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Additional Information

# Can dasymetric mapping significantly improve population data reallocation in a dense urban area?

#### 1. Introduction

As required by national, supranational and federal laws, the vast majority of social variables distributed by statistical agencies are offered aggregated by areal units because of confidentiality issues (see, e.g., UK Parliament 2007; OJEU 2013; USC 2002). This is the case, for example, with census figures, which are only available as geographically accumulated data in order to ensure that it is impossible to identify a particular person, either directly or indirectly in connection with any other published information. In fact, according to the US Census Bureau (2015), before issuing small-area aggregated statistics single records are even "switched with similar records from a neighboring area" and/or randomly perturbed to preserve individuals' privacy.

Social variables are disseminated in a great variety of geographic entities. Some of them are well-established administrative units, such as states, provinces, counties or cities. Others are more arbitrary and easily modifiable, such as state legislative and congressional districts used in US elections (which are redrawn every 10 years following US censuses) or census blocks and precincts (which are instances of the smallest geographical units for which social data are made public).

For the first group of geographic entities, large series of data are increasingly available in developed countries and temporal comparisons and longitudinal studies can be performed directly from the published data. For the second group of spatial units, however, implementing longitudinal analyses is not so simple; the complexity of the problem even growing as the scale of geographical aggregation reduces. In the case of relatively large geographic entities (such as congressional districts), rebuilding the history of the variables of interest is not as a rule so complex, particularly when the data of the smallest geographic units that constitute it (such as precincts or census tracts) are available (Pavía and López-Quilez 2013). The real problem arises when we deal with smaller areas (such as census blocks). In the latter, when the data come from enumerations, surveys or administrative registers with detailed spatial references (like

postal addresses or GPS coordinates), the figures of the target variables corresponding to the new redrawn areas could, after some additional workings, be theoretically restored from the microdata.

Unfortunately, ordinary analysts hardly ever have access to such detailed information and, what's more, in many cases complete geographical indicators are not available (even for statistical producers), as when dealing with historical data or with electoral outcomes or with surveys in which only rough spatial references are collected. Despite this, disciplines like economics, political science, sociology and other social sciences do not shy away from using census figures, unemployment rates or party supports to examine social trends, evaluate policy impacts or test social theory. Hence, it is not surprising that many approaches have been proposed in the literature attempting to overcome the limitations that the modifications of the spatial unit boundaries impose on performing longitudinal studies.

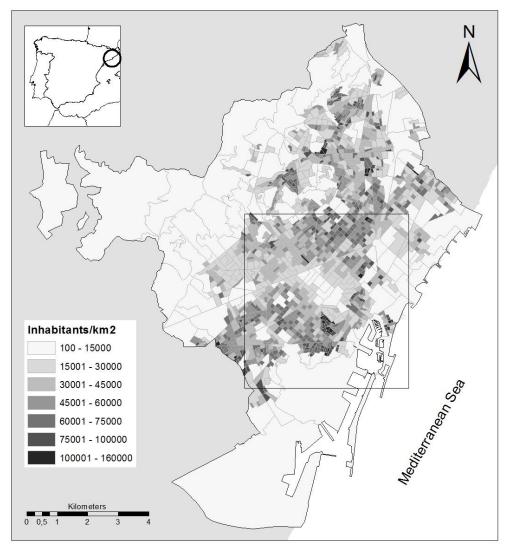
In particular, to solve the problem of reallocating data from a set of geographical administrative units onto another overlapping but non-hierarchical set of spatial units (i.e., in a context where the scale of analysis is fixed and only the shape of the aggregation units is changed), a number of methods of progressively growing in complexity have been suggested over time. They have moved from simple areal weighting interpolation (see, e.g., Goodchild and Lam 1980)—which are equal to performing an Euler-Venn geometric approach—and point-based areal interpolation procedures (e.g., Martin 1989; Bracken and Martin 1989; Fisher and Langford 1995) to more complex strategies based on using dasymetric mapping with ancillary sources of information. Because social data are related to population, dasymetric methods use auxiliary variables to guide in an intelligent fashion the redistribution process (Wright, 1936). Among the variables tested we can find data about land uses (Mennis 2003; Giordano and Cheever 2010), satellite imagery (Robinson et al. 2002; Holt et al. 2004), road networks and night-time lights (Reibel and Bufalino 2005; Briggs et al. 2007; Zandbergen and Ignizio 2010), address points (Zandbergen 2011), the spatial distribution of built structures (Maantay et al. 2007), LiDAR-derived building heights (Sridharan and Qiu 2013), or a combination of spatial methods and the ExpectationMaximization (EM) algorithm (Flowerdew and Green 1994; Gregory and Ell 2005; Sridharan and Qiu 2013).

The first methods are conceptually simple and do not require an in-depth knowledge of spatial methods. They are quite intuitive and can be implemented easily. The dasymetric approaches, nevertheless, are more complex and data demanding and ask for a higher understanding and ability in the use of GIS tools. They entail employing more spatial layers and combining data from more sources. The aim of this work is to examine whether and to what extent a more complex approach is worthwhile in an instance in which *a priori* its usefulness can be put into question.

The rest of the paper is distributed as follows. Section 2 introduces the case study and sets the problem. Section 3 describes the sources utilized and the spatial methods tested in this research. In addition to the baseline approaches of point-areal interpolation and areal weighting, ten additional methods based on dasymetric techniques are analyzed. In Section 4 the different reallocations obtained are compared and their relative merits assessed. Finally, Section 5 summarizes and discusses the findings.

## 2. Case Study

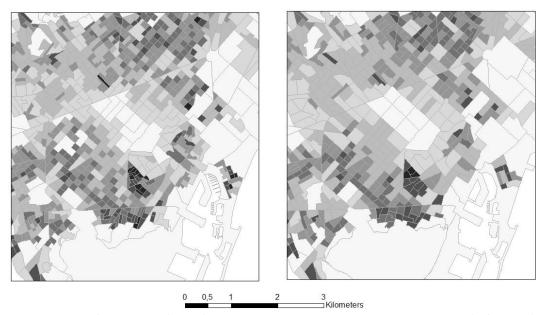
Barcelona is the second-largest city in Spain with more than 1.6 million of inhabitants. Its metropolitan area, with a population of around 4.7 million people, makes up the largest European urban area on the Mediterranean Sea. Barcelona, located on the northeast coast of the Iberian Peninsula (see Figure 1), is a polycentric and spatially complex city (Catalán et al. 2008) with a marked social structure (Broner 2010) that extends for a total area of 170 km², of which 101 are occupied by the city itself and just 48.6 are classified as residential areas.



**Figure 1**. Study area, Barcelona (Spain). Population density for the 2009 census sections (source units) depicted. The square delimits the area displayed in Figure 2.

Barcelona is the focus for the study mainly because it constitutes, from both theoretical and empirical standpoints, an instance in which *a priori* we can expect the dasymetric allocating approaches not to perform any better than the simpler methods. From a theoretical perspective, it could be argued that when units are small enough to be relatively homogeneous with regards to their inner distribution of population and, moreover, when big differences between contiguous units are not expected, areal weighting should be sufficient and that any auxiliary data employed will result in marginal, if any, improvements. Indeed, some previous studies (Brinegar and Popick 2010; Zandbergen and Ignizio 2010) point to this for relatively densely populated areas. Barcelona with an average population density of 16,000 inhabitants per km², which rises to 34,000 if only to residential areas are considered, represents a very densely and

continuously populated urban area (see Figure 1), with a statistically significant positive spatial correlation. Actually, taking the 2009 census sections as spatial units, the Moran index (Cliff and Ord 1981) of Barcelona population density is around 0.08 (p-value: 0.0000), an indicator that population density does not change abruptly at the boundaries of the source units. What is more, the Moran index rises to 0.26 (p-value: 0.0000) when we just consider the core of the metropolitan area (the census sections depicted in Figure 2-left), which comprises more than half the Barcelona total population (just under one million inhabitants). As a measure of urban compactness (Tsai, 2005), this value signals a very highly compact urban form, characteristic of the historic centers of the European cities.



**Figure 2**. Extract of Barcelona (Spain) division in census sections during the 2009 (left panel) and 2010 (right panel). The same zone (area around Catalonia Square, see Figure 1) depicted in both panels. Population densities by census section shaded in the figures.

From an empirical perspective, in urban areas, and mainly in big cities, spatial breakdowns are usually performed by local expert analysts with an in depth knowledge of the region. The boundaries of small areas tend to be established in a thoughtful and intelligent way, respecting area barriers such as rivers, wide avenues, roads or municipalities' boundaries, which can sometimes also act as social barriers. Indeed, in the Barcelona breakdowns the integrity of block buildings are always respected, with the boundaries of the majority of census sections being placed on streets. This, together with the clear spatial patterns that many social variables show, suggests that the

transitions between old and new units should be smooth and consequently begs the question whether in these circumstances dasymetric reallocation techniques would provide significant improvement in accuracy.

This question is pertinent because the majority of previous studies have been focused on big areas less densely populated, where moreover the population is sparsely distributed in the territory. For example, the study area in Eicher and Brewer (2001) encompasses 159 counties, covering four US states, with a maximum population density of 3,424 inhab/km². Mennis (2009) considers Delaware County, an area of 490 km² with 550,864 people in the year 2000 and 144 census tracts. Zandbergen (2011) handles 16 counties in Ohio, comprising 1 million people distributed in 230 census tracts across an area of 19,000 km². And, Reibel and Bufalino (2005) focus their study on Los Angeles County: 9.5 million people in the year 2000 distributed in 10,570 km². A similar picture is found for European studies. For example, Suárez et al. (2008) analyze the population distribution in the Gran Canaria island (Spain)—with 304,000 people distributed in 1,560 km²—, while Goerlich and Cantarino (2013) and Batista et al. (2013) deal with, respectively, the whole of Spain and the whole of Europe.

Every year, and referenced to the first of January, the Spanish Official Statistical Agency (*Instituto Nacional de Estadística*, INE) publishes (among many other variables) the population by age, broken down into five year groups, for each census section. These data come from the Municipal register, where municipality inhabitants are recorded, and can be downloaded free from the INE website (http://www.uv.es). Census section data published refers to the spatial census section breakdown in force in each period. Hence, for Barcelona and at census section level, the official population data were delivered using different breakdowns in 2009 and 2010. Therefore, to link them we would need to express them in the same breakdown.

Once a variable is reallocated from the 2009 breakdown to the 2010 breakdown, a problem emerges when one tries to compare the resulting approximations to the official statistics available for the 2010 breakdown. The official data available for the 2010 breakdown correspond to the first of January of 2010, whereas the reallocated data is referred to the first of January of 2009. In addition to the differences resulting from the

reallocation process, divergences can also occur because of natural and migration annual population movements registered during the year of 2009. Fortunately, on this occasion, single records are available and, under request and on payment, INE agents were kind enough to compute for us the actual data that they would have published if the 2010 breakdown had been in force on the first of January, 2009. In particular, INE agents provided us with a file containing for each 2010 census section the number of people living there the first of January of 2009 broken down into eighteen age groups: 0-4, 5-9, 10-14, 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80-84, and >85. These have been the variables reallocated in our analysis.

We assess whether dasymetric reallocation techniques would provide significant improvement in the case of a densely and continuously populated urban area (taking Barcelona as an ongoing example), by reallocating the population figures available for the 2009 breakdown into the 2010 breakdown and by comparing the resulting estimates to the INE values that would have been observed if the 2010 breakdown had been in force the first of January of 2009.

In addition to performing our scrutiny in absolute values as is usual in the reallocating spatial interpolation literature, we have also applied the reallocating approaches to the variables measured in percentages: calculated as a ratio over the total population of each census section. Many health and social variables are observed as rates or proportions and, therefore, in our opinion more effort should be devoted to finding out how simple and dasymetric interpolation methods would work when faced with the problem of reallocating rates or proportions from a set of geographical administrative units onto another, overlapping but non-hierarchical, set of spatial units.

#### 3. Data sources and methods

In this section, we describe and explain the details of the reallocating approaches assessed in this research. Grouped in pairs, fourteen methods have been tested. The first two pairs of proposals (grounded on point-based interpolation and areal weighting interpolation) are considered simple: they do not require auxiliary variables. They are

used as baseline methods to gauge the value of the other ten alternatives, which are based on dasymetric techniques and therefore are more complex. The point and areal weighting interpolation solutions just require the files providing the polygon attribute tables of the spatial breakdowns to be executed. The dasymetric mapping approaches need additional sources of information to guide the reallocation process: auxiliary variables related to the distribution of the population. In what follows, we first present the ancillary sources and variables employed in the dasymetric approaches, subsequently we introduce the methods considered.

### 3.1. Ancillary sources of information

In addition to data from INE, which provides the census variables analyzed in this research, we have dealt with geographical objects provided by four additional institutions: the City Council of Barcelona (*l'Ajuntament de Barcelona*), the Spanish Geographic Institute (*Instituto Geográfico Nacional*), the European Environment Agency (EEA) and the Spanish Cadastral Agency (*Dirección General del Catastro*). We also examined information from the OpenStreetMap project. However, because in that project data are collected by different teams using different standards, we had to discard it due to its limited adaptability and wide heterogeneity in terms of quality.

CartoBCN is the official website for cartographic information of l'Ajuntament de Barcelona (http://w20.bcn.cat/cartobcn/default.aspx?lang=en), from which many geographic files can be downloaded after registering free. From this source, we obtained (i) the shape (shp) files corresponding to January 2009 and January 2010 section breakdowns of Barcelona (according to which INE official census variables are delivered), and (ii) the Barcelona city street map (Callejero). Callejero contains information on the streets of Barcelona and their intersections. In Callejero, each stretch of road has associated with it, among other information, INE and city council specific codes, the street name and the type of road.

Files corresponding to 2009 SIOSE, LiDAR and MDT05 databases are available in the Download Center (http://centrodedescargas.cnig.es/CentroDescargas/) of the *Instituto Geográfico Naciona*l. Land Cover and Land Use Information System of Spain (SIOSE) is a

unique database of Spain produced at 1:25,000 scale combining topographic maps, satellite imagery, aerial photography and cadastral registers. This database contains information about heterogeneity of land use within any given Spanish polygon (Goerlich and Cantarino 2013). SIOSE comprises 2.5 million polygons with nearby 820,000 different land cover categories, obtained after combining the land cover elements with different weights (Cantarino *et al.* 2014). Each SIOSE polygon contains information about what percentage of the surface in the polygon corresponds to each attribute. Inhabited buildings are identified with four different type attributes in SIOSE. They correspond to compact apartment blocks (A1), isolated apartment blocks (A2), terraced houses (H1), and detached houses (H2). According to Goerlich and Cantarino (2013), taking 1.00 as reference value of the relative population density for a standard spatial unit of detached houses (H2), the values of relative population densities for compact apartment blocks (A1), isolated apartment blocks (A2) and terraced houses (H1) are, respectively, 9.23, 6.67 and 1.83.

Digital files with altimetry information from the LiDAR (which stands for Light Detection and Ranging) cloud of points are distributed in Spain in files of a 2x2 km grid. The download is done by municipality. Point clouds have been captured by flights using LiDAR sensor with a density of 0.5 points/m² and automatically classified and colored by RGB obtained from orthophotos with a pixel size of 25 or 50 cm of the Spanish National Aerial Orthophotography Plan (PNOA). The points, besides elevation, contain information about nine classes of terrain attributes.

MDT05 is a digital elevation model (DEM) with 5 meters resolution, which has been obtained, depending on the map sheet, using one of two main procedures. In Barcelona MDT05 data have been computed by interpolation from LiDAR flights of PNOA by selection of class type "Ground", with sub-metric precision.

The European Environment Agency (EEA) provides the Urban Atlas database. This offers pan-European comparable land use and land cover data for Functional Urban Areas. It can be downloaded (http://www.eea.europa.eu/data-and-maps/data/urban-atlas) for the main European cities, including Barcelona. Available at a 1:10,000 scale with a position accuracy of +/-5 m, its date of production is 2009. The layer called "Urban

Fabric", which classified the polygons into categories according to their percentage of soil sealing or sealing levels, has been the one used in this research. Under the principle that the larger the percentage of soil sealing in a polygon the larger is the percentage of residential area of the polygon, we have considered as proportions of residential areas in each polygon the average of soil sealing of its class: 0.05, 0.20, 0.40, 0.70 and 0.90 for, respectively, the categories "Discontinuous Very Low Density Urban Fabric", "Discontinuous Low Density Urban Fabric", "Discontinuous Medium Density Urban Fabric", "Discontinuous Dense Urban Fabric" and "Continuous Urban Fabric" and "Isolated Structures".

The Spanish cadastral database is an administrative register that is supported by the Ministry of Finance and Public Administration and contains the description of all the rustic, urban and special feature properties in Spain. Cadastral information is a free service (https://www.sedecatastro.gob.es/), but an electronic certificate is required to access it. Shapefile (shp) and alphanumeric (cat) data can be downloaded by municipality. The shp files define the boundaries of the cadastral parcels. The cat files contain information about the use (residential or other types), year of building and living area per dwelling. Shapefile files have no information about the use of buildings or surface of housing, but thanks to cadastral references they can be linked with the data of cat files.

## 3.2. Spatial reallocating approaches

This subsection offers details of the fourteen reallocating approaches considered in our scrutiny. The first two methods (M01-rIDW and M02-rNN) are point-based approaches, the Tobler's approach (M12-TPYC) is rasted-based, and the other eleven methods are vector-based procedures. The point-based methods have been included for the sake of comparison. Besides being inefficient, they do not verify the pycnophylactic condition of volume preservation (Tobler 1979). All the other twelve approaches are volume-preserving (also called mass-preserving). Table 1 displays a summary of the methods with some details regarding the way they have been computed. The auxiliary variables (and sources) employed as well as the acronyms with which we are going to identify them are also included. Following the classical terminology (Goodchild and Lam, 1980),

the units in which the variable of interest is available are called source units (or polygons) and the alternative units where the reallocated data are required are called target units.

**Table 1**. Summary of interpolation methods considered.

		Ancillary information			
Name	Procedure of computation	Source	Variable		
M01. IDW point Interpolation: rIDW.	Target census section (CS) values are obtained from a population density surface constructed applying IDW interpolation on values located in source CS centroids.	No	n/a		
M02. NN point Interpolation: rNN.	Similar to rIDW. The density surface is obtained by Natural Neighbors, instead of IDW.	No	n/a		
M11. Areal weighting interpolation: AW.	Areal weighting interpolation. Weighted sum of source values, with source-target area intersections as weights.	No	n/a		
M12. Pycnophylactic reallocation: TPYC.	Tobler's pycnophylactic interpolation method (Tobler 1979).	No	n/a		
M21. SIOSE residential: SIOSE-RSD	Weighted sum of source values, with source-target intersections, restricted to SIOSE residential areas, as weights.	SIOSE	Residential Area		
M22. SIOSE population: SIOSE-Pop	Similar to SIOSE-RSD, with weights weighted by relative p.population densities (Goerlich and Cantarino 2013).	SIOSE	Relative Population		
M31. Urban Atlas residential: UA-UF.	Weighted sum of source densities, with source-target intersections, restricted to Urban Fabric areas, as weights.	Urban Atlas	Urban Fabric areas		
M32. Urban Atlas residential: UA-SS.	Similar to UA-UF, with Urban Fabric areas weighted by relative soil sealing densities.	Urban Atlas	Urban Fabric soil sealing %		
M41. Volume buildings: UA-VB.	Weights by volume (high) of buildings according to LiDAR, in the Urban Fabric areas defined by Urban Atlas.	U. Atlas LiDAR	LiDAR volume buildings		
M42. Adjusted volume buildings: UA-VBadj.	e Similar to UA-VB, with LiDAR data amended in class "high vegetation".	U. Atlas LiDAR	LiDAR volume buildings		
M51. Road network length: CBCN-L.	Weights determined by the length of the streets, according to <i>Callejero</i> , within source-target intersections.	Carto BCN	streets		
M52. Buffer of streets CBCN-SB.	: Buffer of streets of <i>Callejero</i> , and source-target intersection, restricted to the buffer areas, as weights.	Carto BCN	Area buffer of streets		
M61. Cadastral residential: C-RSD.	Weighted sum of source values, with source-target intersections, restricted to cadastral footprints of residential buildings, as weights.	Cadaster	Footprints of residential buildings		
M62. Cadastral area homes: C-AH.	Weights by total area of housing in cadastral residential buildings.	Cadaster	Dwelling area		

M01-rIDW and M01-rNN are point-based approaches. These procedures (i) identify each source unit with its centroid, to which is assigned the observed value in the source unit, (ii) create a smooth prediction raster surface of the target variable, and (iii) estimate the variable of interest in each target polygon by averaging the estimated surface on it. The implementation of this process requires specifying a grid resolution and an interpolation procedure. We have worked with cells of size 50x50 meters and have used the methods

of inverse distance weighting (IDW) and of natural neighbor (NN) (as default in ArcGis® 10.2) as interpolation procedures.

In addition to being inefficient by reducing all the information of each source area to a point, the point-based approaches are not mass-preserving. That is, these procedures are not reversible: if we go back from target to source units, the outcomes will not coincide with the original values. The other twelve proposed methods are mass-preserving and share approach. All of them use as allocation function an estimator of the form given by equation (1), where  $\hat{P}_j$  is the estimated population in target unit j,  $P_i$  is the population of source unit i, S is the total number of source units and  $w_{ij}$  is the weight assigned to source unit i in estimating the population of target unit j. The methods differ in the way the weights  $w_{ij}$  are computed.

$$\hat{P}_j = \sum_{i=1}^S w_{ij} P_i \tag{1}$$

The areal weighting (M11-AW) approach, also known as polygon overlay (Markoff and Shapiro 1973), is one of the most widely used methods (Goodchild and Lam 1980) and the most popular choice when ancillary information is not available. In this method, the source and target units are overlaid to obtain intersections and the (imputation) weights,  $w_{ij}$ , are determined by the ratio between the area of the intersection between the source unit i and the target unit j and the total area of unit i. Areal weighting is considered a simple method because, as is the case with point-based methods, it does not require any additional data besides source and targets units.

The implicit assumption of the M11-AW approach is that the variable of interest is homogeneously distributed in each source unit, which is quite unlikely. Hence, as an alternative, Tobler (1979) proposed the pycnophylactic method (M12-TPYC) that, maintaining the mass-preserving condition, assumes that the attribute values should not change abruptly at the boundaries of the source units (Kyriakidis 2004). This allows a different value to be assigned to each cell of each source unit, from which  $w_{ij}$  weights are calculated. To preserve the pycnophylactic property, Tobler proposed an iterative procedure, with Mennis (2003) as an alternative. This method is also categorized as a

simple interpolation approach because no ancillary data is used to transfer the variable of interest from the source units to the target units.

The above methods (initially) handled the space of each source unit equally, irrespective of where the population is located and how this is distributed within it. Dasymetric mapping exploits ancillary sources to provide insights on how the population is spatially distributed. The most popular methods track where population is located in each source unit using two-dimensional (2-D) areal measures as auxiliary variables. From richer databases, three-dimensional (3-D) volume measures can be constructed to additionally know how the population is distributed inside of the source unit (Sridharan and Qiu 2013). Examples with one-dimensional (1-D) length measures can also be found in the literature (e.g., Reibel and Bufalino 2005).

The simplest 2-D dasymetric mapping methods are based on a binary classification of land uses: residential and nonresidential (Eicher and Brewer 2001; Holt et al. 2004; Sridharan and Qiu 2013). In the binary approach, nonresidential land use areas are considered unpopulated and consequently omitted (or zero weighted) in the distribution process. Although this refinement is expected to improve population allocations, it is not without flaws. On the one hand, residential areas usually include parts of a landscape (such as roads, footways and yards) where people do not reside. On the other hand, this approach implicitly assumes that within each source unit the population is evenly distributed across its residential areas, which again is seldom true (Maantay et al. 2007). This assumption often results in population underestimates in areas with high-rise buildings and in overestimates in areas with low-rise buildings (Harvey 2002; Sridharan and Qiu 2013). As alternatives, the 2-D multiclass (or polycategorical) dasymetric approaches and the 3-D methods try to amend these misestimates by accounting for the vertical distribution of the population. The 2-D multiclass approaches classify residential areas in more than a class with different population densities and the 3-D methods account for the height, the volume or the total area of the residential buildings. Note that the 2-D multiclass methods could be also be cataloged as 3-D procedures observing relative population densities as heights.

The 2-D binary methods that we have considered maintain quite a resemblance to the areal weighting procedure. In particular, the source and target units are again overlaid but this time restricted to residential areas and the weights,  $w_{ij}$ , determined by the ratio between the corresponding area of the intersection between the source unit i i and the target unit j and the residential area of unit i. That is, the numerator of the ratio is determined by the area of the intersection among the source unit i, the target unit j and the residential land use polygons. The different methods diverge in the source employed to classify polygons as residential. M21-SIOSE-RSD method utilizes SIOSE, M32-UA-UF employs Urban Atlas and M61-C-RSD uses the footprints of the buildings classified as residential in the Cadaster. The details of the method M52-CBCN-SB are discussed in the next paragraph.

In 1-D length methods, it is assumed that the density of population across a unit is directly related to the density of road/power/streetlights network segments across the unit. Hence, under this assumption, the 1-D length methods calculate the weights,  $w_{ij}$ , as the ratio between the length of the network segments within the overlapping area between the source unit i and the target unit j and the total length of network segments within the unit i. The M51-CBCN-L method uses the Callejero road/street network. In Barcelona, however, the boundaries of the majority of census sections are placed on streets, so it is not uncommon (given the one-dimensional nature of street networks) that the majority of Callejero network segments located on street boundaries are completely subsumed in just one census section, when in these cases half of the corresponding length should be apportioned to each side of the street. To amend this, the M51-CBCN-SB method constructs a buffer of 10 meters centered in the road/street lines and proceeds as the other 2-D binary methods do, after viewing the buffer polygons as residential areas. This method could be classified as 1.5-D as it combines a 1-D auxiliary variable with a 2-D strategy.

Land use data are particularly useful as a means to distinguish residential areas from non-residential areas, but 2-D binary methods do not discriminate by residential attributes. In 2-D polycategorical dasymetric procedures, residential areas are divided into different groups (for example, from "Discontinuous Very Low Density Urban Fabric"

to "Continuous Urban Fabric") and a different weight per area unit is assigned to each type according to their relative population density. In particular, denoting by K the number of residential types,  $d_k$  the relative population (or the relative soil sealing surface) of residential polygons of type k and  $w_{ijk}$  the area of the intersection among the source unit i, the target unit j and the residential polygons of type k, we have that the  $w_{ij}$  weights are reached after dividing  $\sum_{k=1}^K d_k w_{ijk}$  by  $\sum_{k=1}^K d_k w_{ik}$ , where  $w_{ik}$  is the area of the intersection between the source unit i and the residential polygons of type k. The M22-SIOSE-Pop method employs the classification in apartment blocks, isolated apartment blocks, terraced houses and detached houses used by SIOSE to identify inhabited buildings and the M32-UA-SS procedures makes use of the five Urban Fabric categories defined in the Urban Atlas database.

The M41-UA-VB, M42-UA-VBadj and M62-C-AH approaches are 3-D methods. They all consider the vertical dimension of residential buildings to compute the weights. The M41-UA-VB and M42-UA-VBadj methods are really close to the methods employed in Qiu et al. (2010) and Sridharan and Qiu (2013), being the M42-UA-VBadj approach a refinement of the M41-UA-VB method.

In the M41-UA-VB method, we consider the residential areas defined by Urban Atlas and construct (LiDAR)-derived residential building volumes as product of LiDAR height measures and LiDAR areas corresponding to points confined in Urban Atlas residential areas and classified as buildings in the LiDAR database. In our approach, a 5x5 m cell is classified as a building when it contains any point classified as building. The relative heights of buildings from the ground are extracted from the LiDAR point cloud taking as reference MDT05 as digital elevation model. Once residential building volumes are calculated, we compute the  $w_{ij}$  weights as the ratio between the total volume of the buildings located within the intersection of the source unit i and the target unit j and the total volume of unit i.

After a random revision of LiDAR point clouds, however, we were aware that many points classified as "high vegetation" in LiDAR should have been classified as building. Hence, to build the M42-UA-VBadj weights we reclassify as building cells those cells that being adjacent to a building cell and being located in an Urban Fabric polygon have a

height higher than 12 meters and occupy a plot with an area of more than 75 m<sup>2</sup>. Afterwards we proceed in the same way as in the M41-UA-VB approach.

The M62-C-AH method follows a different strategy. This approach constructs weights exploiting the more detailed information available to the original unit used by INE to collect population data: the household. Among other issues, the cadaster contains information about the postal address, floor and square meters of each existing property in Spain. Thus, we combine the cadastral shp and cat files to compute in each cadastral polygon its total housing area to then compute the  $w_{ij}$  weights as the ratio between the total housing area placed within the intersection of the source unit i and the target unit j and the total housing area of unit i.

All the spatial computations have been completed in ESRI ArcGIS® 10.2 (ESRI 2014), using its geo-processor ArcPy to create scripts. The calculation routines have been performed with Python 2.7.3 (Python Software Foundation, 2014).

## 4. Assessing allocations

A total of thirty-six variables (eighteen population variables in absolute values and the same number in percentages) have been allocated using fourteen methods for the 1,061 census sections defining the 2010 breakdown (i.e., more than half a million values have been calculated). This section evaluates the closeness of allocations and actual values. The results are quite clear and robust. The same order of preference emerges among groups of methods for all the variables and with all the measures of closeness calculated. The method M62-C-AH, which allocates variables using as ancillary data the total area of the homes in each residential building, is the one producing by far the most accurate results, whereas the point-based methods and the 1-D approach (M51-CBCN-L) are clearly the less accurate.

To evaluate which areal interpolation algorithm generates the most accurate estimates, three classical measures for summarizing the closeness between imputations,  $\widehat{P}_j$ , and actual values,  $P_j$ , in the T target units have been computed for each combination of variable and method: the root of the mean square error (RMSE), the mean absolute

percentage error (MAPE) and the Pearson correlation coefficient (CORR); see Table 2. Although other measures may also be computed—such as the mean squared error (Sadahiro 2000), the adjusted RMSE (Hawley and Moellering 2005) or the value weighted MAPE (Qiu et al. 2012)—the conclusions would have remained the same. RMSE and MAPE statistics are distance measures. Thus, the smaller the RMSE and MAPE distances, the closer allocated values and actual values are. In many applications, however, more important than closely approximating actual values is to dispose of a variable really alike in correlation terms: a variable that may be used in a regression in place of the unobserved variable. In this sense, the closer CORR is to one, the more related (correlated) are allocated and actual values.

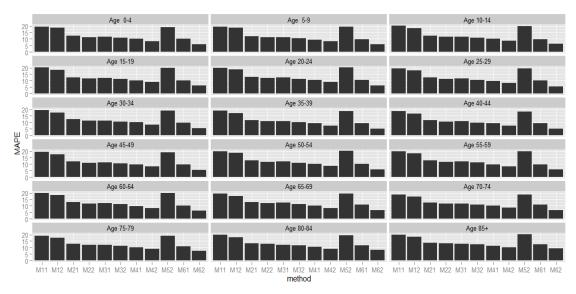
**Table 2**. Closeness measures between imputations  $(\widehat{P_i})$  and actual values  $(P_i)$ .

Description	Acronyms	Equations
Root Mean Square Error	RMSE	$\sqrt{\frac{1}{T} \sum\nolimits_{j=1}^{T} (\widehat{P}_{j} - P_{j})^{2}}$
Mean Absolute Percentage Error	MAPE	$\frac{100}{T} \sum_{j=1}^{T} \frac{\left \widehat{P}_{j} - P_{j}\right }{P_{j}}$
Pearson Correlation	CORR	$\frac{T\sum_{j=1}^{T}\widehat{P}_{j}P_{j} - \sum_{j=1}^{T}\widehat{P}_{j}\sum_{j=1}^{T}P_{j}}{\sqrt{T\sum\widehat{P}_{j}^{2} - \left(\sum\widehat{P}_{j}\right)^{2}}\sqrt{T\sum P_{j}^{2} - \left(\sum P_{j}\right)^{2}}}$

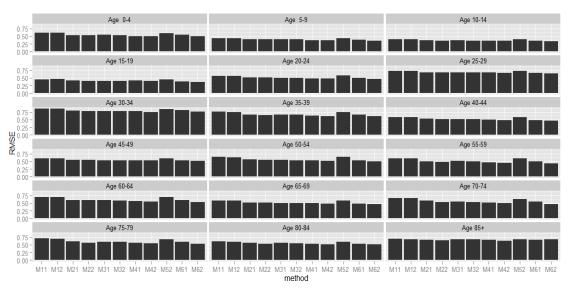
Table S1 (available in the supplementary online appendix) presents, by method and variable, the values of the distances (computed using RSME and MAPE as closeness measures) and of the correlations (calculated using CORR) between the amount of the people (in whole numbers) living in the 2010 census sections at the first of January, 2009 and the estimates obtained from the official values available in the 2009 census sections. The same statistics are displayed in Table S2 (see the online supplementary appendix) for the variables measured in percentages.

From the analysis of the numbers in both tables, several patterns clearly emerge. Firstly, the point-based interpolation strategies, in addition to not being volume-preserving, clearly present the worst approximations. In our application, the M51-CBCN-L method also shows similar figures. Secondly, as a rule, dasymetric methods are clearly preferable to simple methods. Thirdly, within dasymetric approaches, 3-D approaches are

preferable to 2-D procedures. Finally, the differences narrow when we work in relative terms (percentages) instead of in absolute values.



**Figure 3**. Differences by method, measured using MAPE, between the actual and allocated population absolute values for different age groups. The M01, M02 and M51 methods have been excluded to avoid that their large values dominate the scrutiny.



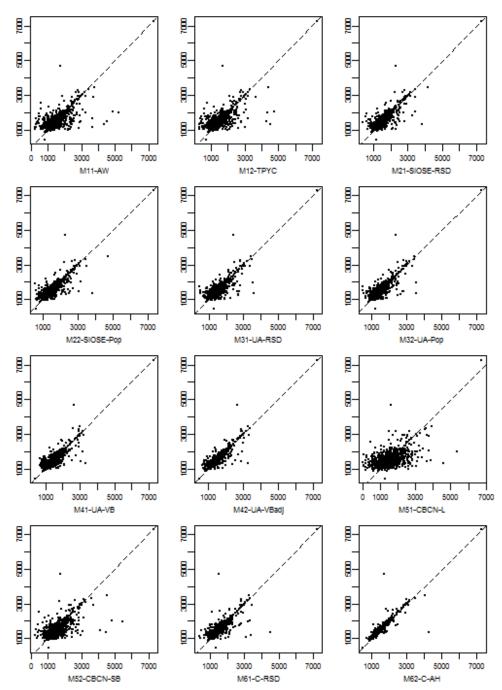
**Figure 4**. Differences by method, measured using RSME, between the actual and allocated population percentage values for different age groups. The M01, M02 and M51 methods have been excluded to avoid that their large values dominate the scrutiny.

To make these patterns more evident, Figure 3 and Figure 4 display by variable, excluding the three worst methods (M01, M02 and M51), some of the numbers

available in Tables S1 and S2. Figure 3 depicts MAPE distances for the allocations in absolute values and Figure 4 portrays RMSE distances for the estimates in percentages.

A closer look at the results even highlights some preferences within the different strategies. Focusing on the allocations of the population figures in whole numbers, the order of preferences we find is: M62 > M42 > M41 > M32  $\ge$  M22  $\ge$  M31  $\ge$  M61 > M21 > M12 > M52 > M11 > M51 > M02> M01. That is, within the 3-D approaches, which are clearly the best as a group, the method M62-CBCN-SB, whose auxiliary variable best approaches the way the population information is originally collected, reveals itself as the procedure generating the most accurate results. Furthermore, we notice that the refinement that method M42-UA-VBadj represents over method M42-UA-VB also yields its fruits.

Likewise, although the differences among the 2-D dasymetric methods are not so evident at first glance, a deeper look reveals a clear pattern among 2-D subgroups. The more complex multiclass 2-D methods (M32-UA-SS and M22-SIOSE-Pop) generate as a rule more accurate results than the binary 2-D approaches. This should not be a surprise, given that in our theoretical disquisition we already note that these methods could even be considered as 3-D. Following multiclass 2-D procedures, the list continues in order of accuracy with the three binary 2-D methods, with the M21-SIOSE-RSD method placed in the third position of the trio, probably consequence of the higher quality of Urban Atlas and Cadaster databases compared to SIOSE database. Among the non-dasymetric approaches, areal weighting procedures are preferable to point-based interpolation, with the Tobler procedure generating better outputs than the simple areal weighting approach. It is interesting to observe that the method M52-CBCN-SB, which can be thought of as dasymmetric 1.5-D, presents levels of accuracy alike to those of the areal weighting approaches. Finally, the 1-D method M51-CBCN-L and the two point-based procedures are clearly placed towards the end of the list.



**Figure 5**. Comparing allocated estimates (horizontal axes) and actual values (vertical axes) of the 2009 total population of Barcelona (Spain) in the 2010 census section breakdown. The distance from the 45° line indicates how far apart allocations and actual values are. The number of data points in each scatterplot is 1,061. Details of the allocating methods can be found in Table 1. The point-based methods have been excluded given their limited quality.

The above conclusions are also manifestly visible observing Figure 5, where comparisons at the level of census section for the total population are displayed for all the methods but the point-based ones. Among panels of Figure 5, the panel corresponding to the M62-C-AH unambiguously stands out as the best and the one corresponding to M51-

CBCN-L unmistakably the worst. The rest of the estimates are in an intermediate position.

When we look at the results of closeness measures corresponding to the allocations of the variables in percentages (Table S2 and Figure 4), the results are not so evident. Nevertheless, although the differences reduce significantly, the order of preference among the methods is almost the same as when we allocate whole numbers. The order of preferences we find in this case is:  $M62 > M42 > M41 > M61 \ge M22 \ge M32 \ge M31 > M21 > M52 \ge M12 \ge M11 > M51 > M02 > M01$ . Despite there being some changes in the relative order of some methods, the general picture remains: 3-D methods are preferable to polycategorical 2-D methods and these again preferable to binary 2-D methods, these are followed by areal weighting and 1-D methods and point-based interpolation procedures are again towards the end of the list.

#### 5. Conclusions and final remarks

The question of reallocating population figures from a set of geographical administrative units onto another set of units is an issue that has received a great deal of attention in the literature. Every other day, a new procedure exploiting a previously unused ancillary source of information is introduced in the literature, claiming that it outperforms alternative algorithms. Given that an accurate location of population is crucial to answering many practical questions of social interest, the introduction in the reallocation process of new auxiliary variables through dasymetric mapping is the route commonly followed. Unfortunately, when the new (usually more complex) methods are applied to a new instance, the improvements achieved are sometimes dubious and just marginal. The tradeoff cost-effectiveness of each solution tends to be case-dependent: it usually depends on both the data and the geography.

In this sense, it is of unquestionable interested to know what approaches would generate satisfactory solutions under each group of circumstances since many studies have shown that different interpolation approaches can yield outcomes with really different implications. To evaluate which areal interpolation algorithm is the most appropriate for a given application, a significant majority of studies have been focused

on large areas with really heterogeneous population densities, the general conclusion being that as a rule more sophisticated methods are worth the extra effort they entail.

From a theoretical point of view, it could be proved that with a target variable uniformly distributed in the territory, areal weighting interpolation would produce perfect allocations. In the extreme opposite case, if people were concentrated in just a bunch of small subareas of the whole space, knowing the geographical distribution of the population would be a necessary condition to (depending on the exact breakdowns) yield accurate interpolations. Obviously, infinite in-between situations are possible. It could be argued, however, that when we work with (relative) homogeneous small source units whose variable to be allocated varies gradually between contiguous units, areal weighting interpolation could be enough and that any ancillary variable employed would yield marginal improvements. The spatial distribution of population in census sections of Barcelona in Spain meets these requirements. We have studied whether in these circumstances dasymetric reallocation techniques would provide significant improvement.

Our study shows that even under the above conditions the most sophisticated approaches clearly produce the better results. In particular, the method that allocates people using as ancillary variable the total dwelling area in each residential building is by far the one yielding the most accurate outcomes. In general, our study shows the 3-D methods generating the better outcomes followed, in order, by the multiclass 2-D procedures, the binary 2-D approaches and the areal weighting and 1-D algorithms. The point-based interpolation procedures are by far the ones producing the worst estimates.

Finally, it should be noted that, although some previous studies suggest that when working with small source units the use of more complex methods may be unnecessary when the variable to be distributed presents a strong spatial correlation (such as in the case of partisan vote proportion distributions), our analysis does not completely support this conclusion. Indeed, despite the differences between the methods showing a clear narrowing (for instance, from using areal weighting to using our best method the MAPE improvement gap reduces from 50% to 20%), we reach similar conclusions when we deal with the variables in percentages as well as in absolute numbers. Further research,

therefore, should be done to gauge to what extent more sophisticated approaches could be helpful in the process of reallocating a rate or a proportion in an urban area. In particular, it would be interesting to find out how the relative performance of simple and complex methods could be affected by the type of variable to be distributed, the final aims of the reallocation process and/or the urban structure under study. A possible line of future research would consist in studying whether spatial measures of urban morphology (such as the Moran index, as a summary of the combined effect of the variable to be distributed and the urban geography) are related to the accuracies of the areal distribution methods.

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## **SUPPLEMENTARY (ONLINE) APPENDIX**

**Table S1**. Closeness of actual and allocated population absolute values by method and variable.

	<b>Table S1</b> . Closeness of actual and allocated population absolute values by method and variable.																			
	Variable: Age Group  Method 0-4 5-9 10-1415-1920-2425-2930-3435-3940-4445-4950-5455-5960-6465-6970-7475-7980-84											-								
N	1ethod																			
	M01		44.7	44.9		44.2						44.6					-	47.1		45.0
	M02		43.3	43.4	43.8	42.7	43.1	43.8	43.5	43.2	43.1	42.9	42.9	43.2	43.9	44.2	45.0	45.7	45.4	43.7
	M11		19.5	20.1	20.0	20.0	19.4	19.4	19.0	18.8	19.4	20.0	19.7	20.0	19.5	18.7	19.1	19.6	19.8	19.5
Error	M12		18.7	18.1	18.3	18.5	17.9	17.5	17.3	16.9	17.6	18.5	18.2	18.5	17.7	16.9	17.5	17.9	18.2	17.9
e E	M21		12.2	12.6	12.7	12.9	12.6	12.5	11.9	11.7	12.1	12.7	12.7	12.7	13.0	12.4	12.7	13.2	13.5	12.6
Square	M22		11.3	11.7	11.9	12.2	11.5	11.3	10.8	10.7	11.1	11.8	11.8	11.8	12.1	11.7	12.2	12.8	13.3	11.7
Sq	M31		11.3	11.9	12.1	12.3	11.7	11.4	10.9	10.7	11.5	12.1	12.1	12.1	12.3	11.6	12.1	12.2	13.0	11.8
Root Mean	M32		10.4	11.0	11.2	11.3	10.7	10.5	10.0	9.8	10.5	11.1	11.3	11.4	11.5	10.8	11.2	11.5	12.4	11.0
Š	M41		9.6	10.1	10.2	10.4	9.8	10.0	9.2	9.2	9.7	10.0	9.7	9.6	10.1	10.2	10.3	10.7	11.4	10.0
00 00	M42		8.1	8.6	8.9	8.9	8.2	8.1	7.5	7.3	8.1	8.6	8.2	8.4	8.3	8.5	8.8	9.0	10.1	8.5
~	M51		39.5	39.9	40.9	41.6	40.1	40.2	42.0	40.0	40.4	40.8	40.3	40.7	40.9	40.3	40.0	40.0	41.0	40.5
	M52		19.3	19.8	19.8	20.2	19.5	19.0	18.7	18.4	19.0	20.1	20.0	19.9	19.6	18.8	18.9	19.6	20.2	19.4
	M61		9.8	10.0	10.2	10.5	10.1	9.9	9.5	9.3	9.6	10.2	9.9	10.3	10.8	10.9	11.0	11.6	12.5	10.3
	M62	5.9	5.9	6.2	6.3	6.2	5.6	5.7	5.1	5.0	5.4	5.9	5.8	6.2	6.6	6.8	7.3	8.0	9.3	6.3
	M01	26.5	23.1	22.3	25.3	37.8	59.0	63.6	53.3	45.5	41.6	37.1	35.0	34.0	27.8	28.5	29.0	22.9	20.5	35.2
_	M02	26.7	23.1	22.0	24.9	36.7	57.7	62.9	53.2	45.5	41.1	36.4	34.2	33.6	27.6	28.4	28.8	22.5	20.1	34.7
Error	M11	23.5	18.7	16.8	18.7	28.9	42.3	45.5	41.7	34.9	32.1	29.2	24.4	23.4	18.2	18.2	18.0	12.9	11.6	25.5
ē.	M12	22.7	18.1	15.4	17.2	27.8	40.7	42.9	39.1	32.3	30.0	27.3	22.6	21.4	16.3	16.0	16.0	11.6	10.7	23.8
Percentage	M21	15.6	12.5	11.3	13.1	21.3	31.5	32.8	28.9	23.8	23.0	20.4	16.2	15.7	12.7	12.6	12.7	9.4	8.9	17.9
cer	M22	15.3	11.8	10.6	12.3	20.6	30.6	31.6	27.9	22.9	21.6	19.2	15.4	15.0	11.9	11.7	12.0	9.5	9.7	17.2
Per	M31	14.8	12.3	11.1	12.2	20.1	30.1	30.7	26.9	22.5	21.2	18.5	15.7	15.4	12.4	12.2	12.5	9.5	9.8	17.1
ıte	M32	14.1	11.7	10.4	11.5	19.6	29.5	29.8	25.9	21.5	20.1	17.6	14.9	14.7	11.6	11.4	11.8	9.2	9.9	16.4
Wean Absolute	M41	13.3	10.5	9.5	10.1	18.4	29.0	29.9	24.5	20.0	18.2	15.5	12.6	12.5	10.0	10.5	11.0	8.8	10.0	15.2
Ab	M42	11.2	9.7	8.7	9.5	17.1	26.3	25.9	21.6	18.0	17.1	14.3	11.5	11.3	8.7	9.2	9.4	7.4	8.8	13.6
an	M51	25.3	21.7	20.5	23.2	36.1	53.5	55.5	48.4	42.0	39.8	37.3	34.9	34.5	26.2	25.7	24.6	18.8	17.1	32.5
Σ	M52	20.7	17.2	16.2	18.6	29.4	42.4	43.3	38.3	32.6	31.8	29.5	25.2	23.7	17.9	17.5	17.3	12.9	11.7	24.8
	M61	13.6	10.8	10.0	11.3	23.6	36.7	35.7	28.7	22.8	20.0	18.2	14.9	14.7	12.2	12.3	12.4	9.4	10.3	17.6
	M62	7.5	6.5	6.1	6.7	19.1	31.2	28.1	20.9	16.3	13.7	11.4	8.4	8.4	6.9	7.1	7.9	7.6	10.0	12.4
	M01	.677	.658	.633	.593	.639	.680	.651	.613	.556	.534	.552	.626	.568	.502	.526	.500	.546	.646	.594
	M02	.661	.630	.613	.583	.638	.667	.622	.586	.531	.528	.549	.620	.552	.482	.498	.475	.530	.633	.578
	M11	.697	.725	.745	.744	.745	.785	.780	.723	.698	.679	.662	.773	.754	.754	.777	.783	.828	.851	.750
_	M12	.711	.737	.776	.777	.765	.802	.803	.751	.730	.708	.695	.804	.793	.802	.827	.827	.859	.872	.780
tior	M21	.840	.856	.869	.856	.853	.875	.875	.846	.831	.805	.810	.890	.872	.861	.878	.877	.898	.906	.861
ela	M22	.844	.870	.884	.873	.863	.882	.884	.856	.844	.827	.829	.902	.884	.878	.896	.891	.896	.889	.872
Pearson Correlation	M31	.854	.859	.872	.876	.870	.886	.889	.866	.847	.831	.843	.896	.877	.866	.886	.882	.898	.885	.871
n O	M32	.865	.871	.885	.888	.876	.890	.895	.874	.858	.846	.857	.906	.886	.881	.900	.894	.903	.883	.881
rso	M41	.878	.893	.904	.914	.891	.894	.895	.886	.875	.874	.890	.934	.919	.912	.915	.908	.911	.882	.899
Pea	M42	.913	.908	.918	.924	.905	.913	.921	.911	.898	.887	.905	.945	.932	.932	.934	.931	.937	.908	.918
_	M51	.651	.651	.652	.635	.616	.673	.691	.651	.606	.560	.515	.569	.520	.560	.611	.646	.672	.709	.622
	M52	.746	.754	.754	.739	.730	.781	.796	.756	.724	.675	.643	.743	.736	.756	.787	.794	.823	.844	.755
	M61	.878	.891	.898	.895	.821	.831	.852	.847	.843	.850	.849	.908	.889	.869	.883	.883	.900	.875	.870
	M62	.961	.958	.960	.962	.882	.877	.906	.915	.915	.927	.939	.971	.963	.957	.960	.951	.933	.881	.934

**Table S2**. Closeness of actual and allocated population percentages by method and variable.

Nethor   N		Variable: Age Group																			
Mor	M	- lethod	0-4	5-9	10-14	15-19	20-24	25-29	30-34					55-59	60-64	65-69	70-74	75-79	80-84	>85	- Mean
Moray   Mora		M01	10.7	9.6	9.9	10.4	10.0	8.5	8.8	8.5	7.2	7.4	9.6	11.0	11.5	11.2	11.8	13.5	14.8	16.6	10.6
M12				9.4	9.7							7.2		10.7					14.5	16.1	10.3
Mail		M11	6.9	6.6	6.6	6.6	6.5	5.5	5.8	5.3	4.6	5.1	6.2	6.7	7.3	7.8	8.4	9.1	10.0	10.7	7.0
May	o.	M12	6.9	6.5	6.6	6.6	6.4	5.5	5.7	5.2	4.5	5.1	6.1	6.6	7.2	7.7	8.4	9.0	9.9	10.5	6.9
May		M21	6.3	6.0	6.2	6.0	6.0	5.1	5.3	4.7	4.2	4.7	5.6	5.9	6.5	7.1	7.6	8.4	9.4	10.2	6.4
May	ıare	M22	6.2	5.9	6.1	6.0	6.0	5.1	5.3	4.6	4.1	4.6	5.6	5.7	6.5	7.1	7.5	8.2	9.2	10.1	6.3
May	Squ			5.9	6.0	5.9	5.8	5.1	5.2	4.6	4.1	4.6	5.5	5.8	6.3	7.0	7.3	8.2	9.0	9.9	6.2
Mail		M32	6.2	5.8	6.0	5.8	5.8	5.0	5.1	4.6	4.1	4.6	5.4	5.8	6.3	7.0	7.2	8.1	9.0	9.8	6.2
M52   7.0   6.7   6.9   6.8   6.8   5.7   5.9   5.5   4.7   5.2   6.4   7.0   7.5   8.0   8.5   9.4   10.4   11.2   7.8     M61   6.1   5.6   5.9   5.7   5.6   4.8   5.1   4.5   4.0   4.5   5.3   5.6   6.1   6.7   7.2   8.1   8.9   9.8   6.8     M62   5.6   5.3   5.5   5.3   5.2   4.5   4.7   4.0   3.7   4.1   4.9   4.9   5.5   6.2   6.5   7.2   8.0   9.3   5.1     M02   7.72   5.38   4.90   5.84   7.60   9.69   1.085   1.002   7.40   7.38   8.58   8.78   9.05   7.16   7.95   8.72   7.76   7.92   7.92   7.92     M11   6.1   6.3   6.40   6.56   6.52   7.34   8.62   7.59   5.81   5.98   6.30   6.05   7.01   5.83   6.58   6.7   6.7   6.67   6.99     M12   5.32   4.06   3.66   4.09   5.15   6.86   7.91   6.64   5.29   5.50   5.67   5.05   5.99   5.22   5.75   6.20   6.67   6.79     M13   5.42   4.02   3.62   4.02   4.99   6.82   7.81   6.66   5.21   5.36   5.94   5.94   5.94   5.94   5.94   5.94     M44   5.92   3.67   3.48   4.00   4.84   6.59   7.76   6.61   5.12   5.32   5.28   4.92   5.86   5.00   5.32   5.89   5.52   5.70   5.52   6.83   5.94    M65   5.44   3.91   3.53   3.89   5.03   6.68   8.15   6.68   8.15   6.68   6.01   6.95   5.70   5.70   5.81   5.84   5.94   5.	Μe			5.7	6.0	6.0	5.7	5.0	5.1	4.4	4.0	4.6	5.4	5.6	6.3	6.9	7.2	7.9	8.8	9.8	6.2
M52   7.0   6.7   6.9   6.8   6.8   5.7   5.9   5.5   4.7   5.2   6.4   7.0   7.5   8.0   8.5   9.4   10.4   11.2   7.8     M61   6.1   5.6   5.9   5.7   5.6   4.8   5.1   4.5   4.0   4.5   5.3   5.6   6.1   6.7   7.2   8.1   8.9   9.8   6.8     M62   5.6   5.3   5.5   5.3   5.2   4.5   4.7   4.0   3.7   4.1   4.9   4.9   5.5   6.2   6.5   7.2   8.0   9.3   5.1     M02   7.72   5.38   4.90   5.84   7.60   9.69   1.085   1.002   7.40   7.38   8.58   8.78   9.05   7.16   7.95   8.72   7.76   7.92   7.92   7.92     M11   6.1   6.3   6.40   6.56   6.52   7.34   8.62   7.59   5.81   5.98   6.30   6.05   7.01   5.83   6.58   6.7   6.7   6.67   6.99     M12   5.32   4.06   3.66   4.09   5.15   6.86   7.91   6.64   5.29   5.50   5.67   5.05   5.99   5.22   5.75   6.20   6.67   6.79     M13   5.42   4.02   3.62   4.02   4.99   6.82   7.81   6.66   5.21   5.36   5.94   5.94   5.94   5.94   5.94   5.94     M44   5.92   3.67   3.48   4.00   4.84   6.59   7.76   6.61   5.12   5.32   5.28   4.92   5.86   5.00   5.32   5.89   5.52   5.70   5.52   6.83   5.94    M65   5.44   3.91   3.53   3.89   5.03   6.68   8.15   6.68   8.15   6.68   6.01   6.95   5.70   5.70   5.81   5.84   5.94   5.	oot			5.5	5.7	5.7	5.5	4.8	4.9	4.3	3.8	4.4	5.2	5.3	6.0	6.6	6.9	7.5	8.5	9.4	5.9
M61   6.1   5.6   5.9   5.7   5.6   4.8   5.1   4.5   4.0   4.5   5.3   5.6   6.1   6.7   7.2   8.1   8.9   9.8   6.5   6.2   5.5   5.3   5.5   5.3   5.2   4.5   4.5   4.0   4.0   3.7   4.1   4.9   4.9   5.5   6.2   6.5   7.2   8.0   9.3   5.3   5.0   6.0   6.0   7.2   7.5	æ			8.5	8.5	8.8	8.8	7.1	7.4	7.0	6.1	6.6	8.1	9.3	10.1	10.2	10.7	11.7	13.1	14.3	9.2
Mol				6.7	6.9	6.8	6.8	5.7	5.9	5.5	4.7	5.2	6.4	7.0	7.5	8.0	8.5	9.4	10.4	11.2	7.2
M01 .823 .549 .504 .607 .792 1.000 1.145 1.055 .755 .764 .897 .914 .924 .739 .824 .906 .776 .810 .825 .826 .807 .916 .925 .725 .725 .838 .490 .584 .760 .969 1.085 1.002 .740 .738 .858 .878 .905 .716 .795 .872 .754 .792 .795 .871 .616 .416 .416 .416 .416 .416 .416 .41				5.6	5.9	5.7	5.6	4.8	5.1	4.5	4.0	4.5	5.3	5.6	6.1	6.7	7.2	8.1	8.9	9.8	6.1
MO2 .772 .538 .490 .584 .760 .969 1.085 1.002 .740 .738 .858 .878 .905 .716 .795 .872 .754 .792 .795 .876 .976 .476 .795 .876 .796 .679 .610 .692 .626 .796 .876 .796 .876 .796 .876 .796 .679 .610 .692 .626 .796 .876 .796 .796 .876 .796 .996 .996 .996 .996 .996 .996 .9				5.3	5.5	5.3	5.2	4.5	4.7	4.0	3.7	4.1	4.9	4.9		6.2	6.5	7.2	8.0	9.3	5.6
M11   M12   M13   M39   M36   M56   M56   M56   M56   M56   M56   M57   M56				.549	.504	.607	.792	1.000	1.145	1.055	.755	.764	.897	.914	.924	.739	.824	.906	.776	.810	.821
M51	_			.538	.490	.584	.760	.969	1.085	1.002	.740	.738	.858	.878	.905	.716	.795	.872	.754	.792	.792
M51 .643 .492 .461 .539 .691 .852 .967 .836 .673 .713 .745 .787 .865 .702 .743 .807 .721 .788 .72  M52 .597 .438 .400 .445 .576 .735 .847 .743 .577 .598 .646 .604 .697 .584 .637 .685 .603 .680 .619  M61 .544 .391 .353 .389 .503 .668 .815 .668 .489 .524 .535 .491 .589 .489 .542 .601 .537 .669 .544  M62 .493 .356 .335 .369 .469 .643 .754 .606 .465 .507 .495 .429 .527 .462 .472 .527 .510 .675 .507  M01 .809 .864 .870 .829 .822 .875 .870 .825 .828 .782 .817 .848 .851 .852 .846 .829 .842 .828 .833  M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844  M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889  M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .907  M21 .911 .920 .926 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .92  M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .92  M41 .920 .933 .930 .916 .925 .942 .940 .937 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .922  M41 .920 .933 .930 .916 .925 .942 .940 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .826  M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .800 .897 .930 .908 .903 .906 .900 .890 .871 .900	iro			.439	.405	.456	.562		.862						.701			.707	.610	.692	.623
M51 .643 .492 .461 .539 .691 .852 .967 .836 .673 .713 .745 .787 .865 .702 .743 .807 .721 .788 .72  M52 .597 .438 .400 .445 .576 .735 .847 .743 .577 .598 .646 .604 .697 .584 .637 .685 .603 .680 .619  M61 .544 .391 .353 .389 .503 .668 .815 .668 .489 .524 .535 .491 .589 .489 .542 .601 .537 .669 .544  M62 .493 .356 .335 .369 .469 .643 .754 .606 .465 .507 .495 .429 .527 .462 .472 .527 .510 .675 .507  M01 .809 .864 .870 .829 .822 .875 .870 .825 .828 .782 .817 .848 .851 .852 .846 .829 .842 .828 .833  M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844  M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889  M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .907  M21 .911 .920 .926 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .92  M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .92  M41 .920 .933 .930 .916 .925 .942 .940 .937 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .922  M41 .920 .933 .930 .916 .925 .942 .940 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .826  M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .800 .897 .930 .908 .903 .906 .900 .890 .871 .900	ge I																			.679	.617
M51 .643 .492 .461 .539 .691 .852 .967 .836 .673 .713 .745 .787 .865 .702 .743 .807 .721 .788 .72  M52 .597 .438 .400 .445 .576 .735 .847 .743 .577 .598 .646 .604 .697 .584 .637 .685 .603 .680 .619  M61 .544 .391 .353 .389 .503 .668 .815 .668 .489 .524 .535 .491 .589 .489 .542 .601 .537 .669 .544  M62 .493 .356 .335 .369 .469 .643 .754 .606 .465 .507 .495 .429 .527 .462 .472 .527 .510 .675 .507  M01 .809 .864 .870 .829 .822 .875 .870 .825 .828 .782 .817 .848 .851 .852 .846 .829 .842 .828 .833  M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844  M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889  M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .907  M21 .911 .920 .926 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .92  M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .92  M41 .920 .933 .930 .916 .925 .942 .940 .937 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .922  M41 .920 .933 .930 .916 .925 .942 .940 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .826  M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .800 .897 .930 .908 .903 .906 .900 .890 .871 .900	nta																				.560
M51 .643 .492 .461 .539 .691 .852 .967 .836 .673 .713 .745 .787 .865 .702 .743 .807 .721 .788 .72  M52 .597 .438 .400 .445 .576 .735 .847 .743 .577 .598 .646 .604 .697 .584 .637 .685 .603 .680 .619  M61 .544 .391 .353 .389 .503 .668 .815 .668 .489 .524 .535 .491 .589 .489 .542 .601 .537 .669 .544  M62 .493 .356 .335 .369 .469 .643 .754 .606 .465 .507 .495 .429 .527 .462 .472 .527 .510 .675 .507  M01 .809 .864 .870 .829 .822 .875 .870 .825 .828 .782 .817 .848 .851 .852 .846 .829 .842 .828 .833  M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844  M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889  M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .907  M21 .911 .920 .926 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .92  M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .92  M41 .920 .933 .930 .916 .925 .942 .940 .937 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .922  M41 .920 .933 .930 .916 .925 .942 .940 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .936  M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .826  M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .800 .897 .930 .908 .903 .906 .900 .890 .871 .900	rce																				.543
M51	. Pe																				.551
M51	lute																				.544
M51	oso																				.531
M61         544         391         353         389         503         668         815         668         489         524         535         491         589         489         542         601         537         669         54           M62         493         356         335         369         469         643         754         606         465         507         495         429         527         462         472         527         510         675         50           M01         809         864         870         829         822         875         870         825         828         782         817         848         851         852         846         829         842         828         83           M02         825         864         872         839         827         883         884         841         822         792         830         857         850         858         857         842         844         829         848           M11         880         906         909         899         898         931         925         903         883         861         890         929         <	η																				.517
M61         544         391         353         389         503         668         815         668         489         524         535         491         589         489         542         601         537         669         54           M62         493         356         335         369         469         643         754         606         465         507         495         429         527         462         472         527         510         675         50           M01         809         864         870         829         822         875         870         825         828         782         817         848         851         852         846         829         842         828         83           M02         825         864         872         839         827         883         884         841         822         792         830         857         850         858         857         842         844         829         848           M11         880         906         909         899         898         931         925         903         883         861         890         929         <	leal																				
M62         .493         .356         .335         .369         .469         .643         .754         .606         .465         .507         .495         .429         .527         .462         .472         .527         .510         .675         .506           M01         .809         .864         .870         .829         .822         .875         .870         .825         .828         .782         .817         .848         .851         .852         .846         .829         .842         .842         .842         .842         .844         .822         .792         .830         .857         .850         .858         .857         .842         .844         .829         .84           M11         .880         .906         .909         .899         .898         .931         .925         .903         .883         .861         .898         .929         .907         .902         .897         .892         .887         .867         .89           M12         .883         .908         .911         .899         .900         .932         .926         .905         .886         .861         .900         .931         .909         .904         .900         .901	2																				
M01 .809 .864 .870 .829 .822 .875 .870 .825 .828 .782 .817 .848 .851 .852 .846 .829 .842 .828 .838 M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844 .829 .844 M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889 M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .905 M21 .911 .920 .926 .919 .916 .940 .937 .927 .904 .883 .921 .951 .933 .923 .924 .918 .904 .875 .91 M22 .913 .925 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .92 M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .925 .936 .934 .924 .930 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .928 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .925 .936 .934 .923 .926 .945 .940 .937 .932 .889 .931 .935 .945 .936 .938 .931 .916 .881 .926 .936 .934 .925 .936 .934 .923 .926 .945 .940 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .938 .823 .864 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .864 .856 .857 .937 .931 .938 .838 .823 .864 .856 .857 .937 .931 .931 .931 .931 .931 .931 .931 .933 .930 .931 .931 .933 .932 .934 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .934 .935 .934 .935 .934 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .935 .934 .934 .935 .934 .935 .934 .934 .935 .934 .934 .935 .934 .934 .935 .934 .934 .934 .934 .934 .934 .934 .934																					
M02 .825 .864 .872 .839 .827 .883 .884 .841 .822 .792 .830 .857 .850 .858 .857 .842 .844 .829 .844 M11 .880 .906 .909 .899 .898 .931 .925 .903 .883 .861 .898 .929 .907 .902 .897 .892 .887 .867 .889 .873 .908 M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .908 M21 .911 .920 .926 .919 .916 .940 .937 .927 .904 .883 .921 .951 .933 .923 .924 .918 .904 .875 .916 M22 .913 .925 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .928 M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .926 M41 .920 .933 .930 .916 .925 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .928 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .928 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .938 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 .858 .857 .862 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .908 .900 .890 .871 .909																					-
M11 880 906 909 899 898 931 925 926 905 886 861 898 929 907 907 902 897 892 887 892 887 867 892 898 M12 883 908 911 899 900 932 926 905 886 861 900 931 909 904 900 895 893 873 900 M21 911 920 926 919 916 940 937 927 904 883 921 951 933 923 924 918 904 8875 91 M22 913 925 930 922 918 942 939 931 908 887 925 955 934 924 937 932 916 882 92 M31 907 922 928 922 921 941 939 926 907 889 929 951 935 928 934 925 936 907 871 925 M41 920 933 930 916 925 942 940 927 911 891 891 932 954 936 928 935 927 909 873 92 M41 920 933 930 916 925 942 940 934 918 890 931 958 940 931 941 931 916 881 92 M42 925 936 934 923 926 945 943 937 923 893 935 961 943 935 945 937 922 891 938 M51 867 881 880 854 841 907 904 881 840 794 860 877 854 856 870 857 838 823 866 M52 887 907 911 903 893 932 928 907 885 860 897 930 908 903 906 900 890 871 900 8871 9																					
M12 .883 .908 .911 .899 .900 .932 .926 .905 .886 .861 .900 .931 .909 .904 .900 .895 .893 .873 .900 M21 .911 .920 .926 .919 .916 .940 .937 .927 .904 .883 .921 .951 .933 .923 .924 .918 .904 .875 .910 M22 .913 .925 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .937 .932 .916 .882 .928 M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .907 .871 .925 M41 .920 .933 .930 .916 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .928 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .924 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .938 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .900																					
M21 .911 .920 .926 .919 .916 .940 .937 .927 .904 .883 .921 .951 .933 .923 .924 .918 .904 .875 .916 M22 .913 .925 .930 .922 .918 .942 .939 .931 .908 .887 .925 .955 .934 .924 .924 .937 .932 .916 .882 .926 M31 .907 .922 .928 .922 .921 .941 .939 .926 .907 .889 .929 .951 .935 .928 .934 .925 .936 .927 .909 .871 .925 M32 .909 .924 .930 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .926 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .928 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .945 .937 .922 .891 .938 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .908																					.901
M32 .909 .924 .930 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .925 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .92 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .93 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	on																				.919
M32 .909 .924 .930 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .925 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .92 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .93 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	lati																				.923
M32 .909 .924 .930 .925 .923 .942 .940 .927 .911 .891 .932 .954 .936 .928 .935 .927 .909 .873 .925 M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .92 M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .93 M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	orre																				.921
M41 .920 .933 .930 .916 .925 .942 .940 .934 .918 .890 .931 .958 .940 .931 .941 .931 .916 .881 .92  M42 .925 .936 .934 .923 .926 .945 .943 .937 .923 .893 .935 .961 .943 .935 .945 .937 .922 .891 .93  M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866  M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	ŭ																				.923
M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	rsor																				.927
M51 .867 .881 .880 .854 .841 .907 .904 .881 .840 .794 .860 .877 .854 .856 .870 .857 .838 .823 .866 M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90	<sup>2</sup> ea																				.931
M52 .887 .907 .911 .903 .893 .932 .928 .907 .885 .860 .897 .930 .908 .903 .906 .900 .890 .871 .90				.881																	.860
1464		M52	.887	.907	.911																.901
92 914. 876. 927 .932 .927 .919 .943 .933 .925 .919 .895 .930 .954 .935 .932 .933 .923 .914 .876 .92		M61	.907	.927	.932			.943	.933	.925	.919	.895	.930	.954	.935	.932	.933	.923	.914	.876	.923
NACO		M62	.924	.940	.938			.948								.940	.949	.942	.923	.873	.934