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Effect of Member Layout on Elastic Buckling Behavior for Square-and-Diagonal Double-Layer Lattice Domes

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Abstract

Double-layer lattice structures composed of the square pattern and the diagonal pattern in top and bottom layers have the member layouts of 4 types as the arrangement of the mesh patterns as shown in figure 1. To design these lattice structures, the relationship between the member layout and the edge of plan must be decided. The mesh patterns in top and bottom layers of "large-square-on-small-diagonal" and "small-diagonal-on-large-square" have been already studied. However, the "large-diagonal-on-small-square" and "small-square-on-large-diagonal" of the double-layer lattice structures have not been sufficiently studied yet. Therefore, in this paper 4 types of the mesh patterns composed of the square pattern and the diagonal pattern are discussed. The effect of member layout on the buckling behavior and stress states is investigated for double layer lattice domes. The other numerical parameter is the loading conditions. In addition, the usefulness of the method to estimate the effective strength by the segment consisting of 3×3 structural units is discussed.

Keywords: double-layer lattice domes, mesh patterns, elastic buckling behavior, member layout, loading condition, strength of structural unit

1. Introduction

Double-layer lattice structures are widely designed as roofs or walls of large span structures, such as gymnasiums, ambulatories, pavilions as well as hangars. As mesh patterns of the double-layer lattice structures, the triangle pattern, the square pattern, the diagonal pattern and the hexagonal pattern are usually used in top and bottom layers.

Double-layer lattice structures composed of the square pattern and the diagonal pattern in top and bottom layers have the member layouts of 4 types as the arrangement of the mesh patterns as shown in figure 1. To design these lattice structures, the relationship between the member layout and the edge of plan must be decided. The mesh patterns in top and bottom layers of "large-square-on-small-diagonal" (denoted as LS-SD) and "small-diagonal-on-large-square" (denoted as SD-LS) have been already studied in some papers [1] ~ [4]. However, the "large-diagonal-on-small-square" (denoted as LD-SS) and "small-square-on-large-diagonal" (denoted as SS-LD) of the double-layer lattice structures have not been sufficiently studied yet.

Therefore, in this paper 4 types of the mesh patterns composed of the square pattern and the diagonal pattern are discussed. The effect of member layout on the buckling behavior and stress states is investigated for double-layer lattice domes. The other numerical parameter is the loading conditions.

To analyze lattice structures composed of a large number of structural units, the continuum treatment is one of the useful methods [5]. The method to estimate the effective strength by using the segment with some structural units is presented [6]. In this paper, the elastic buckling strength of the double-layer lattice domes is estimated by the effective strength of the segment with some structural units.

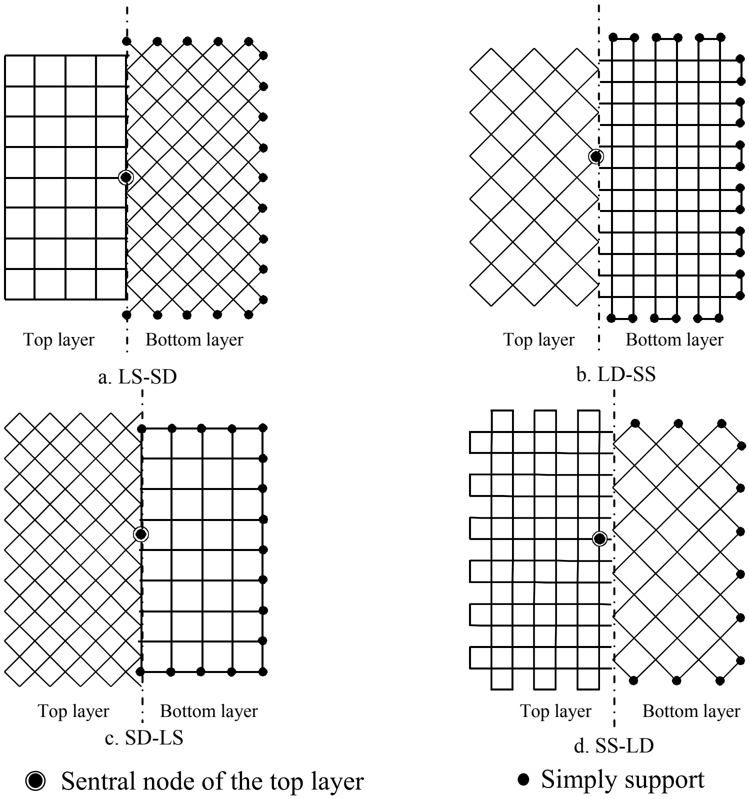


Figure 1. Member layout and support condition

2. Analytical models

In this chapter, analytical models, supporting and loading conditions are described.

2.1. Analytical models

The forms and mesh patterns of four double-layer lattice domes are shown in Figure 2. The size of each analytical model is shown in Table 1. Each top node (bottom node) of the dome is decided by a spherical surface with the same radius. Each analytical model is made by repeating the structural unit with almost the same size to the comparison.

The numerical models are designed by the allowable stress concept with uniform vertically distributed loads. The standard design load is 2.203 kN/m^2 as the dead load and snow load. The mechanical properties of members are shown in Table 2.

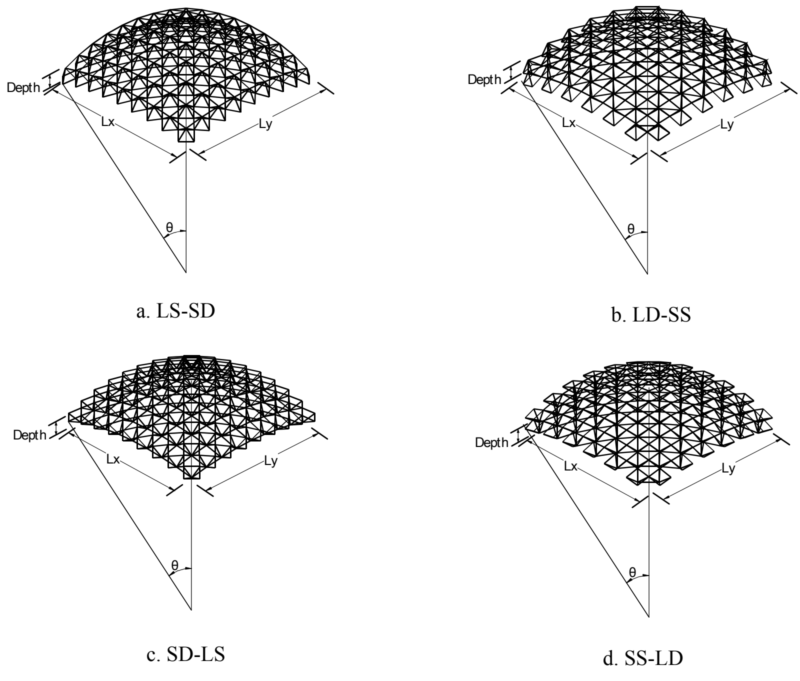


Figure 2. Geometry of the domes

Table 1. Size of each analytical model

Model	Lx (mm)	Ly (mm)	Depth (mm)	θ (°)	Length of members (mm)		
					Top	Bottom	Web
LS-SD	27818	27818	1632	30	3258～3446	2119～2380	2228～2456
LD-SS	28657	28657	1632	30	3192～3482	2195～2369	2180～2383
SD-LS	23576	23576	1632	30	2119～2380	2986～3168	2186～2400
SS-LD	25499	25499	1632	30	2195～2369	2941～3207	2142～2333

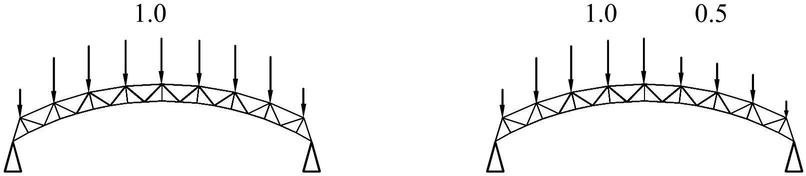
Table 2. Mechanical and section properties of members

Model	Member	Sectional dimension (mm)	Cross sectional ares (mm^2)	Geom etrical moment of inertia (mm^4)	Slenderness ratios
LS-SD	Top	ϕ 89.10×3.2	863.6	7.98×10^5	107 ~ 113
	Bottom	ϕ 60.50×3.2	576.0	2.37×10^5	105 ~ 117
	Web	ϕ 60.50×3.2	576.0	2.37×10^5	110 ~ 121
LD-SS	Top	ϕ 89.10×3.2	863.6	7.98×10^5	105 ~ 115
	Bottom	ϕ 76.30×3.2	734.9	4.92×10^5	85 ~ 92
	Web	ϕ 48.60×2.8	402.9	1.06×10^5	134 ~ 147
SD-LS	Top	ϕ 60.50×3.2	576.0	2.37×10^5	105 ~ 117
	Bottom	ϕ 89.10×3.2	863.6	7.98×10^5	98 ~ 104
	Web	ϕ 60.50×3.2	576.0	2.37×10^5	109 ~ 118
SS-LD	Top	ϕ 76.30×3.2	734.9	4.92×10^5	85 ~ 92
	Bottom	ϕ 89.10×3.2	863.6	7.98×10^5	97 ~ 106
	Web	ϕ 60.50×3.2	576.0	2.37×10^5	106 ~ 115
Young's mondulus E (MPa)			2.05×10^5		
Shear mondulus G (MPa)			7.90×10^4		
Yield stress σ_y (MPa)			2.94×10^2		

The joints of the models are treated as rigid-joints under the assumption that the strength and stiffness of the joints are sufficiently large.

2.2. Supporting and loading conditions

The lower perimeter nodes of the models are simply supported as shown in Figure 1. The analytical models are subjected to two loading cases, which are the uniform vertically distributed load and the asymmetric vertically distributed load respectively as shown in figure 3. Each nodal point in the top chord surface with the occupied area is loaded by the vertical load.



a. Symmetric loading

b. Asymmetric loading

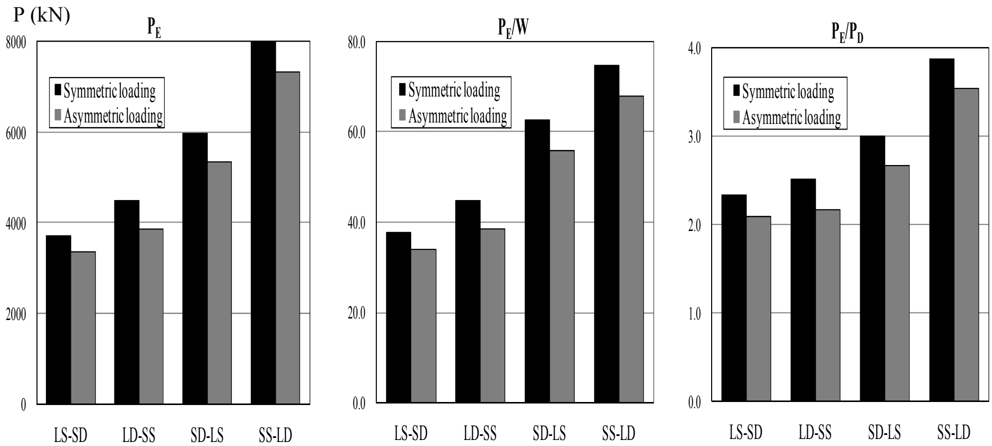
Figure 3. Loading condition

3. Numerical results

From numerical results of the double-layer lattice domes composed of a square pattern and a diagonal pattern, the elastic buckling loads, the load-deflection relationships and the buckling modes are presented as follows.

3.1. Elastic buckling load

The comparison of the elastic buckling loads obtained by elastic buckling analysis is presented in Figure 4. In the figure 4, the elastic buckling loads, the elastic buckling loads against the weight of each model and the elastic buckling loads against the design load are presented in (a), (b), (c) respectively. The sequence of the elastic buckling strength is almost SS-LD>SD-LS>LD-SS>LS-SD for all loading conditions. That is to say, the elastic buckling strength of large-diagonal-and-small-square mesh patterns is bigger than large-square-and-small-diagonal mesh patterns.

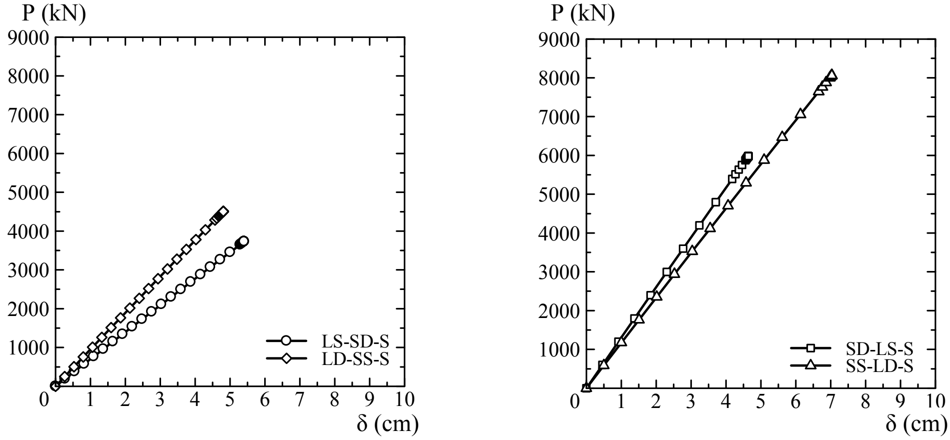


(a) Buckling loads (b) Buckling loads against weight (c) Buckling loads against design load

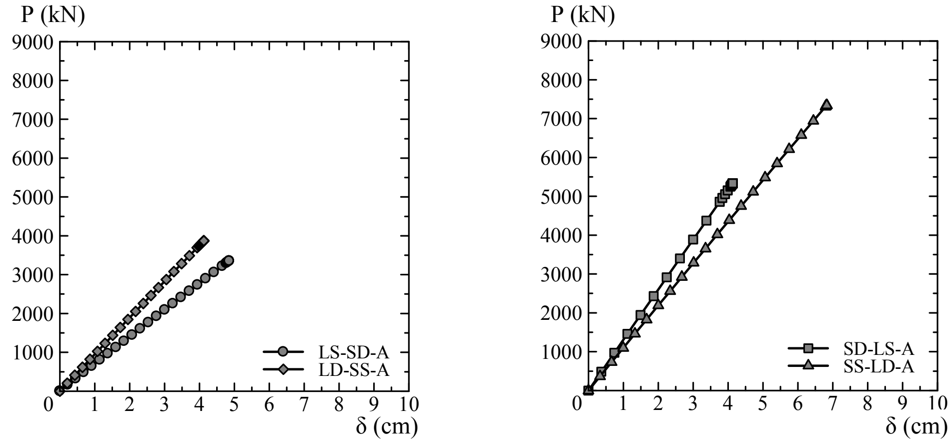
Figure 4. The comparison of elastic buckling loads

3.2. Load-deflection relationship

The relationships of the total load P of analytical model and the deflection δ at the central node of the top layer are presented in Figure 5. The central node of the top layer is the node which indicates the largest vertical displacement in the elastic range as shown in Figure 1. The stiffness of LD-SS type is larger than the LS-SD type, and SD-LS type is larger than the SS-LD type for all loading conditions. In other words, the stiffness of domes being supported at the square pattern is larger in the load-defection curves. The support direction and the member layout direction are identical in case that the supported layer is a square pattern. The arch action by the constituent members yields the large stiffness.



a. Symmetric loading



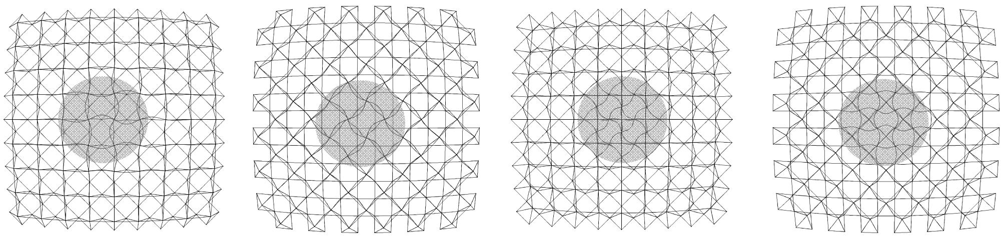
b. Asymmetric loading

Figure 5. Load-deflection relationships

3.3. Buckling mode

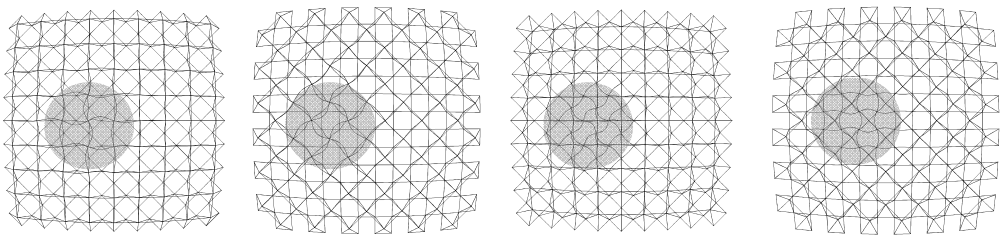
The typical buckling modes are shown in Figure 6. Gray areas in the figure indicate buckling areas. The buckling mode of each model is the horizontal buckling direction of top members for all loading conditions and member layouts.

The buckling areas in the case of symmetric loading appear in the center of analytical models. And the buckling areas in the case of asymmetric loading appear in the side where ratio of load is 1.0.



LS-SD LD-SS SD-LS SS-LD

a. Symmetric loading



LS-SD LD-SS SD-LS SS-LD

b. Asymmetric loading

Figure 6. Elastic buckling modes

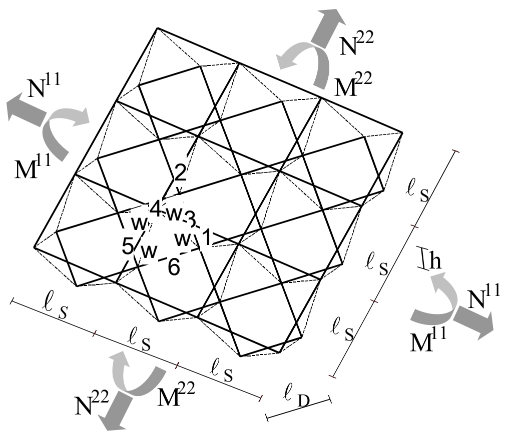
4. Effective strength by the segment and its applications

The effective strength of double-layer lattice structures composed of the congruent units is represented as the strength of the equivalent continuum, and is expressed in terms of the sectional forces on the equivalent continuum, such as stress resultants, stress couples and so on. The effective strength depends on the bucking of members is estimated by using the segment with subdivision 3 units x 3 units as the few in order to consider buckling effect.

4.1. Effective strength by the segment

The segment of the square-and-diagonal double-layer lattice structure, the equivalent stress results and stress couples, and the relations of them and the axial forces of constituent members are showed in Figure 7. The effective strength under bi-axial equivalent bending moments, $M^{11}=M^{22}$, and in-plane forces, $N^{11}=N^{22}$, without equivalent twisting moments M^{12}, $M^{21}$and in-plane shearing forces $N^{12}=N^{21}$is derived from the segment, because the space grid unites have the forces of the members under the stress states on the pin-jointed case.

The effective strength which depends on the bucking of members from the segment is showed in Figure 8. The solid line shows that the effective strength curve of LS-SD (SD-LS) model with the sectional dimension of members. The broken line shows that the effective strength curve of LD-SS (SS-LD) model with the sectional dimension of members.



$$N_1 = N_2 = \ell_S\ N^{11} / 2 \ - \ \ell_S\ M^{11} / h$$

$$N_3 = N_4 = N_5 = N_6 = \ell_S\ N^{11} / 2\sqrt{2}\ + \ \ell_S\ M^{11} / \sqrt{2}h \quad , \quad w = 0$$

Figure 7. Form of segment

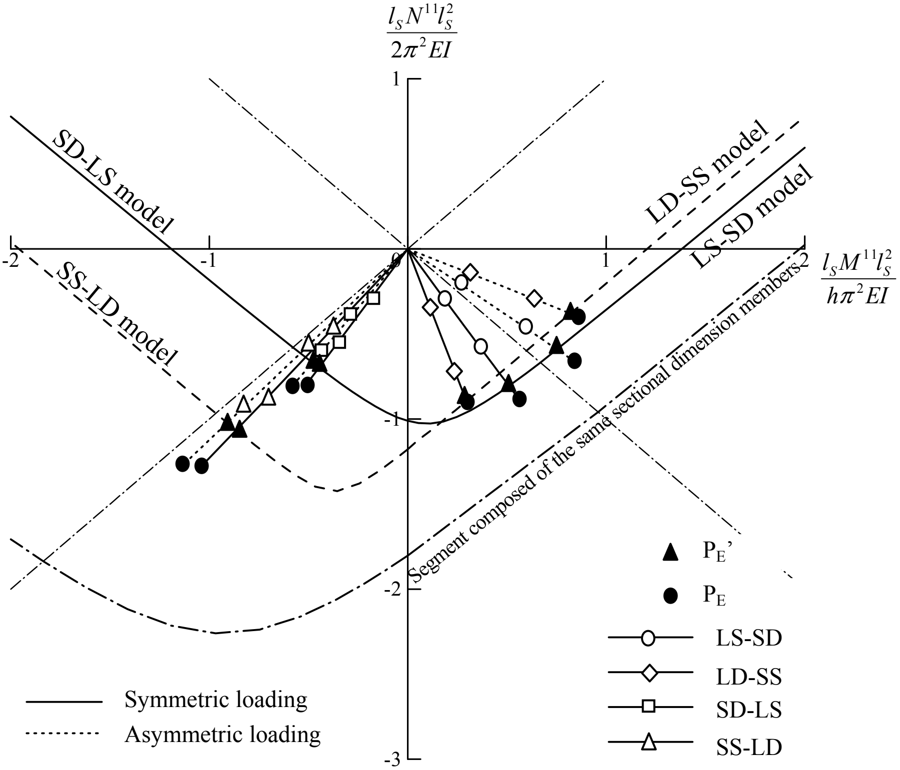


Figure 8. The effective strength of segment

4.2. Elastic buckling loads by effective strength

The elastic buckling loads of the double-layer lattice domes composed of the square pattern and the diagonal pattern are approximately estimated by using the effective segment strength as shown in Table 3. P_E and P_E' in the table represent the exact loads by the numerical analysis and the approximate loads from the effective strength. P_E' is 80% ~ 96% of P_E. Therefore it could be concluded that both loads are almost identical.The reason for the exact loads become smaller than the approximate loads is that the axial force of web members and the torsional rigidity are not considered when the effective strength of the segment is calculated.

The precision of the approximate loads in the case of asymmetric loading becomes worse than symmetric loading. LD-SS type and SD-LS type are predominant in the equivalent sectional axial force of the buckling areas than LS-SD type and SS-LD type, respectively. The equivalent sectional bending moment of the buckling areas in the case of asymmetric loading is larger than asymmetric loading for all member layouts.

Table 3. Comparison of exact loads and approximate loads

Model	Exact loads by numerical ananlyes P_E (kN)	Approximate loads by effective strength P_E' (kN)	P_E' /P_E
LS-SD-S	3727.4	3395.7	91.1%
LS-SD-A	3349.9	2961.4	88.4%
LD-SS-S	4504.2	4360.1	96.8%
LD-SS-A	3871.9	3612.5	93.3%
SD-LS-S	5981.2	5275.4	88.2%
SD-LS-A	5335.0	4417.4	82.8%
SS-LD-S	8061.2	6747.2	83.7%
SS-LD-A	7344.0	5911.9	80.5%

5. Conclusions

The effect of member layouts and loading conditions on elastic buckling behavior for 4 types of square-and-diagonal double-layer lattice domes is examined. In addition, the applicability of the method to estimate the effective strength by the segment consisting of 3×3 structural units is discussed. The important conclusions obtained by this investigation are follows as.

1) The stiffness of domes being supported at the square pattern is larger in the load-defection curves. The arch action by the constituent members yields the large stiffness.

2) The sequence of the elastic buckling strength is almost SS-LD>SD-LS>LD-SS>LS-SD for all loading conditions. In other words, the elastic buckling strength of large-diagonal-

and-small-square mesh patterns is bigger than large-square-and-small-diagonal mesh patterns.

3) For double-layer lattice domes, the approximate elastic buckling loads, which are estimated from the strength of segment, are in good agreement with the results of the elastic buckling analysis. It could be confirmed that the equivalent sectional force of elastic buckling areas of each model is different. Furthermore, the load resistant system is understood by the effective strength of the segment. It may be the useful information for structural engineers.

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