*Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia*
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

# Stability Analysis of a New Type Reticulated Shell

Jiachun CUI, Chengming LI, Lianping YANG, Wei TIAN

Technical Center of Shanghai Xian Dai Architectural Design (Group) Co.,Ltd.
20F, 258 Shimen Er Road, Shanghai, China. 200041
Jiachun_cui@xd-ad.com.cn, chengming_li@xd-ad.com.cn, lianping_yang@xd-ad.com.cn,
Wei_tian@xd-ad.com.cn

## Abstract

The elastic and elastic-plastic overall stability problems are analyzed for the reticulated shell used in 2010 Shanghai Expo-axis, and studies are made to contrast and analyze the calculated results corresponding to different parameters so that the effects on the overall stability of structures caused by the material model assumption and initial geometric imperfections are obtained, and at last the paper presents the deficiency of current specifications when used for the new type reticulated shell and points out some pressing problems.

**Keywords:** new type reticulated shell; nonlinear analysis; initial imperfection; overall stability; ANSYS

## 1 Stability Analysis of the Reticulated Shell

Focus is made mainly on the stability analysis of regular reticulated shell by the data with respect to the structural stability in existing papers and monographs, for the reason that the regular reticulated shell was widely used in last century end because of its graceful modeling, simple system and the ability to cross large span and so on, whose characteristics of its overall stability was obvious and gained wide attention.

However, with the emergence of novel building styles, the application of irregular reticulated shell is more and more common. The stability analysis methods of regular and irregular reticulated shells are the same and the same calculation theory is also used for them. Therefore we can adopt the traditional methods that used for the regular reticulated shell to analyze the stability of the irregular reticulated shell.

The purpose of stability analysis is to determine the critical load when the structure come into unstable equilibrium state from a stable one and corresponding mode shapes. For typical shell structures, stability analysis contains two aspects, that is, the linear eigenvalue buckling analysis and the whole process tracking analysis of "load-displacement" considering the nonlinearity.

### 1.1 Eigenvalue Buckling Analysis

The eigenvalue buckling analysis is used to forecast the theoretical buckling strength of an ideal linear structure, whose advantage is to obtain the critical load and buckling shapes without complex nonlinear analysis, and can provide a reference value for the nonlinear analysis. The commonly used methods of eigenvalue buckling analysis include Subspace method and Block Lanczos method. The governing equation of the eigenvalue buckling analysis can be expressed as

$$\left([K]+\lambda_i[S]\right)\{\psi\}_i=\{0\} \tag{1}$$

Where: $\left[K\right]$ is the stiffness matrix, $\left[S\right]$ is the stress stiffness matrix , $\lambda_i$ is ith eigenvalue ( used to multiply the loads which generated [S]), $\left\{\psi\right\}_i$ is ith eigenvector of displacements.

Since the results gotten by the eigenvalue buckling analysis are usually non-conservative, they cannot be applied to practical engineering.

### 1.2 Nonlinear Stability Analysis

In the progress of loading to the ultimate bearing capacity of structures, it is common to come with a larger deformation and at the same time the stiffness also degenerates. Thus a complete analysis of the stability should take the effects of geometric non-linearity into account, and this time the finite element equation is:

$$\left(K_0+K_\sigma+K_L\right)\Delta u=\Delta P \tag{2}$$

where $K_0$ is the stiffness matrix of structures, $K_\sigma$ is the stress stiffness matrix of structures, $K_L$ is the displacement stiffness matrix, $\Delta u$ is the displacement increment and $\Delta P$ is the load increment.

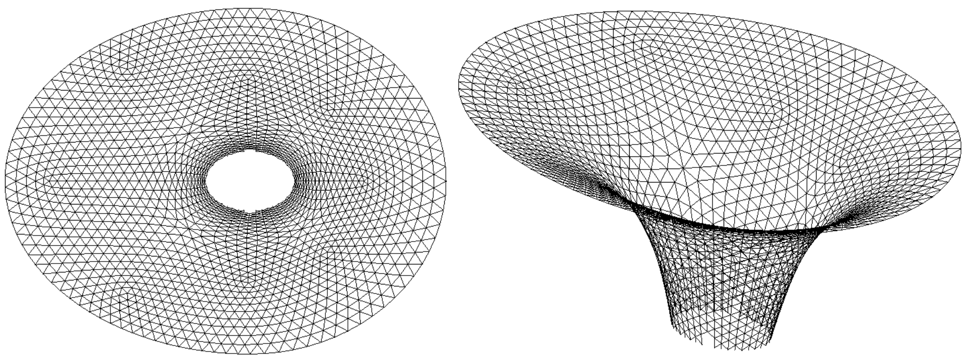
When Equation (2) is solved, the "load - displacement" curve of the whole loading history is obtained. A method applied extensively and with a better effect is the arc-length method.

In addition, as a result of manufacture, installation errors and other factors, there are often some deficiencies in the geometry in the actual structure, thus there are some differences from the ideal structures. Therefore, in the process of non-linear stability analysis it is necessary to consider the impact of the initial geometric imperfections, especially the single-layer reticulated shell structure which is sensitive to imperfections. The initial imperfections are generally in random distribution, but often in the actual process of analysis, the consistent mode imperfection method is applied for an approximate simulation. This method assumes that the initial imperfections are distributed as the lowest order buckling mode, and theoretically it proves that such a distribution is the least favorable distribution in a statistical sense, as a result, each reticulated shell with the random imperfections needs imperfection analysis only once [2].

## 2 Engineering Introduction to a New Type Reticulated Shell

The reticulated shell used in 2010 Shanghai Expo-axis is an irregular geometric surface, whose shape is approximate to a trumpet flower. Its bottom is an elliptical closed curve with a long axis 70.0m and short axis 12.0m; its top is a closed curve consisting of a semicircle and a semi-ellipse and the height of the entire structure is 41.0m. Fig.1 presents the structural arrangement.

The entire structure composition is a single-layer reticulated shell made up of triangle meshes, in which most of the members use the hollow rectangular welded pipes. Solid members of rectangular section are applied to strengthen the confinement effect in the top circle of the structure; the length of the members is within 1.5m to 3.5m, the cross-section height is within 180mm to 500mm and the width within 65mm to 120mm. Therein, the cross-section exterior size of most of members is 65mm×180mm. All the cross-section sizes are listed in Table 1.



（a）the planar graph （b）three dimensional views

Fig.1 The structural arrangement

Table 1 Sizes of member sections (mm)

| Number | Height | Width | Top flange | Bottom flange | Web thickness |
|---|---|---|---|---|---|
| 1 | 180 | 65 | 10 | 10 | 6 |
| 2 | 180 | 65 | 16 | 16 | 6 |
| 3 | 180 | 65 | 0 | 0 | 0 |
| 4 | 180 | 80 | 16 | 16 | 10 |
| 5 | 180 | 80 | 16 | 16 | 10 |
| 6 | 180 | 80 | 16 | 16 | 16 |
| 7 | 180 | 80 | 20 | 20 | 20 |
| 8 | 180 | 80 | 25 | 25 | 10 |
| 9 | 180 | 80 | 25 | 25 | 16 |

| | | | | | |
|---|---|---|---|---|---|
| 10 | 180 | 80 | 30 | 30 | 20 |
| 11 | 180 | 80 | 30 | 30 | 30 |
| 12 | 180 | 100 | 16 | 16 | 16 |
| 13 | 180 | 100 | 25 | 25 | 16 |
| 14 | 180 | 100 | 30 | 30 | 20 |
| 15 | 180 | 100 | 30 | 30 | 30 |
| 16 | 180 | 120 | 34 | 34 | 34 |
| 17 | 215 | 80 | 40 | 40 | 12 |
| 18 | 216 | 100 | 40 | 40 | 20 |
| 19 | 240 | 120 | 34 | 34 | 30 |
| 20 | 290 | 80 | 40 | 40 | 10 |
| 21 | 290 | 100 | 40 | 40 | 20 |
| 22 | 365 | 120 | 40 | 40 | 20 |
| 23 | 400 | 120 | 40 | 40 | 20 |
| 24 | 500 | 100 | 40 | 40 | 16 |
| 25 | 500 | 140 | 40 | 40 | 30 |

**Note:** Section 3 is rectangular.

The loads applied to the structure contain dead load, live load (0.5kN/m$^2$), wind load (considering eight wind direction angles, according to wind tunnel test results), temperature load (40□, -20℃) and seismic load. The material used here is Q345.

As regards to the constraints, the bottom joints are fixed constraint in vertical direction and elastic restraint along the tangential direction and normal direction of the surface and the elastic stiffness is respectively 1.0E+07kN/m and 2.5E+06kN/m.

In the overall stability analysis of structures, two kinds of load combinations including "dead load + live load" and "dead load + live load+ wind load" are performed. It is unpractical both on time and energy to make the whole process tracking analysis of "load-displacement" considering the nonlinearity for all combinations. Therefore, buckling analysis is done first in various combined load cases to get the load factor and corresponding buckling mode, further, to determine the most unfavorable load combination, and then make a whole process tracking analysis under this load combination.

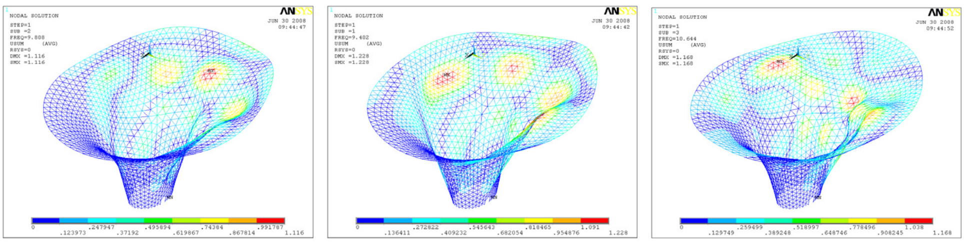
## 3 Buckling Analysis

There are nine load combinations based on "dead load + live load" and "dead load + live load+ wind load", see Table 2. Analyze the nine load combinations and we can obtain the buckling mode of the first three orders of each combination and corresponding load factor.

Table 2 Buckling load factors of the first three orders under the nine load combinations

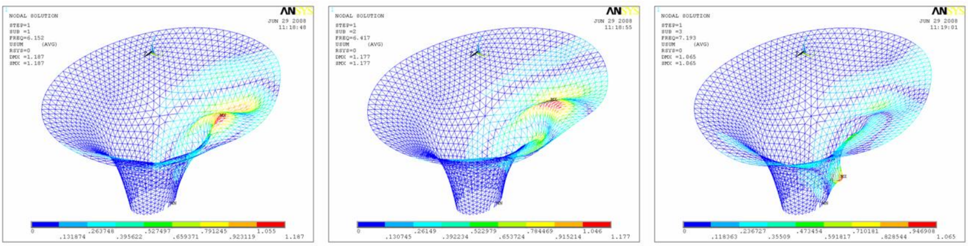
| **Load case** | | **load factor** | | |
|---|---|---|---|---|
| **Number of load case** | **Meanings of load case** | **1st order** | **2nd order** | **3rd order** |
| 1 | D+L | 9.40 | 9.81 | 10.64 |
| 2 | D+L+W0 | 8.97 | 9.51 | 10.00 |
| 3 | D+L+W45 | 8.27 | 8.62 | 8.87 |
| 4 | D+L+W90 | 8.44 | 8.93 | 9.15 |
| 5 | D+L+W135 | 8.29 | 8.79 | 9.48 |
| 6 | D+L+W180 | 8.05 | 8.72 | 9.32 |
| 7 | D+L+W225 | 8.93 | 9.64 | 10.21 |
| 8 | D+L+W270 | 8.23 | 8.59 | 9.38 |
| 9 | D+L+W315 | 6.15 | 6.42 | 7.19 |

As we can see from Table 2, the buckling load factors of the first order under the nine load combinations are all more than 6.0, and therein, the value under the combination "dead load + live load+ wind load (315°)" is the smallest, to be 6.15. The results in Fig. 3 and Fig.4 show that all the buckling modes of the first three orders are overall buckling. As a result, the combination "dead load +live load+ wind load (315°)" can be used for the overall stability analysis.



(a)1st (b)2nd (c)3rd

Fig.2 First three buckling modes under "dead load +live load"



(a)1st (b)2nd (c)3rd

Fig.3 First three buckling modes under "dead load +live load+ wind load (315°)"

# 4 Geometrical Nonlinear Stability Analysis

## 4.1 Geometrical nonlinear overall stability analysis

It is prescribed by Literature [3] that finite element methods considering the geometrical nonlinearity can be used to calculate the stability of reticulated shells based on the assumption that the material keeps linear elastic. Therefore, the paper takes the combination "dead load +live load+ wind load (315°)" as the load case for the overall stability analysis, assumes the material is linear elastic and makes use of "load-displacement" tracking technique to make geometrical nonlinear stability analysis, and the results are presented in Fig.4. In addition, according to Literature [3], it is necessary to take initial geometric imperfections into account to analyze the overall stability of reticulated shells and the maximum value of initial geometric imperfections is recommended 1/300 of the span, and thereby the maximum value of this project can by prescribed as 267.0mm. Based on the consistent mode imperfection method, Fig.5 shows the results when the initial geometric imperfection is taken into account.

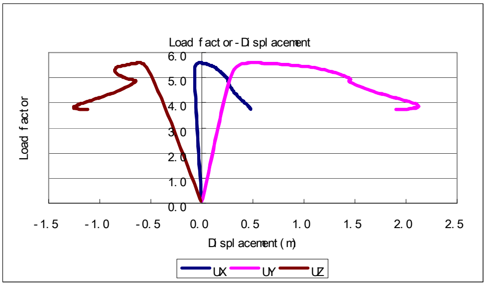



Fig.4 Load-displacement curves considering geometrical nonlinearity

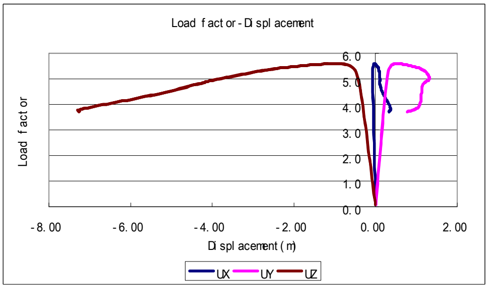



Fig.5 Load-displacement curves considering initial geometric imperfections

From the results in Fig.4 and Fig.5 we can see that when the geometrical nonlinearity is considered, the stable load factor is 5.59, decreased by 9.1% comparing with the results of linear buckling analysis, and when an initial geometric imperfection of 1/300 span is taken into account, the stable load factor is 4.54, going down by 26.2% comparing with the results of linear buckling analysis, which proves that the geometrical nonlinearity plays an important role in the overall stability analysis of reticulated shells and should not be ignored. According to regulations by Literature [3], the stable load factor $K$ can be controlled above 5, and in this project $K$ is 4.54, less than the suggested value of Standard slightly.

## 4.2 Elastic-plastic overall stability analysis

The applied load in the stability analysis is usually some times or even more than ten times of service load and as a result, a part of members fall into plasticity. So the results obtained based on the assumption that the material keeps linear elastic are often too large and non-conservative in the practical engineering. The elastic-plasticity of material is one of the conditions that must be taken into account the stability analysis, and also it is encouraged by Literature [3] to consider the double non-linear full-range analysis. In practical applications, we assume that steel is a bilinear elastic-plastic model.

Here again we take the combination “dead load +live load+ wind load (315°)” as the load case for the overall stability analysis, consider the elastic-plasticity of material and take the buckling mode of the lowest order as the distribution mode when initial imperfections are applied. Now consider the maximum value of initial geometric imperfections is 0mm, 20mm, 40mm, 100mm, and 267mm respectively, to make the tracking analysis of "load-displacement" considering the double nonlinearity. The results are shown by Fig.6 and Table 3.

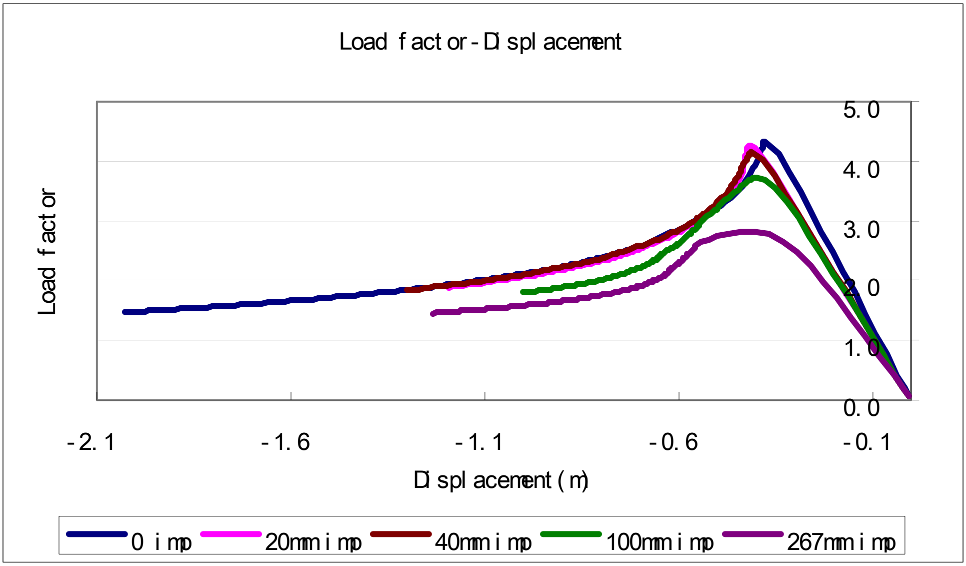



Fig.6 Load-displacement curves considering double nonlinearity

Table 3 Load factor and reduction factor in different conditions

| Conditions | Ultimate load factor | Linear reduction factor compared with linear cases |
|---|---|---|
| Buckling | 6.15 | 0 |
| elastic+0 imperfection | 5.59 | 9.1% |
| elastic +267mm imperfection | 4.54 | 26.2% |
| elastic-plastic +0 imperfection | 4.34 | 29.4% |
| elastic-plastic+20mmimperfection | 4.26 | 30.7% |
| elastic-plastic +40mm imperfection | 4.16 | 32.4% |

| | | |
|---|---|---|
| elastic-plastic +100mm imperfection | 3.72 | 39.5% |
| elastic-plastic +267mm imperfection | 2.83 | 54.0% |

We can see from Fig.6 and Table 3 that when the plasticity of steel is considered, the stable load factor reduces to 4.34, decreased by 29.4% compared with that in linear condition, and with the increase of the imperfection value, the ultimate load case decreases. However as we can see in Table 3, when the initial imperfection increases from 0 to 100.00mm, corresponding ultimate load reduction factor only increases by 10.1%, which shows that the structure is not sensitive to the initial imperfection when the initial imperfection is not very big. But when the imperfection value is relatively big, for example 267.00mm, the corresponding load factor can reduce to 54.0% of that in linear conditions. Hence if the initial imperfection is not properly valued, the results may be not accurate.

## 5 Conclusions

The paper has made the whole process tracking analysis of "load-displacement" of a new type reticulated shell in both elastic and elastic-plastic conditions by using the finite element software ANSYS10.0, and has considered different initial imperfections. The results indicate that the ultimate stable load factor of the structure can reach to 4.54 in the elastic conditions, and when the elastic –plasticity is taken into account, the ultimate stable load factor are still up to 4.16, so it can be regarded that the ultimate stability bearing capacity satisfies the design requirements.

Although there are some related Stands as a basis for the stability analysis nowadays, with the emergence of new types of reticulated shell and the improvement of construction technologies, there are also some pressing problems that must be urgently solved:

(1) How to select the maximum value of imperfections: A suggested value of 1/300 span is prescribed by Literature [3], which is merely applied to specific structural forms such as reticulated spherical shell, cylindrical reticulated shell and ellipsoid reticulated shell, but how to select the value for the new type needs a further study. Even for the regular structures such as reticulated spherical shell, the value is larger than the true one, which causes great difficulty for the practical engineering.

(2) How to determine the stability coefficient *K* considering the elastic-plasticity of material: A suggested value is given by Literature [3] based on the elastic assumption, but the value considering the elastic-plasticity of material is not presented.

The development of reticulated shells has entered a new period when some more novel types of structural system appear; especially the development of structures with free-form surface seems to represent a new trend. With respect to these new types of reticulated shells, how to analyze the overall stability and how to select corresponding parameters are the problems urgent to be solved.

# References

[1] Yin Deyu, Liu Shanwei, Qian Ruojun. Structure design of reticulated shells Beijing: China Architecture & Building Press, 1996.

[2] Shen Shizhao, Chen Xin. Stability of reticulated shells. Beijing: Science Press, 1999.

[3] JGJ 61-2003. Technical specification for latticed shells. Beijing: China Architecture & Building Press, 2003.

[4] Liu Xiao, Luo Yongfeng. Non-linear finite element analysis on stability of complicated large span steel structures. Computer aided engineering, 2007, 9.25-29.