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Membrane structure in the form of Moebius strip

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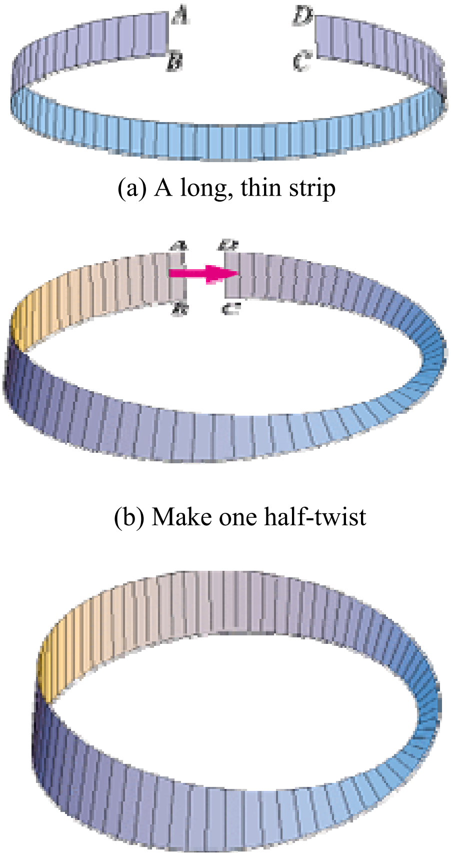
Abstract

Form-finding of membrane surface bordered by Möbius strip is investigated. Möbius strip is a surface with only one side and only one boundary component. In this study, the possibility of adopting the form of Möbius strip as surface shape for membrane structure is studied. The influence of non-orientable characteristic of Möbius strip on the modeling process using finite element procedures is investigated. Non-linear displacement method is used for form-finding analysis. Form and pattern of pre-stress in the resulting membrane surface are studied.

Keywords: Möbius strip/ring, form-finding, membrane structure, nonlinear displacement method

1. Introduction

Möbius strip or Möbius ring is a one sided, one edged loop named after an astronomer and mathematician called August Möbius(1790-1868). A model of a Möbius strip as shown in Figure 1 can be constructed by joining the ends of a strip of paper with a single half-twist. A line drawn starting from the seam down the middle will meet back at the seam but at the "other side". If continued the line will meet the starting point and will be double the length of the original strip of paper. This single continuous curve demonstrates that the Möbius strip has only one boundary. It is a standard example of a surface which is non-orientable.



(a) A long, thin strip

(b) Make one half-twist

(c) A Möbius strip

Figure 1: Simple steps to produce a Möbius strip

Petresin *et al.* [5] has shown that the Möbius strip has a great potential as an architectural form. The form of Möbius strip has been adopted in the creation of sculpture (Friedman [3]), exploration of idea for bridges and for buildings (Séquin [6]). Mahadevan and Keller [4] have represented the Möbius strip as a bent, twisted elastic rod with a rectangular cross-section. Its mechanical equilibrium is governed by the Kirchhoff-Love equations for the large deflections of elastic rods. These are solved numerically for various values of the aspect ratio of the cross-section and an asymptotic solution is found for large values of this ratio. The resulting shape is shown to agree well with that of a band made from a strip of plastic. Choong and Kuan [2] has investigated thin shell surface in the form of Möbius strip with the objective of gaining basic understanding about its structural behaviour. Computational models of the surface are first generated and next analysed using finite element method under self-weight loading condition. Strength and stiffness of the surface are studied under the influence of different support conditions. Numerical results have shown that choice of support condition has a significant influence on the structural performance of the Möbius strip shell surface investigated. From the survey above, it can be seen that surface of Möbius strip has attracted much attention by researchers from various fields.

In this study, the possibility of adopting Möbius strip as surface form for membrane structure has been investigated. Aspects of modeling of surface in the form of Möbius strip, a non-orientable surface, and the form as well as pre-stress pattern of the resulting membrane structures through form-finding analysis are also studied.

2. Generation of Möbius strip membrane surface

A Möbius strip of half-width *W* with midcircle of radius *R* at height $z = 0$ as shown in Figure 2 can be represented parametrically by the following set of equations (Weisstein [7]):

$$X = (R + S\cos\frac{\theta}{2})\cos\theta\,, Y = (R + S\cos\frac{\theta}{2})\sin\theta\,, Z = S\sin(\frac{\theta}{2}) \tag{1}$$

for $\{S{:}-W,W\}$and $\{\theta{:}0,2\pi\}$.

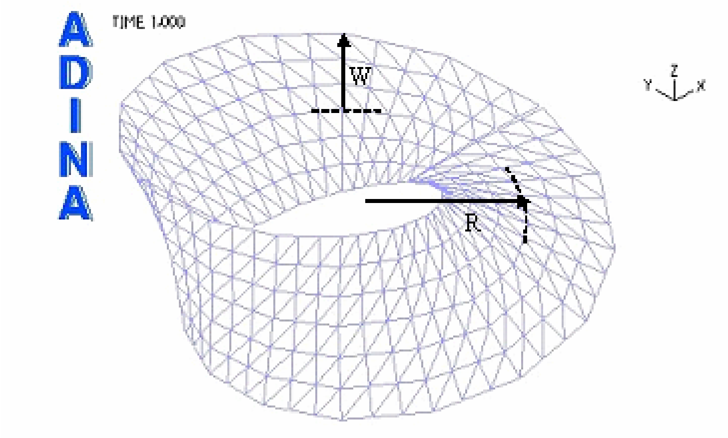


Figure 2: A Möbius strip with midcircle radius *R* and half-width *W*

For this study, the software ADINA has been used for the purpose of model generation. The problem with generation of model in the form of Möbius strip is that it is a single continuous surface involving only one curve along the band edges in a Cartesian space. The surface of a Möbius strip created by Equation (1) (Weisstein [7]) has the normal of the faces at the adjoining edges of the band in opposite direction which makes it impossible to join the band borders thus closing the surface. This problem has been overcome by applying discontinuous surface in the modeling. The solution adopted was to introduce a small gap across the width of the strip as depicted in Figure 3.

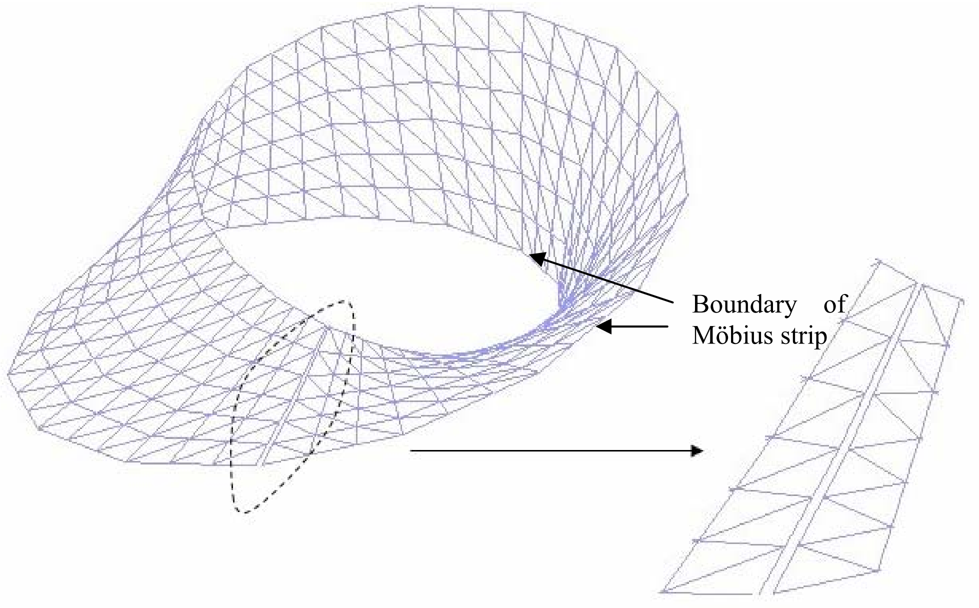


Figure 3: Generation of Möbius strip membrane surface

2.1. Form-finding analysis

A non-linear finite element analysis program for the analysis of membrane structures has been used in this study. The procedure adopted is based on the work by Cheong [1]. 3-node plane stress element has been used as element to model the surface of membrane structure. All x, y and z translation of nodes lying along the boundary edge of the Möbius strip have been restrained. Similarly, all nodes lying on either side of the gap introduced were also restrained from translating in all directions. The member pretension in warp and fill direction, denoted as T_{xx} and T_{yy} respectively is 2000N/m. The shear stress T_{xy} is zero. Two stages of analysis were involved in the procedures of form-finding analysis used. The first stage(SF) is analysis which starts with an initial assumed shape in order to obtained an updated shape. This is then followed by the second stage of analysis(SS) aiming at checking the convergence of updated shape obtained at the end of stage 1. During stage SF, artificial membrane properties with very small values are used. Both warp and fill membrane stresses are kept constant. In the second stage of S1, the actual membrane properties values are used. Resulting warp and fill membrane stresses are checked at the end of the analysis against prescribed membrane stresses. The resultant shape at the end of satge S1 is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses if difference between the obtained and the precribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and precribed stresses have been presented in the form of percentage of stress deviation versus element number. As a first shape for the start of form-finding analysis procedure adopted in this study, initial guess shape is needed. For the generation of such initial guess shape, knowledge of the requirement of anti-clastic nature of membrane surface is used.

3. Numerical analysis

The FE model has been generated using 3-node triangular elements into two cases: Case I: R=10m, W is varied; and Case II: W=4m, R is varied. In Case I, W is increased from W/R=10% to W/R=200%. As for Case II, R/W=10% to R/W=200%. The results of generation are shown in Figures 4-7.

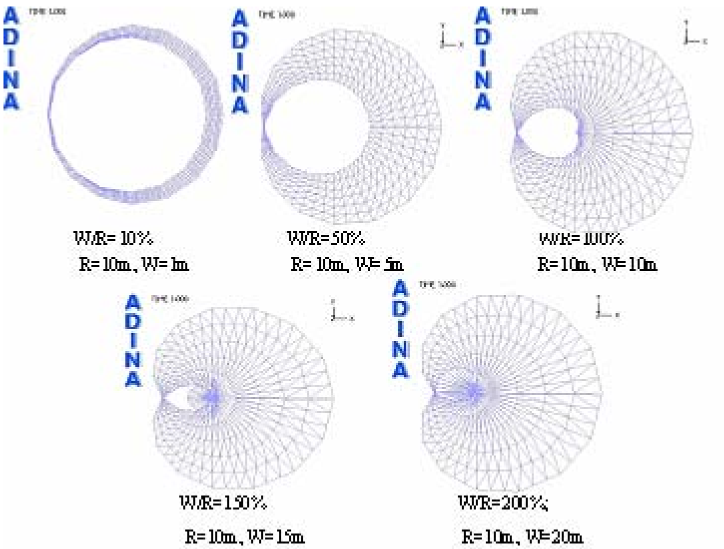


Figure 4: Plan view for models of Möbius strip: Case I

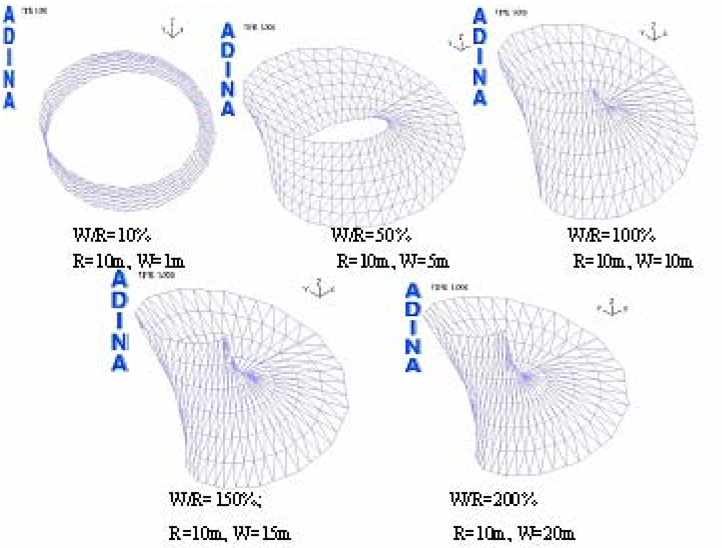


Figure 5: Different views for models of Möbius strip: Case I

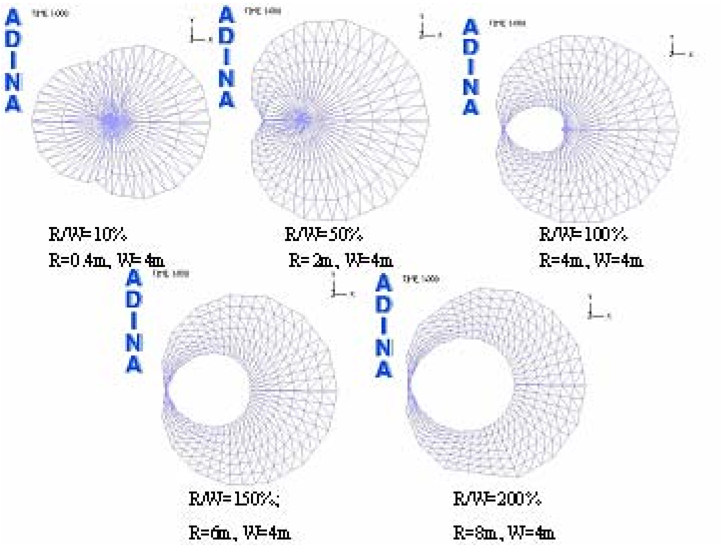


Figure 6: Plan view for models of Möbius strip: Case II

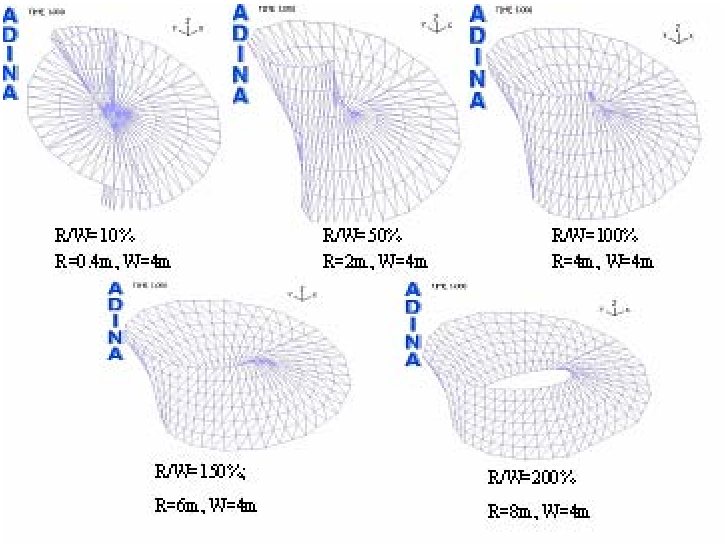


Figure 7: Different views for models of Möbius strip: Case II

For Case I, it can be seen in Figures 4 and 5 that when W is increased, the opening of Möbius strip becomes smaller. When W/R=100%, a sharp point can be seen to form(Figure 4). With increasing W/R ratio from 100% to 250%, the sharp point is seen to grow larger and tends to cover the opening of the generated models. When W/R=200%, the surface of the model starts to overlap the opposite direction of the surface. For case II, it can be seen in Figures 6 and 7 that when R becomes larger the opening center of the Möbius strip becomes larger. Only models with W/R<1 have been considered in this study. Three models of membrane structures have been studied: they are: (R=10m, W=5m), (R=8m, W=4m) and (R=6m, W=4m). Stress deviation at the end of form-finding analysis for the case of (R=10m, W=5m) in Figures 8 and 9, respectively for warp and fill stresses. From Figure 8, it can be seen that the average minimum stress deviation for the elements is -0.87% and average maximum stress deviation for the elements is 3.23% in warp direction.

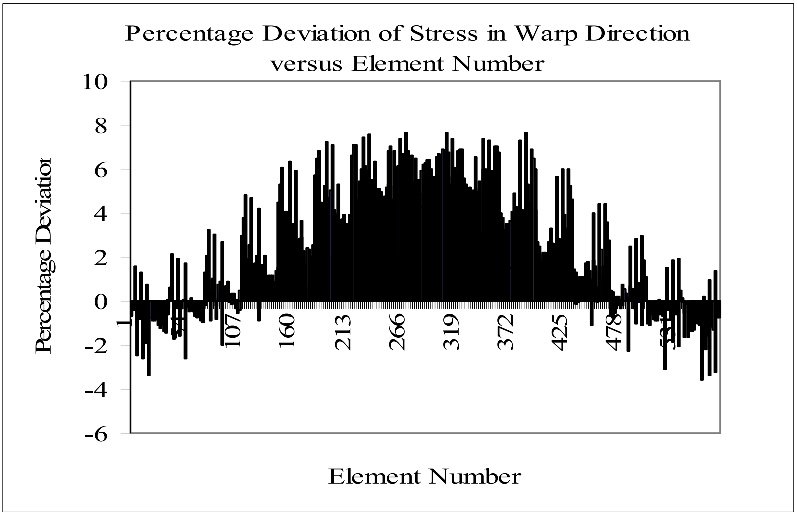


Figure 8: Percentage deviation of warp stress versus element number

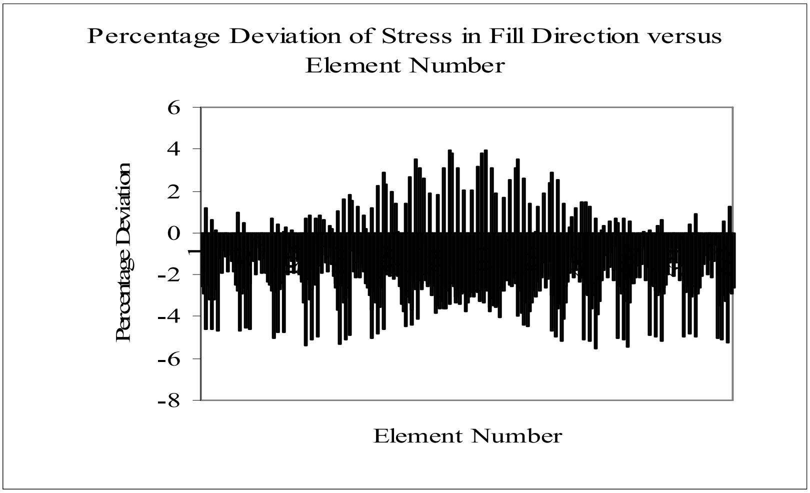


Figure 9: Percentage deviation of fill stress versus element number

From Figure 9, the average minimum stress deviation for the elements is -2.20% and average maximum stress deviation for the elements is 1.40% in fill direction. The ratio of warp to fill stresses obtained is $\sigma_W/\sigma_F = 1.0$. The obtained initial equilibrium surface for the model of Möbius strip is shown in Figure 10.

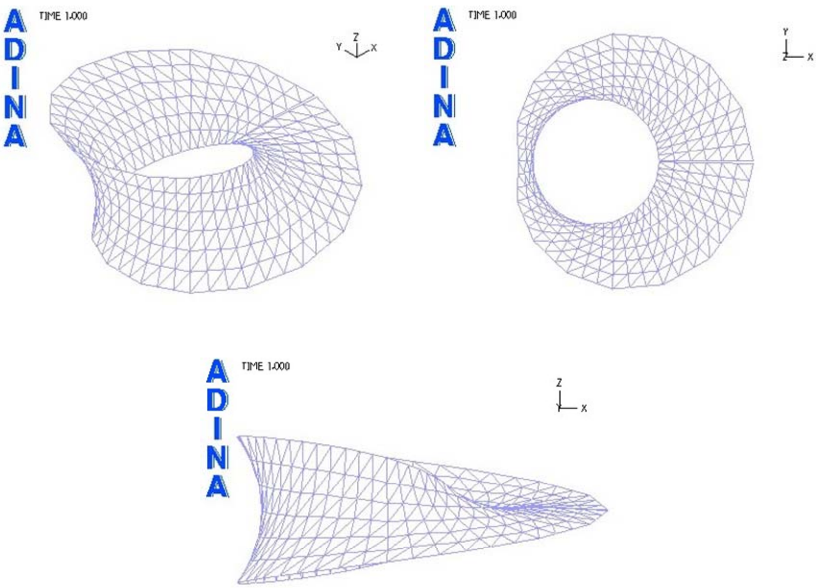


Figure 10: Different views for the model membrane surface in the form of Möbius strip after form-finding analysis (case of R=10m, W=5m)

The same trend as in the case of R=10m, W=5m has been observed in the cases of R=6m, W=4m(R/W=1.5) and R=8m, W=4m(R/W=2). For models with (R=6m, W=4m) and (R=8m, W=4m), the average minimum stress deviation for the elements is -0.81%, -1.55% and average maximum stress deviation for the elements is 3.09%, 2.55% in warp direction, respectively. While the average minimum stress deviation for the elements is -2.16%, -2.41% and average maximum stress deviation for the elements is 1.27%, 1.05% in fill direction, respectively. The obtained warp to fill stress ratio is observed to be $\sigma_W/\sigma_F = 1.0$.

4. Conclusion

Form-finding analysis on membrane structure with surface in the form of Möbius strip has been carried out successfully using the procedure adopted which is based on nonlinear displacement method. Method to overcome modeling problem associated with Möbius strip has been described and proposed. The resulting forms with warp to fill stress ratio approximately equal to one indicate that the obtained initial equilibrium shapes are minimal surfaces.

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