Summary

The classical Nykodym theorem (1933) asserts that a set H of countably additive complex measures defined on a σ-algebra $\mathcal{S}$ which is bounded for each element of $\mathcal{S}$, then H is uniformly bounded, that is, $\sup\{\lambda(A) : \lambda \in H, A \in \mathcal{S}\} < \infty$. It is well known that this theorem fails if we replace the σ-algebra $\mathcal{S}$ simply by an algebra.

Let $\mathcal{A}$ be the algebra of subsets of a nonempty set Ω, and consider the Banach space $ba(\mathcal{A})$ of all real (or complex) finitely additive measures of bounded variation defined on $\mathcal{A}$. A subset $\mathcal{B}$ of $\mathcal{A}$ is said to have the N-property (Nikodym property) if every $\mathcal{B}$-pointwise bounded subset M of $ba(\mathcal{A})$ is uniformly bounded on $\mathcal{A}$. Recall the classical Nikodym-Dieudonné-Grothendieck's theorem which says that each σ-algebra has the N-property. Moreover $\mathcal{B}$ is said to have the strong N-property if for each increasing countable covering $(\mathcal{B}_m)_m$ of $\mathcal{B}$ there exists $\mathcal{B}_n$ which has the N-property. Valdivia proved in 1979 that each σ-algebra has the strong N-property.

The aforementioned Valdivia's theorem motivated to prove that each σ-algebra $\mathcal{S}$ of subsets of Ω has web-N-property, that is, if $(\mathcal{B}_{m_1})_{m_1}$ is an increasing countable covering of ($\mathcal{S}$ and if $(\mathcal{B}_{m_1,m_2,\ldots,m_p,m_{p+1}})_{m_{p+1}}$ is an increasing countably covering of $\mathcal{B}_{m_1,m_2,\ldots,m_p}$, for each natural numbers p, m_i, with $1 \leq i \leq p$, then there exists a sequence $(n_r)_r$ such that $\mathcal{B}_{n_1,n_2,\ldots,n_r}$ has the N-property for every $r \geq 1$.

In this thesis it is proved that nearly all infinite chains in the increasing web $\{\mathcal{B}_{m_1,m_2,\ldots,m_p} : (m_1, m_2, \ldots, m_p) \in \bigcup_s \mathbb{N}^s\}$ are composed of sets that have web-N-property.

In the main result in this thesis it is proved that the algebra $\mathcal{J}(K)$

of Jordan measurables subsets of a compact k-dimensional interval K contained in $\mathbb{R}^k$ has the web-N-property. This result imporves the 2013 Valdivia's theorem stating that $\mathcal{J}(K)$ has the strong Nikosym property, which in turns was a grest improvement of Schachermayer's result od N property of $\mathcal{J}([0,1])$.

In addition, some applications of the obtained results to vector measures are presented.