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Imperfection sensitivity in the buckling of single curvature concrete shells

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Abstract
Depending on their geometry, supports, material and loads configuration, shells can experience a reduction in the buckling load. If this happens, the shell is said to be sensitive to imperfections. Several approximate methods contemplate the phenomenon of instability in concrete shells, above all that which is based on the IASS Recommendations. In the latter there are some curves reflecting the influence of the initial geometric imperfections in the buckling load for simple geometric models, such as the sphere and the cylinder. In recent decades, different revisions of the Recommendations have been carried out referring to the way the value of the imperfections is quantified. However, new curves, which reflect the influence of imperfections in models with different geometries to those mentioned above, have not been stated. In this study, a method similar to that used by Dulácska and Kollár [1, 5] is implemented to determine the imperfection sensitivity factor in the case of shells with geometries such as a spherical dome and barrel vault.

Keywords: imperfection sensitivity, buckling, concrete shell

1. Introduction
The buckling load of shells constructed in homogeneous elastic material sharply decreases with increasing initial imperfection amplitude w_0. This decrease is due to the magnitude of the imperfection itself and the eccentricity e_0 of the compressive force caused by this imperfection. In the case of homogeneous material shells, the stiffness of the shell cross section is practically independent of the eccentricity, so it is sufficient to investigate only the decrease of the buckling load with increasing imperfection alone. However, the plastic deformation, the load bearing capacity provided by the shell wall, and the stiffness of the cracked reinforced concrete cross section are heavily dependent on the eccentricity of the normal force applied. The influence of the imperfection w_0 and that of the eccentricity e_0 of

the normal force may be dealt with separately. However, a relationship between both may be stated (Kollár and Dulácska [5]).

In the late 70s, IASS Working Group No. 5 [3] devised a document of recommendations for reinforced concrete shells. The section on stability analysis includes a procedure based on the results of some previous research, particularly on the work of Kollár and Dulácska [4]. The procedure involves several factors which affect the linear buckling load to take into account geometric imperfection, creep and shrinkage of concrete, cracking, reinforcement and material non-linearity. Scordelis [9] and Medwadowski [8] observed that the method provides conservative results in the case of spherical domes. However, Kollár [6] compared the theoretical predictions with experimental results and found great similarity. The first review of the Recommendations was prepared by Kollár [7]. Years later, Medwadowski [8] went back to revise the initial and the Kollár proposals, presenting some modifications and suggestions.

These analytical solutions have the advantage of using abacuses and simple formulas to obtain the buckling load, but have the disadvantage of predicting this type of behaviour only for certain theoretical cases (sphere and cylinder), not being able to deal with structures with other geometries.

In this paper, the influence of imperfection on the buckling load is studied for the case of shells of different geometries (such as spherical dome and barrel vault) to those studied in the IASS Recommendations.

2. Initial geometric imperfection

2.1. Imperfection sensitivity factor for homogeneous elastic material

The value of this factor is one if the shell is not sensitive to imperfection, and is otherwise less than one. The initial geometric imperfection sensitivity factor ($\rho_{hom} = p_{cr}^{upper}/p_{cr}^{lin}$) is the relationship of the upper critical load (p_{cr}^{upper}) with respect to the buckling critical load for linear homogeneous material (p_{cr}^{lin}). The parameters relating to plain concrete and reinforced concrete shells are denoted by ρ_c and ρ_{rc}, respectively.

The calculation of ρ_{hom} is often difficult. Some cases may be found in technical literature. Figure 1 shows the variation of ρ_{hom} with respect to w_0/e, where w_0 is a measure of the imperfection and e is the thickness of the shell. The curves A, B and C belong to laterally compressed long (L^2/Re = 10000), medium (L^2/Re = 1000) and short (L^2/Re = 100) cylinders, respectively, with L length and R radius. The curve D is for spheres and axially compressed cylinders.

The initial imperfection consists of accidental imperfection $w_{0,accid}$, due to erection inaccuracies, and calculable imperfection $w_{0,calc}$, quantified by the bending theory of shells.

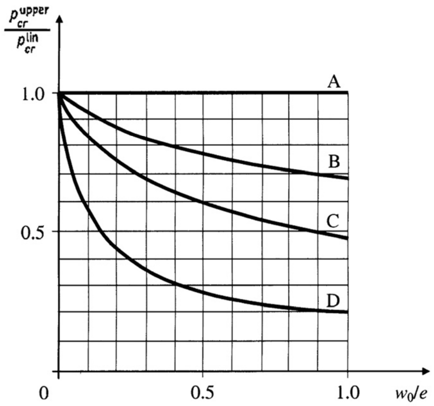


Figure 1: Initial geometric imperfection sensitivity factor ($\rho_{hom} = p_{cr}^{upper}/p_{cr}^{lin}$) in concrete shells, using concrete as homogeneous material (IASS Working Group No. 5 [3])

The maximum values of both imperfections are unlikely to coincide. According to the probability theory, the higher of the following values must be taken to chose the design imperfection $w_{0,design}$ (Kollár and Dulácska [5])

$$w_{0,design} \geq \left\{ \begin{matrix} w_{0,calc} + 0{,}8 w_{0,accid} \\ w_{0,accid} \end{matrix} \right\} \qquad (1)$$

Small deviations that may occur when estimating this design value are considered by using a safety factor. In the case of preferring a more conservative way of designing, both types of imperfections may be taken into account at the same time.

To obtain $w_{0,accid}$, Kollár and Dulácska [5] proposed the following expression

$$w_{0,accid} \approx \frac{R}{3500} \qquad (2)$$

where $R = \sqrt{R_1 R_2}$ if the shell is double curvature, with R_1 and R_2 being the mean principal radii of curvature of the shell. In the case of a sphere, both radii have the same value.

Eq. (2) is an approximate expression to quantify the accidental imperfection of a shell. Using this approach and modelling the shell by finite elements, the influence of the accidental imperfection may be stated by means of a geometric nonlinear analysis, which takes into account the effect of large displacements. However, it must be borne in mind that this equation is only valid for carefully erected shells (by using rigid formwork) and for R between 20 and 80 m approximately with the thickness between 50 and 70 mm.

By the method of the IASS Recommendations, the linear buckling load P_{cr}^{lin} is used as a starting point, and the effect of large deformations is considered by the bending deflection of the shell (w_0'= $w_{0,calc}$ = bending deflection), which may be estimated numerically or experimentally. Then, with the ratio deflection/thickness (w_0'/e) and Figure 1, the upper critical load p_{cr}^{upper} for homogeneous elastic material is obtained. If the geometry of the shell does not match any of the curves shown in Figure 1, and in the absence of further information, the curve D for spheres and axially compressed cylinders must be used.

Since the geometry of the actual shell differs from the theoretical design, if the post-buckling effect is important, the IASS Recommendations propose to reduce the linear critical load in a similar way, i.e., estimating the deviation $w_0'' = w_{0,accid}$ on the surface and using the ratio w_0''/e and Figure 1.

If both effects are present simultaneously, the reduction procedure must be performed at the same time using the geometric imperfection as $w_0 = w_0' + w_0''$.

2.2. Imperfection sensitivity factor for inhomogeneous nonlinear material

So far, the influence of geometric imperfection on the critical buckling load of concrete shells has been studied, with concrete considered a homogeneous and linear material, although in reality this is not true. Creep, plasticity, reinforcement and cracking have an important role in the calculation. In order to quantify these effects in real structures, it is necessary to provide a correction in the sensitivity factor of homogeneous material ρ_{hom} by using several factors to modify the linear buckling load

The method of the IASS Recommendations provides an approximation of the buckling load by applying factors to reduce the bifurcational critical load, also called linear buckling load, for a geometrically perfect shell, that is homogenous and made of linear elastic material. The design buckling load p_d is obtained by applying four factors to the bifurcational load

$$\gamma p_d = \alpha_1 \alpha_2 \alpha_3 \alpha_4 p_{cr} \tag{3}$$

where

γ = factor of safety,

p_{cr} = bifurcational or linear buckling load,

α_1 = imperfection sensitivity factor,

α_2 = creep factor,

α_3 = cracking and reinforcement factor, and

α_4 = inelasticity of concrete factor.

The linear buckling load p_{cr} of the perfect homogeneous shell must be calculated accurately, preferably by a FEM linear elastic analysis. Loads, material, geometry, support conditions and, just in case, the structural elements connected to the shell must be represented in the FE model correctly.

3. Imperfection sensitivity factor for several geometries of shells

3.1. Introduction

The curves of the influence of geometric imperfection in the critical buckling load of shells have only been defined for certain geometrical models (spheres and cylinders). For this reason, in the absence of further information, the IASS recommends to use the safest curve (curve D for a sphere under radial pressure and axially compressed cylinder). However, this approach in most cases is too conservative. Thus, it may be useful to obtain new curves for other geometries in order to have a better approximation of their structural behaviour against imperfection.

In this section, the curves for several geometrical models of single curvature are presented. Previously, validation of the method is performed by obtaining the curves corresponding to spheres and cylinders and by comparing them with the curves of the IASS Recommendations (Tovar [10]).

3.2. Sphere and cylinder. Validation of analysis method

To study the behaviour of the models against accidental geometric imperfection, the critical buckling load for each design is calculated by a linear analysis (p_{cr}^{lin}) and a geometric nonlinear analysis (p_{cr}^{upper}). Relating both results, the imperfection sensitivity factor (ρ_{hom}) is obtained for a given value of w_0''.

Eq. (2) is used to quantify the accidental imperfection in the shell up to values of $w_0/e = 1$ and higher, although this level has not been exceeded in this work in order to state a methodology similar to that in the IASS Recommendations. Each design was analysed for a thickness of 50, 60 and 70 mm; these values being within the range of validity of Eq. (2).

A tolerance of forces and displacements of 0.0001% has been used in the nonlinear analysis in order to obtain results with sufficient accuracy.

A mesh size that leads to a relative error less than or equal to 1% in the critical buckling load has been chosen. It does not lead to an excessively large computational cost and the obtained results have an acceptable error.

The influence of the boundary conditions in the model is not very crucial if the edges are not weaker than the shell itself, since most shells experience local buckling rather than global buckling (Kollár and Dulácska [5]). As a priori buckling behaviour of each shell is unknown, all designs have been analysed with different restrictions (degrees of freedom) at their edges. Figure 2 shows a graphical summary of ρ_{hom} for several thicknesses and boundary conditions.

Using the validated method and following the same criteria stated in this section for the mesh size and the tolerance of forces and displacements, other usual geometrical configurations in the erection of shells are analysed.

3.3. Barrel vault and cylindrical shell

The influence of the imperfection is analysed for several boundary conditions to simulate different behaviour: barrel vault (supported on right edges) or cylindrical shell (supported on curve edges). The results are shown in Figure 3.

3.4. Spherical domes

In this section, the imperfection sensitivity is analysed considering several types of configurations for a spherical dome. Reduced domes are particularly interesting because their arches do not exceed the so-called neutral line, defined by the parallel between compressed parallels (upper cap) and tensioned parallels (lower cup). It is not usual to erect domes reduced more than 36º. Domes with an angle of 90º, 60º, 36º and 20º and a dome on a polygonal base with an angle of 36º have been used in this study (Figure 4).

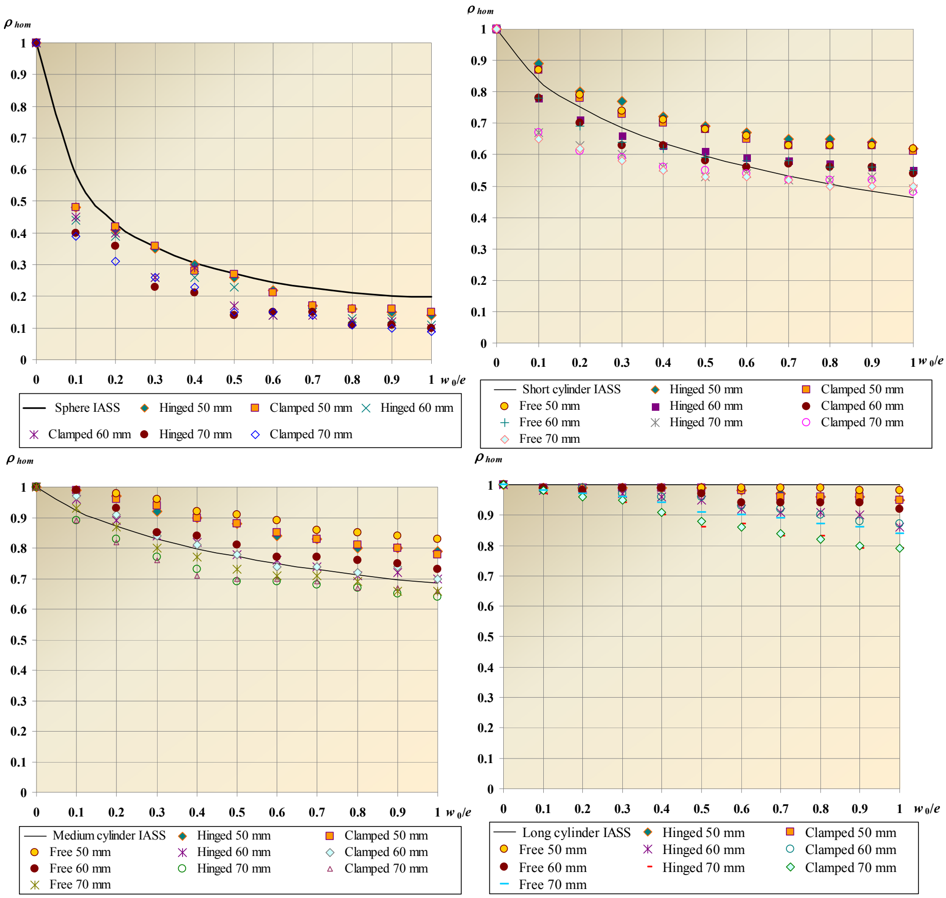
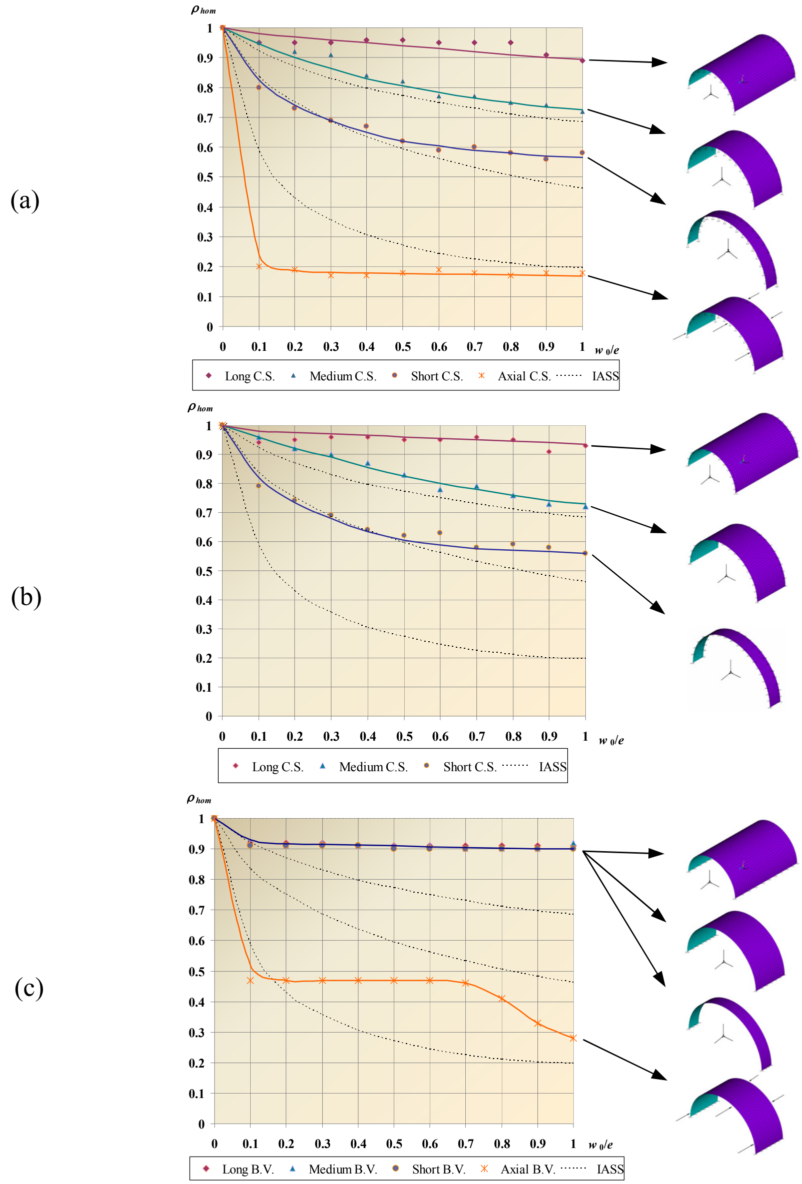


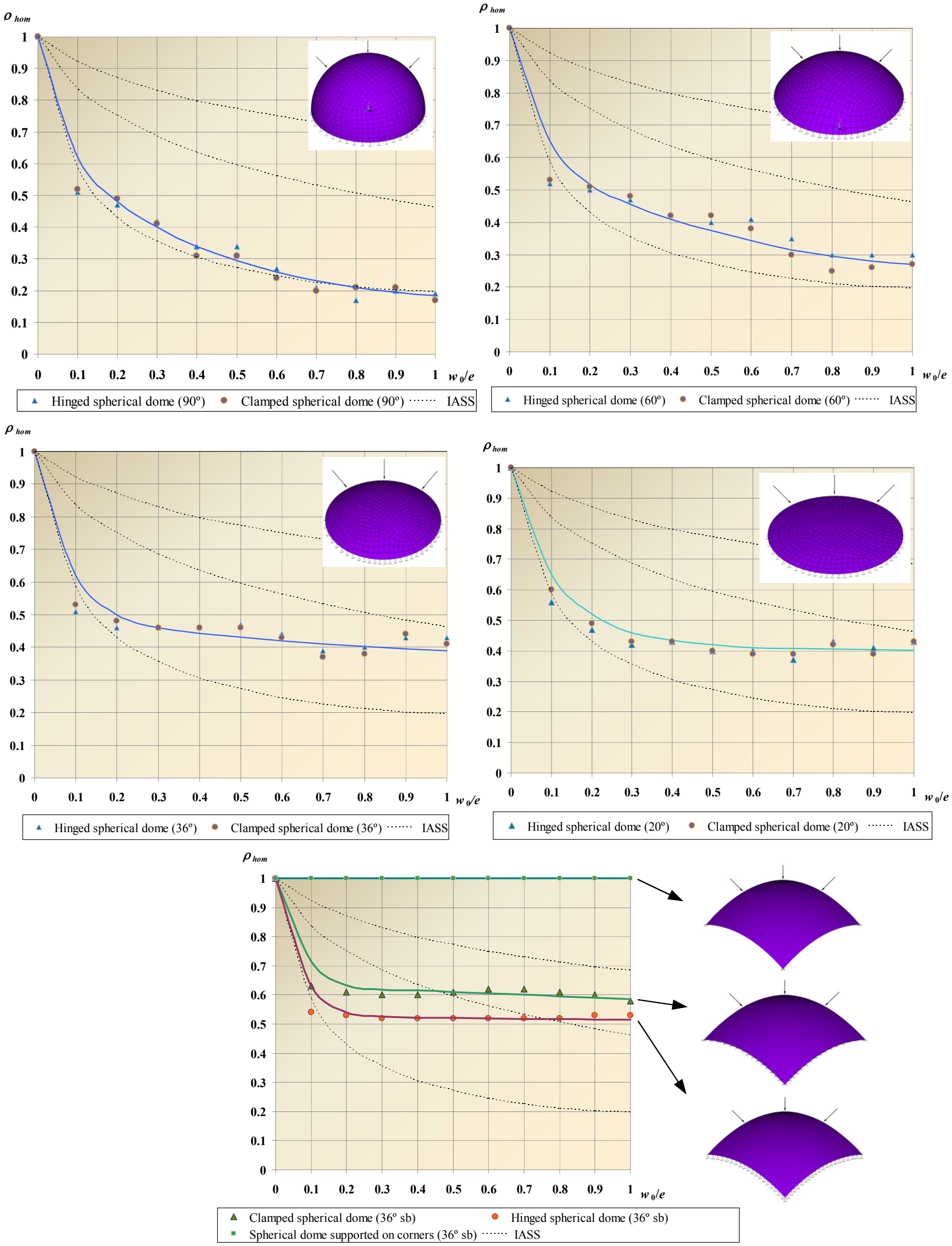
Figure 2: Imperfection sensitivity factor. Sphere and axially compressed cylinder; laterally compressed short, medium and long cylinder. Thicknesses of 5, 6 and 7 cm. Hinged, clamped and free supports

Simple boundary conditions such as those which have been used (hinges and clamps) have little influence on results. However, there are sometimes cases where the critical load of spherical domes is reduced, not only because of imperfections, but also because of the boundary conditions on which they are supported (Gioncu [2]).



w_0 = geometrical imperfection; e = thickness; Axial = axially compressed shell (the rest of the shells are laterally compressed); IASS = curves from IASS [3]

Figure 3: Imperfection sensitivity factor (ρ_{hom}). (a) Cylindrical shells (C.S.) with hinged supports on curve edges. (b) Cylindrical shells (C.S.) with clamped supports on curve edges. (c) Barrel vaults (B.V.) supported on right edges



w_0 = geometric imperfection; e = thickness; sb = square base; IASS = curves from IASS [3]

Figure 4: Imperfection sensitivity factor (ρ_{hom}). Spherical domes

4. Conclusions

The main conclusions of this study can be summarised in the following points:

- Axially compressed models present the most unfavourable imperfection sensitivity factor, with a trend of the curves similar, or even below those in the IASS Recommendations for spheres and cylinders subjected to axial compression. Most models when laterally compressed, usually adopt intermediate values between the short and the long cylinder, except for the cases of spherical domes.
- The behaviour of the spherical dome is mainly conditioned by the angle (taken from the ridge). The worst case is the hemisphere (with values slightly higher than the sphere).
- There are no significant differences in the behaviour against imperfection between hinged and clamped supported shells. However, a reduction of the stiffness at the supports such as allowing horizontal displacement, presents a significant importance, since it causes a significant reduction in the imperfection sensitivity factor.
- In general, a minimum configuration of supports provides an imperfection sensitivity factor near to one. However, significant changes in distribution and the number of supports may cause significant increases in imperfection sensitivity.
- The IASS Recommendations to quantify the influence of geometric imperfection in shells whose behaviour is not known with certainty, by means of the curves for spheres and axially compressed cylinders, is too conservative.

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