Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Deployable folded plate structures – folding patterns based on 4-fold-mechanism using stiff plates

Univ.-Prof. Dr.-Ing. Martin TRAUTZ, Dipl.-Ing. Arne KÜNSTLER*

RWTH Aachen – Faculty of Architecture – chair of structures and structural design
Schinkelstr. 1, 52056 Aachen, Germany
kuenstler@trako.arch.rwth-aachen.de

Abstract

Four-fold-mechanisms, such as the Miura-Ori-pattern, and its possible combinations are investigated by completing the mathematical description of a thin paper model done by Haas [1], with influences resulting from the thickness of the plate material. Additionally, different planning and realisation factors are analysed, focusing particularly on the elasticity of the plate material, the folding pattern, the realistic simulation of the hinge position and its necessity of kinematical freedom, as well as the inner friction resistance, resulting from the deployment of the necessary drive mechanism. Possible combinations of single four-fold-mechanisms to larger patterns featuring these characteristics are investigated, taking into account the previously mentioned conditions for planning and realisation.

Keywords: deployable structure, folded plate structures, 4-fold-mechanism, Miura-Ori-Pattern

1. Introduction

Throughout the centuries deployable structures have been used in the field of architecture. Their application started early, in mobile tents, deployable textile sunscreens in antiques arenas and is nowadays often used for example in temporary roofing of sport arenas. These constructions share the common general intention of an improved building adaptability through the possibility to react to different utilisation requests.

Currently most deployed constructions are based on the use of textile materials as deployable elements or completely rigid building elements, which can be removed entirely. Deployable structures using folded plate constructions are rarely realized despite the fact that it is possible to create wide span high performances structures with enclosing and formative character. The articulated design of the folds furthermore allows the structure to provide kinematical properties. Thus a structure can be designed which combines the advantages of folded plate structures with the possibility of a reversible building element through folding and unfolding.

This idea is not new, but remains an attractive subject, as the multitude of publications on this theme confirm. Lots of different folding patterns had been developed (Figure 1) and verified in thin-paper-models, differing in the number of folds meeting in one knot and the angle arrangements around the knots. By moving paper-models in different ways different deployment possibilities could be assumed (Figure 2).

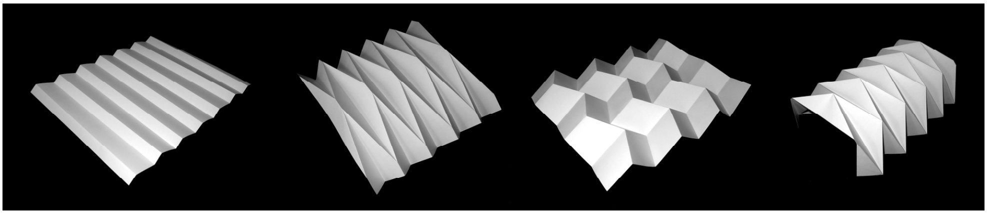


Figure 1: one-fold-, four-fold- and 6-fold-folding-patterns

But in reality, where wide spans have to be taken, the paper-model has to be replaced by a model considering the existing plate thickness and associated stiffness. These considerations lead to several restraints in realizable foldable plate structures of which one important is the feasible compactness in the completely folded state, especially when the structure is supposed to disappear. Hence only three reasonable folded plate structure mechanisms remain: the one-fold, the four-fold and the six-fold-pattern (Figure 1). As the one-fold-mechanism represents a quit easy folding pattern to analyze, the four-fold-mechanism becomes more difficult to handle in regards of the geometric, static and kinematical characteristics.

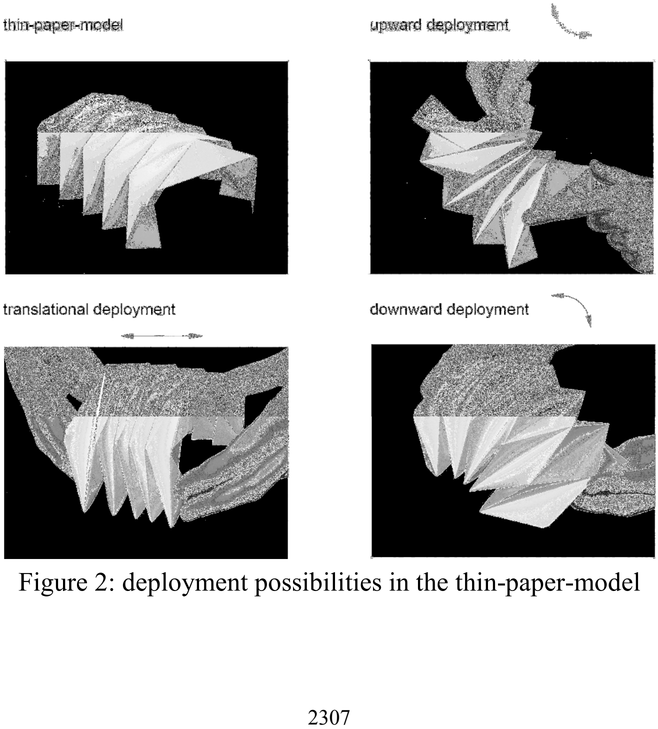


Figure 2: deployment possibilities in the thin-paper-model

2. Geometrical characteristics implicating the plate thickness

The mathematical description of a four-fold-mechanism of thin paper as described by Haas [1] enables to calculate the movement of one fold (vector OD) in dependence on the remaining three folds OA, OB and OC given by the angles α, β, γ and δ between the folds and the opening angle ε (compare Figure 4).

The possible opening angle ε varies for the thin-paper-model between -180° < ε < 180°. Regarding a realistic volume-model with a certain plate thickness the opening angle ε is limited to a range of 0° < ε < 180° (compare Figure 3). Due to the alignment of the hinge on the concave side of the fold to enable a complete folding in one direction, a restriction is created, which disables the folding process beyond the completely unfolded state.

A second important effect on 4-fold-mechanisms arising from the plate thickness is shown in Figure 3. Instead of rotating the plate elements against the hinge axis to provide a deployment without restraints (Piekarski [2]), longitudinal hinge deformations must be allowed to achieve a maximum compactness in the completely folded state.

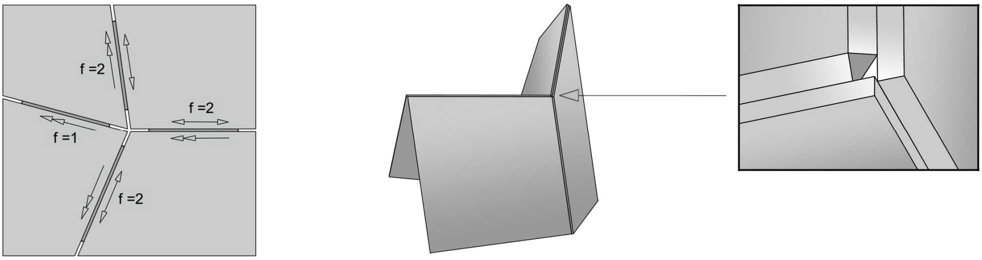


Figure 3: 4-fold-mechanism and its longitudinal hinge deformations

2.1. Mathematical description

The calculation of the vectors OA to OD is independent of the thickness of the plate material. Thus the mathematical description of Haas [1] stays valid. Therefore the effect of translational hinge deformations can be solved by calculating the vector OO’ with the help of vector OC on the one side and by consideration of vector OA on the other side (Figure 4):

$$\overrightarrow{OO'} = c_v \cdot \overrightarrow{OC} + d_v \cdot \overrightarrow{O'D} + t \cdot \frac{\overrightarrow{t_1}}{\left|\overrightarrow{t_1}\right|} = c_v \cdot \begin{pmatrix} \cos\beta \\ \sin\beta \\ 0 \end{pmatrix} + d_v \cdot \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} + t \cdot \frac{\overrightarrow{t_1}}{\left|\overrightarrow{t_1}\right|} \tag{1}$$

$$\overrightarrow{OO'} = b_v \cdot \overrightarrow{OB} + a_v \cdot \overrightarrow{OA} + t \cdot \frac{\overrightarrow{t_2}}{\left|\overrightarrow{t_2}\right|} = b_v \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_v \cdot \begin{pmatrix} \cos\alpha \\ \sin\alpha \cdot \cos\varepsilon \\ \sin\alpha \cdot \sin\varepsilon \end{pmatrix} + t \cdot \frac{\overrightarrow{t_2}}{\left|\overrightarrow{t_2}\right|} \tag{2}$$

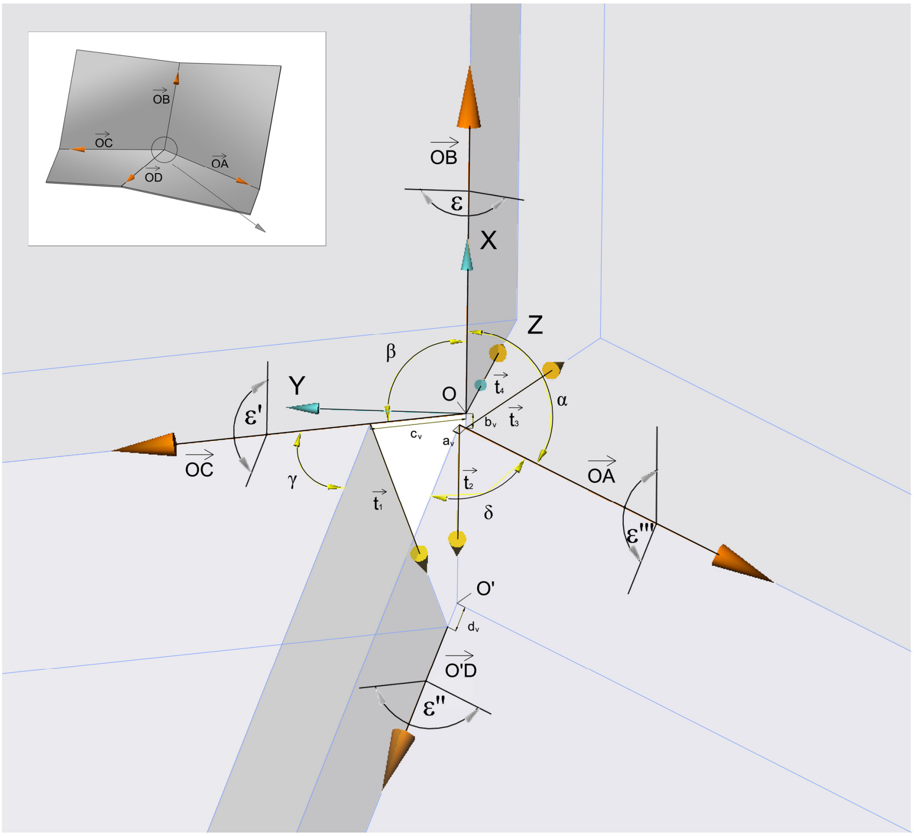


Figure 4: 4-fold-mechanism with assignment of the influencing variables considering the plate thickness

Through equating of both equations it is possible to calculate three of the four unknown translational displacements. Only one displacement must be given to get all other variables.

The diagrams in Figure 5 to Figure 7 show the results for three different four-fold-mechanism with a given plate thickness of $t = 10$ mm and a given displacement of $d_v = 0$mm.

Within a symmetrical folding pattern (Figure 5) only displacements in the folds OA and OC occur and increase dramatically by approximation to the completely folded condition. In the fold OB zero translation occurs.

In comparison, within an unsymmetrical folding pattern, which is also completely foldable and unfoldable (Figure 6), there are translational displacements in all three folds OA, OB and OC. The same occurs in an unsymmetrical folding pattern which is not completely foldable and unfoldable (Figure 7). Hereby it is notable that the translations in folds OA and OC reverse their direction.

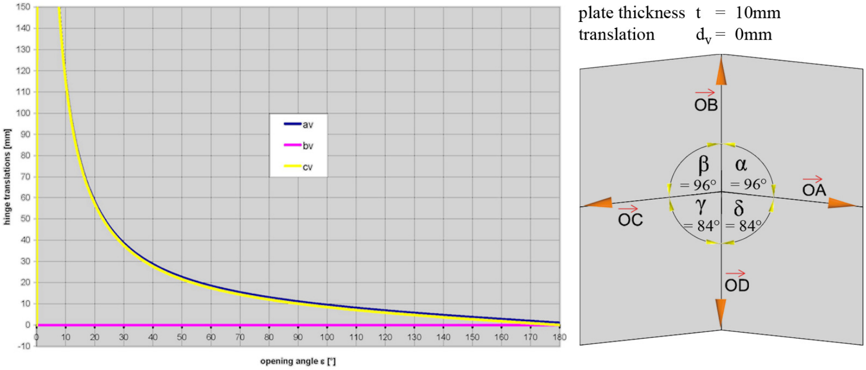


Figure 5: Translations for the simple-symmetric angle arrangement

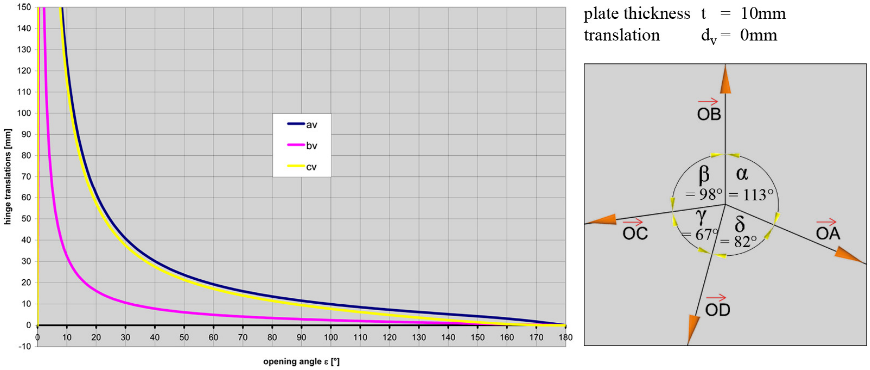


Figure 6: Translations for unsorted angle arrangement

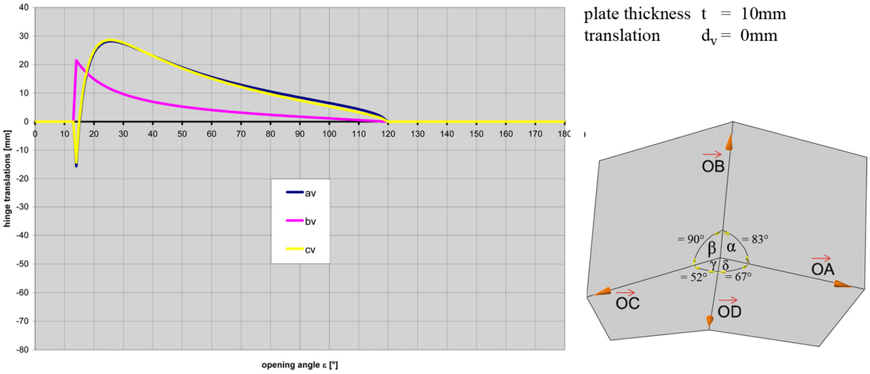


Figure 7: Translations for the unsorted, not fully deployable angle arrangement

2.2. Factors for complete and limited folding and unfolding

In addition to the rules for angle-arrangements around a knot in a thin-paper-model described by Haas [1] resulting in either completely foldable patterns and/or patterns which are completely unfoldable to a plane, the volume-model provides some further restrictions.

Because of the structural limitation of the hinge translation, the assumption of a completely foldable folding pattern is not realistic, especially on the subject of the infinite displacement towards the completely folded condition displayed in Figure 5 and Figure 6. In addition, the variation of the plate thickness necessary because of the structural and static requirements results directly in the increase or decrease of the hinge translation (Figure 8). Therefore a factor must be created evaluating the degree of possible foldability for folding patterns.

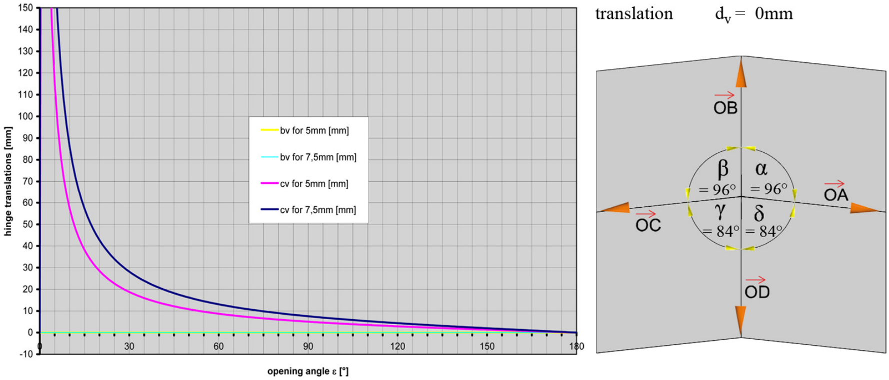


Figure 8: Hinge translations for the simple-symmetric angle arrangement with different plate thickness

Based on the demonstration of Pellegrino and Vincent [4] regarding the influence of the angle-arrangement for the Miura-Ori-folding-pattern on the expansion factor, the influence of the plate thickness can easily be considered with the knowledge of the realistic folding-vectors already described in Figure 4. The distance AC and BD (Figure 9) calculated and based on the maximum values in the unfolded state AC_{max} and BD_{max} result in an expansion factor applicable for any folding-pattern. To be able to compare the volume-model with the thin-paper-model, the folding-pattern dimensions a and b have to be considered. Figure 10 shows a nearly identical curve progression of the expansion factor curves on the right side of the chart, starting at the completely unfolded state for the thin-paper-mode and the volume-model. On the left side the curves start to differ up to a dramatically increase of the volume-model due to its hinge translations. Thus it appears that the realistic degree of expansion for the analyzed pattern is significantly restricted in the direction of the distance AC.

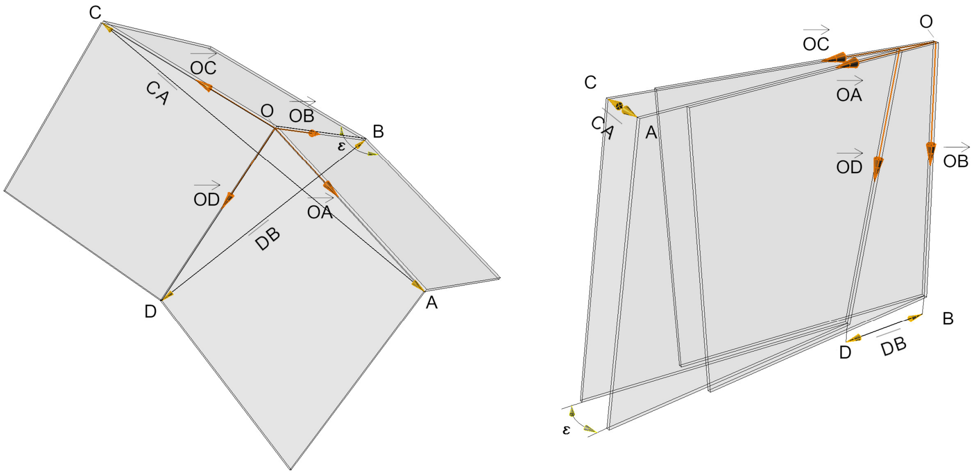


Figure 9: Definition of the expansion factor

But the restriction of the longitudinal hinge translation does not only delimit the folding process in a negative way. Regarding in contrast the unfolding process, it enables a simple and sustainable limitation up to the provided deployment limits. A folding pattern as shown in Figure 5 only envisaged to deploy between an opening angle between 0° < ε < 135° can structurally be limited by restraining the hinge deformations in the folds OA and OC. Thus the unfolded structure becomes independent of supplementary adjustments of the driving mechanism.

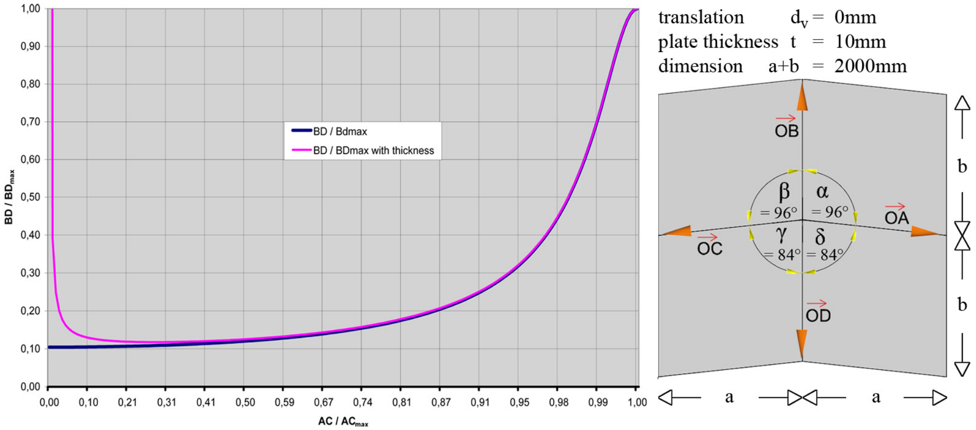


Figure 10: expansion of the simple-symmetric angle arrangement with and without plate thickness

3. Geometrical factors for combinations of 4-fold-moduls

The foldability of a combination of four single mechanisms in a 4-knot-system (Figure 11) depends not only on the foldability of the single mechanism but although on the geometrical dependencies between the single mechanisms, as well for the thin-paper-model as for the model considering the plate thickness.

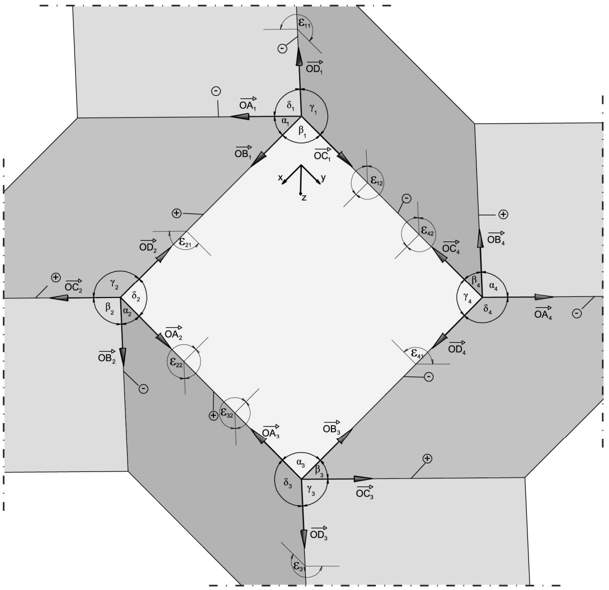


Figure 11: Combination of single 4-fold-mechanism in a 4-knot-system

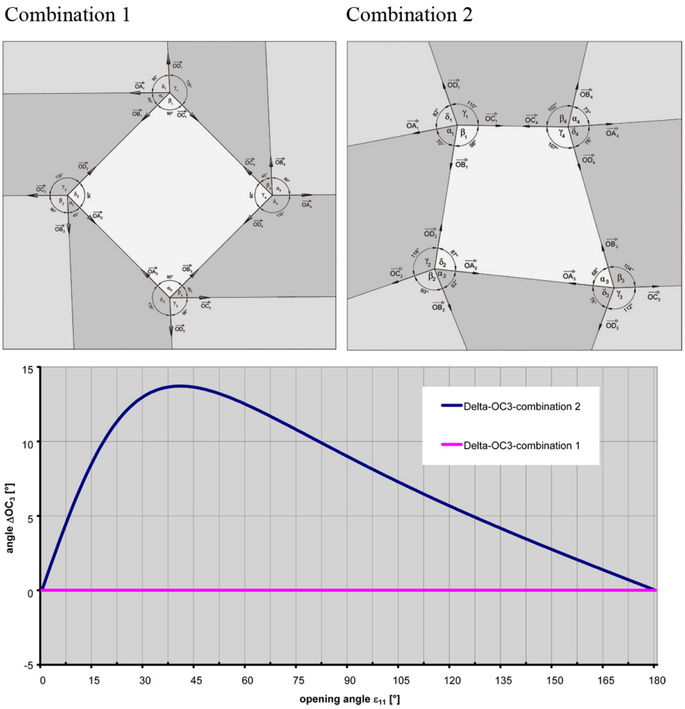
3.1. Mathematical description of the thin-paper-model

A mathematical verification of the deployment capacity of a folding pattern can be effected by two possible ways to calculate the vector OC_3. One is to calculate the vectors in knot 2 in dependence on knot 1 and to later define the vectors in knot 3. The other is to do the same by using the vectors in knot 4 in dependence of knot 1. The results should be identical with every opening angle ε_{11} given. If the results are different, the combination of the chosen single 4-fold-mechanisms is not deployable.

This effect can be demonstrated by calculating the angle between both vectors OC_3, one calculated via knot 2, the other via knot 4 (Figure 13):

$$\Delta OC_3 = \arccos\left(- \frac{\overrightarrow{OC_{3(2)}} \cdot \overrightarrow{OC_{3(4)}}}{\left|\overrightarrow{OC_{3(2)}}\right| \cdot \left|\overrightarrow{OC_{3(4)}}\right|} \right) \tag{3}$$

For a deployable and a not deployable mechanism the calculated angle ΔOC_3 is depicted in Figure 12. In these cases, every single 4-fold-mechanism is completely foldable and unfoldable by itself. Even in the complete folded and the complete unfolded state the combination of the four single mechanisms provides realistic results (the angle ΔOC_3 becomes 0°). But in between these two conditions for the not deployable pattern, the value of the calculated angle differs from zero. The investigated combination is not deployable. For the deployable combination, the angle stays zero for any opening angle ε_{11} given.


Combination 1

Combination 2

Figure 12: Diagram of the angle ΔOC_3

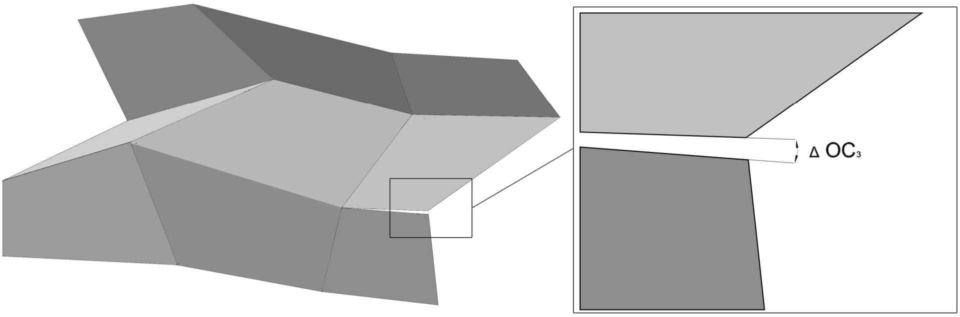


Figure 13: The angle ΔOC_3

3.2. Restrictions while considering the plate thickness

Mathematical descriptions for the hinge translations extended with combinations of 4-fold-mechanisms demonstrate further restrictions in some possible deployable folded plate structures. Figure 14 shows for example the effect of an already as deployable defined folding pattern. While for the simple unsymmetrical mechanism one hinge translation has to be fixed, for the combination in Figure 14 all four translations need to be uncoupled. Thus multiple hinge translations depending on one introduced deformation are possible for one value of the opening-angel ε. Accordingly a driving mechanism must be provided to control this supplementary degree of freedom. Further the expansion of the combination in Figure 14 unfortunately causes successive enlargements of the hinge translations in adjacent folds.

Due to this fact every deployable folded plate structure has to be analyzed in regard of its necessary number of translational hinges around a knot. The mentioned effects are reduced with the use of simpler folding patterns.

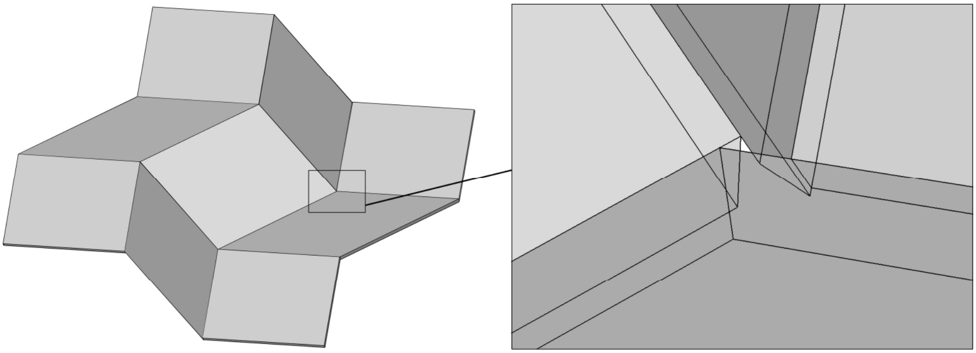


Figure 14: Hinge translations in a 4-knot-combination

4. Statically and kinematical effects

As for every deployable structure the influence of the characteristics resulting from the associated structural design reduce in a certain way the load-bearing capacity. The principle load bearing characteristics of folded plate structures normally include two aspects which cannot be generated. One is the bending resistance between the plates in the area of the fold. This characteristic is disabled due to the articulated hinge construction in the fold. The second missing aspect results from the exigency of translational hinge deformations depending on the chosen folding pattern. Shear forces in the folds resulting from different geometries of the adjacent plates or unbalanced live loads on the structure cannot be transmitted by the hinges. Hence the general load bearing capacity of the structure is reduced, as shown in Figure 15. The deformation of the folded plate structure under dead load results in a self-deployment of the structure and therefore in further hinge translations.

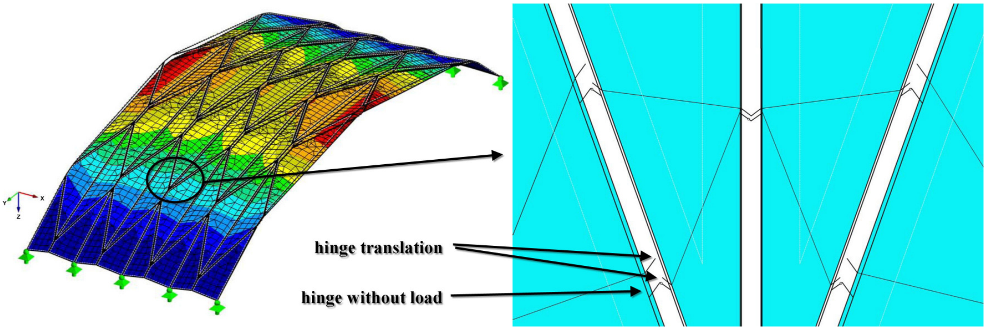


Figure 15: FE-Model - Deformations under dead load

As the maximum deployed state is normally as well the determining statical state, boundaries in the hinge translations can help to minimize the mentioned effects. These boundaries are not only limitations in the hinge translations but also in the hinge rotation.

In contrast to pure statically structures, deployable constructions are supplementary loaded by internal and external forces resulting from the motion of the structure. Additional to the static load-bearing reactions also inner friction forces need to be introduced to the system during deployment. Hinge friction moments resulting from rotation and hinge friction forces resulting from translations can be calculated by setting the sum of energy performances P_i resulting from the driving forces and all internal forces to zero:

$$\sum P_i = \sum F_i \cdot v_i + \sum F_{Ri} \cdot v_{Ri} + \sum M_{Ri} \cdot \omega_i = 0 \tag{4}$$

The determination of the velocities v_i, v_{Ri} and ω_i of the different construction elements can be done by the Polplan and/or using the already calculated vector model (Figure 4). Multi-Body-Programs as used in mechanical engineering can help to determine the frictional forces F_{Ri} and M_{Ri}.

It is obvious that the application on multiple points of applications of the driving force reduce the inner stress ratio resulting from the internal forces.

5. Summary and outlook

In order to reach large structural spans with the least material thickness as possible, deployable folded plate structures need stiff plate materials. The minimization of the gap in the folds, the desire of edged folds and of the least appearance of the hinge construction leads to the necessity of translational hinges to guarantee the architectural design request. The use of new materials such as aluminium sandwich plates persuade by their extreme light weight with coincidental high load-bearing capacity enables an optimized plate thickness and therefore reduces the mentioned associated restraints.

Nevertheless not all 4-fold-mechanisms and their combinations are applicable with the use of stiff plate materials and need to be designed as deployable folded plate structures. Their capability of extreme compactness in comparison to one-fold- and six-fold-mechanisms with simultaneously planar as well as spatial deployment possibilities keeps them fascinating. The variation of the angle-arrangement even allows the arrangement of the load-bearing folds as needed and also a self-stiffening character by designing not completely unfoldable folding patterns. A self-stiffening character can although be achieved by a structural limitation of the hinge translations.

The utilization of elastic plate materials for the entire folding pattern or the selective replacement of stiff plates by elastic ones can increase the number of deployable folding patterns for smaller spans by simultaneous reduction of the load-bearing capacity. This option is also interesting for combinations of six-fold-mechanisms underlying identical restraints and therefore limitations of applicable folding patterns by the use of stiff plates.

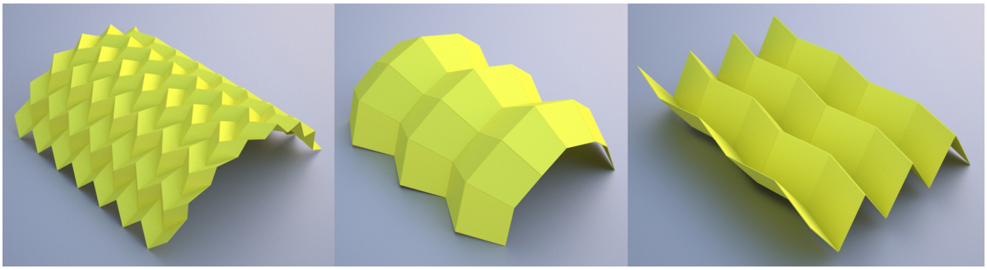


Figure 16: possible combinations of 4-fold-mechanisms

6. References

[1] Haas F., *Geometry and mechanics of hind-wing folding in Dermaptera and Coleoptera*, University of Exeter, 1994.

[2] Piekarski M., Constructional solutions for two-way-fold-deployable space trusses, *IUTAM-IASS Symposium on Deployable Structures: Theory and Application*, Pellegrino S. and Guest S.D., Kluwer Academic Publishers, 2000, 301-310.

[3] Hernandez C.H. and Stephens R., Development of folding aluminum sheet roofs, *Mobile and Rapidly Assembled Structures*, Escrig F. and Brebbia C.A., WIT Press, 2000, 155-164.

[4] Pellegrino S. and Vincent J.F.V., How to Fold a Membran, *Deployable Structures*, Pellegrino S., Springer Wien New York, 2001, 59-75.

[5] Trautz M., *Das Prinzip des Faltens*, (in preparation), Aachen, 2009.