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Shape optimization of small span textile reinforced cementitious composite shells

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Abstract

The property of concrete to be poured into any shape and harden at ambient temperatures makes it the most widely-used material for shells. Using this traditionally brittle material in shells restricts their forms to mostly compression shapes. Often steel reinforcement is still necessary to carry tensile forces occurring under different load combinations and to limit crack formation. A new composite material, textile reinforced cementitious composite (TRC), eliminates this restriction by combining the two most frequently used materials for structural curved shapes: textile and concrete. High volume fractions of flexible fibre textile reinforcement are inserted into the cementitious matrix to produce a material with both high tensile and compressive capacities. In this way, the steel reinforcement is replaced by the textile reinforcement and this flexible reinforcement eases the construction of the shell. More importantly with both compressive and tensile capacities in the shell material, a whole new vocabulary of load bearing structural surfaces is possible. The use of noncorrosive textile reinforcement has another great advantage over steel reinforcement in that the required concrete corrosion cover can be omitted. Consequently, shells have to be made only as thin as structurally necessary, which makes them an interesting option (especially for small span applications). In this paper, a methodology for numerical optimization of a 10 m span doubly curved TRC shell is presented. The optimization process aims at a shell shape that solely experiences tension and compression under selfweight. The initial continuous cylindrical shell shape is discretised into a grid shell and is optimized using a version of dynamic relaxation adapted for gravity loads. The resulting grid shape is then modelled as a continuous shell in the finite element analysis program Abaqus to study the influence of the boundary conditions on the load-carrying behaviour under load combinations of selfweight and asymmetric loads as required by Eurocode 1. The proposed methodology to achieve purely axially loaded shell shapes under gravity loads, produces very slender, force-efficient doubly-curved shells using the combined tensile and compressive capacity of TRC and opens a new realm of structural curved shapes.

Keywords: textile reinforced cementitious composite, shape optimization, small span, shell, dynamic relaxation

1. Introduction

Modern architecture tends towards daring complex curved shapes due to the advent of digital geometrical modelling tools such as CATIA, Rhino etc. With this evolution, the demand for suitable materials that achieve durability through structurally efficient use and appropriate fire resistance, arises. The most common materials for complex curved load carrying surfaces are technical textiles (used for membrane structures) and steel-reinforced concrete. However, both materials have limitations for their application in load carrying complex curved surfaces.

Technical textiles allow for visually light and complex curved pre-stressed shapes, but experience problems of acoustics, fire-safety and thermal insulation. The necessary pre-tension within the membrane requires moreover strong anchorages.

When properly designed, structurally efficient slender steel-reinforced concrete shells can be made to cover spans of more than 30 m. Due to the brittleness of concrete, the design of concrete shells is often restricted to shapes working mainly in compression. The steel reinforcement can only in specific circumstances be omitted; crack formation and tensile forces can still occur under various load combinations. The largest disadvantage of having to implement steel reinforcement in the concrete matrix is the necessity of a concrete cover, which causes an increased dead weight and shell thickness (the minimal concrete cover on both sides amounts to minimally three centimetres, depending on the exposure coefficient described in Eurocode 2 [1]).

Textile reinforced cementitious composites (TRC) can eliminate some of the disadvantages inherent to steel-reinforced concrete, without losing and adding extra advantages. When using dense fibre mats instead of short fibre bundles, a high reinforcement ratio and thus high tensile capacity can be achieved, while steel reinforcement can be eliminated. The reinforcement diameter is three orders of magnitude smaller, which eases the construction of complex curved shells. When using a noncorrosive reinforcement such as glass fibres, the concrete corrosion cover can be omitted and the thickness and thus the self weight of the shell decreased. Whereas for practical reasons, a concrete shell with two layers of reinforcement and the necessary cover cannot be less than 6 to 8 centimetres thick, the thickness of a TRC shell may be exactly that required to resist the applied loads. TRC with its high tensile as well as compressive strength does not only extend the span range where it becomes interesting to apply shells to smaller spans, it also creates the possibility for a whole new series of structural curved shapes.

In [2] the author has demonstrated through case studies that slender small span TRC shells with strongly varying curvature can be designed and built. A prototype 2m span shell only requires a 8 mm shell thickness, whereas a 10 m doubly-curved shell had to be 4 cm thick.

This paper will show that through shape optimization and manipulation of the shell's boundary conditions, this thickness can be reduced further. A 10m span doubly-curved shell is shape optimized for self weight with an adapted version of the dynamic relaxation method for gravity loads. The shape optimization aims at an exclusively axial stress state within the shell and omits any bending stresses. Once the optimal shell shape is found under self-weight, its structural behaviour is studied under different boundary conditions and all required load combinations of dead and asymmetric live load through the FE program Abaqus. The 10m span doubly curved shell requires a nominal thickness of 5 mm to fulfil the strength and buckling requirements prescribed by Eurocode 1 [3] and Eurocode 2 [4]. With these findings, TRC shows to be a new efficient material that can achieve extremely slender structures that experience both tensile and compressive in-plane stresses.

2. Textile reinforced cementitious composite: material behaviour

Cementitious materials have relatively high compressive strength, but very low tensile strength; cracks appear at very low tensile stresses. The addition of short fibres to a fresh cement mixture constrains the crack width and enhances its toughness. However only a limited fibre content – and so limited tensile capacity- can be achieved with this method (Cuypers and Wastiels [5]). For structural applications, steel reinforcement bars are still needed. These steel bars can be eliminated if the fibre reinforcement is able to provide the necessary tensile capacity. According to classic composite modelling, considerable amounts of fibres have to be inserted into the cementitious materials in the direction of the tensile stresses to provide the necessary stiffness and strength beyond the introduction of matrix multiple cracking in the matrix as stated by Aveston, Cooper and Kelly [6]. Glass fibres can be added to cement in high amounts in the form of textiles. These fibres are cheap in comparison to other fibre materials, but are damaged by the alkalinity of cement matrices. Generally, two solutions can be applied to this problem: firstly the use of alkali resistant glass fibres, which does not eliminate but slows down the degradation process (Orlowski et. al. [7]), and secondly the use of a low-alkaline or non-alkaline matrix (e.g. Inorganic Phosphate Cement, IPC [8]). Both material combinations allow the production of very thin textile reinforced cementitious composite (TRC) shells that can bear tensile as well as compressive stresses. TRC can be applied in structural elements and will exhibit similar physical properties as concrete.

For the application in the doubly curved shell under study, a cementitious composite was chosen consisting of an Inorganic Phosphate Cement (IPC, [8]) matrix, and dense randomly oriented E-glass fibre mats impregnated by the hand lay-up technique to allow for high fibre volume fractions. The rheology of the fresh IPC-matrix allows much higher fibre volume fractions (up to 20%) compared with Ordinary Portland Cement fine tuned mortars, which results in a stiffer and stronger cementitious composite in tension. Moreover; while alkali-resistant OPC still degrades with time, previous research has shown that the strength loss with time of E-glass fibres embedded in the IPC-matrix is negligible (Cuypers [9]). Finally, IPC offers also a major building physical advantage over Ordinary Portland Cement (OPC) as it has excellent fire resistance. General material properties of the used composite are given in Figure 1.

			GTR-IPC
Matrix			Inorganic Phosphate Cement
Fibre textile			E-glass fibres
Fibre volume fraction	V_f	%	20
Density	ρ	kg/m³	1900
Initial E-modulus	E1	GPa	18
Poisson coefficient	υ		0.3
Crack stress	σ_{crack}	MPa	7
Design compressive stress	$\sigma_{c,\ max}$	MPa	33
Design tensile stress	$\sigma_{t,\ max}$	MPa	20

Figure 1: Material properties of 20 vol% TRC

TRC behaves linear elastic up to the maximum design compressive stress. In tension however, the material is highly nonlinear. The typical stress-strain behaviour of textile reinforced cementitious composite in tension is illustrated in figure 2. Three stages can be distinguished (Brameshuber [10]):

- Stage I: pre-cracking. At the beginning of loading, the stiffness of the uncracked composite is determined by the "law of mixtures" for linear elastic composites and is equal to the Young's modulus in compression. Since the volume fraction of the glass fibres is 20%, the stiffness in this first stage is mainly determined by the matrix.

- Stage II: multiple cracking. When exceeding the tensile strength of the IPC matrix, the first cracks appear. At the crack face, the whole tension force has to be carried by the reinforcement. As the amount of fibres is larger than the critical fibre volume fraction, the acting load can be carried and the composite does not fail. As the tension force increases, additional cracks occur: due to the frictional bond between filaments and cementitious matrix, forces are transferred in the IPC matrix until its tensile strength is reached again. The cracking distance and the crack width are determined by the properties of the reinforcement, the bond characteristics between reinforcement and matrix, and the tensile failure strain of the matrix.

- Stage III: post-cracking. In this stabilised crack pattern stage, no further cracks occur. As the load increases, the filaments are strained further until their strength is reached. Failing of the specimen can however also happen as a result of fibre pull-out.

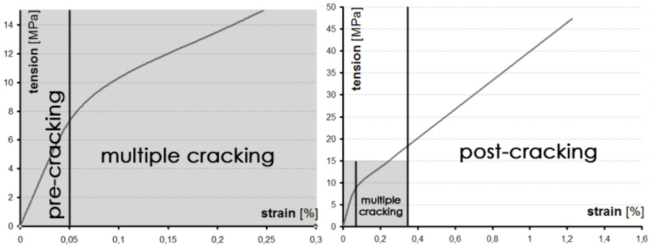


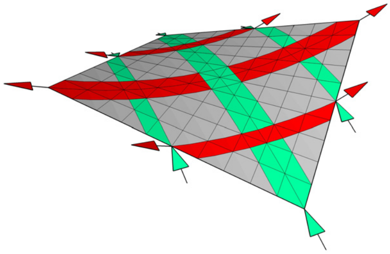
Figure 2: Experimental stress-strain behaviour of 20 vol.% GTR-IPC under uniaxial tensile load

Conclusively, the stiffness of TRC in tension is several times lower than its Young's modulus in compression, and the effect of this specific material behaviour on the design of the doubly-curved shell must be studied. This reduced stiffness in tension can not only have an influence on the load-carrying behaviour of the shell optimised for the same material stiffness in tension and compression, but also on the buckling resistance.

3. Shape optimisation

3.1. Choice of form

The shell shapes of Felix Candela and Jorg Schlaich have been precedents for this TRC doubly curved shell study (Hamburg-Sechlinspforte swimming centre hypar shell, 1967 by Schlaich and Leonhardt as described by A. Holgate in a book on Schlaich's work [11]; Cosmic Rays Laboratory hyperbolic paraboloid shell by Candela, discussed by Garlock et. al. [12]). A hyperbolic paraboloid shell has a doubly curved shape in which a set of parallel lines of parabolic arches can be found, and perpendicular to this a set of parallel suspended parabolas appear. In this way, the hyperbolic paraboloid shell contains within its specific shape the two most efficient means of carrying distributed loads such as selfweight. The concept of parabolic arches and suspended parabolas can be used for straight edged hypar shells like shown in figure 3, but also for hyperbolic paraboloid saddles with curved edges, leading to very aesthetic designs like the Los Manatiales restaurant at Xochimilco, Mexico or the Chapel Lomas de Cuernavaca shown in figure 4, both designed by Candela (Garlock et. al. [12]).



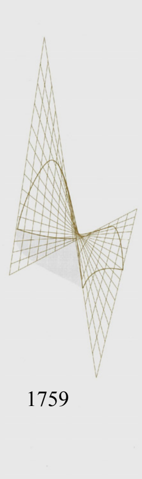


Figure 3: Hyperbolic paraboloid with straight edges: parabolic arches (green) and suspended parabolas (red)

Figure 4 (left and up): Chapel Lomas de Cuernavaca, designed by Candela [12]

Most importantly in the design of very slender TRC shells, is the fact that the two carrying line systems of hyperbolic paraboloids are curved in opposite directions so that each is effective in resisting any change in the shape of the other (A. Holgate, [11]). This characteristic enhances resistance to buckling, which is a significant design criterion for the very thin TRC shells. Moreover the resistance to distortion under asymmetrical loads enlarges, important to carry local accumulations of snow or wind loads, which will have a larger influence on the structural behaviour of slender low selfweight shells relative to the heavier steel-reinforced shells. This increased resistance of doubly curved shells against buckling was exploited by the temporary glass fibre reinforced concrete shell designed by Schlaich for the Federal Garden Exhibition in Stuttgart in 1977. This shell had similar shape and proportions as the 40mm thick Xochimilco restaurant roof spanning 32m, but has a thickness of only 12 mm thick based on buckling resistance for a span of 26 m between supports (A. Holgate, [11]). Even though the shell was planned as a 6 month temporary structure, it stood five years after which it was demolished due to large damage due to creep and brittleness.

3.2. Shape optimization with an adapted version of the dynamic relaxation method under gravity loading

The shape optimisation of the 10 m span doubly-curved shell aims to carry the selfweight of the shell through membrane action only. The dynamic relaxation method turns a static problem into a fictitious dynamic problem and through an iterative process achieves a damped static equilibrium shape. The initial geometry is a 10 m span, 5m high cylindrical shell (Figure 5). The shell is discretised into 11*11 straight bar elements, with an elastic stiffness (EA) of 20 000 N, the lowest possible elastic stiffness for convergence. External gravity loads equivalent to the self weight of a 20 mm thick shell were applied in every node. All nodes on the 2 boundary arches were restrained in the 3 displacement directions. The shape optimisation result, shown in figure 6, is a doubly-curved shell completely in tension. In reality however, the lower boundaries will also be restrained, leading to a shell with arches in compression, and perpendicularly suspended parabolas in tension.

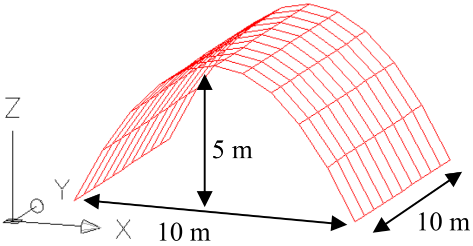


Figure 5: Initial geometry before shape optimisation

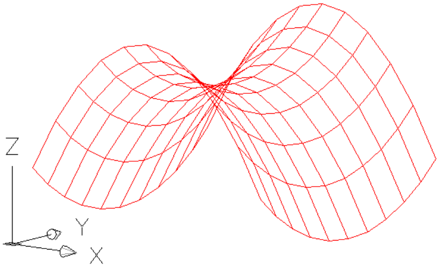


Figure 6: Doubly-curved geometry after shape optimisation

The used version of dynamic relaxation grid-based program also has two disadvantages. Firstly, it does not take the shear stiffness of the shell into account, and secondly the continuous shell must be formed through the discretised grid-structure. In this study, the continuously curving shell is created by drawing curved parabolas through the nodes, and subsequently filling the grid-structure existing of curved links of hanging parabola and arch by the minimal surface areas. Also the curving bottom of the shell is remodelled and cut to obtain a flat connection with the ground. The resulting continuous shell shape is shown is figure 7.

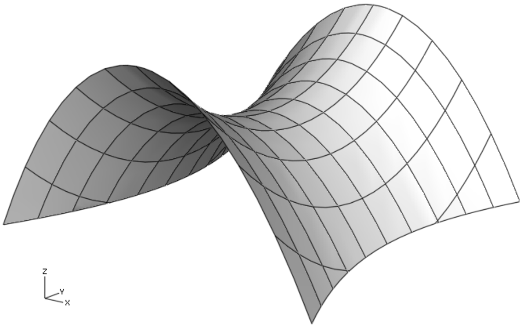


Figure 7: Continuous shell after shape optimization that experiences solely membrane stresses under gravity load

4. Structural analysis of doubly-curved shell using FEM

4.1. Verification: structural analysis under selfweight

The loadbearing behaviour of the resultant 10 m span TRC shell is verified under selfweight, the load for which the shape was optimised. An initial thickness of 20 mm is taken. The shell is calculated with the FEM program Abaqus (version 6.7-1), using 40*40 shell elements (S4R, 4 node doubly curved thin or thick shell) and 9 integration points per section. The material behaviour of TRC is modelled with a simple linear elastic material model, as stresses never exceed the linear elastic limit in tension.

Initially, only the 2 boundary edges at the ground are pinned and the arches are not restrained in order to be able to demonstrate the positive effect of the pinned arches in the following paragraph. The boundary shell edges connecting the shell to the foundation are set to be pinned, so that the shell can rotate with differences in temperature and accommodate some of the thermal stresses. The resulting principal in plane stress pattern in figure 8 shows clearly that the arches are loaded in compression, and that the areas near the arches experience the largest compressive stresses (yellow vectors), however in the perpendicular direction only very small tension stresses occur. The shell mainly works as a cylindrical shell in compression and not as a hyperbolic paraboloid in tension and compression, because there is no resistance against possible tensile forces in the suspended parabola direction. Stresses remain as low as 0.32 MPa in compression, and hardly 0.1 MPa in tension.

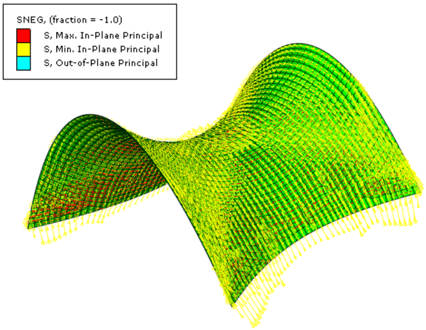


Figure 8: Stress pattern under selfweight; 2 sides pinned, arches free

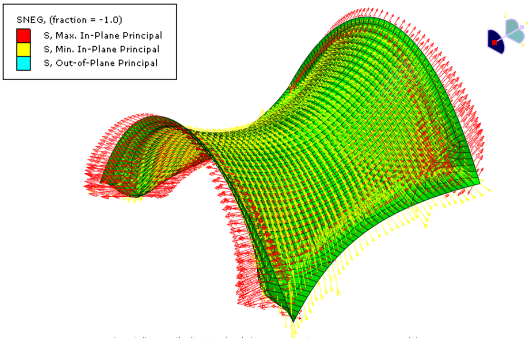


Figure 9: Stress pattern under selfweight; 2 sides and arches pinned

To introduce tension in the suspended parabolas and obtain the stress pattern what the shell was optimized for, the 2 side arches must also be restrained and pinned as was done during the numerical optimization. The resulting in plane principal stress distribution is shown in figure 9. The shell now behaves exactly as how it was optimized carrying the selfweight only by normal forces with compressed arches in the transverse direction and suspended tension parabolas in the longitudinal direction. The maximum tensile stress equals 0.13 MPa, and the maximum compressive stress is 0.15 MPa. These values are half the values of the shell with free boundary arches. The big advantage of the pinned arches will however lie more in its improved buckling behaviour and the way asymmetric loadings are carried as will be shown in the following paragraph. In reality, these boundary conditions can be achieved by a stiff arch which is inclined outwards at its base.

4.2. Structural analysis under asymmetric wind and snow loads

Due to the shell's low selfweight, the relative importance of asymmetrical loading on the structural behavior increases. Therefore, in this paragraph specific attention is devoted to the loadbearing behaviour of the shell under asymmetric wind and snow load. The loads are determined using the guidelines in Eurocode 1 [3] for cylindrical shells. The selfweight, wind and snow loads are then combined applying safety factors and reduction factors (symbolising the reduced chance that the two maximum variable loads occur together) using the prescriptions for ultimate limit state in Eurocode 1. Two ULS load combinations result in the largest stresses: LC1, combination of selfweight, asymmetrical wind load as main variable load, and asymmetrical snow load as secondary variable load (self weight*1.35 + asymmetrical wind pressure*1.5 + asymmetrical snow*1.5*0.5) and LC2, a combination of selfweight and wind only, because the wind load puts a large part of the shell in suction as shown in figure 10 (self weight*1.35 + asymmetrical wind pressure*1.5).

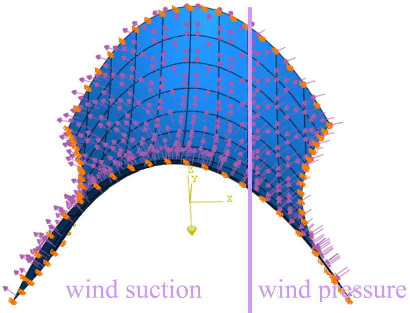


Figure 10: Asymmetrical wind load on shell

Figures 11(a and b) and 12 show respectively the force distribution under LC1 for the case of free edge arches and pinned edge arches. When the edge arches are not restrained, the shell does not carry the loads by the arches and suspended parabolas, but a “cross” of large tensile and compressive stresses appears. Where large tensile stresses occur at the shell’s top surface (figure 11 a), large compressive stresses occur in the shell’s bottom surface (figure 11b) and vice versa. The “crosses” represent thus local zones in bending, which also vary the side of the surface in tension locally. The maximum tensile (7.35 MPa) and compressive (7.39 MPa) stresses occurring under LC1 confirm that the shell does not carry the asymmetrical loads efficiently under these boundary conditions. For LC2 same findings and order of magnitude of maximum stresses are found.

Not only the fact that asymmetrical load combinations are carried structurally inefficient when releasing the edge arches, but also the shell’s reduced buckling capacities will have a large influence on the minimum thickness. For the 2 cm thick shell, the critical buckling load is 9 times the design wind load, however the buckling load diminishes quickly with reducing thickness, such that a 1 cm thick shell would already buckle at the design wind loads and is thus unsafe.

When the side arches of the doubly-curved shell are pinned however, the asymmetrical loads are still carried by tensioned and compressed parabolas like the shell was designed for. The windward side of the shell (on the right at figures 10 and 12) experiences a wind pressure and thus a downward force. In this area, the loads are mainly carried mainly by tensile forces in the suspended parabolas. The leeward side of the shell (on the left and middle at figures 10 and 12) experiences suction. This upward force is carried by the suspended parabolas that are now in compression, and perpendicularly to this, the arches which now experience tensile stresses. Only minimal bending moments occur. The ability of the shell to efficiently adapt its loadbearing behaviour to asymmetrical loading can also be observed in the stresses that remain much lower in comparison with the shell with free boundary arches: maximum 0.85 Mpa in tension and 0.69 Mpa in compression.

The shell's structural behaviour under asymmetrical load combinations is improved by restraining the edge arches and introducing tensile stresses that can be carried by suspended TRC parabolas. At least as important for slender shells however, is that also the buckling behaviour is greatly improved by creating arches in compression and suspended parabolas in tension resisting as such any change in shape of the other. This increased buckling resistance is illustrated by the much higher critical buckling load: with restrained edge arches the shell would buckle at 125 times the design wind load, whereas with free edge arches the shell can only withstand 9 times the design wind load before buckling.

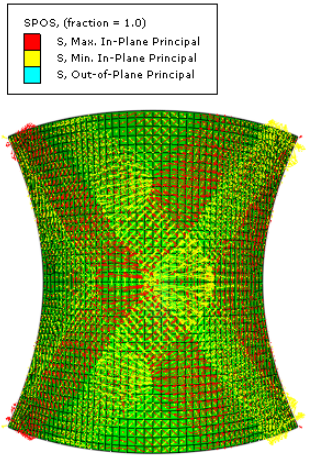


Figure 11a: Stress pattern under LC1; 2 boundary ground curves pinned, arches free, top shell surface

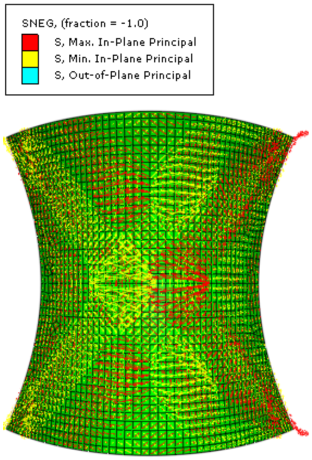


Figure 11b: Stress pattern under LC1; 2 boundary ground curves pinned, arches free, bottom shell surface

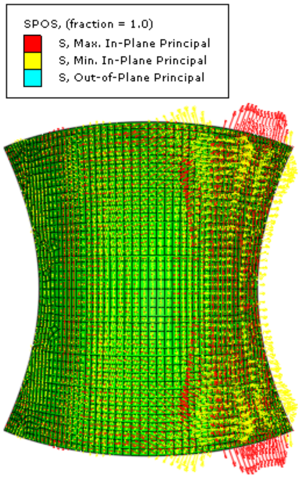


Figure 12: Stress pattern under LC1; 2 boundary ground curves and arches are pinned

As a result of the low stresses occurring in the pinned arch configuration under LC1 and LC2 and the high buckling resistance, the thickness of the shell can be drastically reduced. According to Eurocode 1 [3], serviceability (SLS) and ultimate (ULS) limit state both have to be fulfilled. Under SLS load combination, deflections of concrete structures are limited to span/250. Due to the high structural stiffness of the doubly-curved shell, this is never the determining limit state (deflections remained below 1 mm = span/10000). Under ULS load combinations, the shell has to comply with the tensile and compressive design stress criteria of the considered material (Table 1) as well as resistance to buckling. The doubly-curved TRC shell of only 5 mm thickness proved to fulfil all the criteria.

Due to its very small thickness and in this case the necessity of a stiff inclined arch, the TRC shell can actually better be compared to the membrane structures of today. The largest difference from structural point of view is the fact that TRC can carry tensile as well as compressive stresses. This unique property can be used to create new doubly-curved shell shapes, which through their specific shape and structural behaviour are inherently more resistant to buckling and asymmetrical loads, and can thus be made as slender as 5 mm thick for a span width of 10 m. From building physical point of view, TRC's fire resistance is the largest advantage.

5. Conclusion

In the search of structurally efficient material and shapes this paper investigates the potential of glass fibre textile reinforced cementitious composites in doubly curved shells as an alternative for the existing steel-reinforced concrete shells and membrane structures. Since TRC can resist both tensile and compressive axial stresses and needs no corrosion cover, a 10m span shell shape was studied that takes full advantage of its geometry to resist buckling and loading (by pure membrane action) with a minimum amount of structural material. To ensure that under self-weight the shell works under pure membrane action, shape optimisation under gravity loading is performed on an initial cylindrical shell. In this process, the continuous initial shell is discretised as a grid shell and subjected to an adapted version of dynamic relaxation that takes into account gravity loading due to self-weight (assuming a shell thickness of 20 mm).

The resulting doubly-curved shell has longitudinal suspended parabolas in tension and transverse arches in compression. When analysed in the finite element model Abaqus for load combinations of self weight, snow and wind load, it was found that by fixing not only the lower edges, but also the arch edges, the shell resists these asymmetric loading combinations well and experiences very low stresses (<1MPa). Also the buckling behaviour was greatly improved by the combination of suspended parabolas in tension and compressed arches who prevent any change of shape of the other. The minimum thickness of the doubly curved shell with pinned arches to fulfil the prescriptions in Eurocode 1 and 2 was only 5 mm. It must be stressed that this tensile suspended parabola-compressive arch combination for such thin shells can only be accomplished with a material like textile reinforced cementitious composite which exhibits high tensile as well as compressive strength.

The exploration of TRC as a structural material (with its tensile and compressive capacity) in a double curved shell shows great potential. Upon determination of an optimal shape that functions through membrane action, the shell thickness is solely determined by its buckling behaviour.

Acknowledgement

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