Enhanced Disturbance Rejection for a Predictor-based Control of LTI Systems with Input Delay

Ricardo Sanz [a], Pedro Garcia [a], Pedro Albertos [a]

[a] *Instituto de Automática e Informática Industrial, Universitat Politècnica de València, 46020 Valencia, Spain*

Abstract

In this paper, a new predictor-based control strategy for LTI systems with input delay and unknown disturbances is proposed. The disturbing signal and its derivatives up to the r-th order are estimated by means of an observer, and then used to construct a prediction of the disturbance. Such prediction allows defining a new predictive scheme that takes into account its effect. Also, a suitable transformation of the control input is presented and a performance analysis is carried out to show that, for a given controller, the proposed solution leads to better disturbance attenuation than previous approaches in the literature for smooth enough perturbations.

Key words: Linear systems, Disturbance Rejection, Time delay, Predictive control, Disturbance observer, Tracking differentiator

1 Introduction

The problem considered in this paper deals with possibly open-loop unstable disturbed LTI systems, defined by

$$\dot{x}(t) = Ax(t) + B[u(t-h) + d(t)]$$
$$u(t) = u_0(t) \qquad t \in [-h, 0) \qquad (1)$$
$$x(0) = x_0$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $d \in \mathbb{R}$ is an unknown input disturbance, $h > 0$ is a known and constant input delay, and $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}$ are known matrices.

The Smith Predictor (Smith, 1957), can be considered as the first predictor-based control for open-loop stable linear systems. Later, the same concept was extended for open-loop unstable systems by introducing an h units of time ahead state predictor, (Artstein, 1982; Manitius and Olbrot, 1979):

$$\hat{x}_1(t+h) \triangleq e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}Bu(s)\,\mathrm{d}s, \qquad (2)$$

referred to as the conventional prediction in this paper. The variable $\hat{x}_1(t+h)$ is understood as the projection of the state starting at $x(t)$ driven by the control history $u(t+s)$, $s \in [-h, 0]$. In the absence of disturbances, the feedback law $u(t) = K\hat{x}_1(t+h)$ achieves asymptotic stabilization for any $h > 0$ with a proper choice of K.

However, in a disturbed system, an error is introduced in the prediction $\hat{x}_1$. Since there is always an error between the exact and the approximated predictions, it is not possible to remove constant disturbances even using integral action. Although this is an interesting topic from a practical point of view (Krstic, 2010), only few articles have addressed this problem. In an effort to predict the evolution of the disturbances, adaptive algorithms have drawn the attention of some researchers. For example, sinusoidal disturbances of unknown frequency are identified and rejected in (Pyrkin et al., 2010) for LTI systems with known delay, and more recently in (Basturk and Krstic, 2015) for systems with matched uncertainties (see also the references therein). Also, adaptive schemes are used to estimate and reject constant disturbances for unknown input delay in (Bresch-Pietri et al., 2012), and for known distributed delays in (Bekiaris-Liberis et al., 2013). Other works avoid any a priori knowledge of the disturbance structure. For example, a filtered version of the predicted state (2) is proved to minimize a cost functional involving the disturbance in (Krstic, 2008, 2010).

Recently in (Léchappé et al., 2015), a simple solution is

---

considered, where additional feedback from the difference between the measured, $x(t)$, and delayed predicted state, $\hat{x}_1(t)$, is used to define a new prediction

$$\hat{x}_2(t+h) = \hat{x}_1(t+h) + [x(t) - \hat{x}_1(t)] \tag{3}$$

With this simple modification, it is proved that for a certain class of disturbing signals, the new prediction leads to better attenuation than the conventional one. However, perfect cancellation is only possible for constant disturbances, and the attenuation depends entirely on the characteristics of the disturbance.

The main contribution of this paper is the introduction of a new predictive scheme that takes into account a prediction of the disturbing signal, denoted by $\hat{d}(t+h)$. Such prediction is constructed from estimates of the disturbance and its derivatives up to the r-th order, which are obtained by means of a tracking differentiator. The predicted disturbance is used to define a new state prediction, denoted by $\hat{x}_3(t+h)$, allowing to compensate the effect of the disturbance in the overall system. A performance analysis based on Lyapunov's theory is carried out to prove that the proposed scheme performs better than previous proposals in the literature, in the presence of smooth enough time-varying disturbances, achieving perfect cancellation in some particular cases.

2   Problem statement

Let us consider the system (1). Other than the accessibility to the full state, the following assumptions are taken:

Assumption 1 *The pair (A,B) is controllable*

Assumption 2 *The unknown disturbance $d(t)$ is uniformly bounded by $|d(t)| \leq D_0$ and it is $(r+1)$-times continuously differentiable with $\left| d^{(r+1)}(t) \right| \leq D_{r+1}$, $\forall t \geq 0$*

From (1), it can be seen that the actual projection of the state at time $t+h$ is given by

$$x(t+h) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}B[u(s) + d(s+h)]\,\mathrm{d}s \tag{4}$$

Although (4) cannot be used in practice because the disturbance is unknown, an approximated prediction of the state h units of time ahead for the system (1) can be obtained by computing the conventional prediction (2). From (2) and (4), the prediction error is given by

$$x(t+h) - \hat{x}_1(t+h) = \int_{t-h}^{t} e^{A(t-s)}Bd(s+h)\,\mathrm{d}s \tag{5}$$

In the disturbance-free case, $d(t) = 0$ and it can be seen from (5) that stabilizing $\hat{x}_1$ is equivalent to stabilize x because the prediction is exact. However, when $d(t) \neq 0$, the predicted state $\hat{x}_1$ is corrupted. In such case, if the control law is designed so that $\hat{x}_1$ tends to zero, then x will not tend to zero even for constant perturbations. This fact is illustrated by the following proposition, taken from (Léchappé et al., 2015):

Proposition 3 *The asymptotic convergence of $\hat{x}_1$ to zero implies the asymptotic convergence of x to $\int_{t-h}^{t} e^{A(t-s)}Bd(s)\,\mathrm{d}s$*

For constant disturbances, the prediction $\hat{x}_2$ in (3) avoids this problem as stated by the following proposition, also taken from (Léchappé et al., 2015):

Proposition 4 *For constant disturbances, the asymptotic convergence of $\hat{x}_2$ to zero implies the asymptotic convergence of x to zero.*

However, both predictions share some drawbacks: their accuracy is only determined by the characteristics of the disturbance signal, and perfect cancellation of time-varying disturbances is never possible. In the next section, a new prediction that mitigates these problems, denoted by $\hat{x}_3$, is proposed.

3   Proposed Predictor-based control

Let us assume that a future estimation of the disturbance $\hat{d}(t+h)$, is available. Then, a new predicted state which considers the effect of the disturbance can be computed by

$$\hat{x}_3(t+h) \triangleq e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}B[u(s) + \hat{d}(s+h)]\,\mathrm{d}s \tag{6}$$

The disturbance prediction error is defined as

$$\sigma(t) \triangleq d(t) - \hat{d}(t) \tag{7}$$

From (4), (6) and using the definition (7), the error of the new prediction is given by

$$x(t+h) - \hat{x}_3(t+h) = \int_{t-h}^{t} e^{A(t-s)}B\sigma(s+h)\,\mathrm{d}s \tag{8}$$

Proposition 5 *If $\sigma \to 0$, then the asymptotic convergence of the new prediction $\hat{x}_3$ to zero implies the asymptotic convergence of x to zero.*

Proof. If $\hat{x}_3$ tends to zero, from (8) it can be seen that x tends to $\int_{t-h}^{t} e^{t-s}B\sigma(s+h)\,\mathrm{d}s$, and the proposition follows. $\square$

Therefore, with the proposed predictive scheme, the disturbance attenuation will depend on the accuracy of the

disturbance prediction estimation. To this purpose, the methodology adopted in this paper is based on (Zhong and Rees, 2004) where, for a delay-free system, an estimation of the unknown uncertainties and disturbances is obtained using the system model. Considering the input-delayed system (1), the disturbance can be written as [1]

$$d(t) = B^+ [\dot{x}(t) - Ax(t)] - u(t-h) \qquad (9)$$

which cannot be computed because the state derivative is unknown. However, following the ideas in (Zhong and Rees, 2004), a filtered disturbance can be obtained as $\hat{d}(t) = \mathcal{L}^{-1}\{G(s)D(s)\}$, where $D(s) = B^+[sX(s) - AX(s)] - e^{-hs}U(s)$, and $G(s)$ is a strictly-proper unity-gain filter. In the simplest case, $G(s)$ can be chosen as a first-order low-pass filter.

Notice that because of the input delay, even if the disturbance was perfectly identified, it could not be counteracted until h units of time later, which would lead to poor performance. In this paper, a strictly-proper filter $H(s)$ is designed such that an estimated prediction of the disturbance h units of time ahead is obtained as

$$\hat{d}(t+h) \triangleq \mathcal{L}^{-1}\{H(s)D(s)\} \qquad (10)$$

The underlying idea behind the filter $H(s)$ is to make an estimation $\hat{d}_0(t)$ of the disturbance, and its derivatives up to the r-th order, gathered in

$$\hat{\delta}(t) \triangleq [\hat{d}_0(t),\ \hat{d}_1(t),\ \ldots,\ \hat{d}_r(t)] \qquad (11)$$

which are then used to construct a prediction h units of time ahead by using a truncated Taylor series expansion

$$\hat{d}(t+h) \triangleq \sum_{j=0}^{r} \frac{h^j}{j!} \hat{d}_j(t) \triangleq C_H \hat{\delta}(t) \qquad (12)$$

with $C_H \triangleq \left[1,\ h,\ \ldots,\ \frac{h^r}{r!}\right]$. The following lemma introduces a linear tracking differentiator which is used to prove the main result.

Lemma 6 *Let us consider a signal $\xi(t)$ and its derivatives up to the r-th order gathered in the vector $\Xi \triangleq [\xi(t),\ \dot{\xi}(t),\ \ldots,\ \xi^{(r)}(t)]^T$ satisfying $|\xi^{(r+1)}| < M$, and an estimation $\hat{\Xi}(t) \triangleq [\hat{\xi}_0(t),\ \hat{\xi}_1(t),\ \ldots,\ \hat{\xi}_r(t)]^T$ given by the following dynamic system $\xi(t)$:*

$$\dot{\hat{\Xi}}(t) = \underbrace{\begin{bmatrix} -c_0 & 1 & 0 & \ldots & 0 \\ -c_1 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ -c_r & 0 & 0 & \ldots & 0 \end{bmatrix}}_{A_H} \hat{\Xi}(t) + \underbrace{\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_r \end{bmatrix}}_{B_H} \xi(t) \qquad (13)$$

---
[1] Here $B^+ \triangleq (B^T B)^{-1} B^T$ denotes the pseudoinverse of B

with $c_j = \begin{pmatrix} r+1 \\ j+1 \end{pmatrix} \omega_o^{j+1}$ and $\omega_0 > 0$. Then (13) is exponentially stable and $\lim\limits_{t\to\infty} |\xi^{(j)}(t) - \hat{\xi}_j(t)| \leq \frac{c_{j-1}}{c_r} M$.

Proof. The system (13) can be alternatively expressed as

$$\dot{\hat{\xi}}_j(t) = -c_j \hat{\xi}_0(t) + \hat{\xi}_{j+1}(t) + c_j \xi(t),\ j = 0, 1, \ldots, r-1$$
$$\dot{\hat{\xi}}_r(t) = -c_r \hat{\xi}_0(t) + c_r \xi(t) \qquad (14)$$

Let us denote the estimation error of the j-th derivative of the input signal as $e_j(t) = \xi^{(j)}(t) - \hat{\xi}_j(t)$, which allows to rewrite (14) as

$$\dot{e}_j(t) = -c_j e_0(t) + e_{j+1}(t),\ j = 0, 1, \ldots, r-1$$
$$\dot{e}_r(t) = -c_r e_0(t) + \xi^{(r+1)} \qquad (15)$$

or, in matrix form, $\dot{e}(t) = A_H e(t) + [0_{r-1}\ 1]^T \xi^{(r+1)}$. Notice that A_H has a unique eigenvalue, $-\omega_o$, with multiplicity $r + 1$. Computing the analytic expression for the transfer function of each channel, one can see that $|e_j(s)/\xi^{(r+1)}(s)|_\infty = c_{j-1}/c_r$, which corresponds to the maximum amplification of the input on each channel when $t \to \infty$. $\qquad \square$

Theorem 7 *Let us consider a disturbance $d(t)$ satisfying the Assumption 2 with $D_{r+1} > 0$, a filter given by*

$$H(s) = C_H(sI - A_H)^{-1} B_H \qquad (16)$$

with bandwidth $\omega_o > 0$, and A_H, B_H and C_H as defined in (12)-(13). The disturbance prediction (10) can be implemented through the following dynamic system

$$\dot{\hat{z}}(t) = A_H \hat{z}(t) + [A_H B_H B^+ - B_H B^+ A]\, x(t) - B_H u(t-h) \qquad (17)$$
$$\hat{d}(t+h) = C_H [\hat{z}(t) + B_H B^+ x(t)] \qquad (18)$$

with $\hat{z}(t)$ being an auxiliary variable. Then, the disturbance prediction error (7) is ultimately bounded by $\sigma_\infty \triangleq D_{r+1}\left(\beta(\omega_o) + \frac{h^{r+1}}{(r+1)!}\right)$, where $\beta : (0, +\infty) \to \mathbb{R}^+$ satisfies $\lim\limits_{\omega_o \to \infty} \beta(\omega_o) = 0$.

Proof. Let us apply the tracking differentiator (13) to the disturbance $d(t)$. Using the vector defined in (11), it follows that

$$\dot{\hat{\delta}}(t) = A_H \hat{\delta}(t) + B_H d(t) \qquad (19)$$

Plugging (9) into (19), and performing the change of variable $\hat{z}(t) \triangleq \hat{\delta}(t) - B_H B^+ x(t)$ in (19) and (12), yields (17) and (18), respectively. The transfer function (16) follows directly as a realization of (12) and (19).

3

Now, using (7), (12) and the complete Taylor series representation $d(t+h) = \sum_{j=0}^{r} \frac{h^j}{j!} d^{(j)}(t) + \epsilon_r$, one has that $\sigma(t+h) = \sum_{j=0}^{r} \frac{h^j}{j!} [d^{(j)}(t) - \hat{d}_j(t)] + \epsilon_r$ where ϵ_r is the Taylor remainder that is known to be bounded by $|\epsilon_r| \leq D_{r+1} \cdot h^{r+1}/(r+1)!$. Using the Lemma 6 one can bound $\lim_{t \to \infty} |d^{(j)}(t) - \hat{d}_j(t)| \leq (c_{j-1}/c_r) D_{r+1}$, and thus $\lim_{t \to \infty} |\sigma(t)| \leq D_{r+1} \left(\sum_{j=0}^{r} \frac{h^j}{j!} (c_{j-1}/c_r) + h^{r+1}/(r+1)! \right)$. Using the factorial expression for c_j, the theorem follows with $\beta(\omega_o) = \sum_{j=0}^{r} \frac{h^j}{j!} \frac{(r+1)!}{j!(r+1-j)!} \omega_0^{j-r-1}$. $\quad\square$

Although the previous results regarding the new prediction $\hat{x}_3$ are rather general, in this paper a particular control transformation is also proposed. Since a prediction of the disturbance is already available, a suitable transformation is given by

$$v(t) \triangleq u(t) + \hat{d}(t+h), \tag{20}$$

where $v(t)$ is the new control input to the system.

4 Performance analysis

The Artstein's reduction (Artstein, 1982), is a useful tool to analyze time delay systems as it transforms the original system into a delay-free one. It is easy to show that the reduction of system (1) with the conventional predicted variable $z_1(t) \triangleq \hat{x}_1(t+h)$ leads to

$$\dot{z}_1(t) = A z_1(t) + B u(t) + e^{Ah} B d(t) \tag{21}$$

while the reduced system using the alternative prediction $z_2(t) \triangleq \hat{x}_2(t+h)$, proposed in (Léchappé et al., 2015), is derived as

$$\dot{z}_2(t) = A z_2(t) + B u(t) + B d(t) + e^{Ah} B[d(t) - d(t-h)] \tag{22}$$

Similarly, considering the proposed prediction (6) and the control transformation (20), the reduction with $z_3(t) \triangleq \hat{x}_3(t+h)$ is given by

$$\dot{z}_3(t) = A z_3(t) + B v(t) + e^{Ah} B \sigma(t) \tag{23}$$

An improvement of the proposal is already highlighted by the Proposition 5, that is, the new predictive scheme will cancel time-varying disturbances if $\sigma(t)$ tends to zero. From Theorem 7, $\sigma(t)$ tends to zero if $D_{r+1} = 0$. Hence, constant disturbances can be perfectly canceled for $r = 0$; the same applies for $r = 1$ and disturbances with linear growth; and so on.

In order to evaluate the attenuation for other time-varying disturbances, note that all three reduced systems (21)-(23) have the generic form $\dot{\chi}(t) =$

$A\chi(t) + B\vartheta(t) + g(t)$, that is, a nominal system with a perturbation term. Since the pair (A, B) is controllable, there exists a Lipschitz function $f : \mathbb{R}^n \to \mathbb{R}$ such that the feedback law $\vartheta(t) = f(\chi(t))$ makes the origin of the nominal system $(g(t) \equiv 0)$ globally exponentially stable. Furthermore, if $|g(t)| \leq \bar{g}, \forall t \geq t_0$, there exist a positive constant γ and a finite time $t_1 \geq t_0$ such that the state is ultimately bounded by $|\chi(t)| \leq \gamma \bar{g}, \forall t \geq t_1$. Hence, the following ultimate bounds hold

$$|z_1(t)| \leq \gamma |B| \left| e^{Ah} \right| D_0 \tag{24}$$
$$|z_2(t)| \leq \gamma |B| \left[\left| e^{Ah} \right| h D_1 + D_0 \right] \tag{25}$$
$$|z_3(t)| \leq \gamma |B| \left| e^{Ah} \right| \sigma_\infty \tag{26}$$

for some $t \geq t_1$. In order to obtain the bound on the actual state, the transformation has to be undone. In (Léchappé et al., 2015), it is shown that $|x(t)| \leq |\hat{x}_1(t)| + \eta |B| D_0$, and also that $|x(t)| \leq |\hat{x}_2(t)| + \eta |B| h D_1$, with $\eta = \left| \int_{-h}^{0} e^{As} \, ds \right|$. Similarly, from (8) and Theorem 7, it is obtained that $x_\infty \triangleq \lim_{t \to \infty} |x(t)| \leq |\hat{x}_3(t)| + \eta |B| \sigma_\infty$. Gathering the previous results, the different predictive schemes lead to the following ultimate bounds of the state:

$$x_\infty \leq \left[\eta + \gamma \left| e^{Ah} \right| \right] |B| D_0 \triangleq r_1 \tag{27}$$
$$x_\infty \leq \left[\eta + \gamma \left| e^{Ah} \right| \right] |B| h D_1 + \gamma |B| D_0 \triangleq r_2 \tag{28}$$
$$x_\infty \leq \left[\eta + \gamma \left| e^{Ah} \right| \right] |B| \sigma_\infty \triangleq r_3 \tag{29}$$

Lemma 8 *Consider the conventional prediction $\hat{x}_1$ leading to the bound (27) and the proposed scheme leading to the bound (29). There exists a sufficiently large $\omega_o > 0$ such that $r_3 < r_1$, if*

$$\frac{D_{r+1}}{D_0} \frac{h^{r+1}}{(r+1)!} < 1 \tag{30}$$

Proof. From (27), (29), the condition $r_3 < r_1$ is implied by $\sigma_\infty < D_0$. The previous condition is fulfilled if the observer bandwidth satisfies $\beta(\omega_o) < \frac{D_0}{D_{r+1}} - \frac{h^{r+1}}{(r+1)!}$. Because of the properties of $\beta(\omega_o)$, this is always possible with a sufficiently large $\omega_o > 0$ if (30) holds. $\quad\square$

Lemma 9 *Consider the alternative prediction $\hat{x}_2$ leading to the bound (28) and the proposed scheme leading to the bound (29). There exists a sufficiently large $\omega_o > 0$ such that $r_3 < r_2$, if*

$$\frac{D_{r+1}}{D_1} \frac{h^r}{(r+1)!} < 1 \tag{31}$$

Remark 10 *Let us consider sinusoidal disturbances $d(t) = D_0 \sin \omega t$. From (30), the new proposal can lead to better attenuation than the conventional prediction*
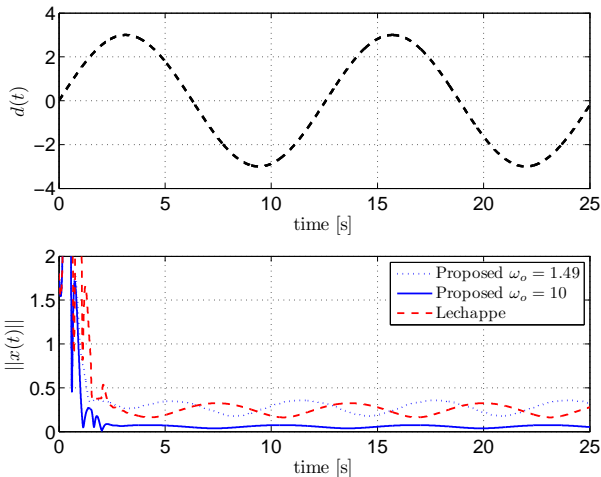
Figure 1. Simulations for $x(0) = [1.5 \quad 1]^T$, $h = 0.5$ s and sinusoidal perturbation

simply with $r = 0$ if $\omega < 1/h$. Similarly, from (31), the proposal in (Léchappé et al., 2015) can be outperformed with $r = 1$ if $\omega < 2/h$. Notice also that in the limit $r \to \infty$, the new prediction improves attenuation for sinusoidal disturbance with arbitrarily large frequency.

5 Numerical validation

In order to validate the bounds derived in the previous section, let us consider the system (Léchappé et al., 2015),

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -9 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t - h) + \begin{bmatrix} 0 \\ d(t) \end{bmatrix} \quad (32)$$

The simulation considers the same scenario that in (Léchappé et al., 2015), with an input delay $h = 0.5$ s, the system starting from $x(0) = [1.5, 1]^T$, a sinusoidal disturbance $d(t) = 3\sin(0.5t)$, and predictor-based control law given by $u(t) = -[k_p, k_d] \hat{x}_2(t + h)$ with $k_p = 45$, $k_d = 18$. The same control law is selected for the proposed scheme by computing (20) with $v(t) = -[k_p, k_d] \hat{x}_3(t + h)$. The observer is calculated according to the Theorem 7 with $r = 1$. In this case, the attenuation can be improved because the condition (31) is fulfilled for $h = 0.5$ s and $\omega = 0.5$ rad/s. The simulation in Figure 1 shows the limit case (same attenuation), along with a larger value $\omega_o = 10$ rad/s (better attenuation).

6 Conclusion

A new predictive scheme to control time delay LTI systems with unknown input disturbances has been proposed. The new predicted variable $\hat{x}_3$, for a given controller, leads to better disturbance attenuation than previous proposals, under some constraints in the disturbance signal. The new prediction also achieves perfect cancellation of a certain class of time-varying disturbances without having any knowledge of their structure.

Acknowledgements

References

Zvi Artstein. Linear systems with delayed controls: a reduction. Automatic Control, IEEE Transactions on, 27(4):869–879, 1982.

Halil I Basturk and Miroslav Krstic. Adaptive sinusoidal disturbance cancellation for unknown LTI systems despite input delay. Automatica, 58:131–138, 2015.

Nikolaos Bekiaris-Liberis, Mrdjan Jankovic, and Miroslav Krstic. Adaptive stabilization of LTI systems with distributed input delay. International Journal of Adaptive Control and Signal Processing, 27(1-2):46–65, 2013.

Delphine Bresch-Pietri, Jonathan Chauvin, and Nicolas Petit. Adaptive control scheme for uncertain time-delay systems. Automatica, 48(8):1536–1552, 2012.

Miroslav Krstic. Lyapunov tools for predictor feedbacks for delay systems: Inverse optimality and robustness to delay mismatch. Automatica, 44(11):2930–2935, 2008.

Miroslav Krstic. Compensation of infinite-dimensional actuator and sensor dynamics. Control Systems, IEEE, 30(1):22–41, 2010.

Vincent Léchappé, Emmanuel Moulay, Franck Plestan, Alain Glumineau, and Abdelhamid Chriette. New predictive scheme for the control of LTI systems with input delay and unknown disturbances. Automatica, 52:179–184, 2015.

Andrzej Manitius and Andrzej W Olbrot. Finite spectrum assignment problem for systems with delays. Automatic Control, IEEE Transactions on, 24(4):541–552, 1979.

Anton Pyrkin, Andrey Smyshlyaev, Nikolaos Bekiaris-Liberis, and Miroslav Krstic. Rejection of sinusoidal disturbance of unknown frequency for linear system with input delay. In American Control Conference (ACC), 2010, pages 5688–5693. IEEE, 2010.

Otto JM Smith. Closer control of loops with dead time. Chemical Engineering Progress, 53(5):217–219, 1957.

Qing-Chang Zhong and David Rees. Control of uncertain LTI systems based on an uncertainty and disturbance estimator. Journal of dynamic systems, measurement, and control, 126(4):905–910, 2004.