Abstract

The present PhD thesis is focused on numerical analysis and computing of finite difference schemes for several relevant option pricing models that generalize the Black-Scholes model. A careful analysis of desirable properties for the numerical solutions of option pricing models as the positivity, stability and consistency, is provided.

In order to handle the free boundary that arises in American option pricing problems, various transformation techniques based on front-fixing method are applied and studied. Special attention is paid to multi-asset option pricing, such as exchange or spread option. Appropriate transformation allows eliminating of the cross derivative term. Transformation techniques of partial differential equations to remove convection and reaction terms are studied in order to simplify the models and avoid possible troubles of stability.

This thesis consists of six chapters. The first chapter is an introduction containing definitions of option and related terms and derivation of the Black-Scholes equation as well as general aspects of theory of finite difference schemes, including preliminaries on numerical analysis.

Chapter 2 is devoted to solve linear Black-Scholes model for American put and call options. A Landau transformation and a new front-fixing transformation are applied to the free boundary value problem. It leads to non-linear partial differential equation (PDE) in a fixed domain. Stable and consistent explicit numerical schemes are proposed preserving positivity and monotonicity of the solution in accordance with the behaviour of the exact solution.

Efficiency of the front-fixing method demonstrated in Chapter 2 has motivated us to apply the method to some more complicated nonlinear models. A new change of variables resulting in a time dependent boundary instead of fixed one, is applied to nonlinear Black-Scholes model for American options, such as Barles and Soner and Risk Adjusted Pricing models, effective numerical algorithm is constructed for a general case of non-constant volatility in Section 3.1. In order to solve resulting equation various finite difference methods are constructed, including explicit, implicit and alternating direction explicit methods. Studying well-known Newton's method for solving nonlinear system allows to propose new modifications of the method. In Section 3.2 the front-fixing method is tested by American option pricing problem with regime switching model. Since in this model there are several regimes, i.e. several optimal stopping boundaries, the front-fixing transformation leads to a system of nonlinear equations that is solved by using explicit finite difference scheme. The stability of the proposed explicit FDM is studied basing on Von Neumann's approach.

Chapter 4 provides a new alternative approach for solving American option pricing problem based on rationality of investor. There exists an intensity function that can be reduced in the simplest case to penalty approach. The model takes into account possible irrational behaviour of the investor. This approach is applied to regime switching model resulting new model that takes into account possible irrational exercise as well as several states of market in Section 4.2. The rationality parameter approach together with a logarithmic transformation allows to construct effective numerical scheme without applying front-fixing method or LCP formulation. For both, vanilla American option and regime switching model, a

family of weighted schemes is constructed. Qualitative properties of the intensity function and numerical solutions are studied.

Chapter 5 deals with multi-asset option pricing. Appropriate transformation allows eliminating of the cross derivative term avoiding computational drawbacks and possible troubles of stability.

Concluding remarks are given in Chapter 6. All the considered models and numerical methods are accompanied by several examples and simulations. The convergence rate is computed confirming the theoretical study of consistency. Stability conditions are tested by numerical examples. Results are compared with known relevant methods in the literature showing efficiency of the proposed methods.