

Análisis del Filtro FPGA en Imágenes de Tomografía Computarizada para la Reducción de Dosis Radiactiva

Estíbaliz Parceró, Vicente Vidal, Gumersindo Verdú, Josep Arnal, Patricia Mayo*

Abstract— Las imágenes de Rayos-X o de tomografía computarizada (CT) pueden contener ruido debido al proceso de adquisición. Este ruido complica sustancialmente el proceso diagnóstico, por lo que será necesario el desarrollo de filtros efectivos. En este trabajo se estudia el comportamiento del filtro Fuzzy Peer Group Averaging (FPGA) sobre una colección de imágenes mamográficas que ha sido previamente contaminada con ruido impulsivo y gaussiano. El objetivo del trabajo es averiguar si FPGA es adecuado para la mejora de imágenes CT obtenidas con una dosis de radiación reducida. Los resultados indican que FPGA se comporta, efectivamente, mejor que el resto de métodos estudiados en este trabajo y por tanto resulta un candidato adecuado.

I. INTRODUCTION

Filtering techniques to improve images, i.e. to detect and correct noise in images, has been a main subject in the last years, particularly in medical image (X-Rays or computer tomography CT methods) where the quality of the image can influence the diagnosis of a disease (for instance, in the detection of microcalcifications in a mammogram).

Besides that a good filter can be employed to improve the output image as a result of using a reduced radiation dose, as in CT images, where exposure to X-Rays is very high.

In this work we compare filtering methods over one mammogram image from de Database of mini-MIAS[1]. The methods studied are the proposed in [2-7] and the method proposed in [8]. In this first study the Peer Group with Fuzzy Metric (PGFM) method and the Non-linear Diffusion method (NDF) were used over a black and white image of a mammogram, that is, in the medical domain. In the second one the method Fuzzy Peer Group Average filter is presented. This filter was used over a set of color images unrelated to the medical domain. The object of this study is to test the usefulness of FPGA in the medical image domain as we did in a previous work [2] and compare the results in terms of quality measures using the same mammogram image employed in the mentioned study, and adding the same amount of impulsive and/or Gaussian noise to be able to compare the methods in an appropriate manner. A good restoration algorithm can lead to the reduction of the radiation dose used as was stated in [9].

Several methods have been studied for image filtering determined by the type of noise to remove, for instance, for Gaussian noise, the methods based on filtering the image in the space or in the frequency domain (see [10] for a review), methods based on solving regularized least-squares problems [11] and methods based on the use of non-linear Diffusion equations [12-21]. In the case of impulsive noise, recent techniques based on the concept of peer group with fuzzy metric which have provided good results in RGB images [22-24].

The paper is organized as follows: Section II explains the different compared methods. The results of the experimental study are shown in Section III and, the conclusions are presented finally, in Section IV.

II. METHODS OF NOISE SUPPRESSION

A. Peer Group and Fuzzy Metric (PGFM)

This method is a two-tiered process. The first one try to detect erroneous pixels and the second step try to correct them. For the detection stage, the fuzzy metric between pixel x_i and x_j is used as described in [22], which is given by the following function:

$$M(x_i, x_j) = \frac{\min\{x_i, x_j\} + k}{\max\{x_i, x_j\} + k}, \quad (1)$$

where $k > 0$.

The value k reduces the non-uniformity or avoids reducing the significance of the data for two different consecutive pairs or distanced vectors. Fuzzy metric (1) is employed in peer group $P(x_i, d)$, where x_i is the central pixel in a window W with size $n \times n$ (in present study, $n = 3$ was considered) and $d \in [0, 1]$. The representation of the $P(x_i, d)$, in mathematical formulation, is as follows:

$$P(x_i, d) = \{x_j \in W: M(x_i, x_j) \geq d\}. \quad (2)$$

The peer group [23] associated with the central pixel x_i of W is the set formed by the central pixel and neighbouring pixels, that are part of the window whose fuzzy distance from x_i is greater than d .

The detection step performs two phases. The first phase calculates the peer group of x_i in W and all pixels that belong to the peer group. It is declared as non-corrupted if the cardinality of the $P(x_i, d)$ is greater than $(m+1)$, where m is a threshold. Otherwise they are labeled as undiagnosed. In the second phase, the pixels labeled as undiagnosed are analyzed. All pixels that belong to the peer group are labeled as non-corrupt if the cardinality of the $P(x_i, d)$ is greater than $(m+1)$, otherwise the central pixel is marked as corrupted.

* Estíbaliz Parceró is with the Institute for Industrial, Radiophysycal, and Environmental Safety, Universitat Politècnica de València, Valencia, Spain (e-mail: esparig@upvnet.upv.es).

Vicente Vidal is with the Department of Informatics Systems and Computing, Universitat Politècnica de València, Valencia, Spain (e-mail: vvidal@dsic.upv.es).

Gumersindo Verdú is with the Department of Chemical and Nuclear Engineering, Politècnica de València, Valencia, Spain, (e-mail: gverdu@iqn.upv.es).

José Arnal is with the Department of Computation Science and Artificial Intelligence, Universidad de Alicante, Alicante, Spain (e-mail: arnal@dccia.ua.es)

Patricia Mayo, Titania (Grupo Dominguis), Valencia, Spain (e-mail: p.mayo@titaniast.com)

The three parameters (k , d and m), which are determined heuristically in the described process take values in a certain range depending on the input image. The value of d depends on the amount and type of noise introduced.

In the correction step, given a x_i previously marked as corrupted, we replace it by the Arithmetic Mean Filter (AMF) [25] of its neighbour pixels (labeled as non-corrupted) in its window W .

B. Non-linear diffusive filter (NDF)

As mentioned in the introduction, a class of image restoration methods is based on the use of non-linear Diffusion equations [12-17]), which appear associated to a variation problem and, may be obtained from the minimization of the appropriate functional. The choice of a particular functional depends upon the specific goal of interest. For example, several diffusive filters, suitable for medical imaging [18], have been obtained from the minimization of the appropriate functional.

Let us consider the functional [19],

$$J(u, \beta, \mu, \varepsilon) = \int_{\Omega} \left[\sqrt{\beta^2 + \left| \vec{\nabla} u \right|^2} + \frac{\mu}{2} (u - I_0)^2 + \frac{\varepsilon}{2} \left(\vec{\nabla} u \right)^2 \right] d\vec{x}, \quad (3)$$

where I_0 is the observed image (with noise), u is filtered image, μ and ε are constant and Ω is a convex region of \mathbb{R}^2 constituting the support space of the surface $u(x, y)$, representing the image. The first term in the functional for $\beta = 1$ represents the area of the surface representing the image, [14], the second term gives account of the distance between the observed image and the desired solution u (Filtered image), and the third term controls the regularity of the solution.

We will consider the minimization problem [13], [14]

$$\min_u J(u, \beta, \mu, \varepsilon) \quad \text{subject to} \quad \frac{\int_{\Omega} (u - I_0)^2 d\vec{x}}{\int_{\Omega} d\vec{x}} = \sigma^2 \quad (4)$$

that is, we search for the image u that minimizes the functional $J(u, \beta, \mu, \varepsilon)$ and presents a variance with respect to the observed image I_0 equal to σ^2 ; σ , the noise standard deviation of the image a priori, is unknown, but it is important to know its value to minimize equation (4). In our work we estimate, σ , by taking the median absolute deviation of the empirical wavelet coefficient of the finest scale and dividing by 0.6745 [18]. For all the images studied, the wavelet was a Daubechey of order 25. This process is the key stone of the non-linear diffusive filter.

For the time discretization, we use a semi-implicit scheme, and for solving the equations we use the alternative additive operator splitting (AOS) [15], [19]. The stopping time selection in the Diffusion equation was proposed by Mrázek and Navara, based on the decorrelation criterion [20].

C. Fuzzy Non-linear diffusion filter (FNLDF)

This technique is the combination of PGFM and NDF method. The sequence of application of the methods is as follows: first PGFM and then NDF. The peer group with

fuzzy metric approach removes the impulsive noise and the Gaussian noise is eliminated by NDF.

D. Fuzzy Peer Group Averaging Filter (FPGA)

This filter performs in two steps, (i) impulse noise detection and reduction, and (ii) Gaussian noise smoothing. Both steps use the fuzzy peer group of a central pixel x_i in a window W of $n \times n$ according to [8] and using a fuzzy metric.

The definition of peer group is based on the ordering of the pixel neighbors with respect to its similarity to the central pixel x_0 .

Let ρ be an appropriate similarity measure between two color vectors. Color vectors $x_i \in W$ are sorted in a descending order with respect to their similarity to x_0 , obtaining an ordered set $W' = \{ x_{(0)}, x_{(1)}, \dots, x_{(n-1)} \}$ such that $\rho(x_0, x_{(0)}) \geq \rho(x_0, x_{(1)}) \geq \dots \geq \rho(x_0, x_{(n-1)})$, where $x_0 = x_{(0)}$. The peer group $P_m^{x_0}$ of $m + 1$ members associated with pixel x_0 is the set

$$P_m^{x_0} = \{ x_{(0)}, x_{(1)}, \dots, x_{(m)} \} \quad (5)$$

In [8], a fuzzy logic-based method is proposed to determine the best number of members \hat{m} of a peer group. The fuzzy peer group of a central pixel x_0 in a window W according to [8] is defined as the fuzzy set $FP_m^{x_0}$ defined on the set $\{ x_{(0)}, x_{(1)}, \dots, x_{(\hat{m})} \}$ and given by the membership function $FP_m^{x_0} = \rho(x_0, x_{(i)})$. Then the best number \hat{m} of members of $P_m^{x_0}$ is defined as the value of $m \in N_W = \{1, 2, \dots, n^2 - 1\}$ maximizing the certainty of the following fuzzy rule.

Fuzzy Rule 1: Determining the certainty of m to be the best number of members for $P_m^{x_0}$

IF " x_m is similar to x_0 " and the accumulated similarity for $x_{(m)}$ is large THEN "the certainty of m to be the best number of members is high".

$C_{FR1}(m)$ denotes the certainty of the Fuzzy Rule 1 for m . Then, $C_{FR1}(m)$ is computed for each $m \in N_W$ and the value which maximizes the certainty is selected as the best number \hat{m} of members of $P_m^{x_0}$, i.e., $\hat{m} = \operatorname{argmax}_{m \in N_W} C_{FR1}(m)$.

The certainty of " x_m is similar to x_0 " is given by the membership function C^{x_0} determined by the similarity measure

$$C^{x_0}(x_{(i)}) = \rho(x_0, x_{(i)}), i = 0, 1, \dots, n^2 - 1 \quad (6)$$

The accumulated similarity for x_m denoted $A^{x_0}(x_{(m)})$ is defined by

$$A^{x_0}(x_{(i)}) = \sum_{k=0}^{i-1} \rho(x_0, x_{(k)}), i = 0, 1, \dots, n^2 - 1 \quad (7)$$

Then, the certainty of " $A^{x_0}(x_{(m)})$ is large" is given by the membership function L^{x_0} defined by

$$L^{x_0}(x_{(i)}) = \frac{(A^{x_0}(x_{(i)}) - 1)(A^{x_0}(x_{(i)}) - 2n^2 + 1)}{(n^2 - 1)^2}, \quad (8)$$

$i = \{0, 1, \dots, n^2 - 1\}$

The product t-norm was used as the conjunction operator and therefore no defuzzification is needed. Then, $C_{FR1}(m) = C^{x_0}(x_{(m)}) L^{x_0}(x_{(m)})$.

The fuzzy similarity function, ρ , used was

$$\rho(x_i, x_j) = e^{-\frac{\|x_i - x_j\|}{F_\sigma}}, i, j = 0, 1, \dots, n^2 - 1 \quad (9)$$

where $\|\cdot\|$ denotes the Euclidean norm and F_σ is a parameter which will be discussed in Section III. This function ρ takes values in $[0, 1]$ and satisfies that $\rho(x_0, x_i) = 1$ if and only if $x_0 = x_i$.

Another fuzzy rule is used to detect impulse noise.

Fuzzy Rule 2: Determining the certainty of the pixel x_0 to be free of impulse noise

IF "accumulated similarity $A^{x_0}(x_{(\hat{m})})$ is large" and " $x_{(\hat{m})}$ is similar to x_0 " THEN " x_0 is free of impulse noise".

In order to compute the certainty of the Fuzzy Rule 2, denoted by C_{FR2} , the certainty of " $A^{x_0}(x_{(\hat{m})})$ is large" is given by L^{x_0} , (defined in (8)) and the certainty of " $x_{(\hat{m})}$ is similar to x_0 " is given by C^{x_0} given by formula (6). The t-norm product is used as conjunction operator and then $C_{FR2}(x_0) = C^{x_0}(x_{(\hat{m})}) L^{x_0}(x_{(\hat{m})})$. This certainty is already computed since $C_{FR2}(x_0) = C_{FR1}(\hat{m})$ and then no additional computation is needed. If the certainty of Fuzzy Rule 2, C_{FR2} satisfies

$$C_{FR2}(x_0) \geq F_t, \quad (10)$$

then x_0 is free of impulse noise else x_0 is an impulse and it is replaced with VMF_{out} [26]. F_t is a threshold parameter with values in $[0,1]$ which will be discussed in Section III.

III. RESULTS

In this section we present the experimental results of filtering a grayscale image taken from the database of mini-MIAS [1] (Fig. 1) with the filters mentioned in the previous section.

We added Gaussian and fixed impulsive noise to the image. In order to measure the resulting quality of the images, we used PSNR and MAE. PSNR (*Peak Signal-to-Noise Ratio*) is used to measure noise reduction and MAE (*Mean Absolute Error*) is used for the preservation of the signal. To define the PSNR, we need to calculate the mean square error (MSE), which for two monochrome images u (Filtered image) and I_0 (Observed image) of size $M \times N$ is defined as:

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left\| I_0(i, j) - u(i, j) \right\|^2 \quad (11)$$

where $M \times N$ is the image size.

Thus, the PSNR is defined as:

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right) \quad (12)$$

where MAX_I is the maximum possible pixel value of the image.

The mean absolute error is given by,

$$MAE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left| I_0(i, j) - u(i, j) \right| \quad (13)$$

We tested the image with different types and amount of noise. The first image contains 0.10 noise density (D) of fixed impulsive noise, the second one contains $\sigma = 0.01$ for generate Gaussian noise and the last one contains a mix of impulsive noise with $D=0.10$ and Gaussian noise with $\sigma=0.01$.

A study was conducted to find the best value of d and m for each case in the PGFM method. Table I shows the best results for each type of noise. Values d and m depend on the type and amount of noise introduced. In the cases which involve the variance, m has the same value (8, all neighbors), otherwise the value is 4. With variance of 0.01, the value of d is 0.92.

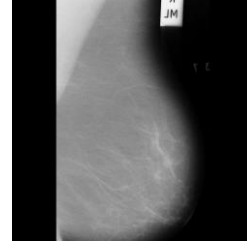


Figure 1. Original Image. 1024x1024

TABLE I. BEST VALUE OF THE PARAMETERS M AND D.

	m	D
$D=0.10$ for fixed impulsive noise	5	0.85
$\sigma=0.01$ for Gaussian noise	8	0.92
$D=0.10$ for fixed impulsive noise with $\sigma=0.01$ for Gaussian	8	0.92

Through a similar process to that used in article [14], there was obtained the value of k whose optimal value is usually 1024 for this type of images.

Another study was conducted to find the best F_t and F_σ parameters for the FPGA method. Table II shows the best results for each contaminated image.

TABLE II. BEST VALUE OF THE PARAMETERS F_t AND F_σ .

	F_t	F_σ
$D=0.10$ for fixed impulsive noise	0.85	475
$\sigma=0.01$ for Gaussian noise	0.85	340
$D=0.10$ for fixed impulsive noise with $\sigma=0.01$ for Gaussian	0.85	350

Once the heuristic parameters k , d , m , F_t and F_σ are determined we will perform a comparative analysis of the performance of the filters FNLDF, PGFM and NDF, and we compare them with FPGA for an image size of 512×960.

Applying the filters to the image with 10% fixed impulsive noise, we obtained the quality of the filtered image from the original shown in table III and figure 2. As we can see, when the image contains only impulsive noise, the best method is FPGA. We can also use the PGFM or the FNLDF method with a little quality difference below the FPGA method. The NDF method does not provide good image quality.

TABLE III. RESULTS OF QUALITY FOR THE IMAGE WITH $D = 0.10$ (FIXED IMPULSIVE NOISE)

	MSE	PSNR	MAE
Filtered image with PGFM	5.5365	40.6985	0.1793
Filtered image with NDF	36.4523	24.3934	9.4899
Filtered image with FNLDF	7.1813	39.5688	0.759
Filtered image with FPGA	4.8948	41.2335	1.1214
Noisy image	1.90E+02	15.3459	12.821

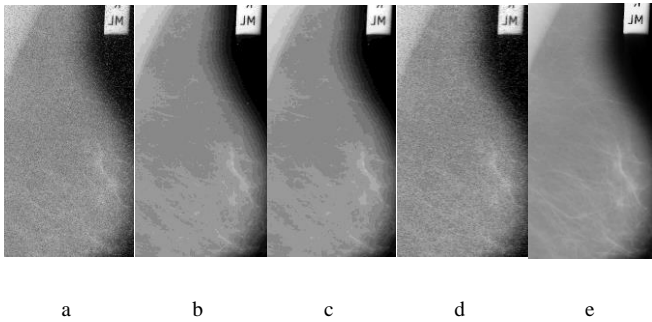


Figure 2. Results for image size 512x960: a) $D = 0.01$ for fixed impulsive noise, b) Filtered image with PGFM, c) Filtered image with FNLDF, d) Filtered image with NDF, e) Filtered image with FPGA

TABLE IV. RESULTS OF QUALITY FOR IMAGE WITH 0.01 OF GAUSSIAN NOISE

	MSE	PSNR	MAE
Filtered image with PGFM	305.8954	23.2751	13.266
Filtered image with NDF	88.5022	28.6613	7.3277
Filtered image with FNLDF	99.7893	28.14	6.9826
Filtered image with FPGA	68.9717	29.7441	6.4761
Noisy image	621.5723	20.1959	19.672

In the case of images only with Gaussian noise, again FPGA outperforms the rest of the methods, the performance of Diffusion method (NDF) and FNLDF have similar results and better than PGFM method. Table IV shows the results and figure 3 shows the resulting image.

For images contaminated with two types of noise (Table V and Fig. 4) shows that the FPGA method is slightly better than FNLDF which is about 4 units PSNR better than the other methods, and with respect to the noisy image is approximately 15 units PSNR better.

IV. CONCLUSIONS

In this paper we present the results obtained by applying the FPGA method and comparing it with FNLDF, PGFM and NDF methods to remove the impulsive noise (fixed), Gaussian and a mix of the two of them on a mammogram obtained from the database mini-MIAS.

If the image contains only impulsive noise (fixed), the best technique is FPGA, although the methods PGFM and FNLDF provide similar results. If the image contains only Gaussian, the best technique for removing noise is FPGA again, followed closely by the NDF and the FNLDF methods. When the image contains the discussed combination of noise, although FPGA get the best PSNR score, the MSE and the MAE scores denote that some improvement could be made. FPGA shows a good behavior in all types of images revealing itself as a good method to be employed in CT images for reduction of radiation dose.

In view of this, the future works will include the study of FPGA over a set of images with a variable amount of radiation dose in order to quantify the improvement. Besides that, and due to the high computational cost of the process, we will introduce high performance computing (GPUs, Multicore, libraries).

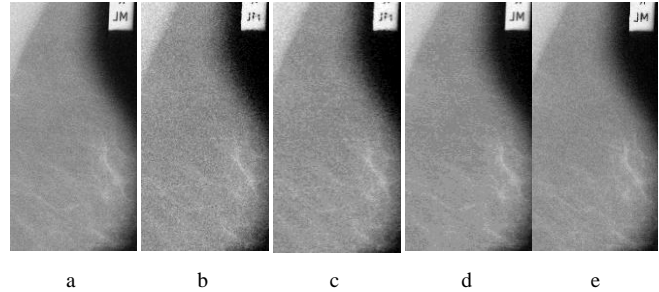


Figure 3. Results for image size 512x960: a) $\sigma = 0.01$ for Gaussian noise, b) Filtered image with PGFM, c) Filtered image with FNLDF, d) Filtered image with NDF, e) Filtered image with FPGA.

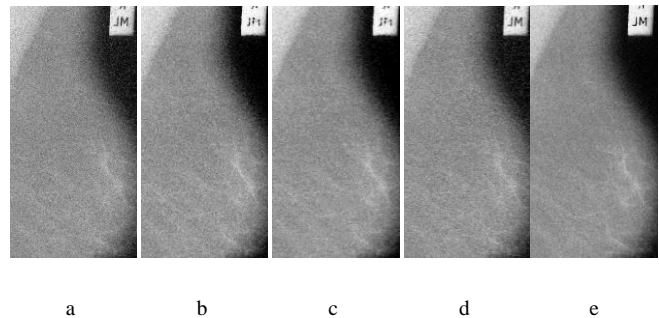


Figure 4. Results for image size 512x960: a) Density $D=0.10$ for fixed impulsive and $\sigma = 0.01$ for Gaussian noise, b) Filtered image with PGFM, c) Filtered image with FNLDF, d) Filtered image with NDF, e) Filtered image with FPGA.

TABLE V. RESULTS OF QUALITY FOR IMAGE WITH (D) = 0.10 AND 0.01 OF GAUSSIAN (FIXED IMPULSIVE AND GAUSSIAN NOISE)

	MSE	PSNR	MAE
Filtered image with PGFM	322.9214	23.0398	13.6645
Filtered image with NDF	299.4993	23.3668	12.5675
Filtered image with FNLDf	103.1628	27.9956	7.3411
Filtered image with FPGA	74.9976	29.3803	6.6121
Noisy image	2.23E+03	14.6383	29.6629

ACKNOWLEDGMENTS

This work was partially funded by ANITRAN PROMETEO/2010/039, the Spanish Ministry of Science and Innovation (Project TIN2008-06570-C04-04), and the spin-off Titania (Grupo Dominguis).

REFERENCES

[1] J. Suckling et al. The mammographic image analysis society digital mammogram database. In *Exerpta Medica*, pp. 375-378. 1994.

[2] M.G. Sánchez, V. Vidal, G. Verdú, P. Mayo and F. Ródenas, "Medical Image Restoration with Different Types of Noise", *IEEE, Engineering in Medicine and Biology Society*, pp. 4382-4385, 2012.

[3] V. Vidal, G. Verdú, F. Ródenas, and P. Mayo, "Restauración de Imágenes Médicas con Diferentes Tipos de Ruido", *Revista de la Sociedad Nuclear Española*, vol. 1, pp. 33-42, 2013.

[4] V. Vidal, G. Verdú, and P. Mayo, "Un Método Híbrido de Restauración de Imágenes Médicas con Ruido Gausiano y/o Impulsivo", *Revista de la Sociedad Nuclear Española*, vol. 1, pp. 98-106, 2011.

[5] Vidal, J. Bataller, J. Arnal, and M.G. Sánchez, "A Parallel Method for Impulsive Image Noise Removal on Hybrid CPU/GPU Systems", *International Conference on Computational Science (ICCS 2013): "Computation at the Frontiers of Science"*, pp. 2504-2507, Barcelona, 2013.

[6] G. Verdú, J. Vidal, B. Jeanine, V. Vidal, and F. Ródenas, "Dose Reduction using Non-Linear Diffusion and Smoothing Filters in Computed Radiography", *12th International Symposium on Radiation Physics*, pp. 120, Rio de Janeiro, 2012.

[7] G. Verdú, J. Vidal, B. Jeanine, V. Vidal, and P. Mayo, "Estimated Radiation Dose Reduction using Non-Linear Diffusion Method in Computed Radiography", *34th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, pp. 1502-1505, San Diego, 2012.

[8] S. Morrillas, V. Gregori and A. Hervas, "Fuzzy Peer Groups for Reducing Mixed Gaussian-Impulse Noise from Color Images", *IEEE Transactions on Image Processing*, vol. 7, pp. 1452-1466, 2009.

[9] M.G. Sánchez, B. Juste, V. Vidal, G. Verdú, P. Mayo and F. Ródenas, "Estimated Radiation Dose Reduction Using Non-Linear Diffusion Method in Computed Radiography", *IEEE, Engineering in Medicine and Biology Society*, pp. 1502-1505, 2012.

[10] J.C. Russ, "The image processing Handbook", CRC Press, 2002.

[11] M. Bertero, P. Boccaci, "Introduction to inverse problems in imaging", *CRC Institute of Physics Publishing*, 1998.

[12] F. Catté, P. Lions, J. Morel and T. Coll, "Image Selective Smoothing and Edge Detection by Nonlinear Diffusion". *SIAM Numerical Analysis*, vol. 29, pp.182, 1992.

[13] L.I. Rudin, S. Osher, E. Fatemi, "Nonlinear total variation based noise removal algorithm", *Physica D*, vol. 60, pp. 259, 1992 .

[14] C.R. Vogel, M.E. Oman, "Iterative methods for total variation denoising", *SIAM J. Sci. Comput.*, vol. 17, 1, pp. 227, 1996.

[15] J. Weickert, B.M. ter Haar Romeny, M.A. Viergever, "Efficient and reliable schemes for nonlinear diffusion filtering", *IEEE Transactions on Image Processing*, vol. 7, 3, pp. 398, 1998.

[16] A. Marquina, S. Osher, "Explicit algorithms for a new time dependent model based on level set motion for nonlinear deblurring and noise Removal". *SIAM J. Sci. Comput.*, vol. 22, 2, pp. 387, 2000.

[17] T. Chan, A. Marquina, P. Mulet. "High-order total variation-based image restoration". *SIAM J. Sci. Comput.*, vol. 22, 2, pp. 503, 2000.

[18] S.L. Keeling, "Total variation based convex filters for medical imaging", *Applied Mathematics and Computation* vol. 139, pp. 101, 2003.

[19] J. Weickert, "Efficient image segmentation using partial differential equations and morphology". *Pattern Recognition*, vol. 34, pp. 1813, 2001.

[20] P. Mrázek and M. Navara, "Selection of optimal stopping time for nonlinear diffusion filtering", *International Journal of Computing Vision*, vol. 52 pp. 189, 2003.

[21] D.L. Donoho, "De-Noising by Soft-Thresholding". *IEEE Transactions on Information Theory*, vol. 41, pp. 613, 1995.

[22] J. G. Camarena, V. Gregori, S. Morillas, A.Sapena, "Fast detection and removal of impulsive noise using peer group and fuzzy metrics", *Journal of Visual Communication and Image Representation*, vol. 19, pp. 20, 2008.

[23] B. Smolka, "Fast detection and impulsive noise removal in color Images". *Real-Time Imaging* vol. 11, pp.389, 2005.

[24] M. G. Sánchez, V. Vidal, J. Bataller, J. Arnal, "A Fuzzy Metric in GPUs: Fast and Efficient Method for the Impulsive Image Noise Removal". In *Proc. ISICIS 2011*, pp. 323-330, 2011.

[25] Ioannis Pitas and Anastasios N. Venetsanopoulos, "Nonlinear Mean Filters in Image Processing", *IEEE transactions on acoustics, speech, and signal processing*, vol. 3, pp.573,1986.

[26] J. Astola, P. Haavisto, and Y. Neuvo. "Vector Median Filters", *Proc. IEEE*, vol. 78, no. 4, pp. 678-689, 1990.