract

Reinforced concrete shells for industrial and civil buildings roofings should carry considerable value of concentrated loads of suspended transport, technological equipment, suspended ceiling and so on. In this paper engineering methods to solve the problems of analysis of bearing capacity dealing with deformed state of reinforced concrete smooth and ribbed shallow shells of positive Gaussian curvature under action of concentrated load are considered. The solution of problems is executed on the basis of a kinematic method of limit equilibrium [1] in nonlinear statement with some possible schemes of failure found experimentally [2]. Round in the plan reinforced concrete shallow shells of positive curvature under the action of the concentrated load applied in the top, fail with formation of complete radial or local failure schemes. In the complete scheme of destruction radial cracks locate at full height of the shell, including a basic ring in which the reinforcement amounts to ultimate tensile strength. Experimental investigation carried out on models and full scale structures have shown that the local failure schemes peculiar to more thin and shallow shells. For this shells the outline in the plan was of no importance if the zone of failure not border upon the contour. The zone of failure is surrounded with the ring crack formed in result of eccentric compression of sections, perpendicular to radial ones. In ring direction near to edge of a failure zone in ultimate state the reinforcement of the shell amounts to ultimate tensile strength. Bearing capacity of ribbed shells considerably grows in comparison with smooth shells. The concentrated load, as a rule, is applied in the places of crossing of longitudinal and transversal ribs of a shell (Fig. 1).

Keywords: carrying capacity, smooth and ribbed shells, concentrated load, nonlinear analysis, method of limit equilibrium, local failure schemes, reinforced concrete, ultimate load, deformed state, plastic yield hinges, model studies, long-time load.

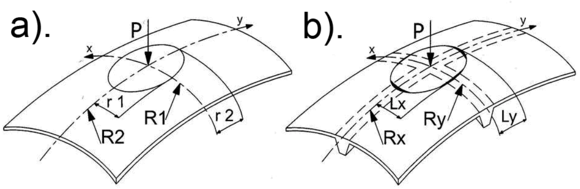

Figure 1: General view of failure zone of shallow shell under action of concentrated load
a) smooth shell; b) ribbed shell.

1. The smooth shallow shells of positive Gaussian curvature

Let's consider for example the plastic deformation and carrying capacity analysis of reinforced concrete spherical shells under action of concentrated load in nonlinear statement. Experimental investigation carried out on models and full scale structures have shown that under concentrated load local failure of the conic form with peak in the place at application of this load is observed.
In this shells the failure zone is limited by a ring cracks with the formation of plastic yield hinges, bear eccentric compression.
For smooth spherical shell with rise f_1 and radius of curvature R_1 (Fig. 1,a) we can find the radius of failure zone using formula:

$$r_1 = 1.1\ \sqrt{R_1\delta} + \mathrm{D} \quad , \tag{1}$$

where: *D* - diameter of the punch for applying of concentrated load.

Let accept for calculation convenience that a surface of the shell is a paraboloid of revolution with thickness δ and rise f_1, described by equation:

$$z = \eta\ (x^2 + y^2), \text{ where } \eta = r_1 / f_1^2$$

In process of deforming of rigid-plastic shell it is possible two principle stage, if we consider this process as succession of limit stage (Fig. 2) [3].

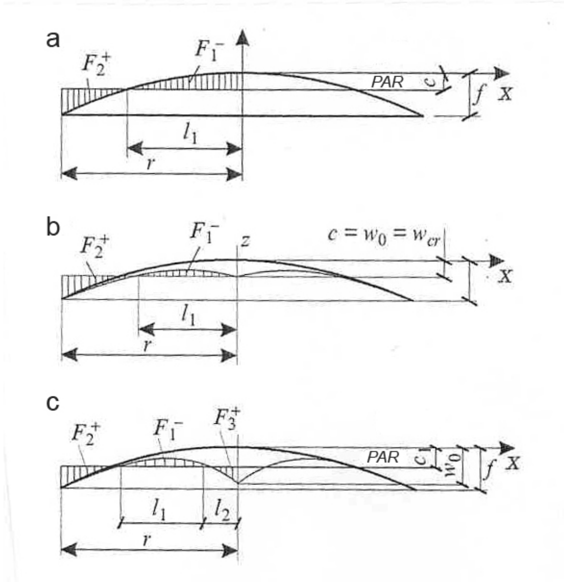

Figure 2: Stages of deforming of shallow shell under action of concentrated load.

The first stage lies in interval from beginning of deforming to a moment when central part of the shell will touch with PAR (plane of axes of rotation) and is characterized by inequality:

$$0 < w_0' t \leq C,$$

here $w_0' t$ - a deflection of the shell; C - the applicata of the PAR (Fig. 2,a).

At the first stage of the deforming the value of limit load may be determined by formula:

$$P_1 = 2\pi(m + n\,t_c) + 2\pi q_s f_1\ [1 + k_1^2 (2k_1 - 3)/(k_1 + 1)^2 -$$
$$- 3w^* (k_1^2 - 2k_1 + k + 1)/(2k + 2)]/3, \quad (2)$$

where $w^* = w_0' t / f_1$

In a limit state with $w^* = 0$, i.e. for ideal rigid-plastic scheme:

$$P_{lim} = 2\pi M + 2\pi q_s f_1 [1 + k_1^2 (2k_1 - 3)/(k + 1)^2]/3, \quad (3)$$

where $M = m + n f_1 [1 - k_1^2 / (k + 1)^2]$.
In (2, 3): $k = R_b(\delta - \acute{\alpha}) + q'_s / q_s$; $k_1 = 1 + n / q_s$;
n and m - linear values of normal forces and bending moment in eccentrically compressed section of the shell in the circular plastic hinge; q_s - linear force in reinforcement of tensile zone is equal $A_s R_s$; A_s – reinforcing steel area and R_s – design strength of tensile reinforcement; q'_s – linear force in reinforcement of compression zone is equal 0,5 $A_s R'_s$; R'_s – design strength of compression reinforcement; R_b – prism strength of concrete; t_c – distance from circular plastic hinge (base of cone) to PAR (the plane of axis of rotation); a' – distance from lower surface of the shell to the reinforcement net.

Considering (2) we can see that when a local scheme of failure is used, increase of deflection involves decrease of value of load P, as in the case of failure of the shell along all its surface, and that means instability of equilibrium.
A load P_{lim} (3) may be named "Upper critical limit load".
Let's find the value of the load P_1', when its mean part will touch with PAR.
Position of PAR in determined by applicata C:

$$C = w_0' t = f_1 k_1 / (k + 1), \ \ and \ \ w_{cr}^* = k_1 / (k + 1) \tag{4}$$

Substituting a value of w_{cr}^* to (3) we shall have:

$$P_1' = 2 \pi M' + 2 \pi q_s f_1 [1 + (k_1^3 - 3 k_1 k - 3 k_1) / (2 + 2k)^2] / 3, \tag{5}$$

where $M' = m + n f_1 [1 - k_1 / (k + 1)]$.

If deflection of the centre of shell $w_0' t > C$ then PAR crosses twice a section of the shell, and second state of deforming begins (Fig. 2,a).

For this state we shall obtain following equation for estimation the value of limit load P_{II}:

$$P_{II} = 2\pi (m + n t_c) + 2\pi q_s [\lambda (k + 1) + \gamma r] / r_1 \tag{6}$$

Here: $\lambda = l_1 (C_1 - a r_1) - \eta (l_1 - l_2)^3 / 3 + a (l_1 + l_2)^2 / 2 + l_2 (\eta l_2 / 3 - a / 2);$

$$\gamma = \eta r^2 / 3 + a r_1 / 2 - C_1; \qquad a = (C_1 - \eta l_2^2) / (r_1 - l_2), \tag{7}$$

where: $l_1 = r_1 k_1 / (k + 1).$

According the deflection $w_0' t$ we shall obtain l_2 and C_1 from equations:

$$l_2 = w_0' t / (2 r_1 \eta) - l_1 / 2; \qquad C_1 = \eta l_2^2 + w_0' t (r_1 - l_2) / r_1 \tag{8}$$

The load P_1'(5) is recommended as designing for spherical shells. The formula (6) for P_{II} are used for calculation of carrying capacity of the shell of positive Gaussian curvature with different main radii of curvature R_1 and R_2 (Fig. 3).

Originally, for smaller radius of curvature (for example, R_1) calculation of bearing capacity of shell P_1' under the formula (5) is conducted. Basic unknowns during calculation of M are the meanings of limiting values of bending moment m and normal forces n on border of a failure zone.
Meanings n determine from a condition of balance of internal forces, thus accept $\xi = \xi_R$. (Here ξ - relative height of a compressed zone of concrete in section, and ξ_R is limiting meaning, at which the destruction of a compressed zone of concrete and tensile reinforcement can come simultaneously). Bending moment m find rather middle axis of section.

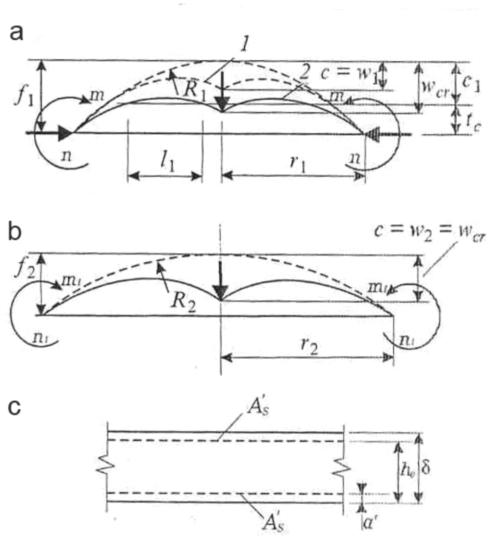

Figure 3: Stages of deforming of shallow shell of positive Gaussian curvature under action of concentrated load: a, b – in directions of radius R_1 and R_2; c – reinforcement in cross section.

The angle of rotation of edge of shell in result of a deflection of centre of shell on value $w_0't = C$ at hinged fastening of a contour would be $\varphi = C / r$. Elastic fixing of edge causes the angle of rotation $\varphi' = \varphi / 2$ [4]. In this case on a contour of failure zone there is the moment $M = i\ \varphi'$, where i - moment from unit angle of rotation of edge of shell.

We shall find the "elastic" characteristics of edge of a shell:

$$i = \beta / (2\alpha^3); \qquad \alpha = 1.306 / (R_1\delta)^{0.5}; \qquad \beta = 0.8\ E_{bl} / R_1^2. \tag{9}$$

According to the normative documents accept usually

$$E_{bl} = 0.75\ E_b\ \varphi_{b1} / \varphi_{b2}, \tag{10}$$

where E_b - initial module of deformation of concrete; φ_{b1} and φ_{b2} - factors, taking into account increase of deformations of structure due to short-term and long-term creep of concrete. Factor 0,75 takes into account heterogeneity of concrete and probable imperfections of thickness of structure. As usual $\varphi_{b1} = 0.85$; $\varphi_{b2} = 2$.

By substituting in expression for meaning C from (4), we shall receive:

$$M = i\ f_1\ k_1 / [2r_1\ (k + 1)] \tag{11}$$

At correctly chosen meaning of height of a compressed zone in considered eccentrically compressed section with small eccentricities the meaning of moments m and M from (11) should coincide. Their coincidence is searched by the step-by-step method using variation of meanings $\xi \geq \xi_R$. Found meanings " $m = M$ " and "n" substitute in (5) for determination of value P_1'.

We similarly find from (5) value P_1' for the shell with radius of curvature of other direction R_2 and rise f_2 (Fig. 3,b). If it will appear, that the deflection of the shell with radius R_2 will be larger than one of the shell with radius R_1 $(w_2 > w_1)$, as a critical deflection of all system is accepted maximum of calculated deflections *(w_2 = w_{cr})*. In this case the limiting deformed state of the shell with radius R_1 is responsed to a stage, resulted on Fig. 3,*a*, when the centre of an shell is located below PAR and it twice crosses the cross section. As it was shown above, in this case, the value of limit load P_{II} should be determined under the formula (6). The same analysis but for a shell with radius of curvature R_2 it is necessary to carry out in case will appear, that the deflection of an shell with radius R_1 will be greater. Receiving the values of limit loads for spherical shells with radii of curvature R_1 and R_2, the value of limit load P for the considered shell, we shall find as their half-sum.

As an example we shall choose smooth reinforced concrete shell (Fig.1,a and 3) and we shall nominate the geometrical and strength-producting properties of the basic variant of the shell structure to value of carrying capacity of which we shall hereinafter compare results of multi-alternative analyses when changing parameters of curvature of surface [6]. For the basic variant radii of curvature of a structure R_1 and R_2 have accepted of 23.4 m and 46.8 m respectively (R_1:R_2 = 1:2) and thickness of shell δ = 6 cm. The shell is reinforced with two layers of welded fabric of wire of ∅4 mm, located with space of 10 cm in both directions (A_s = A_s'= 0.0126 cm^2/cm). The fabrics are located equally spaced 1.5 cm from the top and bottom surface of the shell (a′=1.5 cm, h_o = 4.5 cm Fig. 3,c). The strength of the reinforcement was R_s = 365 MPa. The shell is carried out of concrete with the prism strength of R_b = 14.5 MPa (class of concrete B25 according to Russian Codes).

For the basic variant the analysis has shown the carrying capacity of P = 72.05 kN. The values of a failure zone are given by the dimensions of half-axes of elliptic failure scheme (Fig.1,a) r_1 = 137.84 cm and r_2 = 191.83 cm.

Initial rises of part of a shell in zone of failure (Fig 3,a,b) have been of f_1 = 4.813 cm and f_2 = 4.683 cm. The deflection of point of load application to the moment of failure has amounted to w_{cr} = 4.597 cm, that confirms essential deformation of a surface of shell and necessity of taking into account of a deformed state.

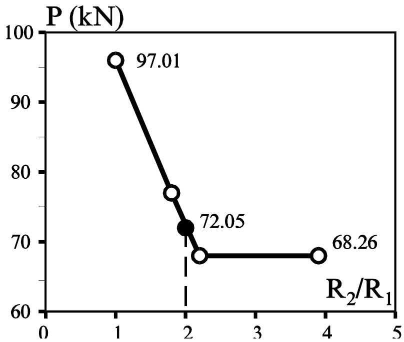

Figure 4: Relationship of carrying capacity of smooth shallow shell on the ratio of radii of curvature.

The diagram of relationship of value of ultimate load P on the ratio of radii of curvature R_2/R_1 (R_1 = const) are shown in Fig. 4, where it can be noted that with increase of R_2/R_1 the carrying capacity of shell decreases almost linearly up to R_2/R_1=2.2, and then it does not change practically. It is connected with that for a shell with R_2=R_1×2.2 in limit state the deflection w_{cr} is found equal to the rise of shell f_2 (Fig. 4). The further increase of curvature radius value R_2 only approaches the behavior of such shell to cylindrical one with radius of curvature R = R_1. It is established by the analyses that the carrying capacity of such cylindrical shell has been of P=68.15 kN, while the shell of positive Gaussian curvature with R_2/R_1=4 had the carrying capacity of P=68.26 kN (Fig.4).

2. The ribbed shallow shells of positive Gaussian curvature

In reinforced concrete ribbed shell of Positive Gaussian curvature concentrated load is applied in the place of crossing of longitudinal and transversal ribs of a shell (Fig.1,b). In the limit state the local failure of the conic form with peak in a place of the application of concentrated load is observed [7]. The failure zone is limited by a ring crack crossing the ribs with formation in them of plastic yield hinges. The sections of ribs in these plastic hinges bear eccentric compression, thus compressed zone of ribs, as a rule, is crushed earlier, than yield in the reinforcement of ribs is reached. In place of the location of concentrated load the plastic hinge will be also formed, however the bending moment plays a more substantial role here and first the lower reinforcement of ribs reaches the yield.

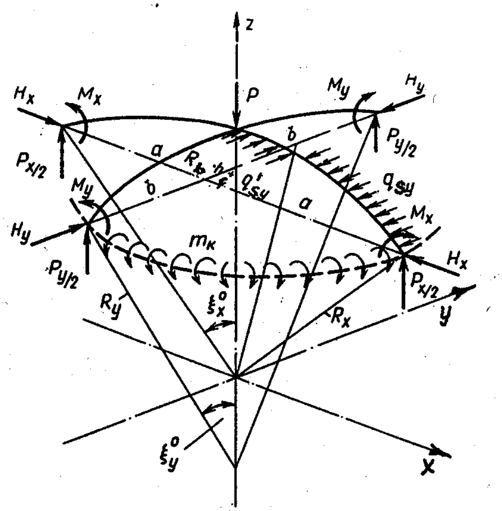

Figure 5: The scheme of forces interaction in the failure zone under concentrated load.

In ribbed shell the main part of the concentrated load is perceived by ribs together with a part of shell plate. The ultimate load can be found from equality of work of external and internal forces on the appropriate displacements in the failure zone. As an example the formula for definition of ultimate value of concentrated load applied to ribbed shell is given. The scheme of forces interaction in the zone of failure under action of concentrated load are shown in the Fig. 5. Location of plastic hinges are determined by angular coordinates ξ_y^0 and ξ_x^0. If one denotes distances from the points of force application to the plastic hinges by l_y and l_x, then angular coordinates in radians can be found from expressions:

$$\xi_y^0 = l_y / R_y\,; \quad \xi_x^0 = l_x / R_x, \tag{12}$$

$$l_y = 1.71\,(4\,I_{ny}\,R_y^2 / \delta_{nx})^{1/2}\,; \quad l_x = 1.71\,(4\,I_{nx}\,R_x^2 / \delta_{ny})^{1/2}\,, \tag{13}$$

where I_{ny} and I_{nx} – inertia moments per unit with the shell cross section. Crossing respectively by planes ZOX and ZOY; δ_{nx} and δ_{ny} – transformed shell thickness in sections, crossing accordingly by planes ZOY and ZOX. When determining the ultimate load value P the strength of two mutually perpendicular section along ribs is considered (Fig. 1,b). It was anticipated in this case, that the rib, being in Y direction, together with a part of shell plate, adjoining it, would perceive a part of total load P_y, while the rib of another direction – P_x [9].

$$P_y = 2\,[2\,R_x^2\,q_{sy}\,(\xi_x^0 - \sin\xi_x^0) + N_y\,R_y\,(1/\cos\xi_y^0 - 1) + R_s\,A_{sy}\,z_{sy} + M_y + m_k\,Y\,] / (R_y\,tg\xi_y^0) \tag{14}$$

Here: R_x , R_y – radii of shell curvature up to middle surface of the shell plate; q_{sy} linear force, perceiving the reinforcing steel located in the tensile zone of the shell plate; A_{sy} – bottom reinforcement of the rib in the sections under concentrated load; R_s – its design strength; z_{sy} – distance from rib reinforcement to the middle of shell plate; M_y – ultimate moment perceiving by the rib in the ring hinge under eccentric compression; m_k – value of the linear bending moment in the ring plastic hinge of shell plate; *X, Y* parameters, taking into account the for and sizes of the ring hinges.

Ultimate load part P_x is defined similarly to (14) with substitution of indices indicating the direction of forces.

Summary ultimate load: $$P = P_x + P_y \quad (15)$$

The deflection value directly does not enter into formulas for P_x and P_y, however all geometrical parameters, and also the values of *M* and *N* corresponding with deformed state of shells in a stage close to failure. The values of *N* and *M* calculate simultaneously with deformations of system by the method of successive approximation to the ultimate value *N* and *M* laying [7]. The analysis of deformed state of the shell in the failure zone make for conditional arches representing of the ribs of shell in the direction of axes X and Y with parts of the shell plate and rises f_x and f_y. In the general cases deformation w_x and w_y are not equal among themselves. For a critical deflection of all system the maximal size of a deflection is accepted. Angular coordinates ξ_y^0 and ξ_x^0 and radii of curvature R_x and R_y recalculate according to the found critical deflection and together with the found ultimate values M_x, M_y, N_x and N_y substitute in the equation (14) and obtain the carrying capacity of the ribbed shells under concentrated load action with regard to deformation of the system by the moment of failure.

The algorithms for program based on this analysis are worked out for personal computers. Using this program the multi-alternative analyses are carried out, in which varied: ratio of radii of curvature R_x / R_y; depth of ribs h_x and h_y, including flange; thickness of flange h_f; reinforcement of ribs A_{sx}, A_{sy}, A'_{sx}, A'_{sy}; reinforcement of ribs in the joint under concentrated load A_{spx}, A_{spy}; specific ultimate tensile forces of the reinforcing wire fabrics in the flange q_{sx}, q_{sy}; strength of concrete R_b [6].

As the basic variants of analysis of ribbed shells of Positive Gaussian curvature with the dimensions of 18x24 m in plane is accepted. Such shells were applies as roof of industrial buildings, in which concentrated load are transferred to shells from suspended crane equipment. Radii of curvature for shell are R_x = 39,01m and R_y = 23,96 m. The rib spacing in direction of X-axis is 3.0 m, and in Y-direction of 6.0 m. The ribs have trapezoidal cross section, their dimensions and reinforcement are shown in fig. 6. The reinforcing bars are accepted of class A-III (according to Russian Codes) with strength of R_s = 365 MPa. The shell is made of concrete with strength of R_b = 14.5 MPa.

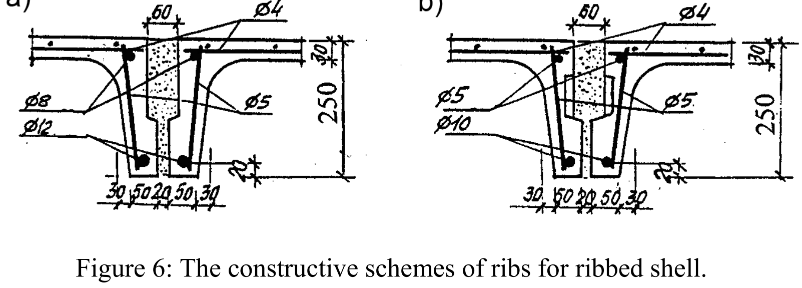

Figure 6: The constructive schemes of ribs for ribbed shell.
a) the longitudinal ribs; b) the transversal ribs.

In result of the computation of carrying capacity of the basic variant of shell the ultimate value of concentrated load P =154.96 kN is received (Fig.7).

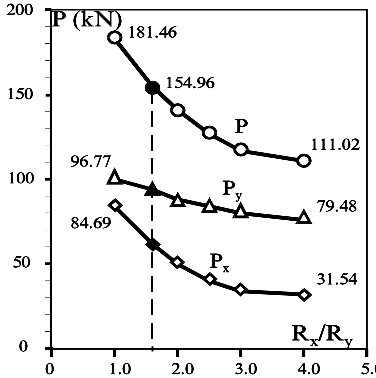

Figure 7: Relationship of carrying capacity of ribbed shell on the ratio of radii of curvature R_x /R_y.

In this case the rib, located in direction of X-axis, with a part of field contiguous to it (flange), supports the part of complete load P_x = 61.56 kN, and the rib of other direction - P_y = 93.39 kN. The dimensions of failure area are determined in distances to plastic hinges: l_x = 300 cm and l_y = 294.7 cm.

The relationships of value of ultimate load P and its components P_x and P_y from the ratio of radii of curvature R_x/R_y from 1 up to 4 are for this shell resulted in fig. 7, where it can be noted that with increase of value R_x/R_y (R_y = const) the carrying capacity decreases almost linearly up to R_x/R_y = 2.5. Further increases of value R_x/R_y, especially in range from 3 up to 4, does not result in appreciable decrease of ultimate load. It is explained by that when $R_x = 3R_y$ the coming of the limit state is accompanied by the shell deflection w under force, equal rise of shell in the failure area in direction of X-axis. For this case the ultimate load is rather close to the carrying capacity of ribbed cylindrical shell with radius $R = R_y$ = 23.96 m and the same geometrical characteristics of ribs and the same reinforcement, the difference between them is 1.7 % only.

3. The influence of long-time load action

Since the estimation of the ribbed shell carrying capacity is done according to upper critical load it is advisable to evaluate possible growth of deflection as a result of long-time load action. Investigation of effect of concentrated long-time load action on reinforced concrete shell carrying capacity was experimentally conducted on reinforced concrete model 2 × 2 m [7]. The model was a ribbed shallow shell with the field 6 m thick and ribs 30 m high and spacings 50 × 50 m. The shell was loaded by means of a lever device with concentrated load applied at the point of ribs intersection. Similar models had been studied and tested till failure under short time effect of concentrated loading. Long-time load P_{1t} = 5,56 kN made up 0,77 at short time failure load P_{st}. By the moment of loading the concrete prism strength was 39,3 MPa, modulus of elasticity E = 26100 MPa. The model was covered with waterproof layer: the test was conduct on constant temperature and humidity of working premises. Model failure occurred on 129 day since the moment of loading. The shell deflection diagram in the point of concentrated load action after loading and in process of long-time deformation during t = 129 days is presented in Fig 8,a.

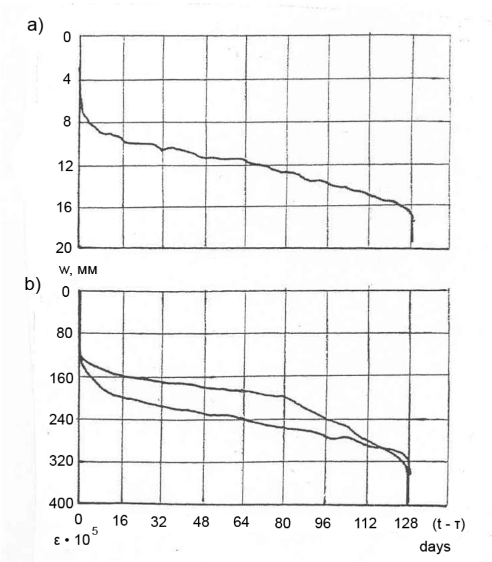

Figure 8: The shell deflection diagram (a) and curve of the time depended strains (b).

Upon the loading shell deflection was 6,8 mm and prior the failure – 19,1 mm, i.e. increased by 2,8 times. As shown in Fig. 8,a, the most significant increase in deflection was observed during the first 17 days upon loading. If the deflection value prior to the moment of failure is to be assumed as 100 %, then upon 17 days the deflection increases by 15,7 %. During a long period of secondary creep (about 109 days) the deflection was monotonously growing and increased by 32 %. Upon 126 days the stage of deflection dramatic growth began. For only 3 days the deflection grew 16 mm to 19,1 mm, i.e. by 16 % with subsequent failure. Relative fibre strain diagrams in section along the loaded rib are given in Fig. 9 (a – upper surface, b – lower surface).

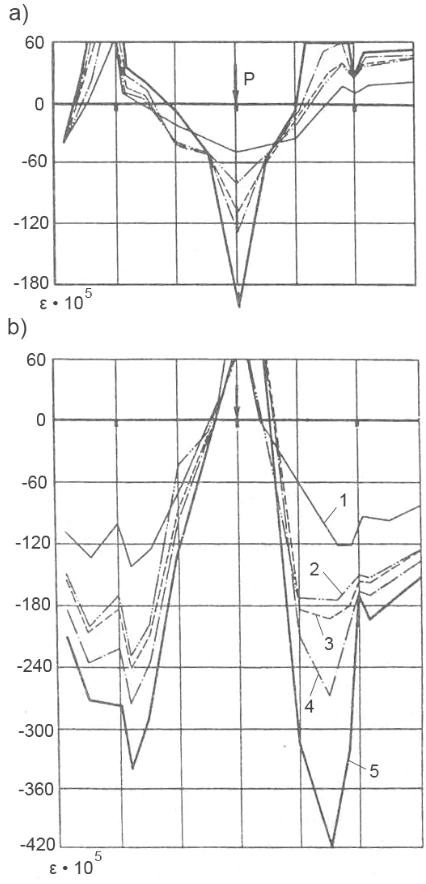

Figure 9: Relative fibre strain diagram in section along the loaded rib.

In Fig. 9,a – curve 1- relative strain at once after loading; 2 – over 52 days; 3 – over 70 days; 4 – over 110 days; 5 – on 128 days. The diagrams have the same nature as under the short-time load test. Maximum values of compression and tension stresses correspond to location of central (under concentrated load) and support (at points of intersection with ring crack) plastic hinges. In support hinges relative strains of compressed concrete on underside of ribs of ribs at the moment of loading of the rest were (120…140) 10^{-5}, and increased like deflections to the moment of failure by 2,3 – 3 times. The curve of the time depended strains is presented in Fig. 8,b.

The model failure diagram is similar to those previously obtained under short-time load test through the model deflection is essentially greater. It becomes understandable with analysis of section stressed states in support plastic hinges of the rib.

Under long-time loading the normal forces in plastic hinges are essentially less than those render short-time loading since the sustained load is only a part of the temporary one. The limit state of the eccentrically compressed section is reached at the expense of moment component primary grown directly depending on deflection growth in the failure zone.

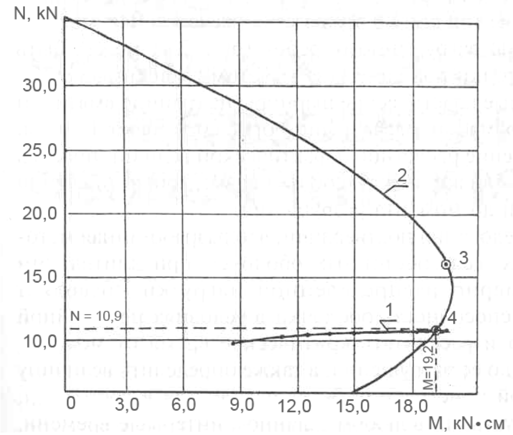

Figure 10: Graphical determination of moment of achievement of ultimate state of rib cross-section on contour failure zone.

It is seen from Fig. 10 that curve 1 reflecting the time-depending growth of moments and normal forces in the rib section in the failure zone has intersected curve 2 representing section strength under eccentric compression, at point with coordinates M = 19,2 kN·cm and N = 10,90 kN. It is characteristic of the given failure that design combination of ultimate moments and normal force corresponds to the case of eccentric compression with considerable eccentricities while under short-time loading rib section failure always occurred with eccentric compression at small eccentricities (point 3 on curve 2, Fig. 10). Comparison of design and actual deflections shows their closeness as well as critical time value prior to the moment of failure. The calculation has indicated that long-time ultimate load corresponding to critical time $t \rightarrow \infty$ equals 4,24 kN that is 0,58 of short-time ultimate load.

4. Conclusion

Conducted researches have shown that smooth and ribbed reinforced concrete shell with surface of positive Gaussian curvature are capable to bear rather heavy concentrated load of the suspended equipment, including crane. The tests on models and full scale shell structures have confirmed local character of failure in a zone of the application of concentrated load. In that case of failure the values of ultimate load being rather close to real one can be received by numerical analyses using the method of limit equilibrium in nonlinear approach. Multi-alternative analyses, carried out by this method have allowed to reveal influence of the different factors, including geometry of surface, thickness of shell and depth of its ribs, strength of concrete and long-time effect of loading on carrying capacity of smooth and ribbed reinforced concrete shells.

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