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Electrical circuit theory

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EDITORIAL
UNIVERSITAT POLITÈCNICA DE VALÈNCIA

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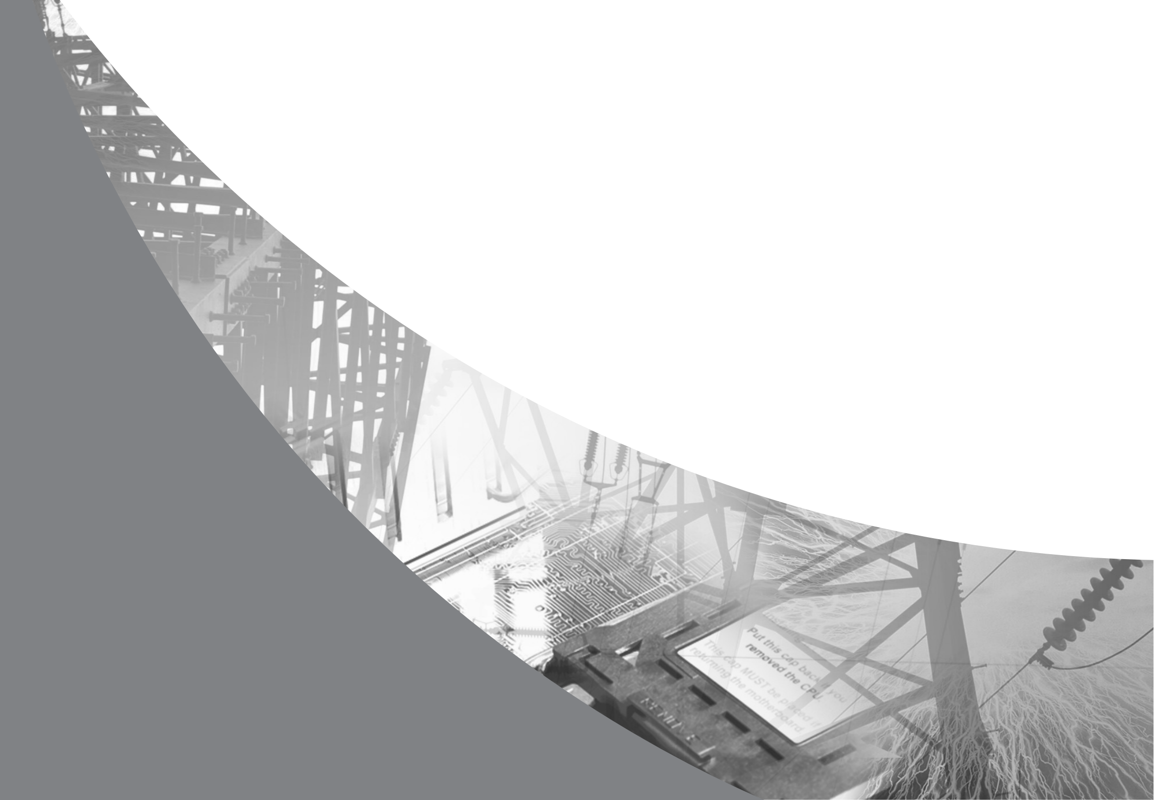

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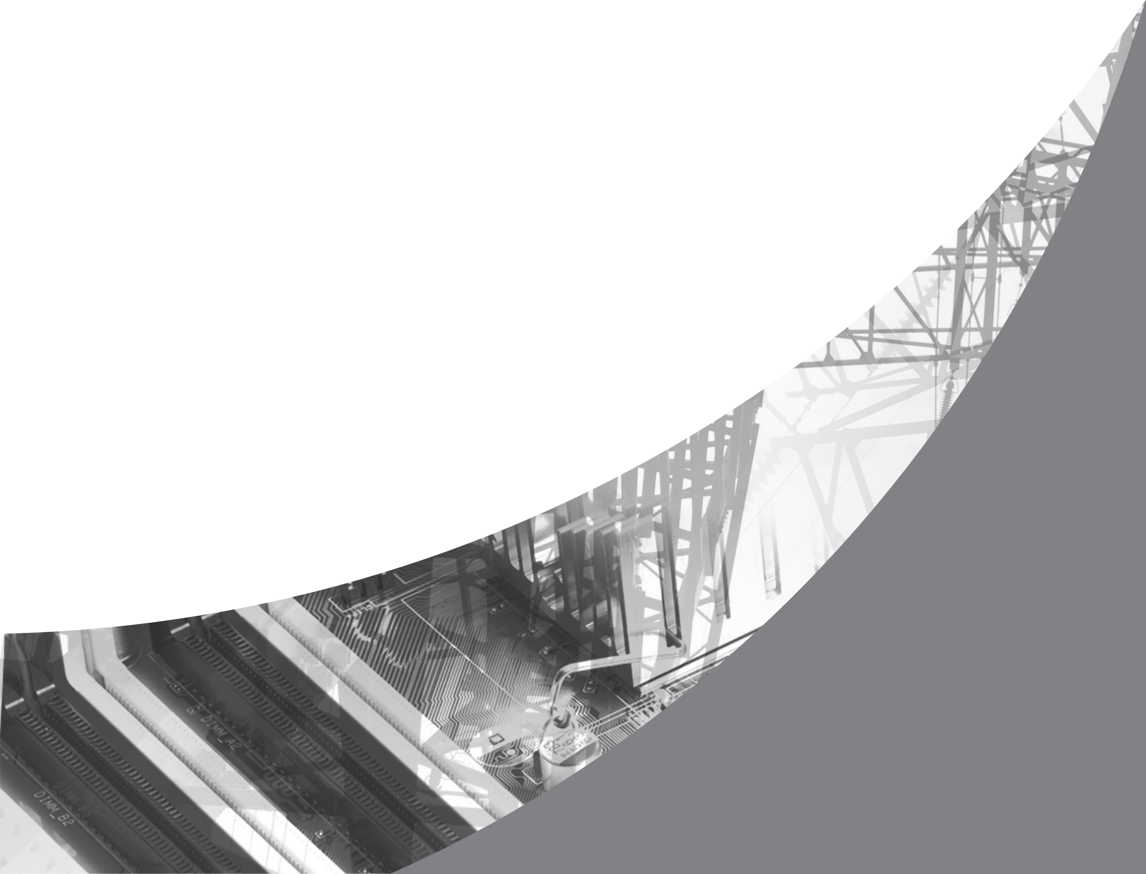


Introduction

The aim of this book is to explain the basis of electrical circuits and networks in order to describe further applications, both in small signal and power. The suitable analysis methodologies and theorems, and its connection with other subjects are described, as well as some typical industrial applications which enhance the electrical circuit theory since a practical point of view.

More specifically, the main objectives of this work can be resumed in six points:

- To Identify and analyze the physical processes that take place in the elements in an electric circuit, associating a model to these processes.
- To compute the temporal response in a circuit under any type of excitation, identifying this response in steady state with constant and sinusoidal excitations and applying Laplace transform to obtain the transient response in linear circuits.
- To utilize properly the concepts of active, reactive, apparent, complex and temporal power in circuits in steady sinusoidal state, identifying the relevance of the power factor and design circuits for its correction.
- To analyze circuits by using node and mesh analysis techniques by applying theorems that allow circuit analysis simplifications when possible.
- To justify the use of three phase systems for power applications computing magnitudes associated to three phase circuits and associating schemes for real and reactive measurement in three phase facilities.
- Finally, to identify the main elements in an Industrial power supply system performing computation in three phase industrial facilities with transformers.

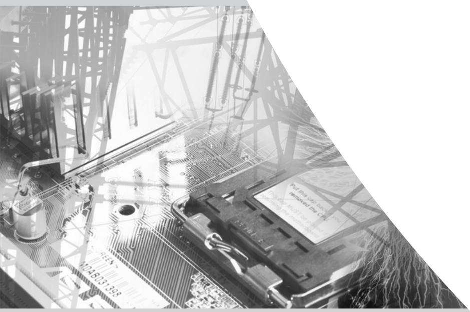
Chapter 1

Fundamentals

Introduction

Review of electric circuits elements

Waveforms



1. Fundamentals

1.1. Introduction

1.1.1 Elemental circuit variables

An electric circuit is defined as a group of elements related among them whose characteristics allow the existence of an electric current. Typically, electric circuits are composed of resistors, capacitors, inductors, transformers and energy sources. The circuits theory is devoted to study the properties of such elements, based on a set of empirical laws commonly accepted called axioms.

There are some basic variables in the study of electric circuits. The **charge** is the property of the matter to attract or repel other matter. According to the attraction or repulsion forces that occur in nature, charges can be positive or negative. The charge is represented by means of the symbol q and it is measured in coulombs (abbreviated as C). The smallest charge in nature belongs to the electron, whose value is equal to $1.6 \cdot 10^{-19}$ C.

The neat amount of charges moving throw a material per unit time is defined as **current**, represented by i and defined as follows:

$$i = \frac{dq}{dt} \tag{1-1}$$

The current measures the flow of electric charge and it is measured in amperes (abbreviated as A), equal to coulombs per second. As there are two types of charge, the direction of current flow will depend on the sign of the charges in movement. Customarily, the net flow of positive charges is considered as the direction of the current flow.

Another important concept, associated with the variation of energy experienced by a charge when passing through a circuit, is the voltage. The **voltage** or electric tension (u) represents the variation of potential energy between two points a and b in a circuit, as represented in Figure 1, when a charge passes from a to b. It is measured in volts (abbreviated as V), equal to joules per coulomb.

$$u_{ab} = -u_{ba} = \frac{dw}{dq} \tag{1-2}$$

The value of the voltage is independent on the path followed by the charge when moving to a to b. In fact, there can be a voltage between two points even if there is not an electric connection between them and, consequently, any current flow does not exist.

The **electric power** (*p)* is defined as the variation of energy during the time, and it is measured in watts (abbreviated as W), equal to joules per second. Mathematically, the electric power is obtained as the product of *u* and *i*, as expressed in (1-3):

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = ui \qquad (1\text{-}3)$$

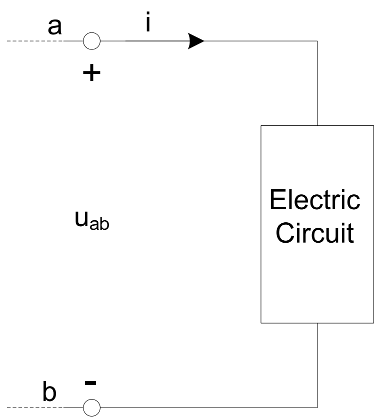


Figure 1. Elemental electric circuit scheme

This power is **absorbed** as positive charges are losing potential and, therefore, some work is being performed inside the electric circuit.

1.1.2 Symbols and units

Electrical engineering, similarly to other science fields, has its own terminology and symbols. Some of the most common magnitudes and symbols, expressed in IS units, are listed in Table 1.

Table 1. Some important magnitudes, their symbols and unit abbreviations

Symbol	Quantity	Unit	Abbreviation of Unit
w	Energy	joule	J
p	Power	watt	W
q	Charge	coulomb	C
i	Current	ampere	A
u, v	Voltage	volt	V/m
E	Electricity field	volt/meter	V/m
B	Magnetic field	tesla	T
H	Magnetic field strength	ampere-turn per metre	AT/m
F	Magnetomotive force	ampere-turn	AT
ϕ	Flux	weber	Wb
λ	Flux lincage	weber	Wb
Z	Impedance	ohm	W
Y	Admittance	siemens	S or ℧
R	Resistance	ohm	W
G	Conductance	siemens	S or ℧
X	Reactance	ohm	W
B	Susceptance	siemens	S or ℧
L, M	Inductance	henry	H
C	Capacitance	farad	F
t	Time	second	s
f	Frequency	hertz	Hz
ω	Radian frequency	radian/second	rad/s
θ, φ	Phase angle	deg or radian	° or rad

Moreover, it is necessary to consider the system of standard decimal prefixes for each unit, since numerical values of the considered magnitudes could range over many orders of magnitude. The use of these prefixes implies that the unit is multiplied by the corresponding power of 10.

Table 2. Standard decimal prefixes

	Multiplier	Prefix	Abbreviation
Multiplier	10^{18}	exa	E
	10^{15}	peta	P
	10^{12}	tera	T
	10^{9}	giga	G
	10^{6}	mega	M
	10^{3}	kilo	k
	10^{2}	hecto	h
	10^{1}	deca	da
Divider	10^{-1}	deci	da
	10^{-2}	centi	c
	10^{-3}	mili	m
	10^{-6}	micro	m
	10^{-9}	nano	n
	10^{-12}	pìco	p
	10^{-15}	femto	f
	10^{-18}	atto	a

1.1.3 Kirchhoff's laws

The differential equations used to study the behavior of electric circuits are based on the work developed by German scientist Gustav Kirchhoff, from which two axioms are defined. These equations are valid whenever the geometry of the studied circuits is much lower than the electromagnetic radiation wavelength (6000 km for 50 Hz)

1.1.3.1 Kirchhoff's Current Law (KCL)

The Kirchhoff's first law states that **the algebraic sum of the currents entering a node is zero at every instant**. Thus, if a group of lines converge to an only node, the following equation must be satisfied:

$$\sum_{k=1}^{n} i_k = i_1 + i_2 + i_3 \ldots + i_n = 0 \quad (1\text{-}4)$$

Figure 2 shows a circuit in that a section composed of 4 nodes is considered. If currents coming in the node are considered as positive and currents coming out the node are considered as negative, the following equations can be stated:

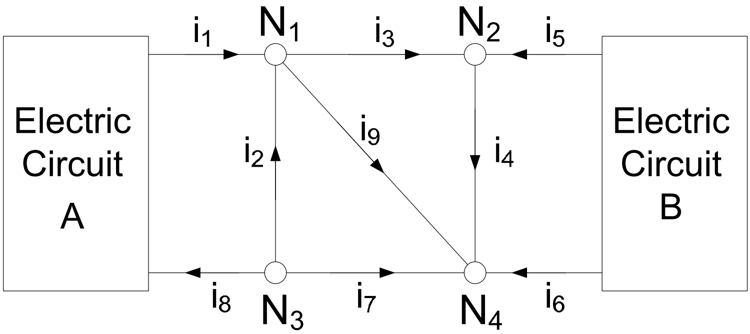


Figure 2. Application of the Kirchhoff's first law

$$i_1 + i_2 - i_3 - i_9 = 0 \quad (1\text{-}5)$$

$$i_3 - i_4 + i_5 = 0 \quad (1\text{-}6)$$

$$-i_8 - i_2 - i_7 = 0 \quad (1\text{-}7)$$

$$i_7 + i_9 + i_4 + i_6 = 0 \quad (1\text{-}8)$$

$$i_1 + i_5 + i_6 - i_8 = 0 \quad (1\text{-}9)$$

Another way to enunciate the Kirchhoff's first law is the following: The sum of incoming currents in a node is equal to the sum of outgoing currents from the same node:

$$\sum i_k^{in} = \sum i_k^{out} \quad (1\text{-}10)$$

Example 1-1

In the circuit shown in Figure 2 we are given i_1 = 3 A, i_4 = -5 A, i_7 = 2 A and i_9 = 2 A. Which is the value of the rest of currents?

SOLUTION: The problem could be also solved by considering certain loops where the KCL can be applied. Consider the loop at N_4 indicated in Figure 3. As i_4, i_7 and i_9 are known, i_6 can be easily evaluated by applying the KCL to node C: $\mathbf{i_6} = -i_7 - i_9 - i_4 = -2 - 2 - (-5) = 1$ A.

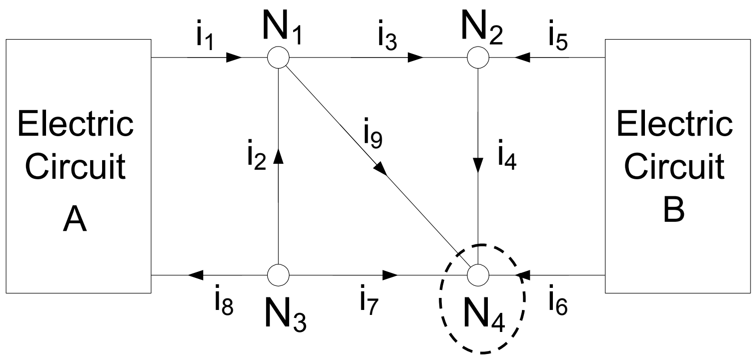


Figure 3. Example illustrating the Kirchhoff's first law (A)

Considering other two loops in the extremes of the grid, as shown in Figure 4, it is possible to evaluate i_5 and i_8: $\mathbf{i_8} = i_1 = 3$ A and $\mathbf{i_5} = -i_6 = -1$ A.

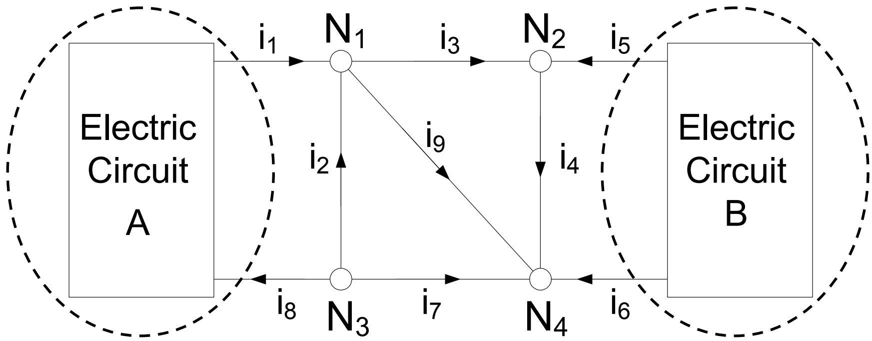


Figure 4. Example illustrating the Kirchhoff's first law (B)

Now we will consider the loop shown in Figure 5 so as to evaluate i_7. If we apply the KCL for this loop, $i_7 + i_1 + i_2$, so that $\mathbf{i_2} = -i_7 - i_1 = -2 - 3 = -5$ A

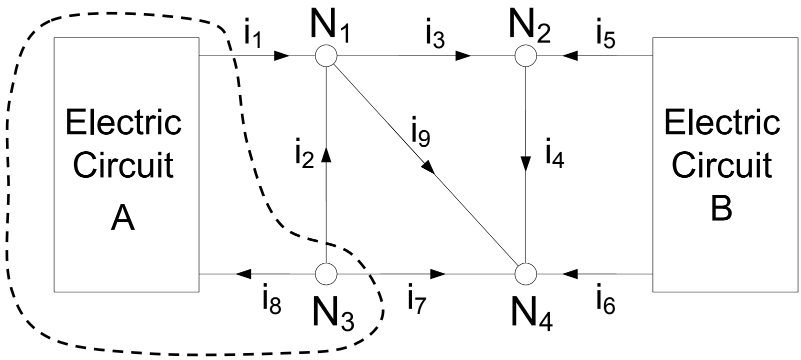


Figure 5. Example illustrating the Kirchhoff's first law (C)

Finally, i_3 can be evaluated by applying the KCL to N2 (see Figure 6): $i_3 + i_5 = i_4$, so that $\mathbf{i_3} = i_4 - i_5 = -5 - (-1) = -4$ A

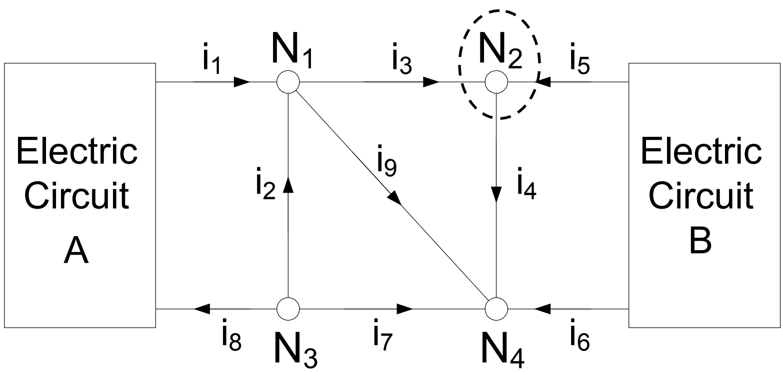


Figure 6. Example illustrating the Kirchhoff's first law (D)

1.1.3.2 Kirchhoff's Voltage Law (KVL)

The Kirchhoff's second law states that **the algebraic sum of all the voltages around a loop is zero at every instant**, being a loop a sequence of devices that forms a closed path. This law is a consequence of defining the voltage between two points as the different of their potential energies.

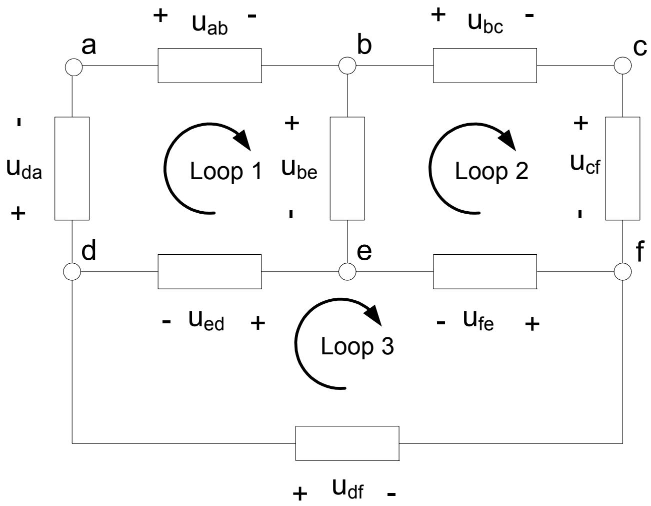


Figure 7. Application of the Kirchhoff's second law

As a consequence, the voltage between any two nodes in a circuit is independent of the way followed to go from one to each other.

Figure 7 shows a circuit composed of three loops defined by six nodes. In order to apply the Kirchhoff's second law, a reference direction to each loop needs to be assigned (turning clockwise in this case). Once the loop direction is fixed, voltages whose reference direction agree with that of the loop will be considered as positive, while they will be assigned a minus sign otherwise. According to those premises, the following loop equations can be stated:

$$\text{Loop 1: } u_{ab} + u_{be} + u_{ed} + u_{da} = 0 \tag{1-11}$$

$$\text{Loop 2: } u_{bc} + u_{cf} + u_{fe} - u_{be} = 0 \tag{1-12}$$

$$\text{Loop 3: } -u_{ed} - u_{fe} - u_{df} = 0 \tag{1-13}$$

Example 1-2

In the circuit shown in Figure 8, U_{ab} = A, U_{da} = B, U_{ce} = C and U_{be} = D. Calculate the value of U_{eb}, U_{bc}, U_{cd} and U_{ac}.

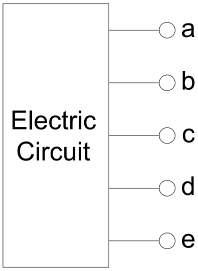


Figure 8. Example illustrating the Kirchhoff's second law

SOLUTION:

$U_{eb} = -U_{be} = -D$

$U_{bc} = U_{be} + U_{ec} = D - C$

$U_{cd} = U_{ce} + U_{eb} + U_{ba} + U_{ad} = C - D - A - B$

$U_{ac} = U_{ab} + U_{be} + U_{ec} = A + D - C$

1.2. Review of electric circuits elements

When studying the properties of any physical phenomenon, models are required in order to understand the behavior of such facts and their parameterization. Such models are an idealization of the reality and, therefore, some simplifications are sometimes required in order to avoid an extreme complexity. In the case of circuit theory, it is common to consider that elements are physical entities that occupy a specific point in the space that do not emit radiation, as well as non-resistive lines to connect the different components of a circuit.

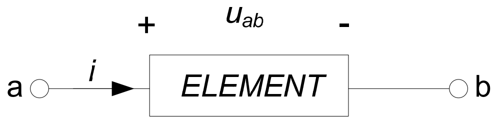
The ideal elements whose combination is able to produce any electric circuit are the following:

- Resistors
- Capacitors
- Inductors
- Coupled inductors

- Voltage sources (dependent and independent)
- Current sources (dependent and independent)
- Transformers

Each one of these elements is characterized by the current going through and the voltage between their terminals (Figure 9), defined by equations that will be detailed in the next paragraphs. Note that the voltage and the current reference directions are conventional.

However, building in the real life a circuit based on the use of ideal elements does not provide us with a realistic model to represent the actual behavior of an electric system. For that reason, real elements are able to be approximated by means of a combination of different real elements. Such combinations can be built according to the accuracy required in the studied electric variables, as well as the external parameters that may affect the proper operation of the system: temperature, humidity, connection quality, etc.



$$u_{ab} = f(i)$$

$$i = g(u_{ab})$$

Figure 9. Ideal element representation in an electric circuit

1.2.1 Energy and power

Considered an electric circuit with 2 accessible terminals, as presented in Figure 1, and taking into account the references established in the figure, the incoming power is given by the following expression:

$$p(t) = u(t) \cdot i(t) \tag{1-14}$$

When $p(t)$ is higher than zero, the power is coming into the circuit (it is consuming energy), while if $p(t)$ is lower than zero, the power is coming out of the circuit (it is supplying energy).

As p(t) represents the variation of the energy with respect to the time, the expression of the energy consumed or supplied by the circuit can be calculated as follows:

$$p(t) = \frac{dw}{dt} \Rightarrow w(t) = w(t_0) + \int_{t_0}^{t} p(\tau)d\tau = \int_{-\infty}^{t} p(\tau)d\tau \qquad (1\text{-}15)$$

It is considered for any physical element that w(-∞) is equal to zero.

1.2.2 Resistors

1.2.2.1 Ideal element

Resistors are two-terminal physical elements which dissipate heat.

The equation which defines the behavior of resistors is known as the Ohm's law, which states that the voltage across a resistor is proportional to the current flowing through it:

$$u(t) = R \cdot i(t) \qquad (1\text{-}16)$$

The letter R represents the resistance, characteristic property of a resistor which measures the opposition of this element to the passage of an electric current. It is measured in ohms (abbreviated as Ω)

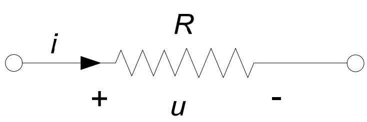


Figure 10. Symbol for a resistor

Every so often in engineering there are devices which do not satisfy the Ohm's law, such as some components of computer, control and communication systems. Consequently, it is important to understand the behavior of this simple element in order to be much better prepared to analyze more sophisticated circuits in the future.

In some cases it is easier to consider not the opposition but how easily electricity flows along a certain path. This property is called conductance, represented by G and calculated as the inverse of the resistance. It is measured in Siemens (abbreviated as S).

$$i(t) = G \cdot u(t) \tag{1-17}$$

Consider a wire made of a metal of resistivity ρ whose section is S_c and length L_c, as shown in Figure 11.

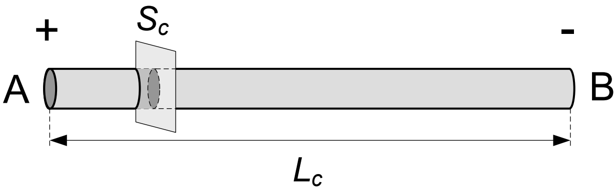


Figure 11. Physical model of a wire

When a voltage *u* is applied between their extremes A and B, an electric field *E* appears, according to this expression:

$$u_{AB} = \int_{L_c} \vec{E}\,\vec{dl} = \int_A^B E \cdot dl \tag{1-18}$$

Such field produces the movement of electric charges, appearing a current density J proportional to this field:

$$\vec{J} = \sigma \vec{E} \;\Rightarrow\; \vec{E} = \rho \vec{J} \tag{1-19}$$

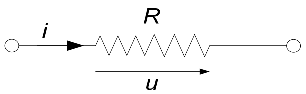
were σ is the electrical conductivity and its inverse, ρ, is the electrical resistivity of the material. Admitting that charges are uniformly distributed along the wire, the following relation could be established:

$$\vec{J} = \frac{i}{S_c} \vec{u} \tag{1-20}$$

were *i* is the current through the wire and S_c is the section, being $\vec{u}$ a unit vector indicating the orientation of u_{AB}. Substituting in eq. (16) and (17) it is obtained the following result:

$$u_{AB} = \int_A^B E \cdot dl = \int_A^B \rho \cdot J \cdot dl = \int_A^B i \frac{\rho}{S_c} dl = \frac{\rho \cdot L_c}{S_c} \cdot i = R \cdot i \qquad (1\text{-}21)$$

Table 3. Summary of main characteristics for resistors

Symbol	Characteristic parameter	Voltage	Current
	$R = \frac{1}{G} = \frac{\rho \cdot L_c}{S_c}$	$u(t) = R \cdot i(t)$	$i(t) = G \cdot u(t)$

Example 1-3

In the circuit of Figure 10, R = 10Ω and u = 20V. Calculate de value of the current flowing through the resistor

SOLUTION: According to (1-16), $i(t) = \frac{1}{R} \cdot u(t) = \frac{1}{10} \cdot 20 = 2A$

1.2.2.2 Real element

Two important deviations arise when this element built in the real life: the impact of the temperature and the effect of the magnetic fields created by the current in the resistor.

Regarding the first one, it is well known that the resistivity of the conducting materials depends on the temperature and, consequently, so it does the resistance R. This is very important in many applications as the resistance is constantly generating heat and it may result in high temperatures.

This relation, when accounted for, is considered linear (resistance vs. temperature) and it is given by the following expression:

$$\rho_{\theta'} = \rho_\theta \left(1 + \alpha_\theta \left(\theta' - \theta\right)\right) \qquad (1\text{-}22)$$

This relation is important so as to evaluate the resistivity and therefore, the resistance, at any temperature if its value is known at another one.

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