

Dodecahedral mutually supported element space structure experimental and numerical modelling investigation: discussion of results

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Abstract

Mutually supported elements (MSE) arranged in closed circuits can create single module lattice type space structures. These may be connected to one another in networks to generate much larger space structures. To broaden the understanding of the structural behaviour of bolted MSE structures an investigation was carried out. A dodecahedral MSE structure was selected for detailed study because of its uniformity: all of the MSE circuits were identical in nature. The overall objective was to determine the predicted behaviour using numerical modelling and compare it to monitored behaviour in the laboratory. Global and Finite element (FE) linear elastic analyses output was compared against displacements and the strain distribution developed in one of the most heavily loaded modules. Global analysis predicted combined shear and torsional stresses were low, implying that the primary action in the circuit elements was due to bending. FE modelling values were found to be very close to those experimentally obtained; the onset of local yielding was indicated as occurring in the areas adjacent to the bolted connection positions. The recorded strains included the effects of axial forces, bi-axial bending, shear and torsion. According to the von Mises ductile material failure criterion, yielding of the material at the monitored locations had not occurred.

Keywords: Mutually Supported Elements, Space Structures, Experimental Investigation, Numerical Modelling, Structural Behaviour.

1. Introduction

Mutually supported elements (MSE) arranged in closed circuits create MSE modules. These modules are also known as reciprocal frames [1] or nexorade fans [2, 3]. MSE circuits may be connected to one another to generate much larger space structures. Such configurations are generally 3-dimensional and non-traditional in form; many of the larger

MSE structures having a similar appearance to woven basket assemblies [4, 5]. Pairs of elements in module circuits are connected together such that they rely on each other for mutual or reciprocal support. There are various ways of connecting circuit elements together: bolting being one of the most simple and effective methods [4 – 9]. Space structures assembled and connected in this way have the potential advantage of eliminating complex ball-joint type connectors traditionally used in lattice type assemblies [6]. Commonly used sloping elements in MSEs space structure configurations however frequently produce complexity in both the overall geometry and structural behaviour due to the creation of eccentricities.

An investigation was carried out to broaden the understanding of the behaviour of space structures composed of bolted MSE circuits. Various MSE configurations were initially evaluated for study. Ease of structure fabrication, assembly, loading and unloading, restraining, and monitoring in the laboratory, modelling the structure numerically, both globally and locally, were factors that were considered in the selection [4].

2. Dodecahedral MSE Structure

A dodecahedral structure was selected for study because of its uniformity: all of the MSE circuits were identical. The rotation method [3, 5] was used to create the configuration. That is, the geometry of an elementary dodecahedron was transformed to an equivalent MSE dodecahedron [6, 7].

The MSE transformed geometry and eccentricity was generated using Formian [10, 11] and AutoCAD [12]. It was scaled and used to select the diameter of the circular hollow sections (CHS) used such that a manageable test structure for the laboratory environment would result [4].

The overall objective of the investigation was to determine the predicted behaviour using numerical modelling and compare this to the observed behaviour monitored in the laboratory. Global and Finite element (FE) linear elastic analyses were carried out of the entire MSE structure and one of the most heavily loaded modules respectively. The output was to be compared against displacements and the strain distribution developed in selected circuits.

2.1. Experimental Investigation

Figures 1 and 2 illustrate the dodecahedral MSE structure, configuration orientation and boundary conditions.

The investigation focused on the structural behaviour of a MSE dodecahedral space structure under applied static loading. The strains were monitored using electrical resistance and mechanical strain (ERS) gauges. Element displacements at selected locations on the structure were also compared. To ensure elastic behaviour, displacements were monitored with linear variable differential transducers (LVDTs) at intervals of loading and unloading. See figures 2 - 4. The test structure was supported and loaded at diametrically opposite vertices. This orientation was found to provide the means by which vertical load could be simply applied via a reaction bar fixed to the laboratory strong floor and the load transfer assembly. Two test sets were carried out; the second set following a reorientation of the

structure. This was carried out to ensure consistency in the recorded output. All the test equipment was calibrated to ensure reliability [4].

A typical MSE circuit consisted of three CHS supporting one another: adjacent elements forming pairs. Single bolts were used to connect the MSE to one another. Clearance holes, through which the bolts passed through, were sized to comply with British Standards [13] and ensured that sufficient tolerance was available for ease of assembly. The eccentricity created between the CHSs is illustrated in figure 4. Saddleback washers were used to ensure that the applied loading was efficiently transferred between the CHS elements as illustrated in Figure 2.

A grade S275 steel with a minimum yield stress of 275 N/mm² was specified for the test structure. Tensile tests carried out on samples of the material gave the average yield stress as 375 N/mm² [4].

The Load transfer assembly consisted of a hydraulic jack and load cell sandwiched between two non-deformable steel reaction plates. One of these plates was located at the top of the assembly and bolted to a steel reaction shaft that passed through the module engagement window and hollows in both the jack and load cell. The other end of the shaft was screwed securely into the laboratory strong floor.

2.2. Numerical Modelling

The numerical modelling was refined in stages. For the predicated overall behaviour, global linear elastic analysis of the entire structure was carried out using GSA v.8.0 [14]. Finite element modelling of one of the most heavily loaded circuits, module, M1 was carried out using ANSYS v 8.1 [15].

It was noted that modelling of the bolted connections between the CHSs would be complex and a number of arrangements were considered. Pinned joints were used to connect adjacent elements to each other forming closed triangulated MSE circuits. Pinned connections when present in three orthogonal directions result in joint behaviour similar to that obtained from rigid connections. The loads were assumed to be transferred between the elements predominately via shear action in the bolts [4].

Two models were considered initially in the FE analysis: the first model was used to give an initial indication of the strain distribution and the required refinement of the geometry in order to produce accurate predictions. The second FE analysis model was the more refined in as much as it included the clearance holes. To represent the contact area of the saddleback washers, three radiating concentric circular areas around the clearance holes were determined on each CHS upper and lower surfaces. Nodes on generated outer, middle and inner rings on the upper surfaces were used to apply the required loading. The inner ring on the clearance holes circumference contained sixteen evenly spaced nodes. These nodes were linked together with ANSYS SHELL63 elements and represented the bolted connection between the CHS cylindrical elements [4].

2.3 Applied Loading

For the test configuration considered the estimated maximum applied load value that would just initiate localised ductile behaviour was determined as 14 kN. This upper limit of applied load would therefore ensure that developed strains remained within the elastic range of the material.

3.0 Results

The displacements, strains and von Mises stresses from both the GSA global and the ANSYS finite element analyses were compared for accuracy and reliability with the results obtained from the experimental investigation. These are now considered briefly as the full results have been previously published [6, 7].

3.1. Modules, M1 and M5 - Vertical displacement, U_z

The behaviour observed from the analysis animations of modules, M1 and M5, indicated that the radial movement at the ends of the elements increases as the distance from the centre of rotation increased. This behaviour was due to the spring stiffness of the support modules. There were three anticlockwise style modules, M2, M3 and M4 both forming and supporting the remote ends of module, M1 as shown in Figure 1. Figure 2 shows the monitored vertical displacement, U_z location.

Figure 5 shows the behaviour and averaged monitored displacement for the upper most loaded module, M1, under the range of the applied static loading and unloading observed and recorded for set 1 testing and module, M5 set 2 testing. A hysteresis curve was found to represent the non-linear elastic displacement behaviour. This behaviour can be attributed to several factors that include the geometry of the module and its load transfer mechanism. The load transfer was via the saddleback washers, and the non-vertical orientation of the bolts. Figure 6 illustrates the clockwise rotational appearance of the vertical displacement, U_z , for anticlockwise style module, M1 [4].

The numerical model types considered gave displacement predictions that were within a range of between (9.3% and 12.0%) higher than that measured [4]. As expected, the refined FEA Model was the nearest in agreement to the measured displacements. See Figure 7.

3.2. Module, M9 – Total module displacement $|U|$

Due to the orientation of the elements in module, M9, shown in Figure 1, the total displacements occurring were measured in the direction parallel to the centroidal axis of each element. It was not possible to precisely isolate the vertical displacement, U_z as out of plane U_x and U_y components were present [4, 6].

A comparison of the total displacement occurring to module, M9 was therefore considered appropriate. Figure 8 shows that the experimentally measured total circuit displacement, $|U|$ and the global GSA analysis predicted value. It can be seen that there is very close correlation between the measured and predicted total module displacements, which are within (11.1%) of each other.

3.3. Strain and Stress Distribution

The strains generated by the applied loading in modules, M1, M9 and M20, shown in figure 1, were monitored. Figure 9 shows the averaged monitored strains. The direct strains were recorded and from these the principal strains and stresses were determined. From the ductile material failure criterion, [16, 17] the von Mises stresses were obtained using Equ (1):

$$\sigma_e = (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{0.5} \leq \text{material yield stress} = 375 \text{ N/mm}^2 \quad (1)$$

Where: σ_1 and σ_2 are principal stresses and σ_e = von Mises stress.

3.1.1. Modules, M1 and M5 - Stresses

Figure 10 shows the experimental and numerical modelling determined von Mises stresses. It can be seen, that the predicted equivalent von Mises compressive stresses are (1.3%) higher than those measured in the laboratory. The equivalent tensile stresses are (13.9%) higher. The refined second FEA model values therefore show close correlation.

Typical module, M5 elements lower surface rosette gauges, (G1) shown in Figures 4 and 9, gave the highest averaged component of strain in tension due to bending and axial forces. Torsion and shear monitored by gauges, (G2 and G3), recorded values of (27.9% and 31.0%) respectively of the peak bending and axial strain values at the maximum applied load. Primary action was in bending and axial force therefore.

Typical M5 elements upper surface rosette gauges shown in Figures 4 and 9 gave similar (to the lower surface) averaged components of strain in compression due to bending and axial forces, torsion and shear.

Module, M5 CHSs lower and upper surfaces von Mises stresses, σ_e were determined as 196.50 N/mm² and 189.30N/mm² respectively. These were found to be within a range of (3.7% to 3.8%) of each other. The FEA predicted difference between the CHS lower and upper surfaces were found to be within a range of (14.3% to 16.7%) of each other. See Figure 10.

Grey, or absence of colour, areas seen in the FE model were where the von Mises stresses were predicted to be higher than the material yield stress of 375 N/mm² [4]. These areas can be seen adjacent to the contact points between the bolts-washer assemblies and the CHSs shown in Figure 11.

3.3.2. Module, M9 - Stresses

The monitored stresses occurring in element, RE11 in module, M9 were low compared to modules, M1 and M20. The CHS upper surface von Mises stress, σ_e was determined as 37.60 N/mm². The difference between the calculated equivalent von Mises stresses and those measured indicated that other MSE circuits in the structure were more likely to achieve material yield beforehand [4, 6, 7].

3.3.3. Module, M20 - Strains

Only the exposed upper CHS surfaces of the elements in module, M20 could be monitored in the experimental investigation due to access restrictions [4]. The averaged module, M20 lower surface strain, measured with single 10mm ERS gauges, at the maximum applied indicated load of 14kN, was 1158 $\mu\epsilon$. The module, M1 averaged lower surface strain at the same applied load was 1035 $\mu\epsilon$. The values of set 1 measured strains were within a range of (10.5% to 11.8%) of each other. This information was used to guide the required monitoring refinement for the set 2 testing.

4.0 Discussion

Increased knowledge of the behaviour of MSE types of lattice structures would assist in their optimisation. The experimental and numerical modelling investigation findings are discussed to facilitate additional future research required to achieve this purpose.

4.1 Experimental Output

The strain gauges were installed after assembly of the test structure. The laboratory recorded tensile and compressive strains in the two most heavily loaded diametrically opposite, geometry identical Modules, M1 and M20, were expected to be alike as prior to any loading being applied. Their initial state of strain was taken as the datum for subsequent readings. The recorded strains were a result of the applied loading only therefore as any strain due to the self-weight and load transfer assembly apparatus had been accounted for.

Many factors could have contributed to the differences in the recorded strains. These included variations in material properties due to the manufacturing process, the locations of CHS welded seams relative to the bolt clearance holes and the strain gauges, and the load distribution via the load transfer plate and bolt head alignments [4, 6,7].

The Von Mises stresses shown in figure 11 being above the measured yield stress of 375 N/mm² and up to the material rupture value, determined experimentally to be between, (394 – 421) N/mm², indicate that either local yielding or localised rupture may have occurred. Further monitoring in the identified locations of predicted high strain would therefore be required to confirm the state of stress.

Anticlockwise style modules used in this investigation require a ‘spring-stiffness’ in the three translational directions in order to represent the observed behaviour [4]. The displacements on plan of the entire dodecahedral structure exhibit an apparent elastic expansion in an outward direction from its centre [4, 7].

The experiment confirmed that the element support stiffness, expected to have a direct effect on the element displacement and distribution of stress, was complex to monitor. This was due to the out-of-plane nature of the element displacements containing U_x , U_y and U_z axes components [4].

Figure 7 shows that there was very close agreement between the measured and the predicted displacements for modules, M1 and M5.

4.2 Global Analysis Output

The behaviour predicted by the GSA analysis showed that anticlockwise style modules exhibited clockwise element displacement, that is, an apparent clockwise rotation of the entire circuits about the centre of the module engagement window. The direct stresses, predicated by GSA for modules, M1 and M20, should have been identical as one module was the rotated mirror image of the other. Considering the calculated values for these two modules, the compressive and tensile stresses were found to be within a range of (2.6% to 2.9%) of each other [4, 7].

For the loading regime, GSA predicted combined shear and torsional stresses were low implying that the primary action was due to bending. The stresses predicted by GSA at the junction of the CHS elements and connecting bolts were well above the yield value of the material used indicating local ductile behaviour with possible local yielding occurring. The maximum-distortion-energy (von Mises) theory, using the principal stresses determined from GSA output occurring at the module, element monitoring locations, for M1, M9 and M20, predicted that the material used had not yielded (failed). The overall stiffness of module, M9 was therefore underestimated, giving conservative predictions. The analysis models used generally underestimated the stiffness of the module element arrangement, giving conservative predictions.

4.3 FE Output

The FE model predicted values were very close to the experimental values. A varying slight over estimation for the predicted state of stress was apparent in the FE models when compared to the experimental values.

The differences between the experimental and FE predicted results were, for the most part, due to the limitations of the numerical modelling. For example, modelling of adjacent element support stiffness and the load transfer mechanism for bolts in clearance holes [4] were complex and therefore difficult to accurately represent.

The observed non-linear elastic behaviour in modules, M1 and M5 was not replicated in the numerical models because no allowance for slip of the bolts in the clearance (oversized to comply with British Standards) holes was made.

The maximum vertical displacement predicted by the FE model was -22.5 mm. The average von Mises stress was predicted to be 191.8 N/mm², compressive, in the CHS upper surface, and 223.8 N/mm², tensile, in the CHS lower surface. The onset of local yielding was predicted to occur in the areas adjacent to the bolted connection positions. See Figure 11.

FE modelling gave very good stress predictions for the top most heavily loaded module. However, due to available computing modelling size restrictions, the entire MSE dodecahedron could not be modelled. The 3-element module, M5 relied on support spring-stiffness data from the global GSA analysis [4].

4.4 Further Studies

The limitations of the experimental investigations and the numerical analyses carried out highlighted areas that require further investigation.

Improved additional FEA modelling of the most heavily loaded module, M1 should be carried out in order to establish the cause for the apparent small differences between the CHS upper and lower surface predicted von Mises stresses seen in Figure 10. More refined FE modelling of the bolted connection load transfer mechanism should be carried out using gap and or spring elements in order to represent more realistically the actual load paths.

Experimental testing with the applied loading increased to cause overall collapse is required in order to establish behaviour at the ultimate load condition.

The structural analysis based on assumed linear-elastic behaviour did not take into account or check if the elements could actually carry the predicted load effects or whether the elements would fail by yielding or buckling.

The concept of plastic theory is well established for steel structures. At higher applied loading, the behaviour would become inelastic. Yielding of the steel would occur at certain points, and plastic hinges would form. The formation of the hinges would cause a redistribution of the load across the structure. Bending moments at other sections would continue to increase until enough hinges formed to create a mechanism, when failure would be expected to occur [18].

High ductility indicates that local deformation is possible without causing fracture Narayanan [19]. The ductility of steel enables small regions that are very highly stressed to yield, thereby relieving this concentration of stress without undue distress to the structure as a whole [20].

For design purposes, a ‘minimum’ yield stress is identified for different steel classifications. The minimum yield quoted for a particular steel used in design, is usually a characteristic value that has a chance, frequently 95%, of being exceeded in any standard tension test. Isolated test results are likely to be significantly higher than the quoted yield stress values given in design standards [21]. It is therefore important to obtain experimentally the strength of the material used so that reliable comparisons can be made.

5.0 Conclusions

This experimental investigation highlighted the difficulties associated with monitoring MSE circuits with arbitrary 3-dimensional space orientation in terms of displacements.

The experimental investigation confirmed that element support stiffness in complex networks composed of MSE circuits should be taken into account at the earliest opportunity in the design process when out-of-plane displacement components are present. Proposed configurations can therefore be accurately monitored.

The strains recorded were complex in nature in as far as they included the effects of axial forces, bi-axial bending, shear and torsion. Analysis of the results was based on the assumption that below the yield stress of the material, determined as 375 N/mm², a linear relationship exists between stress and strain. The maximum applied load of 14kN did not

cause yield of the material at the location of the measured strain according to the von Mises ductile material failure criterion.

The module, M10 CHS upper surface von Mises equivalent stress, σ_e was determined as 37.6 N/mm². Behaviour of the test structure for the range of applied loading was non-linear elastic. This was predominately due to the 3-element MSE-circuits configuration. Hysteresis curves were evident for the observed non-linear elastic vertical displacements of modules, M1 and M5. The maximum average vertical displacement of modules, M1 and M5 was -20.6 mm for the maximum applied load.

The global and FE analysis shows very close agreement with the observed behaviour, monitored displacements and stresses determined from the measured strains in the laboratory test structure.

For the anticlockwise style modules, M1 and M20, the global analysis showed that as the displacement due to the applied load took place, all the elements in these modules exhibited an apparent clockwise rotation of the entire module about the centre of the module engagement windows.

From the observed module, M1 non-linear elastic displacement, it was not possible to confirm whether localised yielding in the areas adjacent to the bolt clearance holes had occurred in the test structure.

Further experimental investigations and non-linear numerical analysis should be carried out in order to confirm the structural behaviour at the ultimate collapse condition.

This experimental and numerical modelling investigation has contributed to a better understanding of MSE-circuit behaviour. This paper is about a particular novel type of MSE structure belonging to the family of lattice structures. A contribution to the understanding of the behaviour of these special structures is therefore being proposed as well as the problems associated with their modelling. Further research is required in order to obtain a more complete picture of behaviour for the numerous MSE configurations possible.

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