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Material Requirement Planning under Fuzzy Lead Times

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Abstract: This paper proposes a fuzzy multi-objective integer linear programming approach to model a material requirement planning (MRP) problem with fuzzy lead times. We incorporate to the crisp MRP model the possibility of occurrence of each one of the possible lead times. Then, an objective function that maximizes the possibility of occurrence of the lead times is considered. By combining this objective with the initials of the MRP model, decision makers can play with their risk attitude of admitting lead times that improve the other objectives but have a minor possibility of occurrence.

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Keywords: Material requirement planning, uncertainty modelling, lead times, fuzzy modeling, multi-objective optimisations.

1. INTRODUCTION

There are many forms of uncertainty that could affect material requirement planning (MRP) systems. Ho (1989) identifies two uncertainty groups: (i) environmental uncertainty, which includes uncertainty in demand and supply; and (ii) system uncertainty, which is related to operation yield uncertainty, production lead time uncertainty, quality uncertainty, failure of production system and changes to product structure. This leads to the development of models for MRP with uncertainty (Mula et al. 2006).

MRP models under uncertainty in demand are the main addressed by the scientific literature through stochastic modelling (Escudero and Kamesam, 1993), fuzzy mathematical programming (Mula et al. 2006; Mula et al. 2007; Mula et al. 2008), safety stocks (Grubbström and Tang, 1999; Mula et al. 2014) or safety times (Wijngaard and Wortmann, 1985). Other approaches can be found in Mula et al. (2006).

With respect to MRP models under uncertain in lead times, it is necessary to highlight the seminal works by Yano (1987a, b, c) based on stochastic lead times and also the works by Dolgui and Louly (2002) and Louly and Dolgui (2004). Other approaches can be found in Dolgui and Prodhon (2007), Dolgui et al. (2013) and Aloulou et al. (2014).

This paper proposes a fuzzy multi-objective decision model for the material requirement planning (MRP) problem. Here, the main contribution is to provide an initial solution methodology to address MRP problems with fuzzy lead times. In order to validate the model, a numerical example is presented to illustrate the proposed solution methodology

2. FUZZY MULTI-OBJECTIVE MODEL FORMULATION FOR MATERIAL REQUIREMENT PLANNING

2.1 Assumptions

The following assumptions have been considered.

- A multi-product manufacturing environment. By the term product we refer to finished goods, components, raw materials and subassemblies structured in a bill of materials.
- A multi-level production systems where the subsets of components are assembled independently.
- A multi-period planning horizon comprised of a set of consecutive and integer time periods of the same length.
- The lead time of a product is the number of consecutive and integer periods that are required for their finalization.
- The inventory of each product (finished good, raw materials and components) is the available volume at the end of a given period.
- The backlog of the demand of a product at the end of a period is defined as the non negative difference between the cumulated demand and the volume of available product.
- The master production schedule (MPS), that specifies the quantity to produce of each finished good in every period of the planning horizon, and the MRP, that provides the net requirements of raw

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materials and components for each planning period, are solved jointly.

- Programmed receptions.
- Production capacity constraints.
- Overtime limits.
- It is assumed that the subcontracted products will be ready just when required without lead time changes.
- Fuzzy lead time for finished goods, components and raw materials.
- Fuzzy lead times are represented by using different values associated with different degree of possibility each one.

2.2 Fuzzy objective functions

Three fuzzy objective functions have been considered: (1) minimizes the total costs over the time periods that have been computed; (2) minimizes the backorder quantities over the whole planning horizon; and (3) minimizes the idle time of the productive resources.

$$Min \quad z_1 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}\left(cp_{it}P_{it} + ci_{it}INVT_{it}\right) + \sum_{r=1}^{R}\sum_{t=1}^{T} ctov_{rt}Tov_{rt} \tag{1}$$

$$Min \quad z_2 \cong \sum_{i=1}^{I}\sum_{t=1}^{T} B_{it} \tag{2}$$

$$Min \quad z_3 \cong \sum_{r=1}^{R}\sum_{t=1}^{T} Tun_{rt} \tag{3}$$

2.3 Constraints

The following constraints have been included.

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} = d_{it} \qquad \forall i \forall t \ (4)$$

$$\sum_{i=1}^{I} P_{it}AR_{ir} + Tun_{rt} - Tov_{rt} = CAP_{rt} \qquad \forall r \forall t (5)$$

$$B_{iT} = 0 \qquad \forall i \ (6)$$

$$P_{it}, INVT_{it}, B_{it}, Tun_{it}, Tov_{it} \geq 0 \qquad \forall i \forall r \forall t \ (7)$$

$$P_{it}, INVT_{it}, B_{it} \in Z \qquad \forall i \forall t \ (8)$$

Constraint (4) is the inventory balance equation for all the products. Constraint (5) establishes the available capacity for normal, overtime and subcontracted production. Constraint (6) finishes with the delays in the last period (T) of the planning horizon. Constraint (7) contemplates the non negativity for the decision variables and constraint (8) establishes the integrity conditions for some of the decision variables.

3. SOLUTION METHODOLOY

Here, an approach to transform the fuzzy goal programming (FGP) into an equivalent auxiliary crisp mathematical programming model for MRP problems is provided. This approach considers non increasing linear membership functions for each fuzzy objective function as follows (Bellman and Zadeh 1970):

$$\mu_k = \begin{cases} 1 & z_k < z_k^l \\ \dfrac{z_k^u - z_k}{z_k^u - z_k^l} & z_k^l < z_k < z_k^u \\ 0 & z_k > z_k^u \end{cases} \tag{9}$$

where μ_k is the membership function of z_k, while z_k^l and z_k^u are, respectively, the lower and upper bounds of the objective function z_k.

The FGP approach by Torabi and Hassini (2008), based on the convex combination of the lower bound for satisfaction degree of objectives and the weighted sum of these achievement degrees, is adopted as the basis of this solution methodology. This FGP programming method proposes that a multi-objective model could be transformed in a single objective model as follows:

Max

$$\lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_k \theta_k \mu_k(x)$$

subject to

$$\lambda_0 \leq \mu_k(x) \quad k = 1,\ldots,n$$

$$x \in F(x) \tag{10}$$

where μ_k represents the satisfaction degree of the k th objective function. $\lambda_0 = \min\{\mu_k(x)\}$ is the minimum satisfaction degree of the objectives. θ_k is the relative importance of the kth objective and γ is a coefficient of compensation.

Then, the equivalent auxiliary crisp mathematical programming model is formulated as follows:

Max

$$\lambda(x) = \gamma\lambda_0 + (1-\gamma)\left(\theta_1 \cdot \frac{z_1^u - z_1}{z_1^u - z_1^l} + \theta_2 \cdot \frac{z_2^u - z_2}{z_2^u - z_2^l} + \theta_3 \cdot \frac{z_3^u - z_3}{z_3^u - z_3^l}\right) \quad (11)$$

subject to

$$\lambda_0 \le \mu_1 \quad (12)$$

$$\lambda_0 \le \mu_2 \quad (13)$$

$$\lambda_0 \le \mu_3 \quad (14)$$

$$0 \le \lambda_0 \le 1 \quad (15)$$

$$0 \le \mu_1 \le 1 \quad (16)$$

$$0 \le \mu_2 \le 1 \quad (17)$$

$$0 \le \mu_3 \le 1 \quad (18)$$

and constraints (4)-(8).

Where z_1, z_2 and z_3 correspond to equations (1), (2) and (3); respectively. z_1^u, z_2^u, z_3^u and z_1^l, z_2^l, z_3^l are their corresponding upper and lower bounds.

We address the fuzziness of lead times by generating several problems instances associated to all possible combinations for product lead times with a possibility degree equal to the minimum possibility degree of all products in each combination. The following solution procedure is proposed:

Step 1: Formulate the original FGP model for the MRP problem.

Step 2: Specify the corresponding linear membership functions for all the fuzzy objectives (upper and lower limits).

Step 3: Determine the corresponding relative importance of the objective functions (θ_k) and the coefficient of compensation (γ).

Step 4: Transform the original FGP problem into an equivalent single-objective mixed-integer linear programming (MILP) form using the Torabi and Hassini (2008) fuzzy programming method.

Step 5: Generate problem instances related to all possible combinations of product lead times values.

Step 6: Solve the proposed auxiliary crisp single-objective model by using a MILP solver for each problem instance and obtain a fuzzy set of solutions.

Step 7: Defuzzify the obtained solution by applying the center of gravity method.

Step 8: Determine the Manhattan and/or the Euclidean distance of each solution to crisp solution.

Step 9: Select the solution with minimum distance to the defuzzified crisp solution.

4. NUMERICAL EXAMPLE

The proposed model has been implemented in the MPL language V4.2. The resolution has been carried out with CPLEX 12.1.0 solver. The input data and the model solution values were processed with the Microsoft Access database (2010). A numerical example (24 instances) to validate and evaluate the results of our proposal is presented.

4.1 Assumptions

- The study considers a finished good (final product) with a product structure composed of two components (Fig. 1).

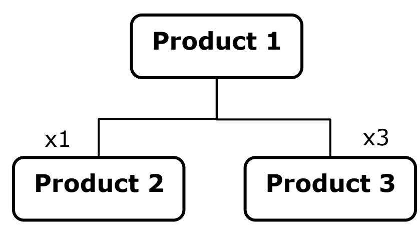

Fig. 1. Product structure.

- The decision variables, P_{it}, $INVT_{it}$ and B_{it} are considered integer.
- A planning horizon of 30 periods has been considered.
- Only the finished good has external demand.
- Firm orders cannot be rejected although backlog for the finished good is considered.
- A single productive resource restricts production: the assembly line.
- Fuzzy lead times are represented by using three different values associated with different degree of possibility each one.

  Product 1: {0/1, 1/0.5, 2/0.2}

  Product 2: {1/1, 2/0.7, 3/0.3}

  Product 3: {3/1, 4/0.8, 5/0.4}

- Fuzzy lead times of component are always higher than or equal to finished good lead times.

The following instances were generated (Table 1).

Table 1. Instances generated (1).

Instance	Lead times	Possibility
I1	{0,1,3}	1
I2	{0,1,4}	0.8
I3	{0,1,5}	0.4
I4	{0,2,3}	0.7
I5	{0,2,4}	0.7
I6	{0,2,5}	0.4
I7	{0,3,3}	0.3
I8	{0,3,4}	0.3
I9	{0,3,5}	0.3
I10	{1,1,3}	0.5
I11	{1,1,4}	0.5
I12	{1,1,5}	0.4

Table 2. Instances generated (2).

Instance	Lead times	Possibility
I13	{1,2,3}	0.5
I14	{1,2,4}	0.5
I15	{1,2,5}	0.4
I16	{1,3,3}	0.3
I17	{1,3,4}	0.3
I18	{1,3,5}	0.3
I19	{2,2,3}	0.2
I20	{2,2,4}	0.2
I21	{2,2,5}	0.2
I22	{2,3,3}	0.2
I23	{2,3,4}	0.2
I24	{2,3,5}	0.2

The following numerical results were obtained (Table 3 and Table 4).

Table 3. Objective functions by instance (1).

Instance	*z1*	*z2*	*z3*
I1	88716.42	967	1047.06
I2	88706.40	1971	1047.06
I3	88697.99	3512	1047.06
I4	88716.42	967	1047.06
I5	88706.40	1971	1047.06
I6	88697.99	3512	1047.06
I7	88716.42	967	1047.06
I8	88706.40	1971	1047.06
I9	88697.99	3512	1047.06
I10	88716.42	967	1047.06
I11	88706.40	1971	1047.06
I12	88697.99	3512	1047.06
I13	88716.42	967	1047.06
I14	88706.40	1971	1047.06
I15	88697.99	3512	1047.06
I16	88716.42	967	1047.06
I17	88706.40	1971	1047.06
I18	88697.99	3512	1047.06
I19	88716.42	967	1047.06
I20	88706.40	1971	1047.06
I21	88697.99	3512	1047.06
I22	88716.42	967	1047.06
I23	88706.40	1971	1047.06
I24	88697.99	3512	1047.06

Table 4. Objective functions by instance (2).

Instance	μ_1	μ_2	μ_3
I1	0.9677	0.9033	0.8547
I2	0.9678	0.8029	0.8547
I3	0.9678	0.6488	0.8547
I4	0.9677	0.9033	0.8547
I5	0.9678	0.8029	0.8547
I6	0.9678	0.6488	0.8547
I7	0.9677	0.9033	0.8547
I8	0.9678	0.8029	0.8547
I9	0.9678	0.6488	0.8547
I10	0.9677	0.9033	0.8547
I11	0.9678	0.8029	0.8547
I12	0.9678	0.6488	0.8547
I13	0.9677	0.9033	0.8547
I14	0.9678	0.8029	0.8547
I15	0.9678	0.6488	0.8547
I16	0.9677	0.9033	0.8547
I17	0.9678	0.8029	0.8547
I18	0.9678	0.6488	0.8547
I19	0.9677	0.9033	0.8547
I20	0.9678	0.8029	0.8547
I21	0.9678	0.6488	0.8547
I22	0.9677	0.9033	0.8547
I23	0.9678	0.8029	0.8547
I24	0.9678	0.6488	0.8547

Fig. 2, Fig. 3 and Fig. 4 show graphically the previous results. From these results, the corresponding center of gravity is obtained for each fuzzy objective function.

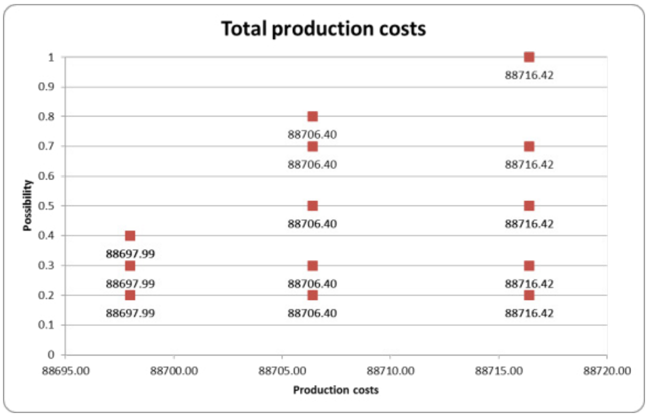

Fig. 2. Graphical results for total production costs.

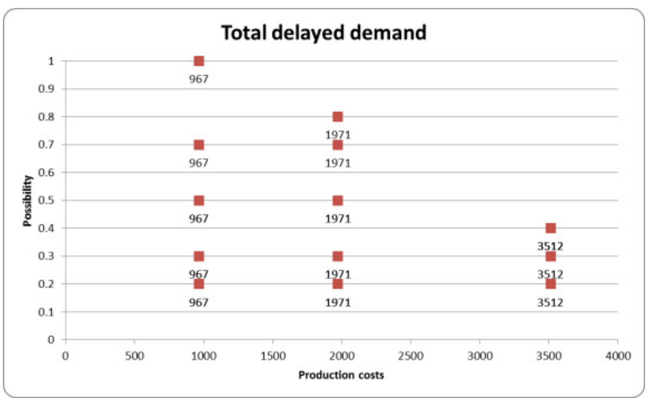

Fig. 3. Graphical results for total delayed demand.

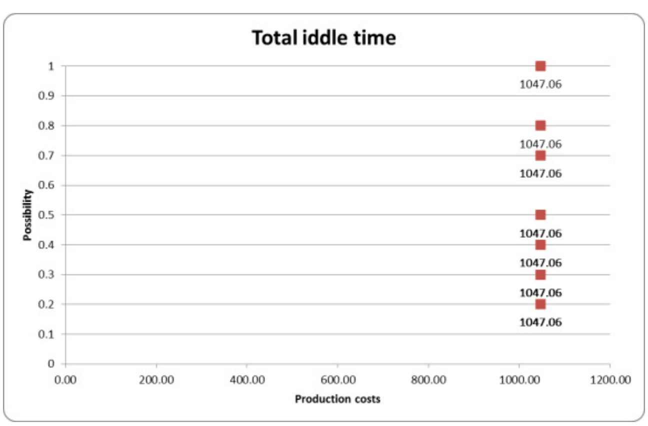

Fig. 4. Graphical results for total idle time.

Table 5 and Table 6 show the fuzzy objective functions solutions and their membership values respectively and the corresponding Manhattan and Euclidean distances with respect to each center of gravity.

Table 5. Fuzzy objective functions solutions.

	Solution 1	Solution 2	Solution 3
z_1	88716.42	88706.40	88697.99
z_2	967	1971	3512
z_3	1047.06	1047.06	1047.06
Manhattan distance	286.30	60.09	595.23
Euclidean distance	289.82	61.62	600.98

Table 6. Membership objective functions values.

	Solution 1	**Solution 2**	**Solution 3**
z_1	0.9677	0.9678	0.9678
z_2	0.9033	0.8029	0.6488
z_3	0.8547	0.8547	0.8547
Manhattan distance	0.0289	0.0061	0.0600
Euclidean distance	7.05E-07	1.44E-09	1.30E-05

Fig. 5 compares fuzzy and crisp solutions.

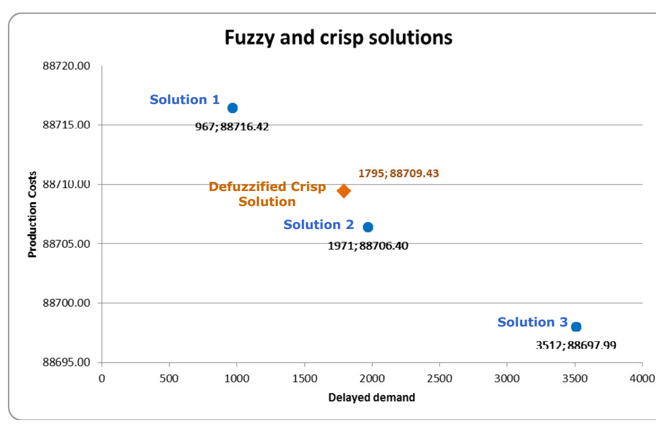

Fig. 5. Fuzzy and crisp solutions.

With this approach, we have obtained all possible set of solutions for each objective function (z_1, z_2 and z_3) and their corresponding center of gravity. Also, for each membership function (μ_1, μ_2, μ_3). It has been proved as an initial method for obtaining crisp solutions with the aim of minimizing the deviations due to uncertain lead times in MRP systems.

5. CONCLUSIONS

This paper has addressed the MRP problem under uncertainty associated with lead times through a fuzzy multi-objective decision model. Multi-objective models are necessary because of the difficulty for companies of defining production parameters as backlog costs or idle time costs, which are used to appear in single-objective traditional MRP models.

For the purpose of solving the multi-objective model, we have proposed a solution methodology based on FGP which considers the lack of knowledge associated to lead times. This proposal has been applied to numerical example with 24 different instances.

The advantages of this proposal are related to: The modelling and establishment of the priorities for production objectives that traditionally are measured through costs estimated with difficulty by companies; and considering different values for product lead times associated to different possibility degrees which provide the decision maker with a broad decision spectrum with different risks levels.

With respect to the limitations of this work, it is important to highlight that related to the defuzzification procedure, we have separately consider each value of the objective functions. Nevertheless, it could be tested that each set of values z_1, z_2, z_3 were addressed as a unique set with an occurrence possibility associated. Other further research proposals are oriented to: (i) Development of a decision support system to systematize the model configuration and running; (ii) exploration of the effect of more complex product structures and to validate the proposed solution methodology in real-world MRP problems; and (iii) propose alternative solution methodologies for the addressed fuzzy problem and compare them with the current proposal; and (iv) comparison with alternative approaches based on parameterization methodologies.

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