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Topology Optimized Adaptive Truss Structures

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Abstract

The aim of this paper is to present a topology optimization algorithm which uses the adaptive design principles to enable the creation of an integral adaptive system. It is shown on numerical examples that it is possible to optimize the topology of adaptive structures under multiple loading scenarios in such a way that structures will be created were in defined degrees of freedom, the deformations are minimized and the corresponding structural weight is significantly reduced in comparison to conventional truss structures. The resulting structures will be compared to the so called Michell-Structures as well and this shows that using adaptivity it is possible to create structures which are even lighter than those well known lightweight truss structures. Furthermore the capability of the optimization algorithm is proven by using a genetic algorithm to generate topology optimized truss structures as well.

Keywords: Adaptivity, Topology Optimization, Truss Structures, Lightweight Structures, Actuators, Interior Point Method, Genetic Algorithms.

1. Introduction

The area of research on adaptive structures for structural systems has developed from the exploration of designing the lightest possible structure by implementing technologies from different disciplines. This has become necessary on one side due to the increasingly complexity in the design of high-rises and wide span structures and on the other side due to the need of environmentally friendly and energy efficient applications of all used materials. Therefore adaptivity stands for the interdisciplinary approach which enables the significant enhancement of the basic and most important features of a structure such as the structural weight as well as the deformations under various loading scenarios. This field of investigation on 'adaptive systems' enables a new understanding of lightweight structures and offers a breakthrough in a new dimension of minimalism.

Adaptive structures are therefore a special form of lightweight structures and are characterized by their ability to internally react to large, unscheduled loadings in order to redistribute the corresponding stresses amongst its components. This enables them to safely carry these loads.

These adaptive systems usually can be described as any structural system which itself is equipped with sensors for monitoring on one side the external loads acting on the system and on the other side, the response of the system due to adaptive manipulation. The response can either be the deformation in defined points or the stress level in selected members depending on the design goal. The sensors transmit their information to a controller unit i.e. a computer which calculates the necessary response in order to fulfil the requirements defined by the designer. The controller transmits this information to the actuators integrated into the structural system [9].

2. Fundamentals on Adaptive Structural Systems

The design principles for adaptive Systems are outlined in Lemaitre and Sobek [2]. The basics, namely the three states of an adaptive System will be explained briefly as follows: The first state is called the passive state which is defined as the state where the system is without manipulation and burdened only with external loads. The activated state is the condition where only the actuators are active. This state is solely considered to be a theoretical state which is necessary to numerically determine the necessary response of the structure in reference to the passive state. Finally the third state is the adaptive state that is defined as the superposition of the passive and the activated state. Summarized the three states and their pertaining to each other can be stated as follows:

Passive + activated = adaptive

Based on these three states and the assumption of a strictly elastic material behaviour, the adaptive normal forces $\mathbf{N}_{adapt}$ and deformations in the degrees of freedom $\mathbf{u}_{adapt}$ of the structure can be calculated through substitution.

$$\mathbf{N}_{adapt} = \mathbf{N}_{passive} + \mathbf{N}_{active} \quad (1)$$

$$\mathbf{u}_{adapt} = \mathbf{u}_{passive} + \mathbf{u}_{active} \quad (2)$$

The actuator locations within the structural system can be determined based on the fact that the introduced stress and deformation states through the actuators have to be linearly independent from each other. Using a greedy algorithm the most effective actuator locations can be determined and their amount can be reduced to a minimal amount (Lemaitre [3]).

3. Topology Optimization of Adaptive Truss Structures

Adaptive truss structures have so far been extensively investigated by Teuffel [8] who presented the high potential of adaptive truss structures for lightweight systems using so called Load Path Management (LPM). This approach considers the controlled and temporally variable adaptation of the characteristics or properties of a structural system and a manipulation of the structural response in the real time. Using these results, the next step was to consider not only the form optimization as used for the LPM but also an optimization of the truss topology to design even lighter and more efficient truss structures that perform equally under different or random loading conditions (Lemaitre [3]). Therefore an algorithm for topology optimization of truss structures is being developed. This algorithm unites an actuator selection algorithm with different approaches on topology optimization routines.

When generally designing a passive structure different objectives have to be satisfied such as minimal structural weight and on the other side maximal stiffness which is needed to secure the best structural performance. These requirements lead to the need of optimizing a structure under different aspects using different techniques. The most general type of such an optimization problem is the so called topology optimization. In this case the only boundary conditions in a design space are the possible supports and the loading point. The goal of this optimization routine is to determine the position, the arrangement and the amount of elements creating the structural system. Michell [4] was the first one you intensively investigated the question what is the idea allocation of the pressure and tension elements within a truss structure. He analytically generated the so-called "Michell-Structures". These structures consist of elements which are positioned along the main stress axis and therefore form a structure which is able to carry the external loadings with the minimal amount of material and deformations. These Michell-structures are until today the reference structures when investigation topology optimized truss structures and represent the lower barrier (figure 1).

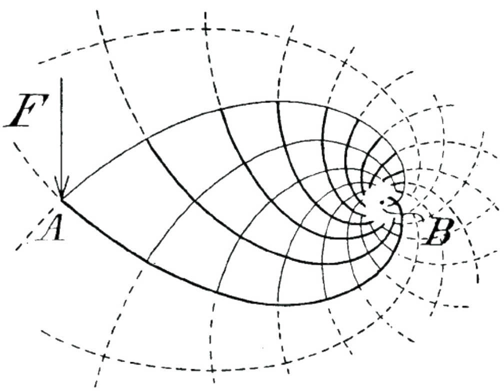

Figure 1: Michell-Structure [4]

This classical topology optimization problem for a truss can be stated as follows: minimize the compliance e.g. the internal potential work that a structural system can absorb as well as the structural weight e.g. the sum of the element voluminous $\mathbf{t}_i$. and minimize while satisfying the boundary condition of being a statically usable truss:

$$\min \frac{1}{2} \cdot \mathbf{p}^{T} \cdot \mathbf{u}$$

s.t.:

$$\sum_{i=1}^{m} t_i = V \quad ; \quad t_i \geq 0 \tag{3}$$

$$\mathbf{K}(t) \cdot \mathbf{u} = \mathbf{p}$$

In order to solve the optimization problem stated in equation 3 two approaches have been established over time. The first one is the mathematical numerical approach which uses search algorithms in order to generate a feasible solution. The second one is the use of genetic algorithms as a search tool. Genetic algorithms mirror the natural evolutionary process and therefore are a very powerful tool to solve highly complex optimization problems.

The mathematical numerical approach has been chosen to develop an algorithm to optimize truss structures since this offers a method which creates one global solution versus using genetic algorithms, which the results have to be verified through repetition since they tend to produce local and global solutions.

In order to use a mathematical optimization routine, the highly complex formulation of equation 3 has to be formulated in a way that there is only a single design variable left and multiple loading scenarios are implemented. Following the results of Brännlund and Svanberg [1] the truss topology optimization problem can be stated as follows:

$$\min \sum_{i=1}^{m} \mathbf{t}_i$$

$$s.t.: \tag{4}$$

$$\phi \sum_{i=1}^{m} \mathbf{t}_i \cdot \mathbf{K}_i - \mathbf{p}^k \mathbf{p}^{k\ T} \succeq 0$$

Transforming the topology optimization problem onto an adaptive system, the boundary condition of maximal reduction of the normal forces as well as the deformations in defined degrees of freedom through the actuator action is added. The topology optimization problem of an adaptive truss structure can therefore be defined as follows:

$$\min \sum_{i=1}^{m} \mathbf{t}_i$$
$$s.t.:$$
$$\min \left\| \begin{bmatrix} \mathbf{N}^{selc} \\ \mathbf{U}_c^{selc} \end{bmatrix}_{aktiv}^{k} \cdot \Delta \mathbf{l}_{aktiv}^{k} - \begin{bmatrix} -\mathbf{N} \\ -\mathbf{u}_c \end{bmatrix}_{passiv}^{k} \right\|^2 \tag{5}$$
$$\phi \sum_{i=1}^{m} \mathbf{t}_i \cdot \mathbf{K}_i - \mathbf{p}^k \mathbf{p}^{k\ T} \succeq 0$$

To solve this problem, the topology optimization algorithm consists of a passive and an adaptive part, which interact through the design space E (figure 2). This is necessary since the system activation is always a response to the passive state. Therefore the passive and the adaptive part have iteratively to be investigated and to be inter-coordinated with each other in order to grant the optimized performance of the adaptive structure.

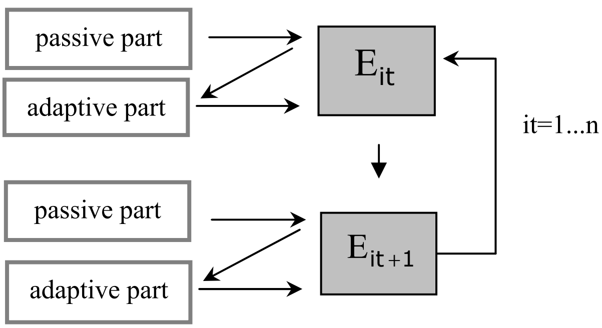

Figure 2: scheme of interaction of the adaptive and the passive part of the topology optimization algorithm

Using the interior point method [7] for the passive state of the structure, the objective of the associated passive part of the optimization routine is to identify the load paths within the design space. In the subsequent step, the system will be activated and the local adaptations for each truss member are calculated. These local adaptations are used as the decision-making parameters for the manipulation of the design space. This optimization routine will be performed repetitively and interactively as long as the global stop criterion is fulfilled.

The schematic workflow of the algorithms is shown in figure 3. In order to investigate and therefore to be able to adjust the critical parameters of the developed optimization, procedure parameter studies have been done in order to determine the ideal point of optimization when the algorithm needs to switch from the passive to the adaptive part. The critical parameter is the so called duality gap, which describes the optimization progress of the interior point method. Other parameters are the degree of discretisation of the design space as well to determine the global stop criterion. These studies and a detailed description of the topology optimization algorithm for adaptive truss structures are published in Lemaitre [3]. The special cases of the pure force- and deformation-adaptation can be implemented as well. The results of the strict force adaption are shown in figure 4 It is demonstrated that the achieved structural weights are starting to be significantly lighter at a height to span aspect of 1,5. Therefore the structural weight frontier represented by the Michell-structures can be undergone by using adaptive elements and the integral design approach as discussed above.

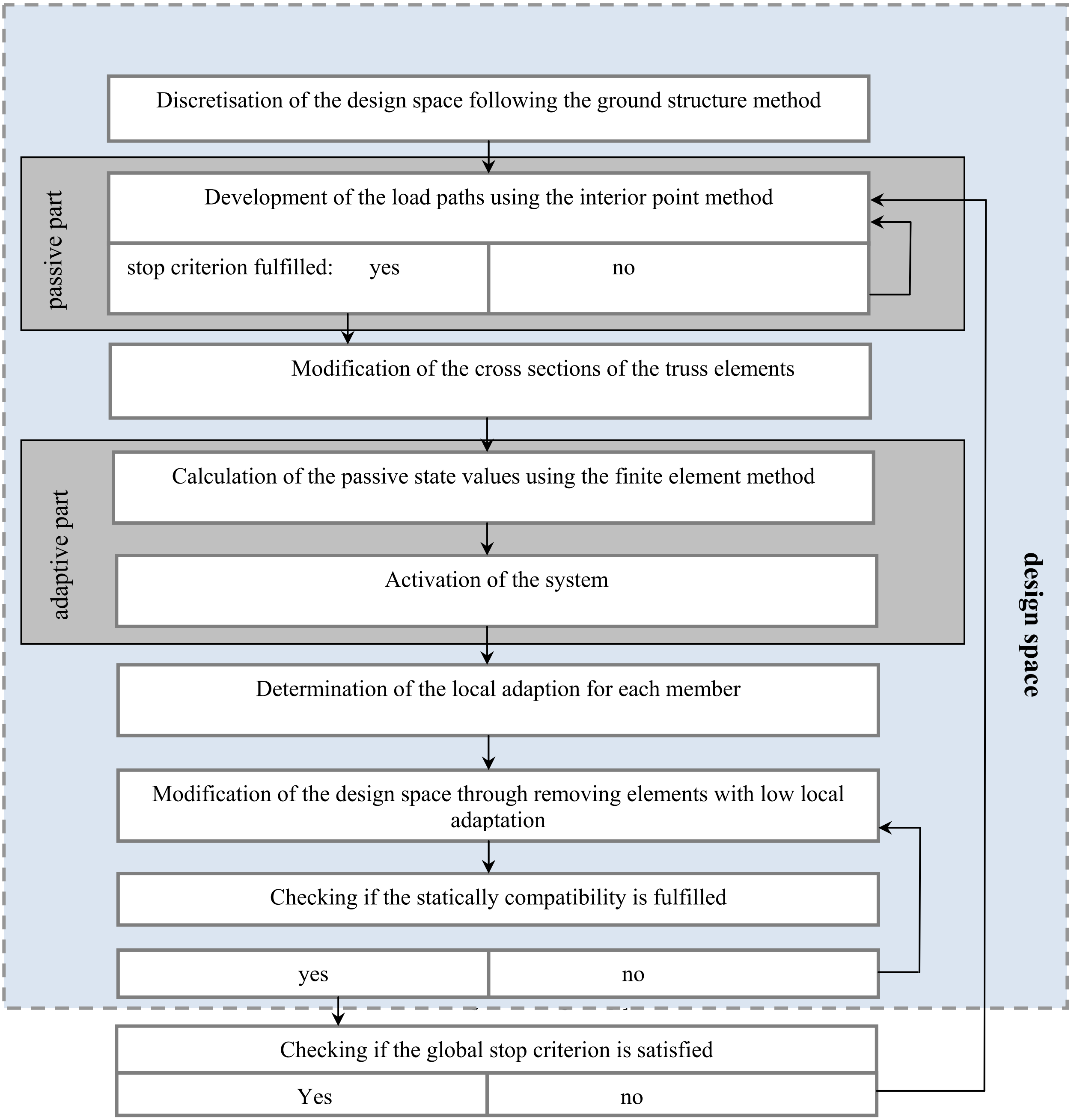

Figure 3: scheme of interaction of the adaptive and the passive part of the topology optimization algorithm

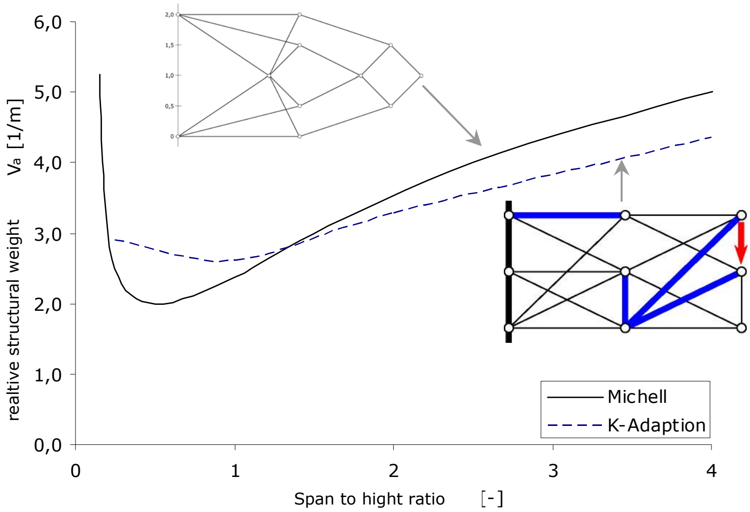

Figure 4: Comparison between the structural weight of Michell-Structures and the topology optimized adaptive truss structures under simultaneous force and deformation adaptation and a single load case

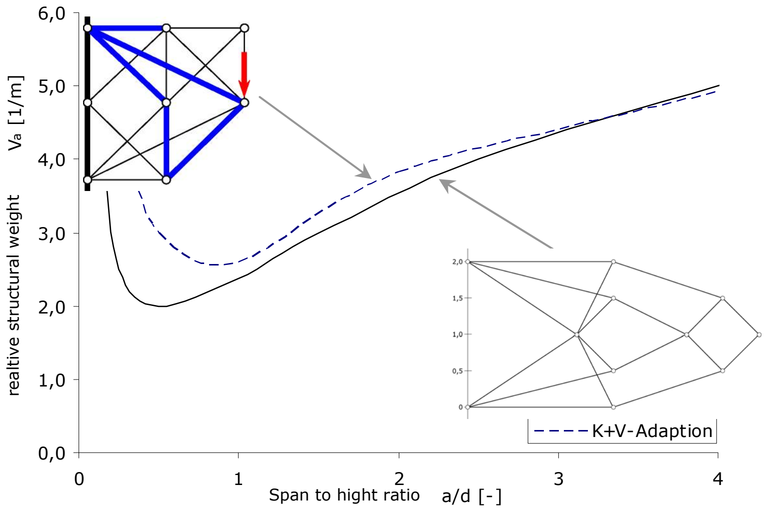

Figure 5: Comparison between the structural weight of Michell-Structures and the topology optimized adaptive truss structures under simultaneous force and deformation adaptation and single load case

Implementing the simultaneous force and deformation adaption shows (figure 5), that those topology optimized adaptive truss structures are not as light as the once generated using the strict force adaption as a target function but they posses almost the same weight as the so called Michell-structures. Furthermore the adaptive truss structures possess a minimal deformation at the defined degree of freedom which in this case is the one at the loaded point in the direction of the external load. When increasing the number of load cases and comparing the resulting structural weight and the corresponding deformations between the adaptive optimized structures and passive structures with the same target function, it shows how the performance and resulting stiffness of those structures is significantly higher (figure 6).

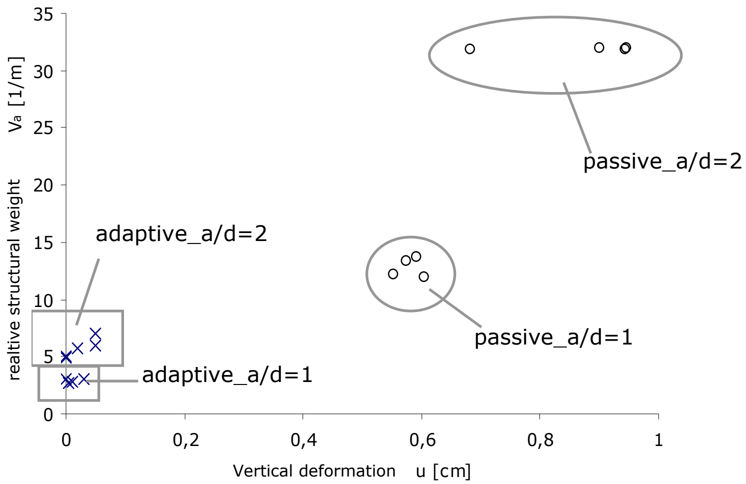

Figure 6: Comparison between the structural weight of Michell-Structures and the topology optimized adaptive truss structures under simultaneous force and deformation adaption and multiple load cases

In order to validate the results of the optimization routine the results have been compared on one side with the structural weights of traditional truss structures which were equipped with actuators and activated (figure 7). It shows that those structures cannot reduce the stress level in such a way, to be even close to the ones of the optimized adapted topologies and hence their structural weight. Additionally there is no direct relation between the ration of span to height which leads to the conclusion that the results are not as predictable and the activation of traditional trusses has to be weighed against the goal which has to be achieved.

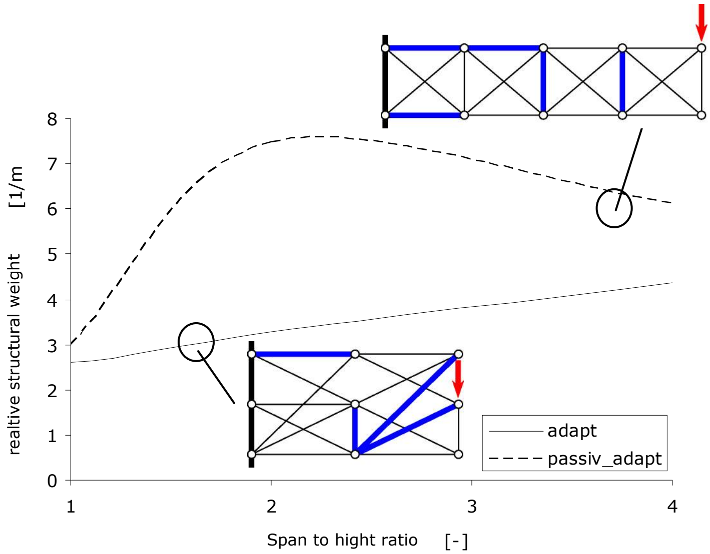

Figure 7: Comparison between the structural weight of the topology optimized adaptive truss structures and conventional adaptive truss structures

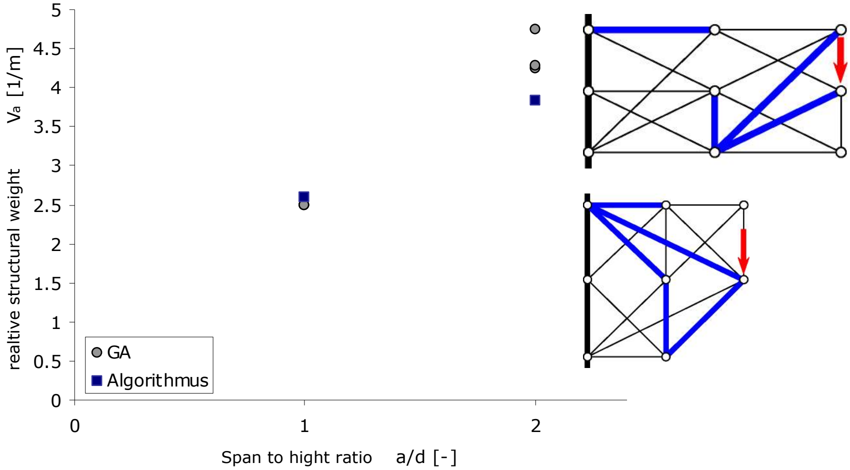

Figure 8: Comparison between the structural weight of the topology optimized truss structures using the presented topology optimization algorithm and a genetic algorithm

In addition, the adaptive design approach was be implemented in a genetic algorithm form in order to verify the results of the topology optimization algorithm (figure 8). It shows that the results from the mathematical numerical approach using the interior point method are verified.

5. Conclusion and Outlook

The above presented has demonstrated how the use of new design approaches can significantly improve the performance of structural systems and open up a wide new field of possibilities. Following these theoretical approaches it will be necessary to validate and test the results on real time models and structures in order to further improve the technology of adaptive structures. Furthermore especially the detailing of the nodal points has to be investigated since they require very special features. On one side they have to be able to allow the necessary deformations of the structure and on the other side they have to possess a certain rigidity to secure the safety of the structure.

Besides these further necessary investigations the use of new materials and design approaches is necessary in order to develop structures which will fit into the new generation of buildings designed to be more resource and energy efficient.

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