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Dutch Maritime Museum: Form-finding of an irregular faceted skeletal shell – Part b

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Abstract
In the context of the search for an efficient structural shape to cover the Dutch Maritime Museum courtyard in Amsterdam, the authors briefly discuss the driving design factors that influenced the earliest glass roof coverings. The trends that have emerged during the late 20th and beginning 21st century in the design of skeletal steel glass shells are exposed. These design developments range from sculptural to geometrical and physical intentions (part a). The discussion of the competition design development of the Dutch Maritime Museum Shell roof by Ney and Partners shows the quest for a structurally efficient catenary form based on a poetic geometrical idea. This paper presents a novel methodology that slightly adapts the catenary shape with the objective of achieving planarity in all the triangulated, quad angulated and pent angulated mesh facets. The challenge of facet planarity is gracefully solved and adds to the elegance, structural efficiency and economy of this design (part b).

Keywords: conceptual design, form-finding, steel shell, planarity facets, historic courtyard, Maxwell reciprocal network.

1. Competition design for a steel glass shell over the NSA courtyard

The Dutch Maritime Museum (Nederlands Scheepvaartmuseum Amsterdam NSA) plans a thorough museum reconversion in the near future. At the moment, the restricted space in the 17th century historic building hinders the movement of visitors. The courtyard needs to be integrated in the museum's circulation space, sheltered from weather and kept to a minimal indoor temperature. An invited design competition was held for a new glass roof that added value to the historic building.

1.1. Initial planar geometry

In the late 17th century, this building (shown in figure 1) was the headquarters of the admirality. It was the instrument and symbol of the Dutch maritime power. The development of this sea faring nation was closely linked to the production of sea charts and the associated sciences such as geometry, topography, astronomy etc. The classical building also uses geometry as a basis for design. The choice for the initial 2D geometry of the glasss roof tells the spectator a story about the building's history and its close relationship to the history of the sea. At the origin of this 2D geometry lies a loxidrome map with 16 wind roses (shown in figure 1). This geometrical drawing is found on sea charts displayed inside the museum. At the intersection of the structural sub elements a LED, with variable color and intensity, is inserted. The cupola's structural mesh should read as a fine line drawing against the sky and becomes a powerful scenographic instrument and a symbolic hemisphere.

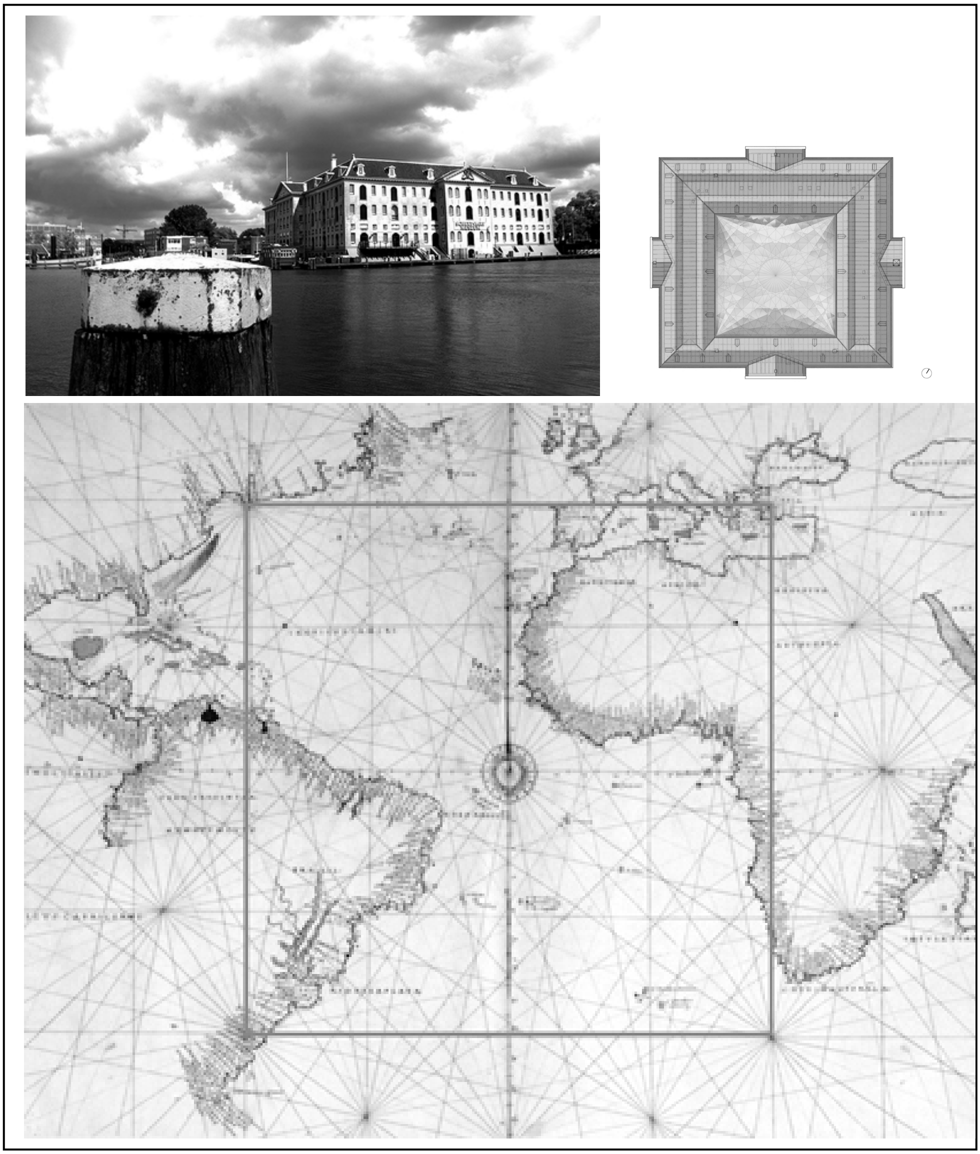

Figure 1: The square courtyard of the NSA will be covered by a steel glass structure whose irregular mesh is based on a loxidrome map with 16 wind roses.

1.2. Physical numerical form and its analysis

Starting from this geometrical 2D mesh pattern an exact 3D shell surface needs to be developed that will hold the shell. The material choice for the skeletal shell is set to steel (taking both compressive and tensile loads). The existing situation imposes the contextual boundary conditions.

- The shell's height cannot appear above the historic building's ridge.
- The courtyard facades can only carry additional vertical loads.
- Any horizontal loads can only be resisted by the four courtyard corners.

The loxidrome 2D map is scaled to the inner courtyard dimensions. One quarter of this 2D grid is modelled with structural elements that have both compressive and tensile load bearing capacity but no bending stiffness. The idea behind the form-finding process is to develop a hanging chain model with only axial loaded members.

The nodes at the boundaries (facades) are restrained in the vertical direction but allowed to move in the direction perpendicular to the façade. The four corner nodes are pinned in all directions. At the intersection of the nodes, the gravity loads due to the self weight of the steel members and glass covering is modelled; this load value differs for most nodes due to the complex 2D geometry of the initial pattern.

An adapted version of dynamic relaxation method with kinetic damping takes into account the contextual boundary conditions, performs the form-finding and results is a funicular 3D cupola with a height of 4.5m shown in figure 2 (ratio height/span = 4.5/34= 1/7). The steel skeletal shell mainly works in compression under self-weight. As to be expected large tensile forces arise in the ring beam framing the shell. The structural elements radiating out from the corners experience the largest compressive forces. Although all boundary nodes can transmit vertical forces onto the facades, the largest vertical reactions are found at the courtyard corners. This event clearly shows that the boundary zones of the shell itself acts as truss along the boundary walls.

After the numerical form-finding the initial geometry of the shell is submitted to a non-linear analysis that might show up instability problems. The shell is submitted to loading combinations of self weight, live load, snow load and wind load. As the cupola should read as a fine line drawing against the sky, all 3 368 elements are dimensioned as steel laths with the same width (40mm) but with variable height (80 – 200mm). The total weight of the steel roof is 90 000kg, the ring beam weighs 32 000kg. A critical linear analysis shows that all elements are realistically loaded far below their critical buckling load. A dynamic analysis finds a eigenfrequency value of 2.46 Hz.

The different analyses show that the shell satisfies all structural criteria. The issue of facet planarity needed for glass panes is dealt with in section 2. This requirement for planarity imposes a slight modification of the geometry of the shell.

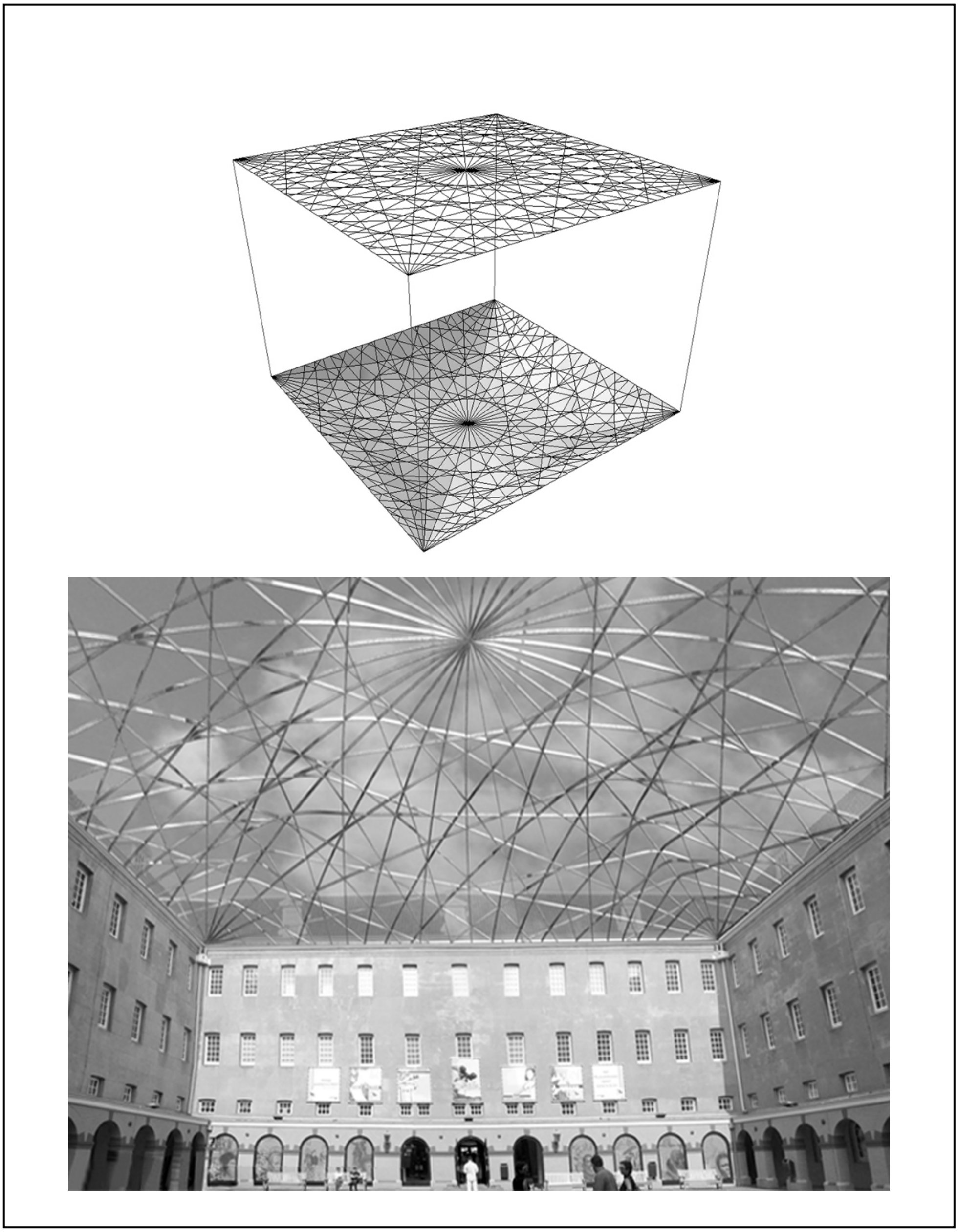

Figure 2: The shape of the shell is form-found to achieve membrane action.

.2. Irregular faceted surface

The plan geometry of the roof is based upon figure 6 in which 16 equally spaced points around a circle are all joined by a total of 120 straight lines. The square plan of the roof itself (figure 3) is the central square part of the circle with only the 4 corner points remaining from the original 16.

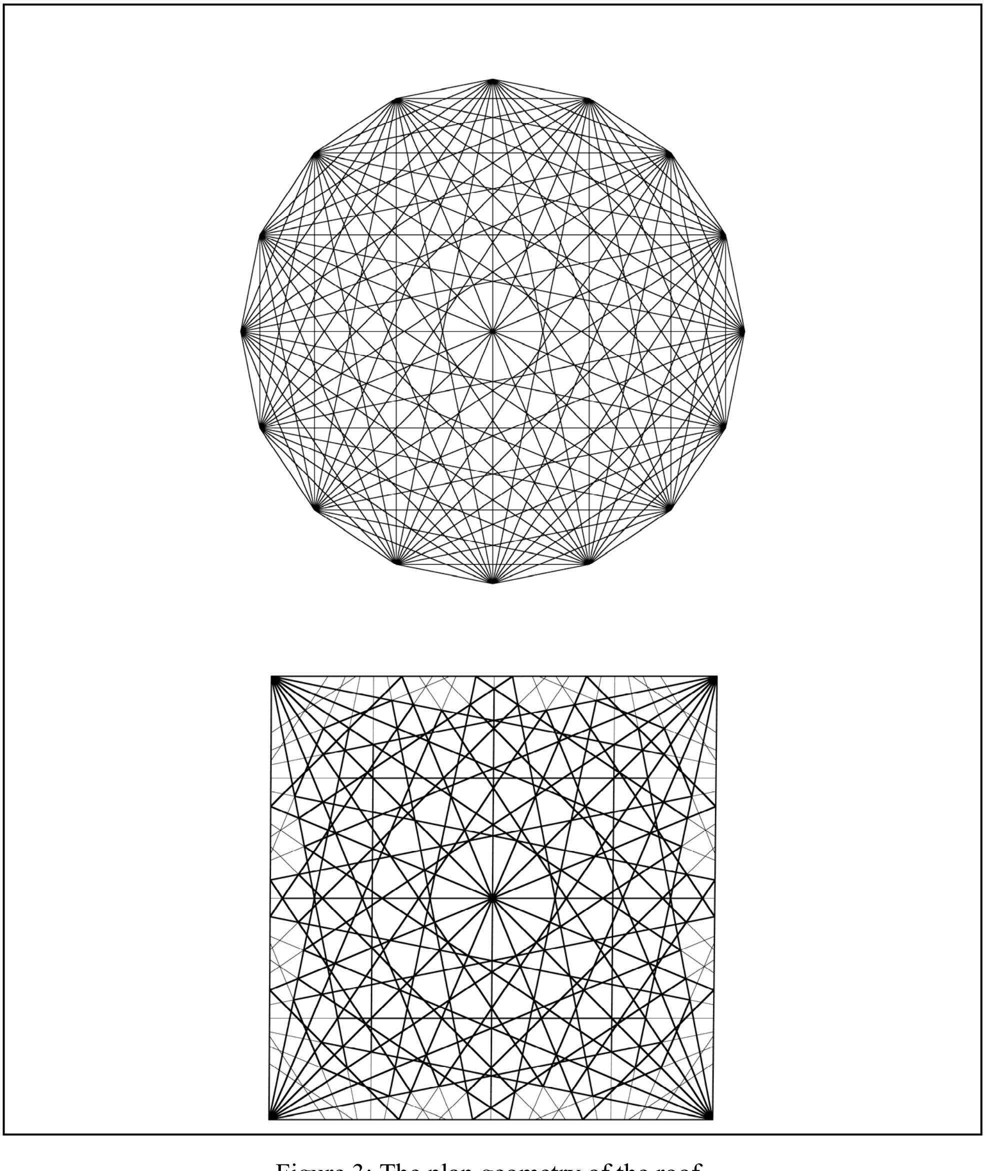

Figure 3: The plan geometry of the roof.

Thus one can calculate (x_i, y_i), the plan coordinates of the i^{th} vertex at which two lines cross. We now need to calculate the heights of the nodes, z_i, so that all the glass facets are flat, even thought the shape of the structure is dome-like as shown in figure 1. Clearly this is only a problem for facets with 4 or more sides since a flat triangle can always be constructed with 3 arbitrary vertices

2. 1. Formulation of the problem

Let one suppose that the equation describing the j^{th} flat facet is

$$z = a_j x + b_j y + c_j \tag{1}$$

If the i^{th} vertex is on the j^{th} facet

$$z_i = a_j x_i + b_j y_i + c_j \tag{2}$$

In order to get the faceted surface to form the dome, we need something to pull it towards the desired shape.

Imagine that the dome were connected to vertical springs at each vertex such that the tension in each spring is equal to

$$s_i\left(z_i - f\left(x_i, y_i\right)\right) \tag{3}$$

This will have the effect of pulling the roof towards the form

$$z = f\left(x, y\right) \tag{4}$$

For the NSA roof $f(x, y)$ was chosen such that

$$\frac{\beta}{z} = \sqrt{\frac{1}{(L-x)^2} + \frac{1}{(L-y)^2}} + \sqrt{\frac{1}{(L-x)^2} + \frac{1}{(L+y)^2}} + \sqrt{\frac{1}{(L+x)^2} + \frac{1}{(L-y)^2}} + \sqrt{\frac{1}{(L+x)^2} + \frac{1}{(L+y)^2}} \tag{5}$$

in which β is a constant, $2L$ is the side length of the square and the origin of coordinates is at the centre of the square. The spring stiffnesses, s_i, are chosen to be proportional to the plan area in the region of each node.

Thus we have the mathematical problem of minimising the 'strain energy' function

$$U = \frac{1}{2} \sum_{i=1}^{\text{last vertex}} s_i \left(z_i - f(x_i, y_i)\right)^2 \tag{6}$$

subject to the constraints

$$z_i = a_j x_i + b_j y_i + c_j \tag{7}$$

for each vertex of each face. The solution was found using Lagrange multipliers. It beyond the scope of this article to describe the process, but some understanding can come from the fact the Lagrange multipliers are the vertical forces that the facets apply to the vertices to prevent the facets becoming bent by the vertical springs.

2.2. Maxwell reciprocal diagram

The problem of finding flat facets to approximate a curved surface is identical to that of finding tensions in a plane prestressed network. This may be a useful concept in that it is easier to imagine forces in a flat network than folds in a surface. The reason that the two problems are identical is based upon the following reasoning.

Let us imagine that the z coordinate of our faceted surface represents an Airy stress function [1], ϕ. The corresponding plane stresses are

$$\sigma_x = -\frac{\partial^2 \phi}{\partial y^2}$$
$$\sigma_y = -\frac{\partial^2 \phi}{\partial x^2} \quad .$$
$$\tau_{xy} = +\frac{\partial^2 \phi}{\partial x \partial y} \tag{8}$$

The curvature of the surface is zero on the facets and concentrated in the folds between them. Thus there is no stress in the areas of the facets and a concentration of stress at the folds. This corresponds to a plane, horizontal prestressed network of struts and ties. Because the dome is convex to the outside the folds on the surface are ties and the upwards folds from the horizontal around the boundary are struts. Thus the problem of finding flat facets is mathematically identical to finding states of prestress in a plane network of struts and ties. This analogy immediately tells us that there can be no fold along the thinner lines in figure 2. The state of stress in a network can be represented graphically using the reciprocal network proposed independently by James Clerk Maxwell and W P Taylor [2-3] In the UK the technique is associated with Bow's notation and on continental Europe with Cremona [4]. Figure 4 shows the reciprocal network applied to the tensions and compressions corresponding to the folds in figure 6. The length of a line represents the change in slope between two facets.

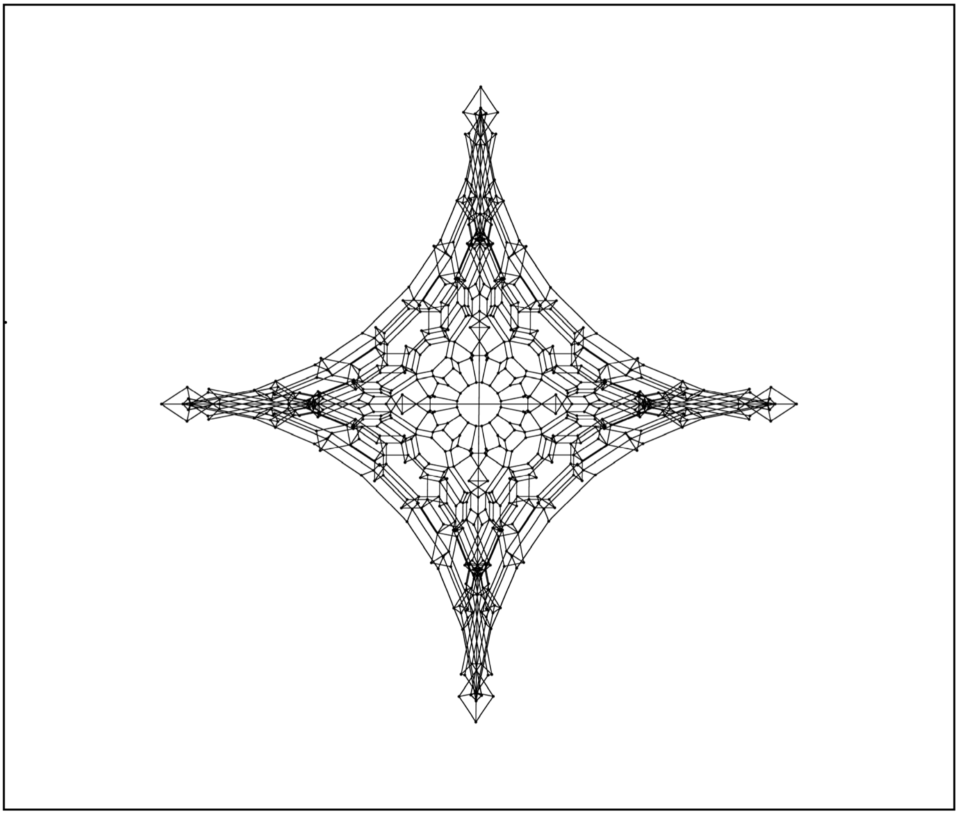

Figure 4: Maxwell reciprocal network diagram for the shell.

3. Conclusion

The presented design won the competition. The skeletal steel shell successfully uses a 2D geometrical mesh idea (the loxidrome chart) to find a 3D physical catenary shape that respects all contextual boundary conditions. The complexity of obtaining planarity in all the quadrangulted and pentagulated facets of the skeletal shell is solved in an analytical "origami" approach. The designer Laurent Ney [5] argues that the freedom of a form lies exactly in the selection of the material and the right boundary conditions.. The shell is found to be both poetic and efficient.

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